DEFINITIONS OF ELECTRON SCATTERING VARIABLES

Rather than try to review the whole field of electron scattering, I will discuss only those experiments in which one or several of the electroproduced hadrons is observed in coincidence with the scattered electron.

But first, by way of introduction, let me say a few words about the single-arm electron scattering data. In this kind of experiment only the outgoing electron is detected. Measurement of the incident and scattered lab electron energies, \( E \) and \( E' \), and the lab scattering angle, \( \theta_e \), fixes the kinematics of the transferred virtual photon (Fig. 1a): its spacelike four-momentum,
\[
q^2 = -Q^2 = -2EE'(1-\cos \theta_e),
\]
its lab energy
\[
\nu = E - E',
\]
and its polarization, both longitudinal and transverse,
\[
\epsilon = \left[ 1 + 2 \frac{Q^2 + \nu^2}{Q^2} \tan^2 \frac{\theta_e}{2} \right]^{-1}
\]
The hadron final state is undetermined except for its total center of mass energy \( W \), which is given in terms of \( Q^2 \), \( \nu \), and the target mass \( M \):
\[
W^2 = s = 2\nu + M^2 - Q^2.
\]
We will take \( Q^2 \), \( W \), and \( \epsilon \) as the most convenient variables for expressing the scattering cross section. We then factor out the well-known effect of the e\( e^- \) vertex and the photon propagator, leaving \( \sigma(Q^2, W) \), the total cross

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section for transverse virtual photons, and $\sigma_L(Q^2, W)$, the cross section for longitudinally polarized photons:

$$d\sigma/d\Omega_e dE' = \Gamma \left( \sigma_T + \varepsilon \sigma_L \right),$$

where

$$\Gamma = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{W^2 - M^2}{2MQ^2} \frac{1}{1 - \varepsilon}. $$

It is also possible of course to express the cross section in terms of the more popular structure functions $W_1$ and $W_2$. These can be related to the virtual photon absorption cross sections:

$$W_2 = \frac{1}{4\pi^2\alpha} \frac{W^2 - M^2}{2M} \frac{Q^2}{Q^2 + \nu^2} (\sigma_T + \sigma_L) \approx \frac{1}{4\pi^2\alpha} \frac{Q^2}{\nu} (\sigma_T + \sigma_L)$$

$$W_1 = \frac{1}{4\pi^2\alpha} \frac{W^2 - M^2}{2M} \sigma_T.$$

The photon cross sections are a more useful representation in the low-$Q^2$ range. The transverse cross section $\sigma_T(Q^2, W)$ approaches the ordinary photoproduction cross section $\sigma(W)$ in the limit $Q^2 \to 0$ ($\sigma_L \to 0$, of course), while $\nu W_2$ vanishes in the photoproduction limit:

$$\lim_{Q^2 \to 0} \nu W_2(Q^2, W) = \left( Q^2 / 4\pi^2\alpha \right) \sigma(W) = 0.$$

**ELECTRON SCATTERING AND PHOTOPRODUCTION RESULTS**

Figure 2 summarizes the main features of the single-arm electron scattering data in terms of somewhat vaguely defined regions in the $Q^2$ versus $W^2$ plot. Except at low $Q^2$ and in the resonance region, the data are consistent with scaling; that is, $\nu W_2$ (and also $W_1$), instead of depending on $Q^2$ and $W^2$, is a function only of the ratio. Fig. 3 illustrates some of the scaling variables that have been used:

$$\omega = (W^2 - M^2)/Q^2 + 1,$$

first suggested by Bjorken,

$$\omega' = W^2/Q^2 + 1,$$

popularized by Bloom and Gilman,

$$\omega_w = W^2/(Q^2 + \omega^2) + 1,$$

suggested by Rittenberg and Rubinstein. It is clear from the figure that they
differ significantly only at low $Q^2$ and $W^2$, where scaling sets in sooner in $\omega'$ than in $\omega$, and where $\omega_W$ allows one to make a smooth connection with photoproduction data.

For $\omega' \gg 5$ we tend to speak of the photon-hadron interaction as "diffractive", because $\sigma_T + \sigma_L$ is approximately independent of $W$ (at fixed $Q^2$) and the neutron and proton cross sections are nearly equal. For $\omega' \ll 5$ the opposite is true, while for intermediate $\omega'$ there is a smooth transition between the two extremes. Below $Q^2 = 1 \omega W^2$ cannot continue to scale, but otherwise the cross section has the same diffractive character as it has in the high-$\omega'$ scaling region.

Indeed the high energy photoproduction cross sections (Fig. 4) are also dominated by Pomeron exchange, the energy dependence for proton or neutron target fitting\textsuperscript{8}

$$\sigma_{p,n}(\nu) = 97 + 55/\sqrt{\nu} \pm 12/\sqrt{\nu}$$

($\sigma$ in $\mu b$, $\nu$ in GeV).

Diffractive two-body channels, such as $\rho^0 N$ and $\omega N$, together comprise about 20% of the cross section, more-or-less independent of energy.\textsuperscript{9} In a vector-meson-dominance picture we can regard these as representing the elastic channel. In fact, the $t$-distribution in $\rho^0$ photoproduction\textsuperscript{10} is quite similar to that in purely hadronic diffraction scattering, elastic $\pi^\pm + p$ for instance. The nondiffractive two-body processes, $\pi N$, $\pi \Delta$, $K Y$, etc., have been thoroughly investigated in photoproduction. They dominate the total $\gamma p$ cross section below 2 GeV but drop off very rapidly, like $1/(W^2 - M^2)^2$. 
DEFINITIONS OF ELECTROPRODUCTION VARIABLES

Now I am ready to discuss coincidence electroproduction. As in the single-arm case we fix the virtual photon mass-squared $-Q^2$ and polarization $\varepsilon$ by observing the scattered electron (Fig. 1b). The total center-of-mass energy $W$ in the virtual-photon-plus-nucleon collision is also fixed; but now we subdivide the hadron final state into an observed hadron plus everything else, thus allowing us to define the $t$ (four-momentum transfer from photon to hadron, or target nucleon to hadron, whichever is appropriate), the missing mass $m_x$ (a fixed quantity in the case of a two-hadron final state), and the azimuth $\phi$ of the observed hadron. Or alternatively we can replace $t$ and $m_x$ with the hadron transverse momentum $p_T$ and $x$, the fractional longitudinal momentum in the center of mass ($x = p_L^{cm}/p_{max}^{cm}$). Notice that the hadron variables are referred to the virtual-photon-plus-nucleon collision; the longitudinal axis and the forward direction are the virtual photon direction and the center of mass refers to the photon plus nucleon. In effect, the electron is forgotten.

As before, we split the measured coincidence cross section into a virtual photon flux $\Gamma$ factor and a reaction cross section for virtual photons.

$$\frac{d\sigma}{d\varepsilon \, dE \, d\phi \, \ldots} = \frac{\Gamma}{2\pi} \bigg[ \frac{d\sigma}{d\ldots} + \varepsilon \frac{d\sigma}{d\ldots} \cos 2\phi + \sqrt{2} \varepsilon (\varepsilon + i) \frac{d\sigma}{d\ldots} \cos \phi \bigg].$$

The first two terms are the main transverse and longitudinal contributions; integrated over hadron directions, they become the $\sigma_T$ and $\sigma_L$ defined before. The other two terms depend on
φ (the first two do not) and average to zero. They correspond to interference between the two transverse polarization amplitudes and between transverse and longitudinal. The interference terms are readily identified experimentally by looking at the φ dependence of the data; however, to separate the main transverse and longitudinal contributions we have to make measurements at different values of φ, keeping all the other variables fixed. Since φ depends mainly on θ_e, this means measurements at widely spaced scattering angles, very difficult, because of the extremely rapid decrease of θ with θ_e. As a consequence, the transverse and longitudinal photon contributions have not yet been separated in any coincidence electroproduction experiment beyond the nucleon resonance region. This is important to keep in mind in the following discussion.

COINCIDENCE ELECTROPRODUCTION EXPERIMENTS

Seven groups working at three accelerators have reported work in this field to the conference. They are listed in Table I. There is a good variety of experimental methods used, from standard spectrometers with high resolution and unambiguous particle identification, through large aperture spark chamber systems, to 4π-solid-angle detection systems, such as a streamer chamber with a liquid hydrogen target inside and a fast cycling hydrogen bubble chamber with triggered pictures. In the latter case the beam is actually a muon beam. In this discussion we will treat muons and electrons as equivalent; "electroproduction" will refer to either one. Fig. 5 shows very approximately the range in Q^2 and W^2 which is
covered well at each accelerator. It is important to note that this is a very small part of the range covered in the SLAC-MIT single-arm experiments, which extends off the plot in both $Q^2$ and $W^2$. The coincidence experiments really are quite difficult and rate limitations in any feasible apparatus make it especially hard to get to high $Q^2$.

