THEORETICAL EXPECTATIONS FOR INELASTIC NEUTRINO-NUCLEON INTERACTIONS

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ABSTRACT

We estimate in this study the contributions of the three structure functions to the double differential cross section for a realistic NAL neutrino spectrum and reasonable theoretical assumptions. Assuming that $\sigma_S \approx 0$, we find that in most of the models the form factors $F_2(x)$ and $F_3(x)$ are separable. We then discuss the theoretical predictions that can be tested with this new experimental information. Most of the theoretical predictions, besides relations among the total cross sections are sum rules. We find that most of the sum rules valid in the deep inelastic region are testable provided that they saturate below 200 BeV. The sum rules, their saturation properties, and relevant references are summarized in the table.

I. THE TOTAL CROSS SECTIONS

The main objective of this article is to summarize some of the most important theoretical results that have been obtained for inelastic neutrino-nucleon scattering and also to discuss the experimental possibilities of testing them at NAL.

The kinematics for the processes have been discussed in many places in as many different notations.[1-5] We recall here[4] that

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi}{EE'} \frac{d\sigma}{d\Omega dE'} = \frac{G^2}{2\pi} \beta(Q^2, \nu) \left(\frac{E'}{E}\right) \left[1 + \frac{\nu}{E'}(L) - \frac{\nu}{E}(R)\right], \tag{I.1}$$

where $\beta = W_2(Q^2, \nu)$ for neutrinos; $(L) = \sigma_L/(\sigma_L + \sigma_R + 2\sigma_S)$ and $(R) = \sigma_R/(\sigma_L + \sigma_R + 2\sigma_S)$ are functions of the two Lorentz invariants:

$$\nu = \frac{q \cdot P}{m} \text{ and } Q = -q^2 = 4EE' \sin\frac{2\theta}{2} . \tag{I.2}$$

The total cross sections occurring in Eq. (I.1) correspond to the total cross sections of a right-handed, left-handed, and scalar current on a proton. They are discussed in detail in Ref. 4 and in the limit $\nu^2/Q^2 >> 1$, $\nu >> M$, they are related to the structure functions as follows:

$$W_1(Q^2, \nu) = \nu\beta(Q^2, \nu) \frac{\nu}{Q^2} \frac{\sigma_L + \sigma_R}{\sigma_R + \sigma_L + 2\sigma_S} , \tag{I.3}$$

$$W_3(Q^2, \nu) = (Q^2, \nu)\, \frac{2M\nu}{Q^2}\, \frac{\sigma_L - \sigma_R}{\sigma_R + \sigma_L + 2\sigma_s}\,. \tag{I.3}$$

The structure function for the antineutrino-induced processes are obtained by the interchanges $\bar{\sigma}_L \leftrightarrow \bar{\sigma}_R$ and will be designated by the superscript bar. The CERN experiment[6] indicates that the data are consistent with the total cross section rising linearly with energy. The best fit with a straight line is

$$\sigma_{tot} = (0.60 \pm 0.15)\, \frac{G^2 M}{\pi} \cdot E. \tag{I.4}$$

This remarkable property follows[4] simply from Eq. (I.1) and the scaling of $\nu\beta \to F_2(Q^2/2M\nu)$ only. To see this we use Eq. (I.1) and integrate over Q^2:

$$\begin{aligned} \frac{d\sigma}{d\nu} &\simeq \frac{G^2}{2\pi}\, \frac{E'}{E} \int_{-0}^{2M} \frac{dQ^2}{\nu}\, \nu\beta(Q^2, \nu) \left[1 + \frac{\nu}{E'}(L) - \frac{\nu}{E}(R)\right] \\ &= \frac{G^2 M}{\pi}\, \frac{E'}{E} \left[1 + \frac{\nu}{E'}\langle L\rangle - \frac{\nu}{E}\langle R\rangle\right] \int_0^1 dx\nu\; \beta(Q^2, \nu), \end{aligned} \tag{I.5}$$

where $\langle R\rangle$ and $\langle L\rangle$ implies the appropriate averages over x have been taken. Then the total cross section is

$$\sigma_{tot} \simeq \frac{G^2 M}{\pi}\, E \left[\int_0^1 dx\; \nu\beta\right] \left[\frac{1}{2} + \frac{1}{2}\langle L\rangle - \frac{1}{6}\langle R\rangle\right]. \tag{I.6}$$

Noting that the values of the last bracket range from one-third to one, we can obtain an upper and a lower bound for the integral:

$$(0.6 \pm 0.15) \leq \int_0^1 dx\; \nu\beta \leq (1.8 \pm 0.45). \tag{I.7}$$

We recall that the corresponding integral for electroproduction has been determined accurately at the SLAC experiments[7,8] and has been interpreted as a measure of the mean-square charge per parton. It is expected that the corresponding integral here will also be one of the first numbers to be determined accurately by the neutrino experiments, and it has a similar interpretation: It is the mean number of non-strange partons per parton. We will return to this point when we discuss the sum rules.

