HADRONIC BRANCHING RATIO OF THE W BOSON

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ABSTRACT

Using scale invariance and SU(3) × SU(3) symmetry W decay into hadrons is related to the high-energy e^+e^- annihilation into hadrons. Quark and parton models for e^+e^- annihilation are then used to estimate the branching ratio $\Gamma(W \to \text{hadrons})/\Gamma(W \to \mu\nu)$. $W \to 2\pi$ is also compared to the total hadron rate.

As experiments are planned to search for the hypothetical W boson, mediator of the weak interactions, it is important to have some reliable estimate of the branching ratio of W into hadrons relative to W into the leptonic modes $\mu\nu$ and $e\nu$. Previous estimates[1, 2] have relied on adding up contributions from the lowest lying two-and three-body hadronic states. It has been noted, however, by Gribov, Ioffe, and Pomeranchuk[3] that it might be possible to relate the hadronic branching ratio of the W via CVC to the hadronic contribution to the e^+e^- cross section. Since colliding-beam experiments should provide information about high-energy e^+e^- cross sections well in advance of the first neutrino experiments at NAL, such a relation might be useful in the search for the W.

We note below that scale invariance as developed by Wilson[4] has some implications for W decay into hadrons and further implies a relation between this decay and e^+e^- annihilation into hadrons in the SU(3) × SU(3) symmetry limit.

The amplitude for W → hadronic state h is

$$M = g\,\epsilon_\mu \langle h | J_\mu | 0 \rangle, \tag{1}$$

where g is the semiweak coupling constant related to the usual Fermi constant G by

$$\frac{g^2}{M_W^2} = \frac{G}{\sqrt{2}}\,, \tag{2}$$

ϵ_μ is the polarization vector of the W, and the weak current is

$$J_\mu = \left(V_\mu^{1+i2} + A_\mu^{1+i2} \right) \cos\theta_c + \left(V_\mu^{4+i5} + A_\mu^{4+i5} \right) \sin\theta_c\,; \tag{3}$$

the total hadronic decay rate of the W is, therefore,

$$\Gamma(W \rightarrow \text{hadrons}) = \frac{g^2}{6M_w}\left(\frac{q^\mu q^\nu}{M_w^2} - g^{\mu\nu}\right)\int e^{iq\cdot x} < 0 \,|\, J_\mu^+(x)\, J_\nu(0) \,|\, 0 > dx, \qquad (4)$$

where $q_\mu = (M_w, \vec{0})$.

Similarly, the cross section for e^+e^- annihilation into hadrons is

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{(4\pi\alpha)^2}{2s^3}\left(p_1^\mu\, p_2^\nu + p_2^\mu\, p_1^\nu - p_1\cdot p_2\, g^{\mu\nu}\right)$$

$$\times \int e^{i(p_1+p_2\cdot x} < 0 \,|\, V_\mu^{e-m}(x)\, V_\nu^{e-m}(0) \,|\, 0 > dx, \qquad (5)$$

where the electromagnetic current is

$$V_\mu^{em} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 . \qquad (6)$$

Scale invariance and current conservation imply

$$\int e^{iq\cdot x} < 0 \,|\, V_\mu^{e-m}(x)\, V_\nu^{e-m}(0) \,|\, 0 > dx = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) a, \qquad (7)$$

for large $q^2 = (p_1 + p_2)^2$ and, therefore,[5]

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{8\pi^2\alpha^2 a}{q^2}, \qquad (8)$$

for large q^2. a is an unknown dimensionless constant. Similarly, scale invariance implies

$$\int e^{iq\cdot x} < 0 \,|\, J_\mu^+(x)\, J_\nu(0) \,|\, 0 > dx = q_\mu q_\nu b - q^2 g_{\mu\nu} c \qquad (9)$$

and, therefore, from Eq. (4)

$$\Gamma(W \rightarrow \text{hadrons}) = \frac{g^2 M_w c}{2}, \qquad (10)$$

combining this with

$$\Gamma(W \rightarrow \mu\nu) = \frac{g^2 M_w}{6\pi} \qquad (11)$$

gives

$$\Gamma(W \rightarrow \text{hadrons}) \,/\, \Gamma(W \rightarrow \mu\nu) = 3\pi c. \qquad (12)$$

The branching ratio Eq. (12) would, therefore, not be expected to increase with increasing M_w.