**NONDIFFRACTIVE TWO-BODY ELECTROPRODUCTION**

Let us begin by discussing the experimental data on the two-body channels. Table II shows a listing of all of the experiments, including those already published or reported at previous conferences. In the nondiffractive category quite a few reactions have been looked at:

\[
\begin{align*}
\gamma \nu p + \pi^+ n & \quad \gamma \nu n + \pi^- p \\
\pi^0 p & \quad \pi^- \Delta^+
\end{align*}
\]

\[
\begin{align*}
\pi^+ \Delta^0 & \quad \pi^+ \Delta^-
\end{align*}
\]

\[
\begin{align*}
\pi^- \Delta^{++} & \\
K^+ \Lambda & \\
K^+ \Sigma^0 & \\
K^+ \Upsilon^0 &
\end{align*}
\]

The data came from DESY, NINA, CEA, and Cornell, most of it already published and reviewed at the Cornell conference. At this conference a Harvard-Cornell group has reported an extension of earlier work on $\pi^+ n$ (Fig. 6) and a DESY group has extended their previous work on the $\pi \Delta$ channels (Fig. 7).

Our understanding of these nondiffractive processes rests mainly on the "Electric Born" model, which I will discuss in terms of the $\pi^+ n$ reaction. The model, elaborated in recent
years by Berends, Schmidt, Devenish and Lyth, among others\textsuperscript{38}, uses the three Born terms (Fig. 8a,b,c) plus dispersion corrections. It successfully accounts for the photoproduction data. In electroproduction the dominant contribution especially at low $t$ comes from the one-pion-exchange pole term (Fig. 8c). This is not surprising, since at high energy the pole is quite close to the physical region. But it does not contribute in forward photoproduction because of an angular momentum conservation argument: a real photon is transverse and therefore has helicity, which it cannot get rid of by knocking a spinless particle directly forward. In electroproduction however, it can be accomplished by longitudinal photons (i.e., zero helicity), and so the pion pole term contributes mainly to $\sigma_L$ and completely dominates the forward cross section.

This is illustrated in Fig. 9 which shows already published data\textsuperscript{30} on the forward $\pi^+\pi^-$ cross section compared with the model, using several hypotheses for the pion charge form factor, the only adjustable parameter in the theory. All the data consistently favor a pion form factor $F_\pi(Q^2)$ which is not far from the nucleon isovector Dirac form factor $F^V_1(Q^2)$ or a simple $\rho$-meson pole $m_\rho^2/(Q^2+m_\rho^2)$. The most striking feature of the data however is the rise in cross section with increasing $Q^2$, eventually followed by a slow decrease. According to the Electric Born model the increase comes entirely from $\sigma_L$ (Fig. 9b). Although there is no direct $\sigma_L$, $\sigma_T$ separation data to confirm this, the model does correctly give the observed longitudinal-transverse interference term (Fig. 9c).
Since the data indicate a ρ-pole form factor for the pion, we might expect a very simple ρ-dominance model\(^\text{39}\) (Fig. 8d) to give a reasonably good representation of the \(Q^2\) dependence of the \(π^+n\) cross section. We write:

\[
σ_T(Q^2, W) = \left( \frac{m_π^2}{Q^2 + m_π^2} \right)^2 \exp(Bt_{\text{min}}) \sigma(W),
\]

\[
σ_L = \xi \frac{Q^2}{m_π^2} σ_T,
\]

with \(t_{\text{min}} \approx -\left( \frac{Q^2 + m^2}{2} \right)^2\).

That is, we assume that \(σ_T\) for \(π^+n\) production is the same as the photoproduction cross section except for the effect of the virtual photon mass on the \(ω\) propagator and the fact that the minimum momentum transfer \(|t_{\text{min}}|\) increases with \(Q^2\) (the \(t\)-slope \(B\) is given in Fig. 6). The longitudinal contribution \(σ_L\) is assumed to be proportional to \(σ_T\) with an extra factor of \(Q^2\) required for gauge invariance\(^3\). The constant \(ξ\), taken from the Electric Born model, turns out to be something like \(ξ=4\). I am not suggesting this as a serious model for \(π^+n\) electroproduction (it certainly fails when applied to the total \(γnp\) cross section), but only as a simple mnemonic which describes rather well the main features of the experimental data and the Electric Born model in the low-\(Q^2\) region covered by the available data.

The same formulas should describe the \(Q^2\) dependence for other nondiffractive channels as well. The values of \(ξ\) may be different, but for all those channels in which a charged pseudoscalar meson is produced, \(ξ\) will be rather large for the same reasons as in the \(π^+n\) case, and the \(Q^2\) dependences will
look very similar. This is in fact confirmed by experiment for all such reactions that have been looked at. Fig. 10 shows an example, $\gamma p + K^+ \Lambda$, exhibiting the same rise then gradual fall with increasing $Q^2$ that we saw in the $\pi^+ n$ case. Since the overall $\gamma p$ cross section is dropping (proportional to $(Q^2 + .4)^{-1}$ according to single-arm experiments\textsuperscript{1}), this means that the meson-pole-dominated nondiffractive processes are becoming a larger fraction of the total cross section as $Q^2$ increases. This is an experimental fact, whether or not the Electric Born model and the $p$-dominance formulas are correct about the reasons for it.

DIFFRACTIVE TWO-BODY ELECTROPRODUCTION

The diffractive two-body final states that have been looked at in electroproduction experiments are $\rho^0 p$ and $\omega p$. There is no a priori reason why our simple $p$-dominance formulas should not work equally well for diffractive processes, so for $\gamma p \rightarrow \rho^0 p$ we write again

$$\sigma_T (Q^2, W) = \left( \frac{m_{\rho^0}^2}{Q^2 + m_{\rho^0}^2} \right)^2 \exp \left( B t_{\omega p} \right) \sigma(W),$$

$$\sigma_L = \xi \frac{Q^2}{m_{\rho^0}^2} \sigma_T,$$

where here the $\sigma(W)$ refers to $\rho^0$ photoproduction, and $\xi$ has a new value.

In the absence of any reliable theoretical guidance in fixing $\xi$, let us appeal to experiment. Of course we still have no $\sigma_L, \sigma_T$ separation data; but the two-$\pi$ decay of the $\rho^0$ tells us something about the relative magnitudes of $\sigma_L$.
and $\sigma_T$. It is a fact that in the $\rho^0$ decay the $\pi^+$ and $\pi^-$ come out along the direction of polarization in the $\rho^0$ rest frame ("polarization" for $\rho$ is here defined the same way as it is for light; it is not the same as helicity). In $\rho^0$ photoproduction there is good evidence for s-channel helicity conservation; that is, the $\rho^0$ preserves the polarization of the incident photon. Fig. 11 shows polar and azimuthal decay distributions for rhos produced by polarized transverse photons. In each plot the pions come out predominantly in the transverse polarization direction, never in the longitudinal ($\cos \theta_H = \pm 1$) or orthogonal transverse ($\psi_H = 90^\circ, 270^\circ$) directions. In electroproduction the evidence is not so direct. A density matrix analysis of the rhos electroproduced in the DESY streamer chamber shows that each matrix element which is proportional to a $\gamma\nu + \rho$ helicity flip amplitude is zero. This at least makes it a plausible assumption that s-channel helicity conservation holds here too. So now we look at the polar and azimuthal decay distributions for electroproduced rhos (Fig. 12). They look rather similar to those from photoproduction except for the presence of some decays in the longitudinal direction ($\cos \theta_H = \pm 1$). Assuming s-channel helicity conservation, we conclude that the electroproduction proceeds mainly through $\sigma_T$ but with some admixture of $\sigma_L$. The data are consistent with $\xi \approx .6$, almost an order of magnitude smaller than in the $\pi^+n$ case. Looking back at the $\rho$-dominance formulas for $\sigma_T$ and $\sigma_L$,
it is clear that $\sigma_T$ with its rapid fall-off in $Q^2$ will dominate the production of rhos, at least for low $Q^2$; and $\rho$ electroproduction will become a smaller fraction of the total $\gamma\nu p$ cross section as $Q^2$ increases. In Fig. 13 I have plotted the available data. Since the cross section depends on $Q^2$, $W$, and $\epsilon^{41}$, it is difficult to plot all the points on a single curve. So I have actually plotted a ratio of ratios, such that if the $\rho$ electroproduction were to remain a constant fraction of the total $\gamma\nu p$ cross section, the data would show a constant ratio equal to one. Instead, $\rho$ production is apparently dying out as $Q^2$ increases (actually the data at different $\nu$ are more consistent when plotted against $Q^2/2M\nu = 1/\omega$). In fact, the simple $\rho$-dominance formulas for $\sigma_T$ and $\sigma_L$ with $\xi = .6$ fit the data remarkably well; the apparent scatter in the data comes mainly from the different values of $\epsilon$ used in the experiments.