The relative magnitudes of the total cross sections vary considerably within the models. In parton models

$$\begin{aligned} &\sigma_{\nu p} \neq \sigma_{\nu n}, \\ &\sigma_{\bar{\nu} p} \neq \sigma_{\bar{\nu} n}, \end{aligned} \tag{I.8}$$

and the ratio $R = \sigma_s/(\sigma_R + \sigma_L)$ depends on the spin of the constitutents within the proton or neutron. Bjorken and Paschos[4] say

$$\sigma_L > \sigma_R, \sigma_{\nu p} > \sigma_{\bar{\nu} p}. \tag{I.9}$$

Drell, Levy, and Yan[9] predict $\sigma_L >> \sigma_R$ leading to

$$\sigma_{\nu p} = \sigma_{\nu n} = 3\sigma_{\bar{\nu} p} = 3\sigma_{\bar{\nu} n}. \tag{I.10}$$

Diffractive models[10] lead to $\sigma_R = \sigma_L$ and also

$$\sigma_{\nu p} \approx \sigma_{\nu n} \approx \sigma_{\bar{\nu} p} \approx \sigma_{\bar{\nu} n}. \tag{I.11}$$

These are some of the challenges for the early experiments. Their determination will distinguish among different models.

II. YIELD ESTIMATES FOR INELASTIC NEUTRINO-NUCLEON EXPERIMENTS

In a realistic situation one may not have a monochromatic neutrino beam. Furthermore, detection of all the final hadronic states is very difficult. Therefore, it is not completely unrealistic to consider a situation with a neutrino spectrum incident on a hydrogen target and demand identification of the final muon with energy E' and at an angle θ. We denote by Y(E) the neutrino spectrum, and we define an effective cross section:

$$\frac{d\sigma^{eff}}{d\Omega\, dE'} = \int_{E' + \frac{Q^2}{2M}}^{E_{max}} Y(E) \frac{d\sigma}{d\Omega\, dE'}\, dE. \tag{II.1}$$

We assume that the three structure functions scale and obtain:

$$\frac{M d\sigma^{eff}}{d\Omega\, dE'} = \frac{G^2}{2\pi^2} E'^2 \int_{\frac{1}{2}}^{\frac{E_{max} - E'}{2E_{max} E' \sin^2 \frac{\theta}{2}}} \frac{d\lambda}{\lambda}\, Y\left(\frac{E'}{1 - 4\lambda \frac{E'}{M} \sin^2 \frac{\theta}{2}}\right) \left\{F_2(\lambda) \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \left[2F_1(\lambda)\frac{(E - E')}{M} + F_3(\lambda)\frac{(E + E')}{M}\right]\right\} \tag{II.2}$$

where

$$\lambda = \frac{p \cdot q}{Q^2} = \frac{M\nu}{Q^2}$$

In estimating the structure functions we make the following working hypotheses:

1. α) The contribution of the vector and the axial-vector currents to $\beta(Q^2, \nu)$ are equal:

$$\beta^V(Q^2, \nu) = \beta^A(Q^2, \nu) \qquad \text{(II.3)}$$

β)

$$\beta_{proton} = \beta_{neutron} \qquad \text{(II.4)}$$

γ) The isoscalar contribution to the electromagnetic $W_2(Q^2, \nu)$ is negligible. These assumptions imply:

$$\beta(Q^2, \nu) = 4W_2^{e.m.}(Q^2, \nu). \qquad \text{(II.5)}$$

2. Motivated by the SLAC result[8] we assume $\sigma_s \approx 0$ which in turn implies

$$F_1(\lambda) = \lambda F_2(\lambda). \qquad \text{(II.6)}$$

This is the result of current algebra[11] with quark commutation relations, and it also follows for any spin-1/2 parton model.