In the SU(3) × SU(3) symmetry limit the leading singularities in the operator product expansions for $V_\mu^{e-m}(x)V_\nu^{e-m}(0)$ and $J_\mu^+(x)J_\nu(0)$ are related. In terms of the unknown constants in Eqs. (7) and (9), this relation is 3a = c = b or

$$\Gamma(W \to \text{hadrons}) = \frac{3g^2 M_W}{16\pi^2\alpha^2}\left[s\sigma(e^+e^- \to \text{hadrons})\right]_{s=M_W^2}; \tag{13}$$

combining this with Eq. (11) gives

$$\frac{\Gamma(W \to \text{hadrons})}{\Gamma(W \to \mu\nu)} = \frac{9}{8\pi\alpha^2}\left[s\sigma(e^+e^- \to \text{hadrons})\right]_{s=M_W^2}. \tag{14}$$

Actually, the SU(3) × SU(3) symmetry assumption used above is almost as good as SU(2) × SU(2) since the contribution from the strange currents is suppressed by $\sin^2\theta_c \approx 1/16$, and the contribution from the isoscalar part of the electromagnetic current is suppressed by 1/3. Thus, we could have derived Eq. (14) (apart from a factor 3/4) in the SU(2) × SU(2) symmetry limit by setting $\theta_c = 0$ and dropping the isoscalar current in V^{e-m}. Therefore, Eq. (13) can be regarded as nearly a result of scale invariance and SU(2) × SU(2) symmetry. We should point out that Eqs. (13) and (14) could be valid even though scale invariance is not. For example, if $3a(s) = c(s) = b(s) \neq$ const.

If we extrapolate forthcoming e^+e^- data to the high-energy region, Eq. (14) provides a relatively reliable prediction of the hadronic to muonic branching ratio of the W. The $e\nu$ decay rate of the W is, of course, expected to equal the $\mu\nu$ rate.

In the absence of any information on the annihilation process in the multi-GeV region, one can contrive various parton and quark models to estimate the ratio Eq. (12). To get an idea of the expected order of magnitude, we give here the results using Eq. (14) and 3 models for $e^+e^- \to$ hadrons given in Ref. 6.

$$\frac{\Gamma(W \to \text{hadrons})}{\Gamma(W \to \mu\nu)} = \begin{cases} 3/2 & \text{spin 1/2 parton model} \\ 3/8 & \text{spin 0 parton model} \\ 1 & \text{quark model.} \end{cases} \tag{15}$$

As an example of an individual hadronic mode, we consider here the 2π channel.

$$\Gamma(W \to 2\pi) = \frac{g^2 M_W}{24\pi}\,|F_\pi(M_W^2)|^2 \cos^2\theta_c\,. \tag{16}$$

If we parameterize the pion form factor by the function $\left[M_\rho^2/(M_\rho^2 + t)\right]^n = F_\pi(t)$, we find for a 10-BeV W boson

$$\frac{\Gamma(W \to 2\pi)}{\Gamma(W \to \mu\nu)} = \begin{cases} 5\cdot 10^{-10} & \text{dipole form factor } (n = 2) \\ 1\cdot 10^{-5} & \text{single pole form factor } (n = 1) \\ 2\cdot 10^{-3} & \text{optimistic value } n = 1/2\,. \end{cases}$$

From these considerations, it seems probable that the hadronic fraction of the W decay will be predominantly multiparticle.

Finally, for convenience we record here the total W width using Eqs. (2), (11), and the quark model result of Eq. (15)

$$\Gamma(W \rightarrow \text{all}) = \frac{G M_W^3}{2\sqrt{2}\,\pi} . \tag{17}$$

For a 10-BeV boson, this implies a width of 1.5 MeV.

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