MUON BEAMS

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ABSTRACT

Various features of muon beams are considered. The crucial question of an estimate of beam halo and background is discussed first. We then consider the possibilities and relationships of muon beams in Area 2. A higher-energy muon beam transport is outlined. Finally, we consider the effects of variations on the nominal beam in Area 1.

I. INTRODUCTION

Past summer studies have suggested muon beams ranging from ideal to almost zero-cost.[1] The beam presented by Yamanouchi[2] was satisfactory to all interested parties in its general characteristics:

decay length:	~ 300 m
shield:	~ 300 m of high-Z material
flux:	$\sim 10^7/10^{13}$ interacting protons
$\Delta p/p$:	±5%
size:	$10 \times 10\ \text{cm}^2$

The detailed implementation in Area 1 as of July 1, 1970, appeared to have several problems concerning flux, operational compatibility with the 15-foot chamber, and especially halo and general background. These problems were related to the decision to bring on Area 1 at 500 GeV. The resulting muon shielding for the neutrino beam grew wider and longer, as did the nominal decay length (~ 600 m).

At present the general characteristics of Area 1 are still not settled. In addition to the probable change to earth shielding, the question of the target box contents is not settled. Consequently, detailed design studies are of limited utility. Instead, we have addressed ourselves to more general questions. First, what can be said about the crucial problem of backgrounds? Second, what are the possibilities for muon beams in Area 2? Third, in view of the 500-GeV capability, what are the possibilities for higher energy muon beams? Finally, what are the effects of some basic variations on the nominal design in Area 1?

II. ESTIMATES OF BEAM HALO

R. Wilson

The basic problem of muon beams is not lack of intensity, but of muons being in a halo around the beam. This problem has not been adequately discussed by anyone. We would clearly like to absorb out any muons in the halo. Since the muon energy discussed is 100 GeV, this can be a hard problem. We must at least make sure that these muons do not enter the muon experimental area in an intensity greater than 10^5 μ/m^3. At this intensity, an anticoincidence can eliminate their effect.

A. Halo of μ Beam Without Focusing

A μ beam without focusing has been suggested in particular by T Yamanouchi and presented to the NAL users in March 1970. Although I have not seen details of this beam, I write down some parameters; they roughly correspond to those proposed by Yamanouchi. In discussing this beam we shall consider a muon energy of 100 GeV.

The π production cross section falls off with a characteristic transverse momentum of 0.16 GeV/c, so that the π production tends to be within an angle ±1.6 mrad The pions are further focused into a parallel beam by a quadrupole doublet. Thus, pions within 10% of the nominal momentum will have a divergence less than 0.16 mrad and come from a source size of about 5-cm diameter. The angle of π-μ decay is 0.3 mrad max, and most of the muons will have close to this angle. We see at once that for a narrow band μ beam, the π-μ decay will dominate the divergence. Over the 600 meters decay distance, 0.3 mrad corresponds to a source size of 18 cm at the end of the tunnel, which will also dominate the source size.

The plan is, then, to have at the end of the decay tunnel a dipole magnet, of aperture 8 × 8 cm bending through 17 mrad as shown in Fig. 1. At this point the muons approximately uniformly fill this aperture and diverge therefrom with the 0.3 mrad divergence. We increase the divergence to 0.4 mrad by adding 2 mean free paths of hadron filter to prevent further muon production further down the channel.

The first bend will select a momentum band of muons given by the aperture in the second bend magnet; if this also is 8 cm, we have ≈ 4/300 for the energy spread.

As the muons approach the second bend, those at high energy will miss the magnet to the left, and be absorbed out by the earth fill, which is approximately enough for absorbing 200 GeV. Those of lower energy will miss the magnet to the right and pass out of the earth fill and out of our experimental interest.

It is vital that the magnetic field be no larger in extent than that actually needed. Then the magnet acts as a collimator. The magnets can be 20 ft long (machine magnets?) and run at 10 kG.

The absorption of mu mesons in earth is about 2 MeV cm^{-2}/g, and $\rho \approx 1.5\ \text{g/cm}^3$. One hundred GeV mesons are absorbed therefore in 3×10^4 cm ≈ 300 m of earth. Provided that the earth shield is properly extended sideways, the 400-500 m of earth shield is clearly plenty.

After the second magnet, low-energy muons will be preferentially on the right of the beam and high-energy ones on the left. However, the beam will expand enough that by the third magnet the energy information is spread over the aperture and muons will be leaving the channel.

At first it was thought to be necessary to bring the beam back into the main beam line to cancel the momentum vector. However, this is likely to be unnecessary, as the geometric expansion of the beam is likely to dominate. Therefore, we can bend the beam in the opposite direction as shown in Fig. 2.

Those muons which because of divergence leave the beam between magnets 2 and 3 will go on straight at magnet 3. However, only after penetrating 2000 m of earth will they scatter through an angle $\sqrt{t/x_0}\,(15/E) \approx 100\sqrt{15/10^5}$ or 17 mrad and equal the bend angle!

These muons will therefore tend <u>not</u> to end up in the beam halo; they will penetrate about 150 m, half their range, and be spread over an area $(0.7\ \text{m})^2$. (See Appendix A.) Since about 3 times as many muons do this as stay in the beam, we can have an intensity (for a 10^7 beam) of:

$$\frac{3\times10^7}{(0.7)^2} = 6\times10^7\ \text{muons/m}^2,$$

but displaced about 3 m to one side of the beam! Since we have about 2 m of spark chambers, this is barely tolerable. We need a larger displacement.

We hope muons passing through the third magnet also pass through the fourth, causing no further problem.

B. Beam Passing Through Coils, and Yoke

In passing through a typical bending magnet (Fig. 3) only a portion of the beam passes through the gap. Since, in this beam, the magnet 3 is irradiated with muons over about 2-3 times its gap area, we must consider what happens to muons which do not enter the gap. In Fig. 3 we draw some typical flux lines.

If this magnet bends to the right, at the top and bottom of the right-hand side the magnet will bend <u>downwards</u> by 17 mrad and on the top and bottom of the left-hand side, upwards by 17 mrad. These muons will clearly go above and below the experimental apparatus. If this magnet is magnet 3, not many muons will strike the return yoke, but there will be many which hit the coils. These will be bent intermediate amounts in accordance with the field. If bent less than 10% of the full 17 mrad, they will miss the next magnet gap and hit the return yoke. Therefore, if muons hit an area about 20% of the area of the gap they can be bent almost the right amount. These will be dangerous "halo" muons.