3. There is no reliable way for estimating $F_3(\lambda)$. We use in the numerical calculations the prediction[3] of the "quark-parton" model:

$$F_3(\lambda) = 3.257\, M \cos^2\theta_c \left(\frac{1}{1 - \frac{1}{2}\lambda}\right)\left[\ln(4\lambda - 1) - 2\left(1 - \frac{1}{2\lambda}\right)\right]. \qquad \text{(II.7)}$$

This expression lies somewhere in the middle of the functional forms that have been discussed in the literature. For a comparison we list the asumptotic behavior of $F_3(\lambda)$ as $\lambda \to \infty$ in the different models.

α) field-theoretic model[9]	$F_3(\lambda) \to \nu$
β) Regge model[10]	$F_3(\lambda) \to \nu^{\frac{1}{2}}$
γ) parton model[3]	$F_3(\lambda) \to \ln \nu$
σ) other models[12]	$F_3(\lambda) \to$ Const.
ϵ) Regge plus duality[10] (Harari)	$F_3(\lambda) \to 0$

The final input for the numerical estimates is the neutrino spectrum. We present in this section estimates for two different spectra. The first is a spectrum for a 500 BeV NAL with a decay length of 1400 meters and an earth shield of 1400 meters. The spectrum is given in terms of the number of neutrinos/BeV/m^2/10^5 incident protons, and it has been calculated by Nezrick.[13] Figure 1 gives the double differential cross section as a function of the muon energy E' and the muon angle θ. These cross sections give measureable counting rates, which can be calculated easily for specific experiments. In separating the structure functions, one would like to know the relative contributions of the three form factors. Figure 2 gives the ratio of the contribution of $F_3(\lambda)$ to the contribution of $F_2(\lambda)$. We note that the ratio increases with angle, as it is expected, and in some places it is of order unity. Therefore, we conclude that separation of $F_3(\lambda)$ should be an attainable goal for NAL. Separation of $F_1(\lambda)$ is harder. For the same kinematic region as in Figs. 1 and 2, its contribution to the cross section in comparison to $F_2(\lambda)$ has the following bounds:

$$0.01 \lesssim \frac{\text{Contribution of } F_1(\lambda)}{\text{Contribution of } F_2(\lambda)} \lesssim 0.25. \tag{II.8}$$

Figure 3 corresponds to a different neutrino spectrum and energy range:

$$Y(E) = 10.0e^{-0.23E} \text{ neutrinos/BeV/m}^2\text{/3278 protons},$$

with $5 < E < 25$ BeV. This spectrum is to a good approximation the neutrino spectrum calculated for a 100-BeV NAL[14] with a shorter decay length and a steel shield. We observe the same general dependence on energy and angle. In the same figure we have also plotted, for comparison, the cross sections for inelastic Compton scattering[15] for the same kinematic region, produced, however, with an incident bremsstrahlung spectrum. We note that for a fixed angle, the neutrino-induced cross sections fall much slower with E' in contrast to the Compton terms. This is due to the fact that neutrino-induced processes have a weaker dependence on Q^2, and they should give measurable rates at larger values of this variable.

In case that the energy of the incident neutrinos can be accurately determined, one can repeat these calculations using Eqs. (I.1), (I.3), and (II.2). The relative values of the contributions of the scaling functions can be obtained from the expression in the curly bracket in Eq. (II.2) and the assumptions (II.6) and (II.7).

It appears that neutrino experiments at small angles can determine $F_2(\lambda)$. Extension of the experiments to larger angles will be sensitive to $F_3(\lambda)$, provided that this structure function is at least as large as the prediction of the parton model.

III. SUMMARY OF NEUTRINO AND ANTINEUTRINO SUM RULES

In the last two years intensive theoretical investigations on inelastic neutrino-nucleon interactions have produced a large number of predictions. The most important results, which at the same time are the easiest to test experimentally, are several sum rules summarized in Table 1. The sum rules are listed in order of theoretical rigor proceeding from left to right. We have listed in the first four rows the assumptions involved in deriving them, then the region were they should be valid, and finally we discuss the possibilities of testing them in future NAL experiments. To be more precise, we discuss several of them in detail.