The SLAC bubble chamber group$^{23}$ has also observed $\omega$ electroproduction (Fig. 14). The cross section shows the same decrease with $Q^2$ as observed in $\rho^0$ production.

There has been a good deal of interest lately in the shape of the $t$-distribution in $\rho$ electroproduction. It has been suggested$^{42}$ that the effective interaction radius of the virtual photon will shrink as the photon becomes more spacelike and less able to act as a vector superposition of hadronic states. If so, this should show up as a flattening of the $t$-distribution for diffractive processes as $Q^2$ is increased at fixed $W$. 

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The most extensive data on $\rho^0$ electroproduction, at least in terms of statistics, is the Cornell missing mass experiment\textsuperscript{35}. Electron-proton coincidences are observed and the $\rho^0$ (including $\omega$) is defined by a cut in the missing mass distribution. Fig. 15 shows the cross section in this mass cut $d\sigma/dt \, d\phi$ versus $t$ at fixed $W$ for two different $Q^2$. The data from a number of such measurements at various $Q^2$ and $W$ have been fitted to the form $e^{Bt}$; the slopes $B$ are plotted in Fig. 16. Although the values of $B$ obtained in electroproduction do not show a very pronounced trend with $Q^2$, they are all much lower than the values ($B = 6$ to $8$ GeV$^{-2}$) reported for $\rho^0$ photoproduction at these energies\textsuperscript{9}.

But before we conclude that the virtual photon is shrinking, it is important to examine the data more closely. Fig. 17 is the distribution in missing mass of the data from which one of the $t$-distributions in Fig. 15 was obtained. At $m_X^2 = 0$ you see the missing photon peak from radiative ep scattering. Around $m_X^2 = 0.6$ GeV$^2$ you see the peak due to $\rho^0$ and $\omega$.

Within the mass cut there is also a sizeable nonresonant contribution as well. In Fig. 17 only the events at low $t$ are plotted; Fig. 18 shows the corresponding mass plot for a higher $t$ range (note the change in vertical scale). Here the $\rho^0$ and $\omega$ have almost disappeared relative to the background. Obviously, the $t$-distribution in Fig. 15 is a composite of $\rho^0$, $\omega$ and background, the background having a much flatter dependence on $t$. But as we have just seen, as $Q^2$ increases the $\rho^0$ is dying out.

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So it is not hard to account for Figs. 15 and 16 as the increasing competition of the flatter background with the steeper $\rho$ as $Q^2$ increases, instead of as a variation in the $\rho$ steepness.

Suppose we turn instead to the experiments in which the $\rho^n$ is observed directly in the $\pi^+\pi^-$ yield. Fig. 19 shows the DESY streamer chamber data compared with photoproduction data. Here again the $\rho$ is defined by a mass cut, but in the $m_{\pi\pi}$ spectrum (see Fig. 20). The photoproduction data have been treated in exactly the same way in order to make a fairer comparison.

The $t$-distributions seem to show a flattening trend with increasing $Q^2$, but even here the background is important. In the highest $W$ range in the DESY experiment the average $W$ is still only about 2.3 GeV, which is not far enough above the $\pi\pi$ threshold to be clear of significant overlap between the $\Delta^{++}$ and the $\rho^n$ in the $\pi^+\pi^-$ Dalitz distribution (Fig. 20).

Why not fit the $\rho$ and the background as a function of $t$ in the DESY and Cornell experiments, to extract the effect of the $\rho$ alone, as is done in photoproduction analyses? First, this requires high statistics in the data, which even for the Cornell experiment is not yet comparable with photoproduction data. Second, various plausible hypotheses for the parametrization of the $\rho$ and the background yield significant differences in the $t$-distribution. In fact, this problem is still far from solved in the photoproduction case. Perhaps the background separation can be done, but so far none of the electroproduction experimenters have been bold enough to try it.
In the SLAC experiments at higher $W$ one might hope to see a cleaner $\rho^0$ signal in the $\pi\pi$ spectrum. This may be the case in the bubble chamber data $^{23}$ (at $\hat{W} \approx 2.7$ GeV) of Fig. 21, but the number of events is not sufficient for any conclusions as to a change in $t$-slope from low $Q^2$ to high $Q^2$. The number of events in the SLAC spark chamber experiment $^{22}$ is enough (Fig. 22) to show a statistically significant trend in the $t$-slopes with increasing $Q^2$ (Fig. 23). Unfortunately, however, as $Q^2$ increases the average $W$ in their data decreases, and in photoproduction the slope is already known to decrease with decreasing $W$. $^{45}$ So here again, the data are inconclusive.

I think the most charitable statement one can make about the $\rho$-slope data is that they all favor a flattening of the slope with increasing $Q^2$. A skeptic, however, would say that there's no real evidence for it yet, and in this case I'm a skeptic. Even if there were such an effect in the $t$-slope, photon shrinkage is not the only possible explanation. Longitudinal and transverse production may have different $t$-dependences $^{46}$ or non-diffractive $\rho^0$ production may become important, for instance. $^{47}$
It is also interesting to look at the t-distribution for missing mass states beyond the rho. Fig. 24 shows Cornell data\(^1\) for the cross section for \(\gamma_V p \rightarrow p X\) as a function of \(m_X^2, t,\) and \(Q^2,\) with \(m_X^2\) in the range \(1\) to \(2\) GeV\(^2\). There is no discernable dependence on \(m_X^2,\) but the t-distribution certainly gets flatter as \(Q^2\) increases. The SLAC bubble chamber data\(^2\) show the same effect (Fig. 25) expressed in terms of transverse momentum instead of \(t.\) These inclusive data may in fact be much better evidence for photon shrinking than the \(\rho^0\) data.

While we are looking at the proton inclusive process (see Table III for a bibliography of inclusive experiments), let us see what we can learn from the longitudinal momentum distributions. In Fig. 26 are plotted the normalized invariant cross sections as functions of \(x\) for \(W \approx 2.8\) GeV, \(p_T \approx 0,\) and \(Q^2 = 0\) and \(1.2\) GeV\(^2\). First we note that the photoproduction and electroproduction data are indistinguishable. Secondly, protons are produced mainly backward in the center of mass. This is not surprising since this is the proton fragmentation region. However, the parton model with spin-\(\frac{1}{2}\) partons predicts an increasing probability for a proton to come out forward as \(Q^2\) increases. There is no evidence for this yet. In photoproduction the continuation of the proton \(x\) distribution to \(x \approx -1\) shows a pronounced peak corresponding to \(\rho^0\) and \(\omega.\)
production (Fig. 27). The fact that this peak goes away as $Q^2$ is increased is accounted for by the rapid decrease in the cross section for diffractive two-body processes.

Our inquiry into the changing steepness of transverse momentum distributions with increasing $Q^2$ can be extended to the pion inclusive process, $\gamma_Y + p \rightarrow \pi^\pm + \text{anything}$. Fig. 28 shows Cornell data for $\pi^+$ in two $x$ cuts in the central region and for three $Q^2$, plotted against $p_T^2$. All the data fit the same form $\exp(-9p_T^2)$; there is no variation in average transverse momentum with $Q^2$. However, if we look in the forward hemisphere (Fig. 29) there seems to be a consistent tendency, both in $\pi^+$ and $\pi^-$, for the $p_T^2$ distributions in electroproduction to be flatter than in photoproduction. It makes one a little suspicious, though, to see the effect setting in so suddenly between photoproduction and the lowest $Q^2$ interval in electroproduction.