In addition to the difference in bending, each halo muon will pass through a great deal of copper or iron absorber. For a 20-ft bend magnet at 100 GeV/c this will scatter:

$$\begin{aligned} \langle \theta^2 \rangle &= \left(\frac{20\text{ ft}}{1.36\text{ cm}}\right)\left(\frac{15}{10^5}\right)^2 \\ &= 9 \times 10^{-6}. \end{aligned}$$

This angle of 3 mrad is small compared with the bend angle, but in 170 meters distance between magnets it will spread the particles out by 50 cm. (Note that some of these will go straight forward and therefore spread out by this much the muons which leave the beam pipe.) The ionization loss is 10 GeV.

Thus only 20% of these enter an aperture 15 cm × 15 cm in magnet 4, giving an overall "halo" of about 4%.

C. Modified Yamanouchi Beam

Figure 2 shows a desirable modification to enable the mu beam to miss the bubble chamber and hopefully to operate when the bubble chamber or counter neutrino experiments operate.

Unfortunately this beam destroys one advantage of the dipole arrangement planned by Yamanouchi (and described in SS-43 by T. Kirk and in a proposal to the AGS in October 1968). By using the dipoles to return to the original beam line the system is nondispersive. In the modified arrangement a further momentum selection is obtained.

The problems with this further selection are less than at first sight. Already at the third magnet the beam distribution is dominated by the divergence and no longer do we have (as at the second magnet) the low-energy particles primarily on the left-hand side.

Provided that we keep a larger aperture for the final bend magnet, we can probably keep the whole momentum divergence inside the fourth magnet.

D. Field Lens

In using the modified Yamanouchi beam it seems appropriate to consider a pair of quadrupoles near magnet 2 to invert the momentum vector and also to cut back the divergence of the beam. This I have not yet calculated. It clearly will help the halo.

E. Shielding Due to Magnetized Iron

In discussing the passage of the edges of the beam through the coils and yoke of a magnet it becomes apparent that we might make a special clean-up magnet. We sketch the face of such a magnet in Fig. 4. Their use was first suggested for muon beams by Lederman (AGS experiment μP II).

The sign of the current can be so arranged that particles out of the beam can be bent further away. This bending overcomes multiple scattering. A 20-ft magnet at 18 kG bends through 30 mrad.

The top and bottom of such a magnet seem good; they bend particles away from any area (except insofar as they cancel the bending in a dipole bending magnet of Fig. 3.) The left and right may also help.

Yamanouchi has suggested that the sign of one or more of these magnets be arranged to bend muons back into the beam. The problem here is energy loss; the 20-ft magnet loses 10% of the muon energy. If we lined the whole muon beam pipe with such a magnet we would be well off. A muon striking the magnet at an angle of 0.4 mrad (the divergence angle) would come out at an angle 0.4 mrad after a distance $0.8/30 \times 20$ ft = 0.5 ft, and penetrate only one or two mils! and have suffered an energy loss of only 200 MeV. A muon would have only two such scatters all the way down the beam run.

An "ideal" muon channel could then consist of an iron tube 0.1-in. thick, magnetized in a circular direction. This tube would have to be in segments to allow the coil to be wound thereon. The coil which would be a thin aluminum strip must also be very thin to avoid multiple scattering and energy loss, but this will not be too bad; either it scatters into the iron and comes out again, or away from the iron into the beam.

Although I cannot envisage a 400 meter long magnet of this sort in the first instance it is worth considering for the last stages of a beam, maybe just before the final bend and momentum analyzing magnet.

F. Multiple Scattering for Containment of μ Beam

Muons striking the walls of the channel will scatter; if they scatter away from the channel they will be lost, if towards it, they will come back into the beam. This effect is similar to the total external reflection used in slow neutron collimators.

The multiple scattering in iron or copper reaches 0.8 mrad (to bring the muon back into line) after about 18 inches, a little more than a magnet by itself would do.

G. Exact Calculations

Yamanouchi has a program to follow π mesons and mu mesons down a transport system and to include the decay. This, as presently run, gives the intensity at any chosen distance down the transport system. We need to modify this to find the number, energy, and angle of the particles leaving this transport system--particularly at the later stages. Then we can follow them to find out the halo.

III. A PRELIMINARY FSTIMATE FOR A μ BEAM IN AREA 2

F. Pipkin and R. Wilson

The high flux π beam (at 3.5 mrad) in Area 2 is a possible place for a μ beam. It satisfies some of the requirements: a high flux, a large angular aperture, and a large $\Delta p/p$.

This is a preliminary estimate pending a more detailed study. The beam, as of June 16, 1970, has the following properties:

Production angle	3.5 mrad
Solid angle	1.9 μsr
$\Delta\theta_h$	±0.5 mrad
$\Delta\theta_v$	±1.2 mrad
$(\Delta p/p)_{max}$	±3%.

At 200 GeV 10^{13} protons incident, 100 GeV, π^+ we find from the yield curves compiled by Awschalom and White (NAL FN-191, June 9, 1969) 8×10^7 π^+ in $\Delta p = 200$ MeV/c.

In 6×100 GeV/100 = 6 GeV we find 2×10^9 π. Now the π-μ decay angle is, at 100 GeV, 0.3 mrad so that we lose about 1/3 of the particles horizontally and 10% vertically, during π-μ decay.

We expect the π mesons to decay all along the length of 1300 ft ($\simeq$ 400 m) to the final focus. At 100 GeV, $\gamma = 100/0.140 \simeq 700$ and the lifetime $= 2.6\times700\times10^{-8}$ sec $- 1.8\times10^{-5}$ sec. The π mesons cross 400 m in 4/3 μsec $= 1.3\times10^{-6}$ sec. So:

$$\frac{1.3\times10^{-6}}{1.8\times10^{-5}}\ \pi \text{ mesons decay} \simeq 7\%.$$

Of these, we will catch perhaps 5% in the $\Delta p/p$ of the transport system. (The spectrum is uniform from 100 GeV to 60 BeV and we pick up about 1/2 $\Delta p \sim 3\%$ of 100 BeV $\sim$ 3 BeV.)

