AN ESTIMATE OF THE PRODUCTION OF BOUND MAGNETIC MONOPOLE STATES

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ABSTRACT

Because of the strong attraction between a monopole and antimonopole produced as a pair, there is a strong likelihood that a pair which is produced will tend to become bound. In a nonrelativistic model, it is estimated that the ratio of bound-state to free-pair production is 10^{10} for production of a low-lying state and decreases rapidly for higher lying states. This prediction is independent of the production mechanism of the monopoles.

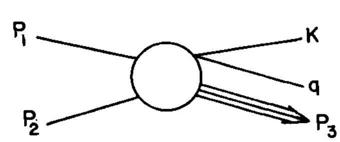
It has been suggested[1] that the production of free monopole pairs may be greatly suppressed with respect to the production of a bound monopole-antimonopole state (which, following S. Frankel, we will call monopolonium). The physical reason for this suggestion can be seen quite easily in the following way: consider a monopole-antimonopole pair with some small separation r_0. Then the force between monopoles is

$$F = g^2/r_0^2 ,$$

where g^2, the magnetic charge, is given by $g^2e^2 \sim 1$, and hence the force tending to bind the monopoles is very strong.

In addition to the energy needed to create the monopoles, a large kinetic energy will also be needed if the monopoles are to be able to separate from each other and be seen as free particles. Thus, one would see monopolonium states at energies just past the threshold for monopole production, and free monopoles would be copiously produced only at much higher energies. In this note, we will try to quantify the conclusion and make a numerical estimate of the relative probability of free-pole production vs monopolonium production near threshold.

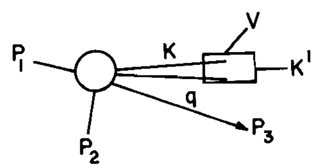
Let us begin our calculation by considering a reaction



(1)

in which a projectile (P_1) strikes a target (P_2) producing hadrons (P_3) and a free monopole-antimonopole pair with momenta K and q. Let the amplitude for the reaction be $A(P_1, P_2, P_3, K, q)$. In this reaction, the final-state interaction between the poles is neglected.

Now consider what happens when the final-state interaction is included. We now have a reaction like



(2)

where V represents the interaction between the poles, and K^1 is the momentum of the final state of the pole-antipole system. The amplitude for this process will be $C(P_1, P_2, P_3, K, q, K^1)$.

In order to compare these two processes, it will be necessary to make some simplifying assumptions.

Assumption I: In reaction (2), the monopoles are on mass shell and, in fact, can be treated as free particles which are created, travel a while, and then interact. In this case,

$$C(P_1, P_2, P_3, K, q, K^1) = A(P_1, P_2, P_3, K, q)\, B(K, q, K^1), \qquad (3)$$

where B represents the amplitude which described the final monopole interaction.

Assumption II: The poles move slowly, so that the amplitude B can be calculated nonrelativistically, i.e.,

$$B_n(K, q, K) \approx \left\langle \Psi(\rho) \,\middle|\, V(\rho) \,\middle|\, e^{i\,(\overline{K-q})\cdot\rho} \right\rangle , \qquad (4)$$

where $\rho = r_1 - r_2$ = separation of the poles, $V(\rho) = g^2/\rho$ is the potential between the poles, and $\Psi(\rho)$ is the final state of the pole pair. Note that since we are dealing with a Coulomb interaction, Ψ is known exactly,[3] and Eq. (4) is good to all orders in g.

These two assumptions mean that the calculation presented here has, at most, a heuristic value, since the interaction between the monopoles is so strong that treating the intermediate monopoles in (2) as free is surely not justified, and the lifetime of monopolonium is so short[1] that the description of the state by a Coulomb wave function is probably not justified. We have, however, gained the enormous advantage of making the ratio of bound to free production independent of the mechanism for creating the monopole pair, as we shall see below. Thus, our calculation will depend only on the known force between monopoles and the known bound states of the system and not on the assumptions about the coupling of the poles to hadrons or other particles.

The amplitude B depends only on $\overline{K-q}$. If we assume that A depends only on $\overline{K+q}$, and is independent of $\overline{K-q}$, then the relative probability of making monopolonium in a state n instead of a free pair is

$$R_n = \frac{\int_{\text{phase space}} |A|^2 \, dP_1 \ldots d(K+q) \int |B_n|^2 \, d^3(\overline{K-q})}{\int_{\text{phase space}} |A|^2 \, dP_1 \ldots d(K+q) \int d^3(\overline{K-q})} = \frac{\int |B_n|^2 \, d^3(\overline{K-q})}{\frac{4}{3}\pi \, |\overline{K-q}|^3_{max}}, \qquad (5)$$

where $(K-q)_{max}$ is the maximum value allowed for $|\overline{K-q}|$ by kinematics.

The problem reduces to calculating B_n. We note already that we expect a large enhancement of bound-state production because of the factor g^2 which appears in the potential. In addition, the bound-state wave function depends on g in a complicated way, but we may expect further enhancement if we note that the matrix element in Eq. (4) is actually like an expectation value of $1/\rho$. Because of the strong binding between the poles, the final-state wave function will be concentrated in a very small spatial extent (for example, we shall see that the first Bohr orbit is at least 137 times as small as in hydrogen), so that contributions from small ρ values will become more prominent, further enhancing the bound-state production.