Longitudinal distributions also show contrasts between photoproduction and electroproduction. Fig. 30 shows normalized invariant $\pi^+$ cross sections for $W \approx 2.6$ GeV and $p_T \approx 0$ at $Q^2 = 0$, $1.15$, and $2.0$ GeV$^2$ from spectrometer experiments at DESY$^{50,12}$ and Cornell$^{18}$. In the forward direction near $x=1$ we can see the effect of the increasing prominence of the $\pi^+n$ channel as $Q^2$ increases. The shoulder in the photoproduction distribution between $x \approx .4$ and $.9$ disappears in electroproduction. In Fig. 31, showing the SLAC bubble chamber data$^{23}$, this is seen as the effect of the disappearance of the $\rho^0$ and $\omega$. 
The same happens in the π^- distributions (Fig. 32, 33, 34).

The π^+/π^- ratio, which for the W range of these experiments lies between 1 and 1.5 in photoproduction, increases in electroproduction. When x is close to one the increase can be understood in terms of the increasing prominence of the π^+n channel, which has no π^- counterpart. The disappearance of the diffractive channels, which contribute 1.0 to the charge ratio, may explain the increase in the ratio from x=.4 to .9. It may be more instructive to compare π^+ and π^- plotted against missing mass (Figs. 35 and 36). The curve in Fig. 36 is identical to the one in Fig. 35 passing through the π^+ data. Although this behavior of the charge ratio may be explainable in terms of the known behavior of the two-body channels, it is hard to see why it should persist at higher W in view of the rapid decrease in the nondiffractive two-body cross sections. But it does seem to persist up to W~4 GeV, as shown by the SLAC spark chamber data in Fig. 37.

Inclusive K^+ data (Fig. 38) show an increase in the relative contribution of K production with increasing Q^2. This may be the effect of the increase in the two-body channels that we have already noted. In fact when the K^+ + anything data is plotted against missing mass (Fig. 39), there is good evidence that most of it comes from two-body processes K^+Λ, K^+Σ^o, KΥ*; a good number of the known Υ* states are represented in the spectrum. Whether the explanation is the Electric Born model, or K. Wilson's more general argument, or both, we can only speculate.
MULTIPLICITY

Another way of seeing the charge ratio effect is to look at the prong distributions that the bubble chamber experimenters always give us (Fig. 40). There is a clear tendency for the proportion of events with one charged hadron prong to increase with $Q^2$ while the three-prong events decrease. Among the one-prong channels are $\pi^+ n$, $\pi^+ \Delta^0$, $K^+ \Lambda$, and similar meson-exchange dominated reactions, all of which we know to be increasingly important in electroproduction. The diffractive processes, $\rho^0 p$ and $\omega p$, are of course seen in the bubble chamber as three-prong events. So at least at these low energies there is no mystery in the prong distributions. As a consequence, though, the average charged hadron multiplicity has to decrease with $Q^2$, as indeed it does (Fig. 41).

TWO PUZZLES

All of the trends in the inclusive distributions with increasing $Q^2$—the changing $x$ distributions for pions, the increase in kaon production, the increasing charge ratio in the forward direction, the increase in the one-prong events, the decrease in the three-prong events, and the decreasing multiplicity—can be correlated with the known behavior of the two-body channels. This explanation, however, leaves us with two puzzles.

First, if the contribution of the single-charged-meson-exchange processes, like $\pi^+ n$, is becoming so important with
increasing $Q^2$, why is $\sigma_L/\sigma_T$ so small in the overall inelastic scattering cross section? According to the Electric Born model, or just appealing to the effect of angular momentum conservation on the one-pion-exchange amplitude, the increase with $Q^2$ comes mainly from the longitudinal photons. How can we avoid a contradiction? Perhaps the Electron Born model is wrong. Perhaps the angular momentum argument is not so relevant, since it applies only at $0^\circ$; or maybe the pion pole term is not dominant. Perhaps the single-arm data on $\sigma_L/\sigma_T$ are wrong. Actually, since the overlap between the region in $Q^2$ vs. $W$ where $\sigma_L/\sigma_T$ is measured and the region where the coincidence experiments have been done is rather small, the contradiction is not yet serious; but it will become serious if the trend in the coincidence data continues to higher $Q^2$.

Second, how do we maintain the explanation in terms of two-body channels at higher energies $W$, where the average multiplicity certainly increases and the effect of the two-body channels becomes less and less significant? Perhaps the new channels that open up are mainly quasi-two-body, $\pi^+N^*$, $K^+\gamma^*$, etc., behaving in the same way with $Q^2$ as do the low-energy channels.

If all the effects that we have seen—increasing forward charge ratio, decreasing average multiplicity, and so on—turn out to be transitory, disappearing at higher $Q^2$ and $W$ beyond the range of the present coincidence experiments, then the puzzles will of course disappear.
Consider the list of observed phenomena in Table IV. Can it be accidental that increasing $Q^2$ at fixed $W$ has all the same qualitative effects on the hadron final states as decreasing $W$ at fixed $Q^2$? Maybe it is an accident, but let us speculate. Suppose that it is a general principle which holds for all $Q^2$ and $W$. Assuming that it is, consider some property of the hadron final states, average charged multiplicity, for instance. Now imagine drawing a line or curve through all points in the $Q^2$, $W^2$ plane (see Fig. 42) for which the multiplicity has the same value. These lines will have positive slope, except for one vertical line at $W^2=M^2$ (elastic scattering) where the final state is always a single proton. Each line will intersect the $Q^2=0$ axis at whatever value of $W^2$ is appropriate for that multiplicity. The result may very well look like Fig. 42. Multiplicity will decrease as one goes from one line to the next by increasing $Q^2$ or decreasing $W$ (or both). Actually the lines may be curves, or the lines of constant multiplicity may not be the same as the lines of constant diffractive/nondiffractive ratio and so on. But if we ignore for now the possible complications, it appears that we have in effect a kind of scaling variable (compare Figs. 42 and Fig. 3). A simple form which has the required features is

$$\omega^* = \frac{W^2-M^2}{Q^2+b^2} + 1 = \frac{2M^2+b^2}{Q^2+b^2}.$$ 

This has some resemblance to the $\omega_W$ scaling variable; of course for high $Q^2$ and $W$ all the scaling variables are the same.
Some evidence for similarity of the final states along lines of constant $\omega$ is provided by the comparison of two missing mass spectra for $\gamma p \rightarrow \pi^+ X$ measured by the Harvard-Cornell group (see Figs. 35, 43) at the same $\omega$ but different $Q^2$. The two-body channels and continuum have exactly the same relative magnitude in the two spectra. What we are saying is that some global properties of the final states—for instance, those listed in Table IV—may scale, although not necessarily the details of the cross sections for each individual channel (new channels keep opening up as $\omega$ increases). Perhaps there is some kind of duality principle at work. Are the $\pi^+ n$, $\pi^+ \Delta$, $K^+ \Lambda$, processes in low energy photoproduction dual to the parton-like behavior at the same $\omega^*$ but at high $Q^2$ and energy?

Is there any reason for such a scaling variable, beyond the empirical fit to data? The virtual photon can be thought of as being absorbed by a fraction $1/\omega$ of the proton. In Feynman’s picture this fraction (a parton) recoils elastically, though perhaps undergoing some final-state interaction. In the Chou and Yang picture this $1/\omega$ fraction of the proton is pulverized by the virtual photon: the spacelike photon must pick up a piece of the proton in order for the photon to become timelike and fragment into real particles. More precisely, the ability of the virtual photon to fragment into a state of mass $m_f$ is determined by the value of $2\sqrt{Q^2 + m_f^2}$ One way to see this was suggested by Ioffe: the virtual photon can propagate virtually as a hadron state of mass $m_f$ only for a
distance set by the uncertainty principle at \(2\nu/(Q^2+m_f^2)\).

Another way to see it is to recall that the minimum momentum transfer required to produce a state of mass \(m_f\) is

\[|t_{\text{min}}| \approx [(Q^2+m_f^2)/2\nu]^2.\]

Multiplying by \(M\) to make it dimensionless, \(2\nu/(Q^2+m_f^2)\) becomes very similar to \(\omega_w\) or \(\omega^*\).

Let me in summary try to give you my picture of what is happening in deep inelastic electron scattering. At high \(\omega\) (whatever scaling variable you prefer) the photon, real or virtual, interacts mainly through its hadronic "constituents" (Fig. 44a). The fraction \(1/\omega\) of the target proton which interacts violently with the photon is small, so that the final state consists of the vector meson fragments of the photon coming off forward with net charge zero, and the proton remaining behind, perhaps fragmenting. The interactions are mainly diffractive, with multiplicities and transverse momenta typical of hadronic collisions.