So the total number of μ mesons at the end will be about $2\times10^9\times2/3\times7/100\times5/100 \simeq 5\times10^6/10^{13}$ protons. These will be μ^+ at 100 GeV, with 4 times as many protons. At 140 GeV, the yield of π^+ is 10 times smaller; μ^- mesons will be less by a factor 4 at both energies.

The aperture of the last quadrupole is 2-in. horizontal × 5-in. vertical; however, the last bend is 4 in. × 2 in. horizontal. The beam presumably gets through these.

A. Pion Filter

The best place for the pion filter is the last pair of quadrupoles. At 100 GeV, we will scatter by 1 mrad in this pion filter. This is 200 ft from the final focus, so we expand about 2 in. in this distance. The region around the beam pipe here can be filled with iron.

$$\text{Total thickness} = 200\ \text{ft} \times 12 \times 2.5 \times 8 = 48{,}000\ \text{g/cm}^2,$$

which will stop 100-BeV μ mesons in the halo and certainly scatter them out of the way.

B. Beam Size

The divergence of the beam due to the π/μ decay is zero when the π decays at the focus and greatest far from the focus. The maximum distance from the focus is 150 ft, so 0.3 mrad expands the beam $0.3/1000 \times 150$ ft at the focus or 1/2 in.

The focus of the beam is supposed to be ±0.12 in. width and ±0.06 in. height, but I expect this neglects the $(\Delta p)^2$ aberrations. Anyway we can expect quite a small beam, apart from the multiple scatterings.

C. Proper Calculation

In the above, I assume Δp $\Delta\theta$ are independent. They probably are not, and there is an ellipse. This will get the number down about $\pi ab/4ab = \pi/4$.

D. μ Background for Other Experiments

I note that most of this mu beam intensity will exist whether or not a special mu beam is set up. Shielding around the beam line should reflect this. I note that the present design has the mu beam leaving the mu shield half way along its length. This may be too little shielding.

E. Summary

μ^+ intensity	100 GeV	3×10^6	$/10^{13}$ ρ at 200 GeV
"	140 GeV	3×10^5	
μ^- "	100 GeV	7×10^5	
"	140 GeV	7×10^4	

Yet to calculate:

1. What happens to off-direction muons?
2. Run through proper calculations to include Δp, $\Delta\theta$ correlations, etc.

IV. A POSSIBLE COMBINED μ AND e, γ FACILITY IN AREA 2

R. Wilson

The high-intensity low-resolution beam at 3.5 mrad has been suggested as a possible electron beam and as a possible muon beam. The former is the subject of a memo by Morrison in early July and the latter in the preceding section.

Since both μ beams and e beams are used primarily to study electromagnetic interactions, there seems a possibility that some of the apparatus for the experiments might be common. (To paraphrase some of the proposals, a facility might be provided which can study both sets of experiments.) This memo is to explore this.

A. γ-Ray Experiments

The first experiments with the electron beam that are proposed are tagged γ-ray experiments (Proposals 24 and 25). After the electron beam, a tagging magnet is required, with a set of tagging counters. The experiments then require only a modest set of counters for the total cross section and the inelastic γ-ray scattering experiments proposed.

In the distant future, the proponents of γ-ray experiments envisage more complex arrangements; these have been foreseen in a summer-study report B. 9-68-49 by R. Wilson, a report by C. A. Heusch in the Berkeley Summer Study 1966, and by proposals by D Caldwell to CEA in 1962, but they have not yet reached the NAL proposal stage. These envisage a wire-spark-chamber dipole-magnet spectrometer to study photoproduction of forward vector mesons and so forth. As we shall see, this is exactly the apparatus proposed by two of the muon-beam experimental proposals.

B. μ Beam Experiments

There are 4 experimental proposals (Proposals 5, 26, 29, and 33). Of these, that by Chen and Hand (26) is a limited proposal, involving limited apparatus, about on a par with the e, γ experiments noted above. The others propose modest (by NAL standards) spectrometers. Those proposed by Perl (Proposal 5) and Wilson (Proposal 29) are almost identical (a clear case of great minds thinking alike!); that by Mo (33) is larger and concentrates on the details of the vertex.

Both types of spectrometer can clearly be useful for studying muon interactions; it is the point of this memo that the identical spectrometers can be very useful for γ-ray experiments also. The spectrometers proposed by Wilson and Perl are almost identical to those discussed by the many authors of summer-study reports for γ-ray interactions. That of Mo and Selove is different, but a spectrometer of this sort has been proposed but never built, for a tagged γ-ray beam at CEA by Frisch.

C. Conclusion

The beams for an electron/tagged γ-ray experiment or a μ experiment are quite different. The apparatus for the γ-ray experiments proposed--which will presumably be the first ones--is small and specialized to the experiment. The apparatus for 3 of the 4 proposed muon-beam experiments is large enough so that it is not desirable to move it all the time. There is probably a long program possible for muon experiments with such apparatus, but there is also a possibility that such apparatus could be used for tagged γ experiments. However, such experiments are not formally proposed and to that extent must be considered hypothetical.

V. A SPECULATIVE ACHROMATIC MUON TRANSPORT FOR HIGH ENERGIES

R. Wilson

The beam transport here proposed will work quite well for any muon energies above 100 GeV/c. It consists of a straight iron beam pipe which is magnetized. The cross section is shown in Fig. 5.

The muons in a muon beam transport system will initially have a divergence governed by the angle of π-μ decay. The angle of the pion production is greater than the angle of π-μ decay, but the pions can be made--over a limited range of momenta--parallel because the initial source is small. A divergence could appear because of the dispersion due to different muon momenta, but again, over a reasonable range of momenta of 10%, the dispersion may be canceled.