The Adler[1] sum rule

$$\int_0^\infty d\nu\,(\bar{\beta} - \beta) = \int dx^4 e^{iq\cdot x} \left\langle p \left| \left[J_0^+(x), J_0(0) \right] \right| p \right\rangle \delta(x_0)$$
$$= -\frac{3}{2}(B - 3Y)\sin^2\theta_c + 2T_3(1 + \cos^2\theta_c), \qquad \text{(III.1)}$$

involves the difference of two form factors with the Pomeranchuk contribution canceling, thus leaving the ρ as the leading trajectory. This guarantees the convergence of the sum rule. The equal time commutator has been evaluated in current algebra in terms of the Cabibbo angle θ_c, and the quantum numbers of the target: baryon number, B, hypercharge, Y, and the third component of isotopic spin, T_3. We emphasize that in checking this sum rule, we test a convergence hypothesis and the value of an equal-time commutator. For small values of Q^2 [say $Q^2 = 0.1\ (\mathrm{BeV})^2$] Adler and Gilman[16] have studied the saturation of the sum rule, and they concluded that the sum rule should be satisfied to within a few percent at a maximum energy $\nu \sim 5$ BeV. For high incident energies and at large momentum transfers Q^2,

$$\frac{d\bar{\sigma}}{dQ^2} - \frac{d\sigma}{dQ^2} \rightarrow 1.3\times 10^{-38}\,\frac{\mathrm{cm}^2}{M^2}, \qquad \text{(III.2)}$$

provided that the sum rule saturates for a value of $\nu << E$. This is a measurable cross section provided that the difference does not extend over a very large region of ν. For instance, if the difference between the form factors is evenly distributed for $Q^2/2M < \nu \lesssim 200$ BeV then the difference between β and $\bar{\beta}$ is only ~30%, and in such a case one will not be able to make a definite statement, considering the experimental errors. This suggests a study of the saturation of the Adler sum rule as a function of Q^2.

The Bjorken sum rule,[17] on column 3, reduces to the Adler result provided[18] that $\sigma_s << \sigma_R + \sigma_L$. In this case, the only difference arises in the derivation of this result, since it involves the evaluation of an equal-time commutator in the quark model and also invokes the validity of the first Bjorken-Johnson-Low limit, that we now explain. The Bjorken-Johnson-Low theorem[19] states that in the $q_0 \to i\infty$, $|\vec{q}|$ - fixed limit the following expansion holds:

$$\int dx^4 e^{-iq\cdot x} T\left[J_\mu^+(x) J_\nu(0)\right] = \frac{-i}{q_0} \int dx^4 e^{-iq\cdot x} \delta(x_0) \left[J_\mu^+(x), J_\nu(0)\right]$$

$$- \frac{1}{q_0^2} \int d_x^4 e^{-iq\cdot x} \delta(x_0) \left[\frac{\partial J^+\mu(x)}{\partial t}, J_\nu(0)\right] + 0\left(\frac{1}{q_0^3}\right). \quad \text{(III.3)}$$

If this expansion holds only to order $1/q_0$, it is known as the first BJL limit, while its validity to order $1/q_0^2$ is known as the second BJL limit.

The third sum rule has been obtained by Gross and Llewellyn-Smith,[3] and it is of great interest because it involves the interference of the vector and the axial-vector current. It is unique for the neutrino experiments. Thus, intuitive arguments can be misleading since we are investigating a completely new structure function in a new kinematic region. The predictions of the models confirm this observation since they vary from zero in the diffractive model[10] (Harari) to infinity in the field-theoretic model. In the parton model the following relation holds:[4,20]

$$\bar{\beta}(\bar{L} - \bar{R}) + \beta(L - R) = 3(\bar{\beta} - \beta), \quad \text{(III.4)}$$

and the number of events contributing to this result are three times the events for the Adler sum rules.

The next two sum rules have been derived only in the parton model by Bjorken and Paschos.[4] The assumptions are stated in Table I where the equal-time commutator cannot be evaluated except in specific models. The expression contained in the table has been calculated in the parton model, with the notation discussed there in detail.[15,4] Within the model this sum rule has a physical meaning: It is the mean number of non-strange partons per parton. Under the assumption that σ_s is very small again, as the SLAC data indicate, the sum rule reduces to:

$$\frac{1}{2}\int_0^1 dx\, \nu(\bar{\beta}+\beta) = \left\langle \frac{1}{N}\left[(N_n + N_{\bar{n}})\cos^2\theta_c + (N_p + N_{\bar{p}}) + (N_\lambda + N_{\bar{\lambda}})\sin^2\theta_c\right]\right\rangle. \quad \text{(III.5)}$$

$$\approx 0.76 \text{ in the quark-parton model.}$$

The slope of the total cross section given by the CERN experiment[6] clearly indicates that the integrals can be determined since they are larger than $\approx 0.6 \pm 0.15$. If, in addition, one assumes that vector and axial contributions are equal,[4] and we average over proton and neutron then:

$$0.6 \pm 0.15 \lesssim \frac{1}{2}\int dx\; \nu(\bar{\beta}+\beta) \lesssim 0.72 \pm 0.06. \quad \text{(III.6)}$$

The agreement between theory and experiment is impressive, but inconclusive because of the ambiguities in the theoretical interpretation of the data.