At low \(\omega\) the hadronic state of the virtual photon does not last long enough to be significant (Fig. 44b). The photon interacts as a point particle, probing the internal structure of the proton, and transferring its momentum to a sizeable fraction \(1/\omega\) of the proton. What comes out forward is most likely a single particle, usually carrying the charge of the proton. At the moderate \(\omega\) values covered in the present coincidence experiments, the forward particle is often a \(\pi^+\) or \(K^+\) meson, but I would guess
that as \( \omega \) approaches one (elastic scattering) it is more and more likely to be a proton. In either case, the remaining fragments of the proton have net charge zero. The interactions are mainly nondiffractive and the multiplicities are low. The final states are determined by the constituents of the proton. Since the photon behaves here like a point particle, transverse momenta tend to be larger.

Fortunately, there are ways we can test these speculations experimentally. The study of the hadron final states should be carried to higher \( Q^2 \) to see if the trends in multiplicity, forward \( \pi^+/\pi^- \) ratio, transverse momenta, etc. continue—also to higher \( W \) to see if the inclusive distributions still reflect the behavior of the two-body channels. A \( q_L, q_T \) separation measurement in one of the non-diffractive channels, say \( \pi^+n \), should be made. We should continue the search for forward protons to lower \( \omega \) values. We should check to see if the nucleons which recoil with low energy in the lab are more likely to be neutrons as \( \omega \) decreases. All of these experiments are likely to be done in the next few years, at the existing electron accelerators and also here at N.A.L., so by the time of the next conference we should know whether there is any truth in what I have been saying.
<table>
<thead>
<tr>
<th>PAPER NO.</th>
<th>AUTHORS</th>
<th>INST.</th>
<th>ACCEL.</th>
<th>$E_{\text{max}}$</th>
<th>TECHNIQUE</th>
<th>REACTIONS</th>
</tr>
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<tr>
<td>810</td>
<td>J.C. Alder et al.</td>
<td>DESY</td>
<td>DESY</td>
<td>6.5</td>
<td>2 spectrometers</td>
<td>$ep-e_{\pi}^{\pm}X$, $eK^{\pm}X$, $epX$</td>
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<td>658</td>
<td>I. Dammann et al.</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5.4</td>
<td>2 spectrometers</td>
<td>$ep+e_{\pi}^{\pm}\Delta$</td>
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<tr>
<td>754</td>
<td>I. Dammann et al.</td>
<td>&quot;</td>
<td>&quot;</td>
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<td>2 spectrometers</td>
<td>$ep+e_{\pi}^{\pm}X$</td>
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<tr>
<td>639</td>
<td>V. Eckardt et al.</td>
<td>&quot;</td>
<td>&quot;</td>
<td>7.2</td>
<td>streamer ch.</td>
<td>$ep+e_{\pi}^{\pm}X$</td>
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<tr>
<td>640</td>
<td>V. Eckardt et al.</td>
<td>&quot;</td>
<td>&quot;</td>
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<td>streamer ch.</td>
<td>$ep+e_{\pi}^{\pm}\Delta^{++}$, $ep^{o}p$</td>
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<td>815</td>
<td>C.N. Brown et al.</td>
<td>Harvard, Cornell</td>
<td>Cornell</td>
<td>10</td>
<td>2 spectrometers</td>
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<td>C.J. Bebek et al.</td>
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<td>&quot;</td>
<td>10</td>
<td>2 spectrometers</td>
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<td>E. Lazarus et al.</td>
<td>Cornell</td>
<td>&quot;</td>
<td>10 (magnet + spark ch.) &amp; spectro.</td>
<td>$ep+e_{\pi}^{\pm}X$</td>
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<tr>
<td>942</td>
<td>E. Lazarus et al.</td>
<td>&quot;</td>
<td>&quot;</td>
<td>10</td>
<td>&quot;</td>
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<td>949</td>
<td>L. Ahrens et al.</td>
<td>&quot;</td>
<td>&quot;</td>
<td>11.8</td>
<td>&quot;</td>
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<td>440</td>
<td>J.T. Dakin et al.</td>
<td>SLAC</td>
<td>SLAC</td>
<td>19.5</td>
<td>magnet + spark ch. (+ sc pipe)</td>
<td>$ep+e_{\pi}^{\pm}X$, $ep^{o}p$</td>
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<tr>
<td>?</td>
<td>J. Ballam et al.</td>
<td>&quot;</td>
<td>&quot;</td>
<td>16($\mu^{-}$)</td>
<td>H$_2$ bubble ch.</td>
<td>$\mu+\mu_{\pi}^{\pm}X$, $\mu pX$</td>
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*Not including experiments in the nucleon resonance region $W < 1.8$ GeV, or single-arm experiments.*
<table>
<thead>
<tr>
<th>ACCEL.</th>
<th>REF.</th>
<th>REACTIONS</th>
<th>$Q^2$</th>
<th>$W$</th>
<th>$-t$ (or $\theta_{CM}^*$)</th>
<th>$\phi$</th>
</tr>
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<tr>
<td>DESY</td>
<td>I. Dammann et al. 13</td>
<td>$ep\rightarrow e\pi^+\Delta^0$ $\pi^+\Delta^+$</td>
<td>0.2/0.8</td>
<td>2.1/2.3</td>
<td>&lt;0.6</td>
<td>all</td>
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<tr>
<td></td>
<td>V. Eckardt et al. 15</td>
<td>$ep\rightarrow e\pi^-\Delta^++ep^0\pi$</td>
<td>0.3/1.5</td>
<td>1.3/2.7</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td>C. Driver et al. 25</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>0.1/0.9</td>
<td>2/2.5</td>
<td>&lt;0.15</td>
<td>all</td>
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<td></td>
<td>C. Driver et al. 26</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>0.1/0.9</td>
<td>1.7/2.7</td>
<td>all</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C. Driver et al. 27</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>0.3/1</td>
<td>1.5/2.1</td>
<td>&gt;100°</td>
<td>$\pi^+\pi^/3$</td>
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<td>C. Driver et al. 28</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>0.1/0.9</td>
<td>1.9/2.6</td>
<td>180°</td>
<td>all</td>
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<tr>
<td>NINA</td>
<td>A. Sofair et al. 29</td>
<td>$ep\rightarrow e\pi^-\Delta^+$ $ep^0\pi$</td>
<td>0.7</td>
<td>1.93</td>
<td>&lt;0.3</td>
<td>$\pi$, ±π/2</td>
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<td></td>
<td>C. N. Brown et al. 30</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>0.17/1.2</td>
<td>2/2.3</td>
<td>&lt;20°</td>
<td>-π, 0</td>
</tr>
<tr>
<td></td>
<td>C. N. Brown et al. 31</td>
<td>$ed\rightarrow e\pi^0NN'$</td>
<td>0.4</td>
<td>2.15</td>
<td>0°</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C. N. Brown et al. 32</td>
<td>$ep\rightarrow e\rho^0$</td>
<td>0.2/1.4</td>
<td>1.8/2.5</td>
<td>&gt;145°</td>
<td>-0</td>
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<tr>
<td></td>
<td>C. N. Brown et al. 33</td>
<td>$ep\rightarrow eK^+\Lambda(\Sigma^+)$</td>
<td>0.2/1.2</td>
<td>0.3</td>
<td>-π, 0</td>
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<tr>
<td></td>
<td>C. N. Brown et al. 34</td>
<td>$ep\rightarrow e\pi^-\Delta^+$</td>
<td>0.4</td>
<td>2.15</td>
<td>0°</td>
<td>0</td>
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<td>CORNELL</td>
<td>C. N. Brown et al. 17</td>
<td>$ep\rightarrow e\pi^+n$</td>
<td>1.2/2</td>
<td>2.1/2.7</td>
<td>-.2</td>
<td>-π/2</td>
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<td></td>
<td>D. E. Andrews et al. 35</td>
<td>$ep\rightarrow e\rho^0$</td>
<td>0.3/1.2</td>
<td>3</td>
<td>1/1.5</td>
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<td>SLAC</td>
<td>E.D. Bloom et al. 36</td>
<td>$ep\rightarrow e\rho^0$</td>
<td>0.1/1</td>
<td>3.7</td>
<td>0.15/0.76</td>
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<td>J.T. Dakin et al. 22</td>
<td>$ep\rightarrow e\rho^0$</td>
<td>4.3/5.2</td>
<td>&lt;0.8</td>
<td>all</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J. Ballam et al. 23</td>
<td>$\mu\rho+\mu\rho^0$</td>
<td>0.15/1.5</td>
<td>2/4</td>
<td>&lt;0.6</td>
<td>all</td>
</tr>
</tbody>
</table>