The angle, therefore, is (30/pc) radians/MeV or 0.3 milliradians for 100(GeV/c) muons. When muons at this extreme angle penetrate the iron pipe, they will bend and leave the iron pipe at this same angle after a length:

$$\ell(\mathrm{cm}) = 2\ (pc)\ \theta/(0.3\ B)\ \ (\mathrm{MeV\ rad/kG})$$

$$\ell(\mathrm{cm}) = 60/0.3\ B\,(\mathrm{kG}) = 12.5\ \mathrm{cm\ for}\ B = 16\ \mathrm{kG}.$$

The muons penetrate a thickness $t = \ell\theta/\pi = 10^{-3}$ cm. During this length of bending, they will also scatter through an angle given by:

$$\theta_{\mathrm{projected}} = \frac{15\ (\mathrm{MeV})}{pv\ (\mathrm{MeV})}\sqrt{\frac{\ell}{x_0}} \qquad x_0 = 1.8\ \mathrm{cm\ for\ iron}$$

$$\frac{\theta_{\mathrm{scattered}}}{\theta_{\mathrm{bend}}} = \frac{15}{pc}\,\frac{pc}{60}\sqrt{\frac{\ell}{x_0}} = \frac{\sqrt{7}}{4} = 0.65\ \ (\mathrm{max\ divergence\ muon}).$$

If we consider muons at less than this extreme angle, they will be bent less and pass less time in the iron. The multiple scattering, however, will only decrease as the square root. The multiple scattering will, therefore, dominate for very small angles of incidence, and bending will dominate for those muons which have the maximum dispersion.

$$\frac{\theta_{\mathrm{scatt}}}{\theta_{\mathrm{bend}}} = \frac{15}{pc}\sqrt{\frac{\ell}{x_0}}\cdot\frac{pc}{4.8}\,\frac{1}{\ell} = \frac{3.1}{\sqrt{\ell x_0}}\,.$$

The effect of the multiple scattering can be easily seen by qualitative arguments. Figure 6 shows the trajectories of muons with and without multiple scattering. The scattering can be seen, in this projection, to increase or decrease the divergence angle. After N collisions we will have $\theta = 0.3$ mrad $\sqrt{1 + 0.4N}$. If we have a length of beam transport of L kilometers and a beam pipe of radius R_{cm}, we will have, at a divergence θ, approximately N collisions with the wall with:

$$N = \left(\frac{4}{\pi}\right)\frac{100{,}000L}{2R}\,\theta = \left(\frac{4}{\pi}\right)\frac{100{,}000L}{2R}\,\frac{3}{10^4}\,\sqrt{1+0.4N}\,.$$

The factor $4/\pi$ is to allow for the fact that the muon trajectory, projected onto the cross section of the pipe, is a chord and not a diameter. [As multiple scattering increases the angular momentum of the beam (see below) this factor increases. The increase is not allowed for.] This equation is plotted in Fig. 8.

Angular Momentum

There is also an effect due to multiple scattering out of the plane shown in Fig. 6. This will add to the angular momentum of the muons about the beam direction.

Since the π mesons all come from what is essentially a point target, the angular momentum is imparted to the muons by the π-μ decay process.

If the π-μ decay takes place in a pipe, or region, 5-cm radius, a typical angular momentum imparted by the π/μ decay is 90 MeV/c.

Each multiple scattering gives a projected angle of 2 mrad at 5 cm from the center, giving about 100 MeV/c. After N collisions with the wall, the angular momentum is then approximately $100\sqrt{N+1}$ MeV/c.

To first order this does not affect the angle in the projection onto the beam pipe cross section since the angular momentum around the center is increasing with N at about the same rate as the transverse momentum through the center. It, therefore, does not affect the considerations of the preceeding section to first order.

Energy Loss

The equations above are equations in which the muon momentum cancels. The number of collisions per km increases inversely with momentum, but the necessary length of beam transport also increases with momentum. But the energy loss prevents this beam pipe being used at low energies.

The energy loss of relativistic muons is about 2.4 MeV cm^2/gm. After N collisions with the wall, the muons will have lost $2.4\,N\ell\rho$ MeV.

At 100 GeV/c, this becomes:

$$2.4 \times 4 \times 12.5 \times 8 \text{ MeV/km} \approx 1000 \text{ MeV/km or } 1\%/\text{km}.$$

This is tolerable.

At 50 GeV/c, this becomes:

$$2.4 \times 8 \times 12.5 \times 8 \text{ MeV/km} \approx 4\%/\text{km}.$$

This is marginal.

Construction of the Muon Pipe

The field of 16 kG was chosen because ordinary annealed soft iron gives B = 16 kG with H ≈ 5 oersted. To achieve 5 oersted we need:

$$\int H\,d\ell = 4\pi i/10 \text{ (i in amps).}$$

So, for a 5-cm radius pipe $\ell = 31$ cm, $\int H\,d\ell = 150$ gauss cm, $i = 150$ ampere turns. We consider winding a coil of thin aluminum strips of about 1/40 mm thickness and stuck to the iron pipe with insulating cement. If we had one turn, the cross-sectional area would be 0.08 cm^2. In a length of 1 km, the resistance becomes:

$$2 \times 2.65 \times 10^{-6} \times 10^{5}/0.08 \text{ ohms} = 6.6 \text{ ohms.}$$

The power consumed = $i^2R = 1.5 \times 10^5$ watts; this is spread over 1 km and needs no special cooling. In practice, we would wind this in 1-cm strips around the circumference, making 200 ohms per km. Again, we would probably cut the magnet into 10-meter sections, making 2 ohms each with 5 amps.

At $\theta = 0.3$ mrad, the muons will pass through a length of aluminum $2 \times 10^4/3 \times 1/400$ cm ≈ 12 cm. The multiple-scattering angle in this thickness is less than that in the iron by a factor of $\sqrt{(\ell/x_0)} \approx \sqrt{3} = 1.7$. In principle, we should correct the previous calculations.

Since we only need a very thin layer of magnetized iron and since the tangential component of H is continuous, we could use a cobalt steel as a liner and use $H = 1$ oersted or less with a consequent 25-fold reduction in power.

The magnet alignment is critical. We need to keep the surface polish good to 1/2 mil (half the penetration of a mu meson in the surface layer). Also, the angle must be good to better than 0.3 mrad in a distance between collisions with the wall (1/4 cm). This is much less than the alignment of the SLAC beam pipe, but our pipe is hopefully buried in the ground. This problem is the same as that for the neutral beams; I here assume a 10-cm diameter pipe is "floated" inside a 20-cm diameter buried pipe and aligned from above ground.

Special Beam Pipe with Large Angle Reduction

After many collisions down a long beam pipe (perhaps 4 km with a radius of 6 cm), $N = 60$.

It may be desirable to remove all muons with $\theta > 1$ mrad from the beam. This could be done by a double pipe system as shown in Fig. 7. The inner pipe, 0.1-mm thick, has a field in such a direction that muons return into the pipe; the outer, thicker, pipe bends muons completely out of the beam, into heavy-earth shielding. The thickness of the outer pipe can be 1 cm giving a bending of about 30 milliradians.