The sum rule in the last column in the table involves the V-A interference term, and it is very convergent. There are two additional sum rules to be found in Ref. 4 which have not been included in Table I. If one wants to classify the sum rules in order of the possibilities of checking them in early experiments, one should start with number 4, then proceed with 3 and/or 5 and finally with 1 and 2.

To complete the picture, we finally discuss a sum rule valid in the limited region $Q^2 \lesssim m_\pi^2$. This is a rigorous result[21] that follows from the assumptions of current algebra:

1. A no-subtraction hypothesis
2. The equal-time commutator $\delta(x_0)\left[J_0^{\alpha}(x), J_n^{\beta}(0)\right] = i\epsilon^{\alpha\beta\gamma} J_\eta^{\gamma}(x)\delta(\bar{x}) + S^{\alpha\beta}$.
3. PCAC.

The result can be stated in terms of the photoproduction amplitude:

$$\frac{1}{\pi}\int_0^\infty \left[(\sigma_L - \sigma_R) + (\bar{\sigma}_R - \bar{\sigma}_L)\right]\frac{d\nu}{\nu} = -4\frac{g_A}{g_r}\left[D(\gamma^+ p \to \pi^+ p) + D(\gamma^- p \to \pi^- p)\right]^{I=1},$$

where

$I = 1$ means isovector photons

$g_A(0) = 1.18$ is the nucleon axial-vector form factor

g_r = the pion nucleon coupling constant, and

$D(\gamma^\pm p \to \pi^\pm p)$ = the CGLN amplitude[22] for the forward photoproduction of $\pi^\pm$ meson by charged photons evaluated at the threshold. The right-hand side can be calculated using the phase shift analyses of the low-energy data. An estimate[23] of the right-hand side in the narrow resonance approximation gives ~20.8 mℓb. The left-hand side is totally unknown.

IV. CONCLUDING REMARKS

It is very important from the theoretical point of view to obtain accurate measurements for:

1. Total cross sections: $\sigma(\nu p)$, $\sigma(\bar{\nu}p)$, $\sigma(\nu n)$, and $\sigma(\bar{\nu}n)$ with a good determination of the slopes.

2. Separation of $W_2(Q^2, \nu)$ and $W_3(Q^2, \nu)$

3. Accurate separation of σ_s with an accurate upper bound in case it turns out to be small.

For the small Q^2 region, one certainly needs light nuclei (preferably hydrogen and deuterium) while for the large Q^2 region, heavier nuclei are also acceptable, but they also require a study of the A-dependence. Good knowledge of the energy and the flux is, of course, very important.

In conclusion, we emphasize that the importance of these data lies in their ability to probe small distances within the proton and also in their ability to distinguish among the numerous models.

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REFERENCES

[1] S. L. Adler, Phys. Rev. 143, 1144 (1966).

[2] J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3] D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 269 (1969).

[4] J. D. Bjorken and E. A. Paschos, Phys. Rev. D1, 3151 (1970); E. A. Paschos, Proc. of the Boulder Conference on High Energy Physics (Colorado Associated University Press, Boulder, Colorado, 1969), pp. 103-129. The work on neutrinos presented in this talk was performed in collaboration with J. D. Bjorken.

[5] A. Pais, Proc. of the Wisconsin Conference on Expectation for Particle Reactions at the New Accelerators, 1970. This review talk covers elastic scattering in addition to the inelastic processes.

[6] I. Budagov et al., Phys. Letters 30B, 364 (1969).

[7] E. Bloom et al., Phys. Rev. Letters 23, 930 (1969).

[8] R. E. Taylor, Proc. of the IV International Symposium on Electron and Photon Interactions at High Energies, Daresbury Nuclear Physics Laboratory, 1969.

[9] S. D. Drell, D. Levy, and T-M. Yan, Phys. Rev. 187, 2159 (1969).

[10] H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969); H. Harari, Phys. Rev. Letters 22, 1078 (1969) and 24, 286 (1970).

[11] C. Callan and D. Gross, Phys. Rev. Letters 22, 156 (1969).

[12] D. R. Palmer, University of Colorado Preprint, 1970.