*a Not including experiments in the nucleon resonance region $W < 1.8$ GeV.

b The aperture in $\phi$ is strongly correlated with the $\rho$ decay angles.
TABLE III

INCLUSIVE ELECTROPRODUCTION EXPERIMENTS

<table>
<thead>
<tr>
<th>ACCEL.</th>
<th>REF.</th>
<th>REACTIONS</th>
<th>$Q^2$</th>
<th>$W$</th>
<th>$x$</th>
<th>$P_T$</th>
<th>$\phi$</th>
</tr>
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<tr>
<td>DESY</td>
<td>J.C. Alder et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>1.16</td>
<td>2.63</td>
<td>.1/1</td>
<td>.0</td>
<td>-</td>
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<tr>
<td></td>
<td></td>
<td>$e^-K^+X$</td>
<td></td>
<td></td>
<td>.5/1</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$epX$</td>
<td></td>
<td></td>
<td>-.6/1</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>I. Dammann et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>1/6</td>
<td>2.1/2.8</td>
<td>.4/1</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td>V. Eckardt et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>.3/1.5</td>
<td>1.8/2.7</td>
<td>all</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td>NINA</td>
<td>A. Sofair et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>.7</td>
<td>1.93</td>
<td>.1/1</td>
<td>.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$epX$</td>
<td></td>
<td></td>
<td>.1/1</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>CORNELL</td>
<td>C.J. Bebek et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>1.2/2.0</td>
<td>2.1/2.7</td>
<td>0/1</td>
<td>&lt;.1</td>
<td>~π/2</td>
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<tr>
<td></td>
<td>E. Lazarus et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>.3/1.2</td>
<td>3.0</td>
<td>-.1</td>
<td>all</td>
<td>~π</td>
</tr>
<tr>
<td></td>
<td>E. Lazarus et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>.3/1.2</td>
<td>3.0</td>
<td>-.8</td>
<td>all</td>
<td>~π</td>
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<tr>
<td></td>
<td>L. Ahrens et al.</td>
<td>$ep+e^-\pi^+X$</td>
<td>.4/2</td>
<td>2.5/4</td>
<td>-.8</td>
<td>all</td>
<td>~π</td>
</tr>
<tr>
<td>SLAC</td>
<td>J.T. Dakin et al.</td>
<td>$ep+e^-\pi^+X^a$</td>
<td>.5/2.5</td>
<td>2.4/5.2</td>
<td>0/1</td>
<td>all</td>
<td>all</td>
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<tr>
<td></td>
<td>J. Ballam et al.</td>
<td>$e^-\mu^+\mu^+X$</td>
<td>.15/1.5</td>
<td>1.8/4</td>
<td>all</td>
<td>all</td>
<td>all</td>
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</tbody>
</table>

$^a$Forward hadrons are not identified, but are analyzed as if they were pions.
<table>
<thead>
<tr>
<th>Property</th>
<th>With Increasing $Q^2$ (Fixed $W$)</th>
<th>With Decreasing $W$ (Fixed $Q^2$)</th>
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<tr>
<td>Nondiffractive/Diffractive</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>$&lt;p_T^2&gt;$ for backward protons</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>$&lt;p_T^2&gt;$ for forward $\pi^\pm$</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>1-prongs/total</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>3-prongs/total</td>
<td>decreases</td>
<td>decreases</td>
</tr>
<tr>
<td>forward $\pi^+/\pi^-$</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>average multiplicity</td>
<td>decreases</td>
<td>decreases</td>
</tr>
</tbody>
</table>
REFERENCES

1. The high energy single-arm data have been reviewed recently by H. W. Kendall, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies, edited by N. B. Mistry (Cornell Univ., Ithaca, New York, 1972). More recent work by the MIT-SLAC group, A. Bodek et al., was contributed at this conference (paper #816). The data in the resonance region have been reviewed by R. Wilson, ibid., and papers have been submitted to this conference by M. Köhberling et al. (#548) and J. C. Alder et al. (#809, #961).

2. See C. W. Akerlof et al., Phys. Rev. 163, 1482 (1969) for definition. In all of the data presented here, c is very nearly unity.


4. The choice of \( \Gamma \) is not unique except in the limit \( Q^2 = 0 \), where \( \sigma_T \) must approach the photoproduction cross section. The convention used here (due to L. N. Hand, Phys. Rev. 129, 1834 (1964)) may not be the ideal choice theoretically, but it has gained the widest usage.


7. V. Rittenberg and H. R. Rubinstein, Phys. Letters 35B, 50 (1971). Our definition for \( \omega_w \) given above is a special case which assumes \( M_w^2 = M^2 + a^2 \) (M is the proton mass).
F. W. Brasse et al., Nucl. Phys. B39, 421 (1972) have fit the data for \( \omega w_2^2 / \omega_2 \) from inelastic electron scattering and photoproduction with a single function of \( \omega_2 \), with \( a^2 \approx 0.4 \) GeV\(^2\) and \( M_2 \approx 1.4 \) GeV\(^2\).


10. See ref. 9, p. 204, 205.

11. The azimuth \( \phi \) is taken with respect to the virtual photon axis, the usual convention being that \( \phi = 0 \) in the electron scattering plane on the incident beam side of the virtual photon line. The opposite convention is used in refs. 13, 14, and 25, 28, causing a reversal in the sign of \( d\sigma_1 / d\phi \).


14. I. Dammann et al., paper #754.

15. V. Eckardt et al., paper #639.
16. V. Eckardt et al., paper 640.
17. C. N. Brown et al., paper #815.
18. C. J. Bebek et al., paper #952.
20. E. Lazarus et al., paper 942.
21. L. Ahrens et al., paper #949.
23. J. Ballam et al., SLAC preprint contributed to this conference.

39. This model in one form or another has a long history. It was applied to the overall $\gamma p$ cross section by J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969). The predicted overall $\sigma_L/\sigma_T$ was much larger than the experimental limits (ref. 1) at large $Q^2$. The model was applied to vector meson electroproduction by H. Fraas and D. Schildknecht, Nucl. Phys. B14, 543 (1969). The model has recently been generalized to include higher-mass vector intermediate states by J. J. Sakurai and D. Schildknecht, Phys. Letters 40B, 121 (1972), UCLA preprint 72/TEP/57 (paper #283 contributed to this conference), and SLAC-PUB-1094 (paper #623), and by M. Böhm, H. Joos, and M. Krammer, paper #380. See also D. Schildknecht, Springer Tracts in Modern Physics 63, 57 (1972).


41. Also, since refs. 35 and 36 are $\gamma^* p \rightarrow p +$ missing mass experiments, the $\omega$ events are included in the $\rho^0$ cross section. So in calculating the ratios for these data points in plotting Fig. 13, I have used the combined $\rho^0$ and $\omega$ photoproduction cross sections for $\sigma_c(W)$.

43. This background is also certainly present in the missing-mass experiment of E. D. Bloom, et al. (ref. 36).


45. A tabulation of slopes $B$ obtained by fitting $p^2$ photoproduction data to $d\sigma/dt = Ae^{Bt}$ is given by P. Joos, Compilation of Photoproduction Data Above 1.2 GeV, DESY-HERA 70-1, p. 138.

46. See for example the diffraction dissociation model of rho electroproduction, G. Kramer and H. R. Quinn, DESY preprint 72/23.

47. See for example H. Goldberg, paper #20, contributed to this conference.


49. H. Burfeindt, et al. (DESY), paper #661 contributed to this conference.

50. W. Struczinski, et al. (Aachen, DESY, Hamburg, Heidelberg, München), paper #668 contributed to this conference.
51. K. G. Wilson, Phys. Rev. Letters 27, 690 (1971). This version of the parton model predicts a kaon multiplicity of about \( \frac{5}{2} \) at \( Q^2 \sim 1 \text{ GeV}^2 \). The data do not yet support this.

52. Berkeley, group A, quoted as "private communication" in ref. 23.

53. Although the absolute threshold for each higher multiplicity channel occurs at a fixed \( W \) independent of \( Q^2 \), the minimum momentum transfer required to produce the final state in each channel increases with \( Q^2 \) (\( |t_{\min}| = [(Q^2 + m_p^2)^2/2\nu]^2 \)), suppressing the production at low \( \nu \), and thus postponing the effective threshold to higher \( W \).