The power requirements of the second pipe are the same as for the inner pipe since $\int H\,d\ell$ is the same and is independent of thickness.

Neutrino Beam

One of the problems with a neutrino beam is the number of high-energy muons produced; these may penetrate the shielding if it is not thick enough. Low-energy muons are no problem because their range is less.

The fact that high-energy muons can easily be kept in a muon channel suggests a partial solution to this problem. The decay tunnel for the neutrinos becomes also a muon channel; at the end of the decay tunnel the muons are bent out of the way. After the bend, the muon channel is magnetic in a different way, as shown in Fig. 10. In this arrangement muons to the right of the channel bend back into the pipe; those to the left (or top or bottom) bend out of the pipe and away from the neutrino beam. There are no fast muons leaving the decay channel. If the channel is long, for a high-energy neutrino beam, the muon channel can be lined by a second iron pipe bending the few escaping fast muons through a large angle.

This is illustrated in Fig. 9. We see that the system particularly matches the neutrino beam proposed by Barish et al. Although the intensity is low for low-energy neutrinos, the shielding requirements are reduced by a large factor over the conventional procedure.

VI. EFFECTS OF VARIATIONS

J. Tenenbaum

Yamanouchi's nominal beam is illustrated in Fig. 11. The sizes and distances are as of August 1970 and differ slightly from Section I of this report.

The basic idea is to extract muons through a hole in the proton dump of the wide band neutrino beam. The beam is bent out around the muon shield and then back to the original neutrino line. A pair of quadrupoles focusing point to point is placed within the target box to focus the beam on the muon hole. It is assumed that these quadrupoles are slightly cocked to keep the hole on the quadrupole axis and yield production angles of 0.5 mrad. Conflicts with other elements in the target box have not been considered further. This beam is labeled nominal in Table I. The resulting muon flux for this beam and the following variations have been calculated using the AGS muon beam Monte Carlo program.[3]

All calculations are for a muon energy of 100 GeV, a proton energy of 200 GeV, and a circular aperture for the quadrupoles and drift spaces. The Trilling formula for secondary particle production is used.[4] A nominal 4-in. diameter aperture was assumed for all elements except the decay pipe which was 18 in. Use of a 2 × 4 in. machine dipole cut the rates an additional 33%. A pion filter is assumed just after the decay region so that the dominant source of muons is the decay region.

At the upstream end the first variation was to turn off the focusing quadrupoles. The result was a two order of magnitude loss of intensity. This loss is understandable because of the very long decay path of 600 m.

The next variation was to change the focusing from "point to point" to "point to parallel." The resulting moderate change is understandable as follows. For point to point focusing getting a larger number of muons into the entry aperture is partially cancelled out by the larger effective emittance. Fewer enter the hole point to parallel, but once in they are more successful in traversing the system.

The final front end variation is to consider the effects of a narrow band pion beam. Such a beam was simulated by replacing the focusing quadrupoles by a six magnet system consisting of two quadrupoles (operating point to parallel) and four dipoles. The upstream portion of this beam was tuned for 100 GeV and its parameters were taken from the proposal of Barish et al.[5] The absolute rate appears to decrease by a factor of 3 as does the muon flux striking the downstream wall of the tunnel. However, the spatial distribution of the background muons is much sharper about the beam aperture. This collimation of the background should aid in its attenuation but must be studied further. The improvement on backgrounds of dumping the protons in the target box is harder to evaluate.

The remaining variations concerned the downstream portion of the system. First, the transverse dimensions were halved. For fixed dipole apertures this increases the momentum acceptance of the first bend and almost doubles the rate. The final variation was to change the direction of the second bend at the downstream end of the neutrino shield. This layout is illustrated in Fig. 2. It is dictated by the operational desire to decouple the muon area from the bubble chamber. This variation appears to have little effect on the results except to increase the momentum dispersion at the target. The implication is that the geometric spreading in the beam dominates the momentum dispersion produced at the first bend and further intensity losses do not occur.

We have not studied the effects of adding quadrupoles in the downstream portion. To some extent all of these beams can be improved in intensity and dispersion by combination of focusing doublets and field lens. Aside from increased distance considerations, a maximum of six quadrupoles is likely to be needed. Our two major conclusions are:

1. Two bends in the same direction are satisfactory.
2. A narrow band front end changes the background distribution and source locations. Its ultimate effect is coupled with the details of the muon shield and require further study.

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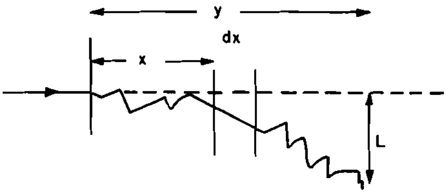
[5] B. Barish et al., Neutrino Physics at Very High Energies, National Accelerator Proposal 21, 1970.

Table I.

	Description	Number of accepted muons in Monte Carlo program	Muons per 10^{13} interacting protons
1.	nominal Yamanouchi beam	458	6.1×10^6
2.	focusing quadrupoles off	1	6.8×10^4
3.	point to parallel focusing	12	4.3×10^6
4.	narrow band pion beam	30	2.0×10^6
5.	transverse dimension halved	169	11.1×10^6
6.	both bends in the same direction	426	5.7×10^6

APPENDIX A. MULTIPLE SCATTERING OF A MUON ON SLOWING DOWN

We want the displacement of a muon on going through the shield.



Consider scattering in an element dx:

$$\langle \theta^2 \rangle = \frac{15}{E}\frac{dx}{x_0} \qquad \text{E in MeV, } x_0 \text{ is rad. length} \simeq 30\ \text{g/cm}^2.$$

So contribution to L at a position Y is:

$$\langle L^2 \rangle = \left(\frac{15}{E}\right)^2 (Y - x)^2 \frac{dx}{x_0}\,.$$

Energy $E = E_0 - xE_0/R$ (R = range of muons) where x lies between 0 and R.