[13] F. Nezrick (National Accelerator Laboratory), personal communication, 1970.

[14] D. Carey et al., A Comparison of Neutrino Fluxes Expected at the High Energy Accelerators in 1973, National Accelerator Laboratory Internal Report TM-221, March 2, 1970.

[15] J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969)

[16] S. L. Adler and F. J Gilman, Phys. Rev. 156, 1598 (1967).

[17] J. D. Bjorken, Phys. Rev. 163, 1769 (1967).

[18] F. J. Gilman, Phys. Rev. 167, 1365 (1968).

[19] J. D. Bjorken, Phys. Rev. 148, 1467 (1966); K. Johnson and F. E. Low, Progr. Theoret. Phys. Suppl. [37-38], 74 (1966).

[20] C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

[21] E. A. Paschos, Phys. Rev. D1, 3138 (1970).

[22] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[23] This estimate was performed by F. J. Gilman and E. A. Paschos (unpublished).

Table I. Neutrino-Antineutrino Sum Rules.

	1	2	3	4	5
Sum rule	$\int_0^\infty d\nu(\bar{\beta} - \beta)$	$\int_0^\infty d\nu[\bar{\beta}(\bar{R} + \bar{L}) - \beta(R + L)]$	$\int_0^\infty d\nu[\bar{\beta}(\bar{L} - \bar{R}) + \beta(L - R)]$	$\int_0^1 dx\,\nu[\bar{\beta}(\bar{R} + \bar{L}) + \beta(R + L)]$	$\int_0^1 dx\,\nu[\bar{\beta}(\bar{L} - \bar{R}) - \beta(L - R)]$
Convergence	O.K. Leading Regge trajectory $\alpha_\rho = 0.5$	O.K. Leading Regge Trajectory $\alpha_\rho = 0.5$	O.K. Leading Regge Trajectory $\alpha_{\omega,\phi} = 0.5$	Converges even for the Froissart bound	Converges even for the Froissart bound
Commutator	$\delta(x_0)[J_0^+(x), J_0(0)]$	$\delta(x_0)[J_x^+(x), J_x(0)]$	$\delta(x_0)[J_x^+(x), J_y(0)]$	$\delta(x_0)[\partial J_x^+/\partial t\,(x), J_x(0)]$	$\delta(x_0)[\partial J_x^+/\partial t\,(x), J_y(0)]$
Value of the commutator in models	Current Algebra $-\frac{3}{2}(B - 3Y)\sin^2\theta_c + 2T_3(1 + \cos^2\theta_c)$	Quark, Parton $-\frac{3}{2}(B - 3Y)\sin^2\theta_c + 2T_3(1 + \cos^2\theta_c)$	Quark, Parton $4B + Y(2 - 3\sin^2\theta_c) + 2T_3\sin^2\theta_c$	Parton $2\left\langle \frac{1}{N}[(N_{n'} + N_{\bar{n}'})\cos^2\theta_c + (N_{p'} + N_{\bar{p}'}) + (N_{\lambda'} + N_{\bar{\lambda}'})\sin^2\theta_c] \right\rangle$	Parton $2\left\langle \frac{1}{N}[-(N_{n'} + N_{\bar{n}'})\cos^2\theta_c + (N_p + N_{\bar{p}}) - (N_{\lambda'} + N_{\bar{\lambda}'})\sin^2\theta_c] \right\rangle$
Other assumptions	None	First BJL Limit	First BJL Limit	Second BJL Limit	Second BJL Limit
Range of validity	$Q^2 \geq 0$	$Q^2 >> M^2$	$Q^2 >> M^2$	$Q^2 >> M^2$	$Q^2 >> M^2$
How well can we check it?	$d\bar{\sigma}/dQ^2 - d\sigma/dQ^2 \underset{E\to\infty}{\to} 1.3\times10^{-38}\,cm^2/M^2$ Testable if it saturates below the multi-hundred BeV region.	For $\sigma_s << \sigma_R + \sigma_L$, it can be measured as accurately as the Adler sum rule.	In the parton model $\bar{\beta}(\bar{L} - \bar{R}) + \beta(L - R) = 3(\bar{\beta} - \beta)$. Thus, this measurement should be 3 times the Adler sum rule.	For $\sigma_s << \sigma_R + \sigma_L$, it has been shown in CERN experiments $\geq 1.20 \pm 0.30$.	Reliable estimates do not exist
Comments	For $Q^2 = 0.1\,(BeV)^2$, it is expected to saturate at $\nu = 5$ BeV. This suggests testing the sum rule at intermediate values of Q^2.	For $\sigma_s << \sigma_R + \sigma_L$, it reduces to the Adler sum rule.	A unique sum rule for neutrino reactions. Therefore, intuitive arguments could be misleading. Model predictions vary widely.	The EASIEST sum rule to test. Parton value = 1.52 CERN experiment with p = n and V = A = 1.44 ± 0.12	Can be related in the parton model to the electroproduction data.
References	1, 16, and 18	17 and 18	3 and 20	4 and 15	4