54. This definition for \( \omega^* \) is premature and should not be taken seriously in its present form. More data on the hadron final states are needed before we can decide what is the most appropriate form (if any) for a scaling variable.


1. (a) Diagram for the single-arm inelastic electron scattering process $e+N \rightarrow e+\text{anything}$. (b) Diagram for the inclusive virtual photon process $\gamma_V+N \rightarrow \text{observed hadron}+\text{anything}$. 
2. The $Q^2, W^2$ plane, showing the main features of the inelastic electron scattering data in various regions. The boundaries of the regions are actually not as precisely defined as they appear in the diagram. The data actually extend well beyond the range of this plot (see ref. 1).
3. The $Q^2, W^2$ plane, showing lines of constant $\omega, \omega', \text{and } \omega_w$. 

\[ \omega = \frac{W^2 - M^2}{Q^2} + 1 \]

\[ \omega' = \frac{W^2}{Q^2} + 1 \]

\[ \omega_w \approx \frac{W^2}{Q^2 + a^2} + 1 \]

\[ a^2 \approx 0.4 \text{ GeV}^2 \]
4. Total hadronic cross sections for $\gamma p$ and $\gamma d$ as functions of laboratory photon energy $\nu = E_{\gamma}$. Data are from ref. 8.
5. The $Q^2, W$ plane, showing the regions covered well in the coincidence electroproduction experiments from the various laboratories (see Table I). The single-arm scattering experiments (ref. 1) extend well beyond the limits of this diagram.
6. Data from the Harvard-Cornell group (C. N. Brown et al., ref. 17) on the reaction $\gamma p \rightarrow \pi^+ n$: the $t$ distributions for $W = 2.14$ GeV, $Q^2 = 1.195$ GeV$^2$ and for $W = 2.66$ GeV, $Q^2 = 2.02$ GeV$^2$ at $|\phi| = \pi/2$. The lines have no special significance.
7. Data from the DESY spark chamber group (I. Dammann et al., ref. 13) on the reactions $\gamma p \rightarrow \pi^\pm \Delta$: the $Q^2$ distributions for $W = 2.23$ GeV and $t-t_{\text{min}} = -0.04$ GeV$^2$. 
8. (a,b,c) Amplitudes contributing to the process $\gamma p \to \pi^+ n$
in the Electric Born model. (d) Rho-dominance model for $\pi^+$
electroproduction.
9. (a) Data from the Harvard group (C. N. Brown et al., ref. 30) on the forward cross section for the reaction $\gamma p \rightarrow n^+ n$ compared with the Electric Born model (F. A. Berends and R. Gastmans, ref. 38) for several choices of the pion form factor ($F_\rho = m_p^2/(Q^2 + m_p^2)$). (b) The same Electric Born model curve as in (a), assuming $F_\rho = F_1^\rho$, and showing the contributions from transversely and longitudinally ("scalar") polarized photons. (c) The data and model compared for the longitudinal-transverse interference term $D = d\sigma_L/d\Omega_T$. In this figure $k^2 = -Q^2$. 

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10. $Q^2$ dependence (called $-k^2$ in this figure) of the forward cross section for $\gamma p \rightarrow K^+\Lambda$ at $W = 1.85$ to 2.50 GeV. Data are from the Harvard group (C. N. Brown et al., ref. 33).
11. Rho decay angular distribution in the helicity system without background subtraction, for rhos produced by 4.7 GeV transversely polarized photons. The curves are proportional to $\sin^2 \theta_H$ and $(1+P_y \cos 2\theta_H)$. Data are from the SLAC, Berkeley, Tufts collaboration (ref. 40), using a monoenergetic back-scattered laser beam in a hydrogen bubble chamber.
\[ \gamma_{\nu} p \longrightarrow p\pi^+\pi^- \]

<table>
<thead>
<tr>
<th></th>
<th>(Q^2)</th>
<th>(W)</th>
<th>(M_{\pi\pi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESY (Eckardt et al.\textsuperscript{15})</td>
<td>(.3/1.5)</td>
<td>(2.0/2.7)</td>
<td>(.65/8.4)</td>
</tr>
<tr>
<td>SLAC (Ballam et al.\textsuperscript{23})</td>
<td>(.5/2.0)</td>
<td>(2/4)</td>
<td>(.65/9)</td>
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<tr>
<td>SLAC (Dakin et al.\textsuperscript{22})</td>
<td>(.5/2.5)</td>
<td>(2.4/5.2)</td>
<td></td>
</tr>
</tbody>
</table>

12. Rho decay angular distributions in the helicity system without background subtraction, for electroproduced \(p^0\). Data are from refs. 15, 23, 22.
\[ \frac{\sigma_p(Q^2, W, \varepsilon)}{\sigma_{\text{tot}}(Q^2, W, \varepsilon)} / \frac{\sigma_\gamma p}{\sigma_{\text{tot}}(W)} \]

13. The \( p^+ \) electroproduction cross section relative to the \( p^+ \) photoproduction cross section, divided by the total electroproduction cross section relative to the total photoproduction cross section, plotted against \( Q^2/2M_p \). If the \( p^+ \) cross section were a constant fraction of the total, independent of \( Q^2 \), the data points would equal unity. Data are from refs. 15, 23, 36, and 35.
$\gamma \nu p \rightarrow p \omega$

\[ \mu^- p \rightarrow \mu^- p \pi^+ \pi^- \pi^0 \]

$Q^2 \geq 0.15 \text{ (GeV/c)}^2$

$Q^2 \geq 0.5 \text{ (GeV/c)}^2$

- 118 Events
- 48 Events

(a)

14. Data from the SLAC bubble chamber group\textsuperscript{23} on $\omega$ electroproduction.
15. Preliminary data from the Cornell group (Ahrens et al., ref. 21) on the t-distributions in $\gamma p \rightarrow px$ for $0.425 < m_x^2 < 0.775$ GeV$^2$, $W = 3$ GeV, $\theta = 0$ and two values of $Q^2$. The cross section includes contributions from $\rho^0$, $\omega$, and nonresonant background.
16. Preliminary data from the Cornell group (Ahrens et al., ref. 21) for the slopes B obtained from a fit of the form $e^{B t}$ to the cross section for the reaction $\gamma \nu p \rightarrow px$ with $0.425 < m_x^2 < 0.775$ GeV$^2$. The cross section includes contributions from $\rho^0$, $\omega$, and nonresonant background.
17. Preliminary data from the Cornell group (Ahrens et al., ref. 21) for the missing mass spectrum in $\gamma p \rightarrow px$, including only events with low momentum transfer $t$. The mass cut used for the data plotted in Fig. 15 is indicated by the vertical lines.
10. The missing mass spectrum corresponding to the same measurement as in Fig. 17, but including only events with higher momentum transfer $t$. 

$$\gamma p \rightarrow pX$$

Cornell (Ahrens et al.\textsuperscript{21})

Preliminary Data

$Q^2=1.1$ GeV$^2$, $W=3$ GeV

$0.525 < t < 0.725$
19. t-distributions for $\rho^0$ production. Solid points are electroproduction data from the DESY streamer chamber (Eckardt et al., ref. 15). Open circles are photoproduction data from the SLAC-Berkeley-Tufts bubble chamber experiment (refs. 40, 44) with the same $m_\pi$ mass cut used in analyzing the electroproduction data.
20. Projections of the Dalitz plot for electroproduced $p\pi^+\pi^-$, showing the overlap of $\Delta^{++}\pi^-$ and $p^0 p$. Data are from the DESY streamer chamber group (Eckardt et al., ref. 15).
21. The t-distribution for \( \gamma p \rightarrow p \pi^+ \pi^- \) events with \( 0.65 < M_{\pi\pi} < 0.9 \) GeV, in two \( q^2 \) intervals. The average \( W \) is about 2.7 GeV. In the lower left corner is a plot of the dipion mass distribution. Data are from the SLAC \( \mu \) p bubble chamber experiment (Ballam et al., ref. 23).
PRELIMINARY DATA