So total L^2 is given by:

$$\langle L^2 \rangle = \int \left(\frac{15}{x_0}\right) \frac{(Y - x)^2 dx}{E_0 - xE_0/R)^2} \int \frac{R^2 15^2 x^2 dx}{E_0^2 x_0 (R - Y + x)^2}$$

$$= \frac{15^2 R^2}{E_0^2 x_0}\left[R - Y + x - 2(R - Y)\log(R - Y + x) - \frac{(R - Y)^2}{R - Y + x}\right]$$

$$= \frac{15^2 R^2}{E_0^2 x_0}\left[Y - 2(R - Y)\log\frac{R}{R - Y} - \frac{(R - Y)^2}{R} + R - Y\right].$$

$\sqrt{\langle L^2 \rangle}$ is tabulated for the parameters below:

$R \simeq 5 \times 10^4\ \text{g/cm}^2 = 300\ \text{m}$

$x_0 = 30\ \text{g/cm}^2 \simeq 20\ \text{cm earth} = 0.2\ \text{m}$

$E_0 = 10^5\ \text{eV}.$

In particular for Y = R:

$$\langle L^2 \rangle = \frac{29 \times 10^6 \times 225}{0.2 \times 10^{10}} \simeq 2.7\ \text{m}^2,$$

$$\sqrt{\langle L^2 \rangle} \simeq 1.5\ \text{m}.$$

So the muons scatter appreciably.

APPENDIX B. BIBLIOGRAPHY

1. Previous Reports

Beams - Section 6, Vol. I. National Accelerator Laboratory 1969 Summer Study.
Earlier references are in J. H. Christenson, "Some Simple Muon Beams," National Accelerator Laboratory 1969 Summer Study, Vol. I, p. 356.

Experiments - Section 3, Vol. IV National Accelerator Laboratory 1969 Summer Study.
Earlier references are in J. H. Christenson, "Muon Beams Compared," National Accelerator Laboratory 1969 Summer Study, Vol. I, p. 365.

2. Experiments Submitted

NAL Proposal 5, M. Perl et al, SLAC
NAL Proposal 26, K. W. Chen and L. N. Hand, Michigan State and Cornell
NAL Proposal 29, Richard Wilson et al, Harvard University
NAL Proposal 33, L. W. Mo et al, Chicago and Pennsylvania

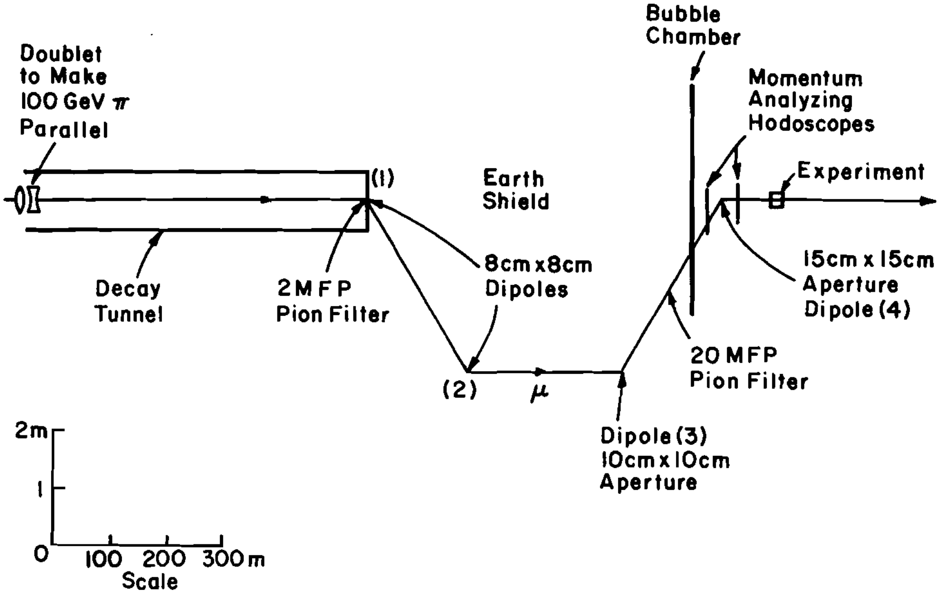


Fig. 1. Yamanouchi beam, March 1970.

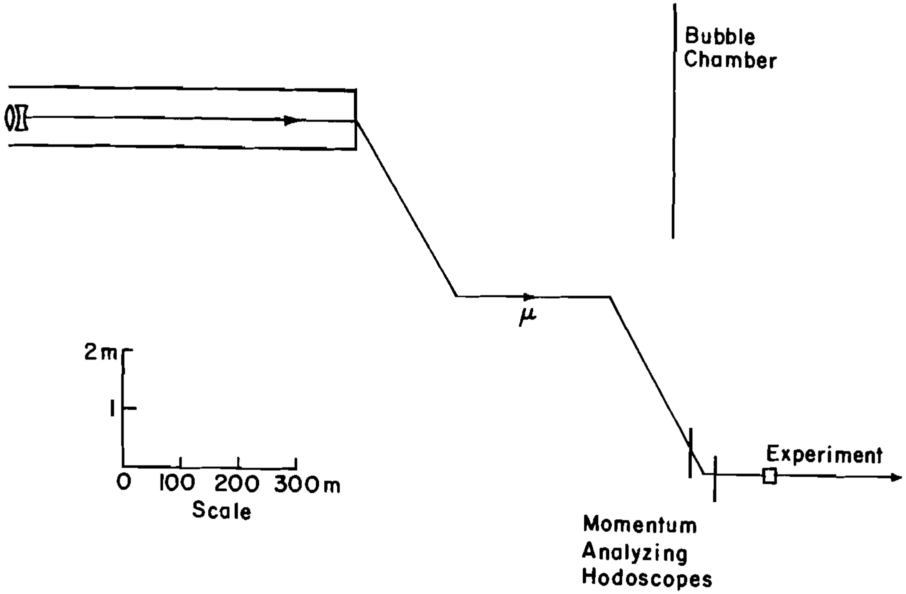


Fig. 2. Modified Yamanouchi beam.