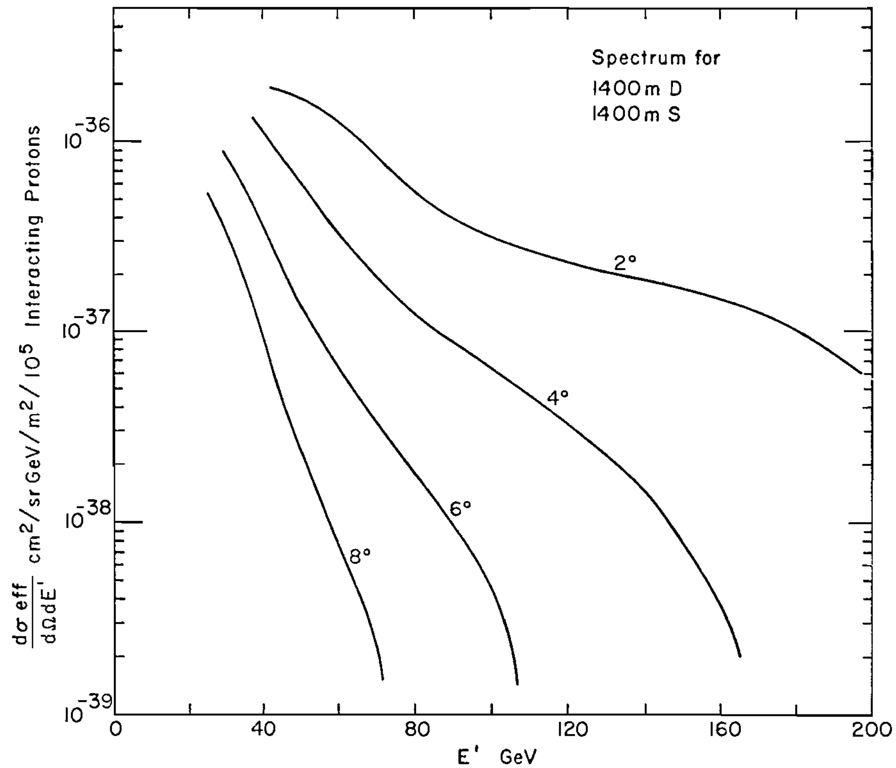

Fig. 1. Effective differential cross section for a 300-BeV neutrino spectrum. The curves correspond to different laboratory angles.