With this expectation in mind, we turn to an explicit calculation of the ratio of bound-state to free-pair production. The Coulomb wave function is

$$\Psi_{n\ell m} = a^{-3/2} \frac{2}{n^2} \frac{\sqrt{(n-\ell-1)!}}{[(n+\ell)!]^3} x^\ell e^{-x/2} L^{2\ell+1}_{n-\ell-1}(x) y(R),$$

where L is the Laguerre polynomial, the Bohr radius is

$$a = \frac{2}{mg^2},$$

and

$$x = \frac{2\rho}{na},$$

and m the mass of the monopole; then, if we let

$$\bar{\xi}_n = \frac{na}{2}(\overline{K-q})$$

and expand the plane wave in spherical harmonics,

$$\left\langle \Psi_n \middle| V \middle| e^{i(\overline{K-q})\cdot\bar{\rho}} \right\rangle = \left(\frac{na}{2}\right)^2 g^2 \frac{2}{n^2} \sqrt{\frac{(n-\ell-1)!}{[(n+\ell)!]^3}}\, a^{-3/2}\, 4\pi(i)^\ell\, Y_{\ell m}(\hat{\xi}_n)$$

$$\times \int x^{\ell+1} e^{-x/2} L^{2\ell+1}_{n-\ell-1}(x)\, J_\ell(\xi_n x)\, dx. \qquad (7)$$

At this stage the problem is finished since we have expressed the ratio R_n in terms of an integral over known functions. To get a quick numerical estimate, we can make a few approximations which will allow us to do the integral exactly. These should not be confused with the assumptions about the monopole production which we made earlier which are essential to the model we are proposing. The following assumptions are made just to simplify the integral so the numerical integrations will not be necessary.

First, let us confine our attention to S waves in the Coulomb wave function. This means that when we sum over final states we will omit values of ℓ greater than 0 and underestimate the true value of R (defined by $R = \Sigma_n R_n$). Also, we note that in the limit of large K,

$$L_K^1(x) \to K e^{x/2}\, \Gamma(2)\, [x(1+n)]^{-1/2}\, J_1(\sqrt{4x(1+n)}).$$

Since the detection of monopolonium will probably be made by looking for its decay from a high state to the ground state (which is probably unstable against annihilation), it makes sense to use this expression in Eq. (7). Then, using the well-known relation[5]

$$\int_0^\infty J_{2\nu}(a\sqrt{x})\, J_\nu(bx)\, dx = 1/b\, J_\nu(a^2/4b),$$

we find

$$\left\langle \Psi_n \middle| V \middle| e^{i(\overline{K-q})\cdot\bar{\rho}} \right\rangle = \sqrt{n\pi a}\, \frac{n}{n!}\, g^2 \frac{1}{\xi_n^2}\, J_0\left(\frac{n}{\xi_n}\right). \qquad (8)$$

Inserting this expression into Eq. (5) and carrying out the integrals over phase space, we find

$$R_n = 6\pi \left(\frac{m}{|K-q|_{max}}\right)^2 \frac{g^6}{n(n!)^2}\, (1 + \cos^2 y_{min} + \cos 2y_{min}), \qquad (9)$$

where we have written

$$y_{min} = \frac{2}{a|K-q|_{max}}$$

and have used the fact that $y_{min} >> 1$ to evaluate the phase-space integrals.

The expression in parentheses in Eq. (9) cannot be smaller than 1 or greater than 3, and the ratio $m/|K-q|_{max}$ cannot exceed 1 by too much before the nonrelativistic approximation with which we started the calculation would break down. Thus, we will in our order of magnitude calculation replace both of these by unity.

Then, using the predicted value of g, we find

$$R_n \sim 6\pi\,(4\pi)^3 \frac{(137)^3}{n(n!)^2} \tag{10}$$

The probability that a monopole pair will, by recombination, become an excited state of monopolonium is then[6]

n	R_n
1	2×10^{10}
2	4×10^{9}
3	2×10^{8}
4	6×10^{6}
5	7×10^{4}
6	1×10^{3}

From these results we can draw two conclusions:

1. Because of the strength of the final-state interaction between monopole and antimonopole, it is much more likely that monopolonium will be produced than a free pair,

2. The probability of creating excited states of monopolonium decreases rapidly with the principal quantum number, and by $n \sim 8$, the bound and free production are roughly equivalent.

The consequence of this conclusion for monopole searches is obvious. The enormous enhancement of monopolonium production over free-pair production means that searches for evidence of monopolonium (that is, for indirect evidence for the existence of monopoles) should be given much more attention than they have heretofore received, and some thought should be given to just how such a system would react and how one could detect its decay products.

REFERENCES

[1] M. A. Ruderman and D. Zwanziger, Phys. Rev. Letters 22, 146 (1969).

[2] The most readable presentation of the basic ideas of monopole properties is J. Schwinger, Science 165, 757 (1969) and earlier references are contained therein.

[3]A. Messiah, Quantum Mechanics, (North Holland Publishing Company, 1961), Ch. XI.

[4]M. Abramowitz and I. A. Stegun, Handbook of Mathematical Function, (National Bureau of Standards, 1966).

[5]I. S. Gradshteyn and I. M. Rhyzhik, Tables of Integrals, Series, and Products, (Academic Press, 1965).

[6]In order that the reader not concern himself greatly with the high n approximation that led eventually to Eq. (9), we note in passing that an evaluation of Eq. (7) for the case of the ground state of monopolonium ($n = 1$, $\ell = m = 0$) gives

$$R_1 = \frac{9\pi}{2} g^6 \left(\frac{m}{|K-q|_{max}} \right)^2 ,$$

which is essentially the same as Eq. (9) evaluation for $n = 1$. Thus, the approximations which simplified the integrals should not have a large effect on the results--certainly not as large an effect as the factor g^6 which is the main thing with which we are concerned. In the table, R_1 is evaluated from the above, and for $n > 1$, Eq. (10) is used.