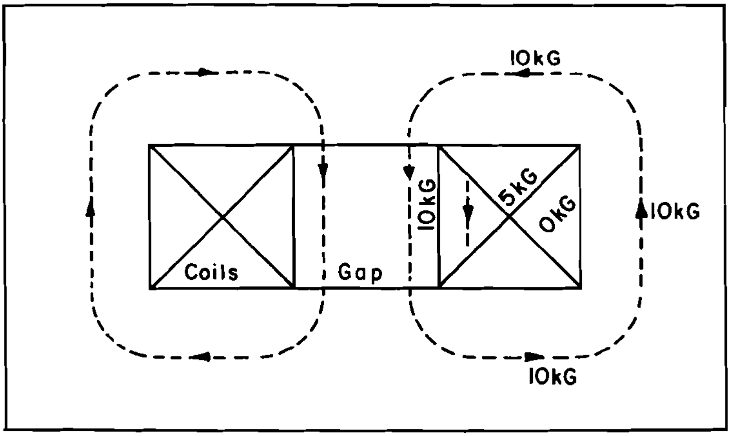


Fig. 3. Face of typical muon dipole bending magnet.

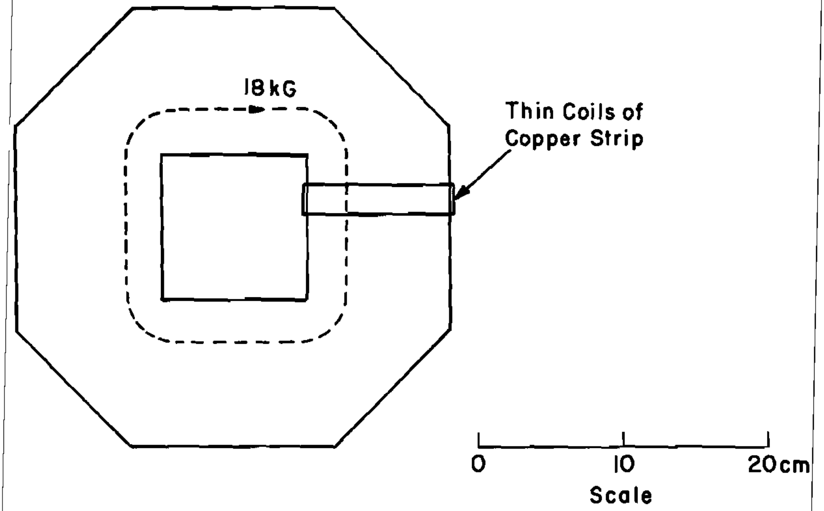


Fig. 4. Clean-up magnet.

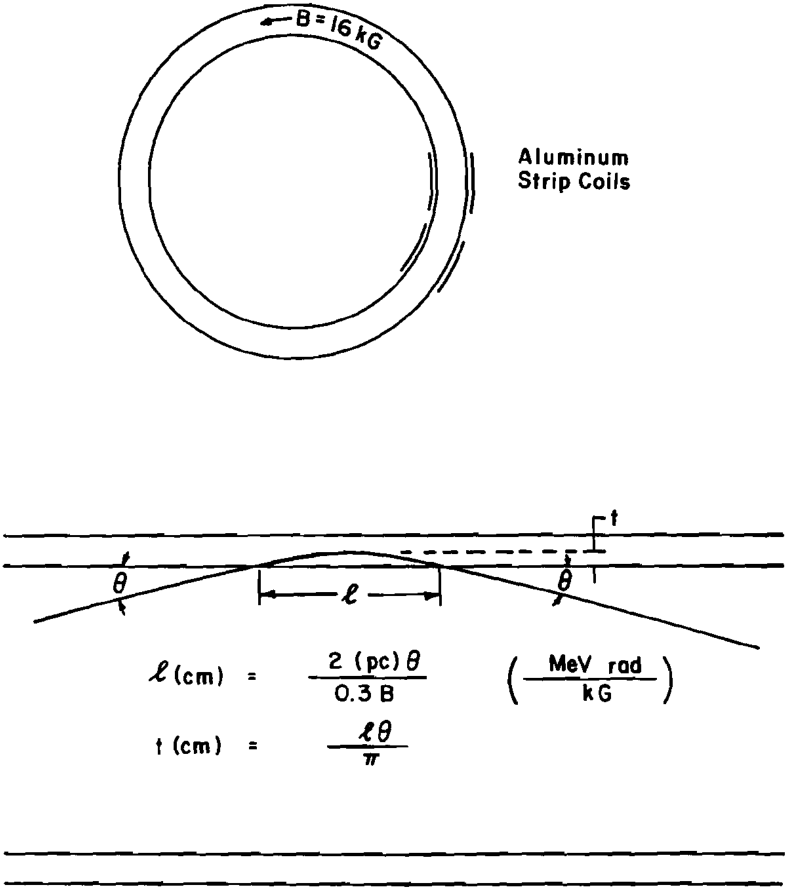


Fig. 5. Muon beam pipe.

Fig. 6. Illustration showing effect of multiple scattering on muon trajectory.

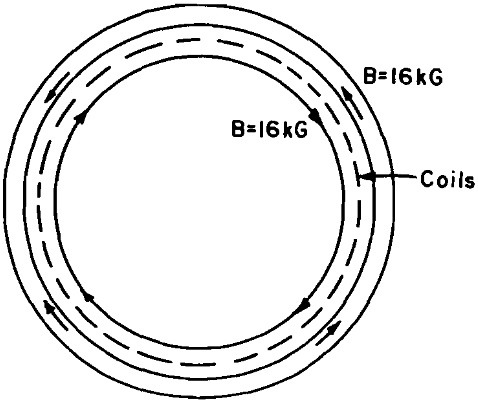


Fig. 7. Double pipe system.

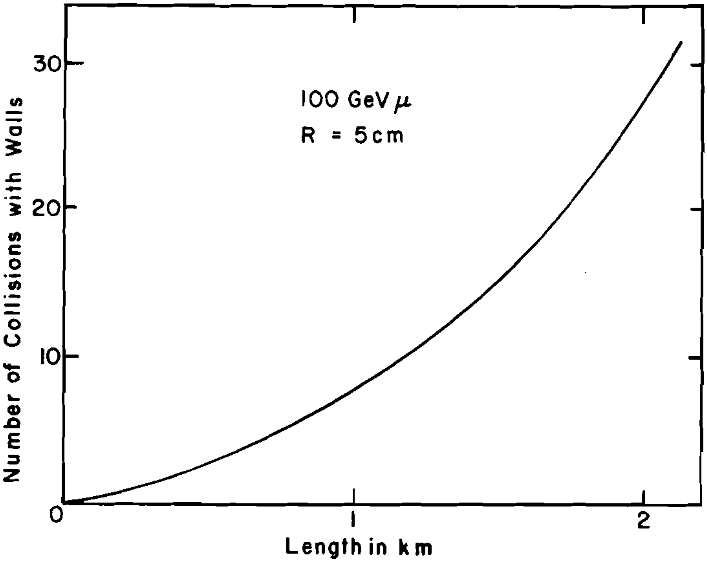


Fig. 8. The average number of collisions of muons with the walls of the suggested muon beam pipe, as a function of its length.