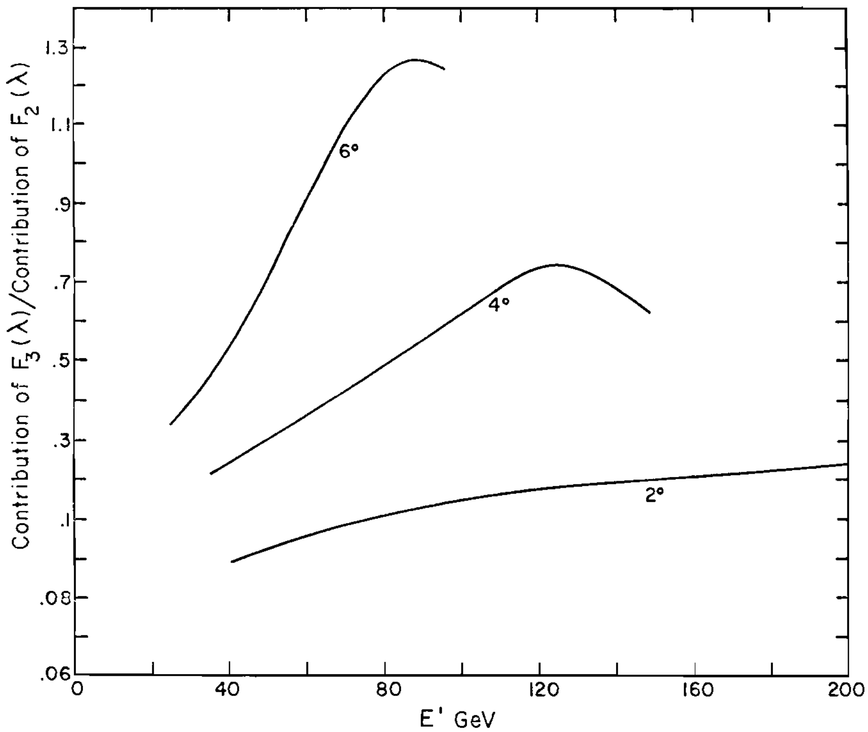

Fig. 2. Relative contributions of $F_2(\lambda)$ and $F_3(\lambda)$ to the cross sections shown in Fig. 1.

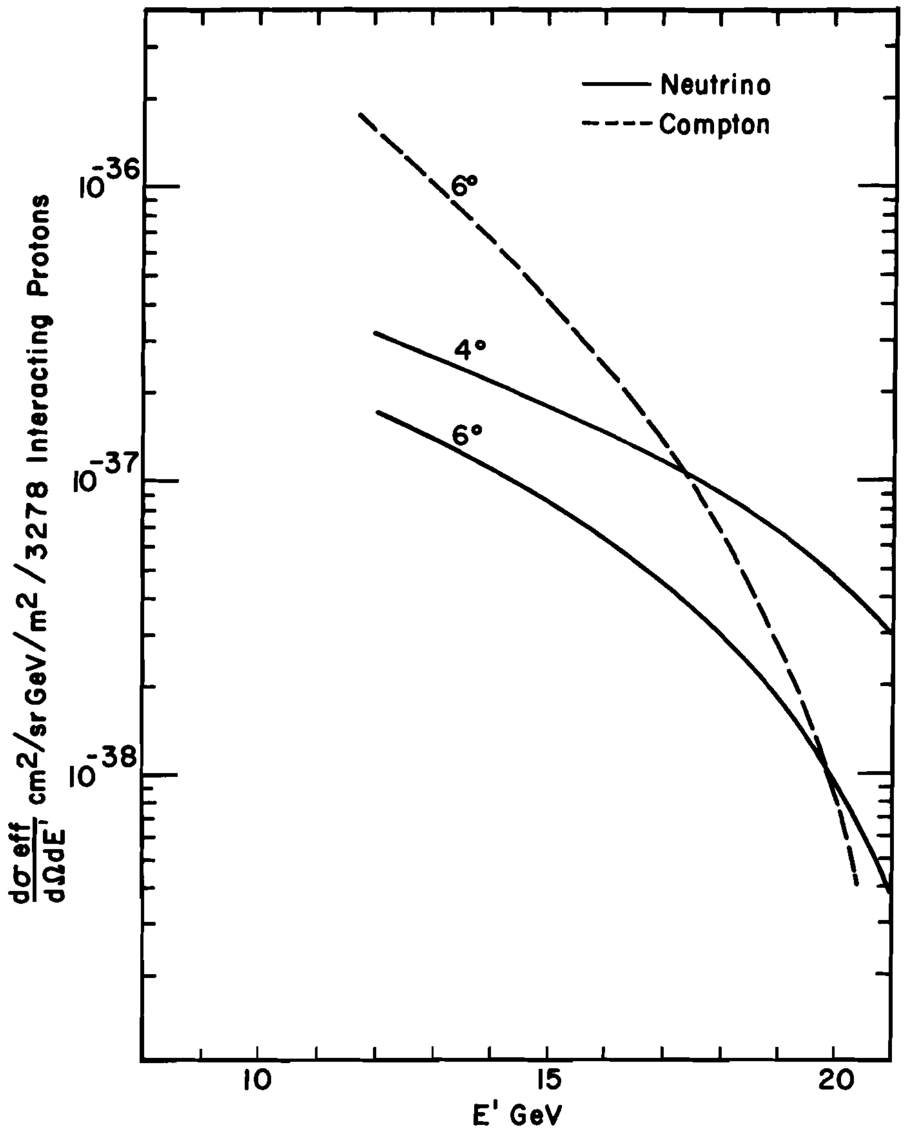

Fig. 3. Effective differential cross section for a E_{max} = 25 BeV neutrino spectrum. The 6° Compton curve corresponds to a bremsstrahlung spectrum with 25 BeV maximum energy The Compton curve is taken from Ref. 15 with an arbitrary overall normalization.