22. The \( \pi \pi \) mass distribution from \( \gamma N \rightarrow p + \pi^+ \pi^- \) for all \( Q^2 \) and \( W \) covered in the SLAC spark chamber experiment (J. T. Dakin et al., ref. 22).
23. The t-distributions in three $Q^2$ intervals for the $\gamma p \rightarrow p \pi^+\pi^-$ events shown in Fig. 22 (J. T. Dakin et al., ref. 22).
Plots of $\frac{d\sigma}{d m_{x}^{2} dt d\phi}$ for the reaction $\gamma p \rightarrow pX$, as a function of $m_{x}^{2}$ and $t$ for three values of $Q^{2}$. The units along the axes are the same for the three plots: $m_{x}^{2}$ and $t$ in GeV$^2$, cross sections in mb/GeV$^4$ (logarithmic scale). Data are from the Cornell group at $W = 3$ GeV (E. Lazarus et al., ref. 20).
The slopes $B$ obtained by fitting the invariant inclusive cross section for $\gamma p \rightarrow p\Lambda$ with $E \, da/d\eta = \exp(-Bp_T^2)$, for backward protons ($-1 < x < -0.5$). Data are from the SLAC bubble chamber group (Ballam et al., ref. 23).
26. Plot of the normalized proton inclusive cross section

\[ \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{d^3p} (\gamma p \rightarrow pX) \] as a function of the fractional center-of-mass longitudinal momentum x for \( p_T \approx 0 \). Solid points are photoproduction, open points are electroproduction. Data are from DESY (refs. 49, 50, 12) and Cornell (ref. 20).
27. The normalized proton inclusive cross section \( \frac{1}{\sigma_{\text{tot}}} \) 
\( E \frac{d\sigma}{d^3p} (\gamma p \rightarrow p X) \) integrated over \( p_T^2 \), as a function of
\( x \). Photoproduction data are from the SLAC-Berkeley-Tufts collaboration (ref. 44),
electroproduction data from the SLAC bubble chamber group (ref. 23). For forward \( x \) values
(beyond the hatch marks) the data are contaminated with \( n^+ \) events. Note the effect on the photoproduction data of excluding the \( p^+p \) and \( \omega \) events.
26. The $\pi^+$ inclusive cross section $E \frac{d\sigma}{d^3p} (\gamma, p \rightarrow \pi^+ X)$ plotted against $p_T^2$ for two $x$ values in the central region and for three values of $Q^2$. Data are from the Cornell group (Lazarus et al., ref. 19).
29. The normalized inclusive cross section \((1/\sigma_{tot}) E \, d\sigma/d^3p\) 
\((yp - \pi^+x)\) plotted against \(p_T^2\) for two forward \(x\) ranges.
Photoproduction data (\(W \approx 2.6\) GeV) are from the DESY streamer chamber (Struczinski et al., ref. 50); electro-
production data (\(W \approx 2.65\) GeV) from the DESY spark chamber group (Dammann et al., ref. 14). At the top of each column
the photoproduction data are compared with the line which best fits the electroproduction data.
30. The normalized $n^+$ inclusive cross section $(1/\sigma_{\text{tot}})$

$E \cdot d\sigma/d^3p (\gamma p \to n^+ X)$ as a function of $x$ at $W = 2.6 \text{ GeV}$, $p_T = 0$.

Photoproduction data are from Struczinski et al. (ref. 50); electroproduction data from Alder et al. (ref. 12), and Bebek et al. (ref. 18).
The normalized inclusive cross section \( \langle 1/\sigma_{\text{tot}} \rangle \) \( d\sigma/d^3p \) \((p^+ + X)\) integrated over \( p_T^2 \) and plotted against \( x \).

Photoproduction data are from the SLAC, Berkeley, Tufts collaboration (ref. 44); electroproduction data are from the SLAC bubble chamber group (ref. 23). Note the effect on the photoproduction data of excluding the \( \omega\) and \( \omega^-\) events.
$\gamma_{V}^{p} \rightarrow \pi^{-} X$

- $Q^{2} = 0$ DESY$^{50}$
- $Q^{2} = 1.15$ DESY$^{12}$
- $Q^{2} = 2.0$ Harvard-Cornell$^{18}$

$W \approx 2.6$ GeV

$P_{T}^{2} \leq .03$ GeV$^{2}$

32. Same as Fig. 30, except $\pi^{-}$ inclusive instead of $\pi^{+}$. 
33. Normalized π^- inclusive cross section \( \langle 1/\sigma_{\text{tot}} \rangle \) E \( d\sigma/d^3p \) \((\gamma p \rightarrow \pi^- X)\) integrated over \( p_T^2 \) and plotted against \( x \).

Data are from the DESY streamer chamber (Struczinski et al., ref. 50, for photoproduction, and Eckardt et al., ref. 16, for electroproduction). The dashed line shows the effect of excluding \( p^+p^- \) events with \( m_{pp} < 1 \) GeV.
\[ \gamma_P \rightarrow \pi^\pm (\text{anything}) \]

1.8 < \( W < 2.5 \) GeV
\( (W) = 2.1 \) GeV

2.5 < \( W < 4.0 \) GeV
\( (W) = 3.1 \) GeV

* elastic \( \rho^0 \) and \( \omega \) excluded

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34. Same as Fig. 31, except \( \pi^- \) inclusives instead of \( \pi^+ \).
$\theta^* \lesssim 9.6^\circ$

$W = 2.66$ GeV

$k^2 = -2.02$ GeV$^2$

$\omega = 4.07$

$\gamma^*_\nu + p \rightarrow \pi^+ + MM$

35. Missing mass spectrum for $\gamma^*_\nu p \rightarrow \pi^+ X$ in the forward direction at $Q^2 = -k^2 = 2.02$ GeV$^2$ and $W = 2.66$ GeV. Data are from the Harvard-Cornell collaboration (Bebek et al., ref. 18). The curve is drawn free-hand through the data.
36. Missing mass spectrum for $\gamma p \rightarrow \pi^- + X$ for the same experimental conditions as in Fig. 35. Data are from the Harvard-Cornell collaboration (Bebek et al., ref. 18). The curve is same as the curve in Fig. 36, passing through the $\pi^+$ data.
37. Plus/minus ratio for hadrons electroproduced in the center-of-mass forward hemisphere. In addition to forward pions and kaons there is some contribution from backward protons misinterpreted as forward pions. Data are from the SLAC spark chamber experiment (Dakin et al., ref. 22).
\[ \gamma p \rightarrow K^+ X \]

\[ W = 2.63 \text{ GeV} \]

- \[ q^2 = 1.16 (\text{GeV}/c)^2 \] (Ref. 1)
- \[ q^2 = 0 \] (This experiment)

Normalized inclusive \( K^+ \) invariant cross sections \((1/\sigma_{\text{tot}}) (d^2 \sigma/d\Omega dp)/\sigma_{\text{tot}}\) at \( p_T = 0 \), plotted against \( p_{\text{lab}} \) (corresponding to \( 0.5 < x < 1 \)). Dark circles are photoproduction data from Burfeindt et al. (ref. 49); open circles are electroproduction data from Alder et al. (ref. 12).
39. Preliminary missing mass spectrum for $\gamma p \rightarrow K^+ X$ for $Q^2 = 2.0 \text{ GeV}^2$, $W = 2.67 \text{ GeV}$. Data are from the Harvard-Cornell collaboration (Bebek et al., ref. 18). The kaon-proton separation (by time-of-flight) is not perfect at the lower missing masses, so there is some proton contamination in the spectrum.
40. Prong distributions for electroproduced charged hadrons, as a function of $Q^2$ in a fixed range of $W$. Data are from the SLAC bubble chamber group (Ballam et al., ref. 23). Also shown are photoproduction data (ref. 40) for comparison.
Average charged hadron multiplicity plotted against $Q^2$ for three fixed $W$ intervals. Data are from the SLAC bubble chamber experiment (Ballam et al., ref. 23). Comparison photoproduction data are from refs. 40, 52.
42. The $Q^2, W^2$ plane showing lines (dashed) along which average charged hadron multiplicity is conjectured to be constant, with multiplicity decreasing as one passes from line to line in the direction photoproduction $\rightarrow$ elastic. Similar (or identical) lines may define fixed forward charge ratio or any other global property of the hadron final states.
43. Missing mass spectrum for $\gamma p \rightarrow \pi^* + MM$ in the forward direction at $Q^2 = -k^2 = 1.195 \text{ GeV}^2$ and $W = 2.14 \text{ GeV}$. Data are from the Harvard-Cornell collaboration (Bebek et al., ref. 18). The value is essentially the same as for the data of Fig. 35. and the curve is the same curve (except for scale) as in Fig. 35.
Diagram to illustrate the qualitative features of deep inelastic scattering for low and high values of the scaling variables (see text).