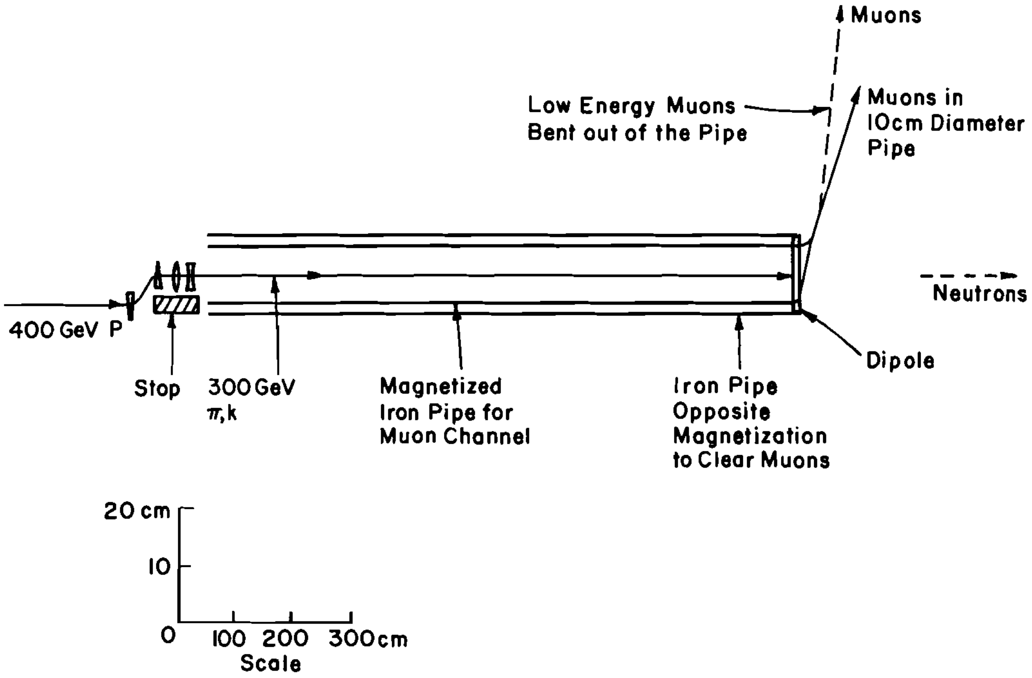


Fig. 9. High-energy muon/neutrino channel.

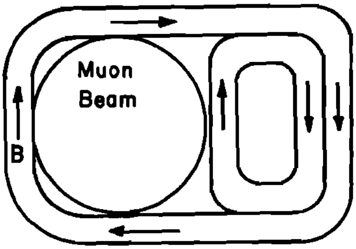


Fig. 10. Channel to allow muons into shield on one side only.

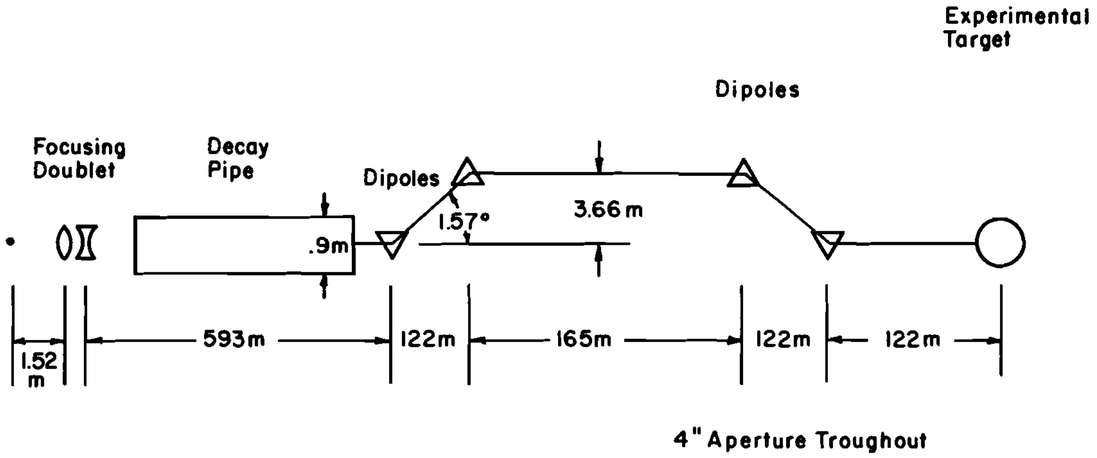


Fig. 11. Nominal beam used in calculations.

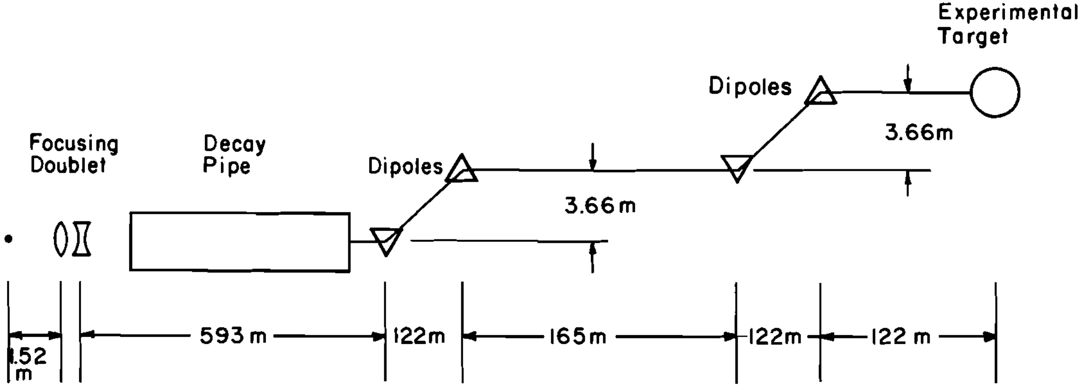


Fig. 12. Double bend used in calculations.