

MUONS AND ELECTROMAGNETIC INTERACTION AT NAL
 A THEORIST'S VIEW

Yung-Su Tsai
 Stanford Linear Accelerator Center

ABSTRACT

Some of the interesting things one would like to learn using e^\pm , μ^\pm , and γ beams derived from a 500-GeV proton beam from NAL are discussed. Topics discussed include:

- A. μ -e relative locality and radiative corrections
- B. Excited state of μ
- C. Heavy leptons and W^\pm bosons
- D. Inelastic μ (or θ) + p scatterings
- E. Production of ρ by a virtual γ from proton.

The amazing thing about NAL is that even though it is a proton machine, it will produce enough high-energy e^\pm , μ^\pm , and γ beams to make unique contributions to the fields of very high energy electromagnetic interactions until SLAC becomes a long-duty-cycle machine at energy equal to 200 GeV or more. I would like to summarize some of the interesting things one would like to learn using e^\pm , μ^\pm , and γ beams from NAL.

A. μ -e Relative Locality and Radiative Corrections

According to quantum electrodynamics, a photon is coupled to an electron current or a μ current in the lowest order in e by

$$eA_\mu \bar{u}(p_2) \gamma_\mu u(p_1), \quad (1)$$

without any form factors. Furthermore, an excited state of e or μ does not exist:

$$\gamma + e \not\rightarrow e^*, \quad (2)$$

$$\gamma + \mu \not\rightarrow \mu^*. \quad (3)$$

By comparing $e + \text{nucleus} \rightarrow e + \text{anything}$ and $\mu + \text{nucleus} \rightarrow \mu + \text{anything}$ at the same four-momentum transfer squared q^2 and energy transfer ν (but not necessarily at the same incident energy or scattering angle), one can test whether e is more local than μ or vice versa, even though this comparison does not test whether both μ and e currents are local separately as given by Eq. (1). The only theoretical complication involved is the question of uncertainties in the radiative corrections, especially noninfrared parts of the two-photon exchange diagrams and the noninfrared parts of the interference

terms between the bremsstrahlung from the lepton and hadron currents. Fortunately, both of these two effects can be checked by comparing μ^+ with μ^- (or e^+ with e^-). From the point of view of radiative corrections, muons have definite advantages over electrons because the radiative correction due to internal bremsstrahlung for electrons is roughly the same as that given by two external radiators with one placed before and one after the scattering, each of thickness¹

$$t = \frac{3}{4} (\alpha/\pi) \left[\ln \left(-q^2/m_e^2 \right) - 1 \right] \quad (4)$$

radiation lengths. The ratio of the probability of μ internal bremsstrahlung to that of e internal bremsstrahlung is given by

$$\left[\ln \left(-q^2/m_\mu^2 \right) - 1 \right] / \left[\ln \left(-q^2/m_e^2 \right) - 1 \right], \quad (5)$$

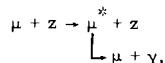
which is roughly 1/2.75 for $-q^2 = 10 \text{ GeV}^2$. The muon bremsstrahlung in the target (called the external bremsstrahlung or straggling effect) is reduced by a factor of²

$$\left(m_e/m_\mu \right)^2 (z/z+1) (\ln 2 E/m_\mu) / \ln (183 z^{-1/3}), \quad (6)$$

compared with the electron bremsstrahlung in the target. This shows that in order to minimize the radiative corrections, the target thickness in the e nucleus scattering should not be much greater than the one given by Eq. (4), whereas for muon scatterings the radiative corrections are practically independent of the target thickness, hence a very long target can be used. On the other hand, this also shows the major difficulty of the muon experiments, namely muon beams, cannot be collimated easily by a simple slit because the slit won't stop muons. When the final states of hadrons are measured and its center-of-mass energy (called missing mass) is known, the radiative corrections simplify greatly; one does not have to worry about the radiative tails due to the excitation of smaller missing-mass states as in the single-arm experiment.¹

B. Excited State of μ

Whether an excited state³ of μ exists [see Eqs. (2) and (3)] can be tested easily by Primakoff-like effect:



where z is a heavy nucleus to supply a Coulomb field for the conversion.

C. Heavy Leptons and W^\pm Bosons

The existence of a muon is a puzzle in physics. The question arises naturally if there are other leptons in nature besides e , μ , and their associated neutrinos ν_e and

ν_μ . From analogy to e and μ , we expect the heavy lepton to obey the laws of quantum electrodynamics in its interaction with a photon and decay via weak interactions:

$$\ell^- \rightarrow \nu_\ell + \mu^- + \bar{\nu}_\mu,$$

$$\ell^+ \rightarrow \bar{\nu}_\ell + \mu^+ + \nu_\mu,$$

$$\ell^- \rightarrow \nu_\ell + e^- + \bar{\nu}_e,$$

$$\ell^+ \rightarrow \bar{\nu}_\ell + e^+ + \nu_e.$$

Perl et al.⁴ have investigated the existence of heavy leptons by photo-pair production. The trouble is that even if the heavy leptons exist, they are likely to decay rapidly without being detected directly. They detected none but were able to give a lower limit to its mass as a function of life time. Obviously a better way to discover the existence of heavy leptons is to measure their decay products.

The long duty-cycle and high-energy γ beam from NAL is uniquely suited for investigating the existence of charged heavy leptons (ℓ^\pm) and weak vector mesons (w^\pm) by photo-pair production. The calculations for both processes

$$\gamma + \text{nucleus} \rightarrow \ell^+ + \ell^- + \text{anything}^5$$

and

$$\gamma + \text{nucleus} \rightarrow w^+ + W^- + \text{anything}^6$$

exist. W. Y. Lee⁷ proposed an ingenious scheme to detect the charged leptonic decay products from these two processes to minimize the background. The calculations of the energy-angle distributions of the decay products from these processes have not been done but can be done very accurately in terms of two form factors of the inelastic e-nucleus scattering, masses and magnetic moments of ℓ^\pm and w^\pm , and their branching ratios into the leptonic modes. Because the spin density matrices for ℓ^+ and ℓ^- (or w^+ and w^-) are correlated in the production, the calculations will be very complicated, but the technique of such a calculation is known. (See, for example, the calculation of e^+ and μ^- energy-angle correlation from

$$e^+ + e^- \rightarrow w^+ + w^- \rightarrow e^+ + \nu_e + \mu^- + \bar{\nu}_\mu$$

by Y. S. Tsai and A. C. Hearn.⁸) It should be pointed out that the production rate of $\gamma + \text{nucleus} \rightarrow w^+ + w^- + \text{anything}$ depends very sensitively upon the anomalous magnetic moment k of w^\pm as pointed out by Berman and Tsai.⁶ We also expect the energy and angle correlations between e^+ and μ^- (or e^- and μ^+) are very sensitively related to the sign and magnitude of k of w^\pm as indicated by the calculation⁵ of $e^+ + e^- \rightarrow w^+ + w^-$

mentioned above. Hence once the existence of w^\pm is confirmed, one can determine its magnetic moment. There is an intriguing possibility that the T noninvariance in the weak interaction is due to the electric dipole moment of w^\pm (Saltzman⁹). If T is violated, we expect to see an asymmetry proportional to $\vec{p}_\gamma \cdot (\vec{e}^+ \times \vec{\mu}^-)$ in the interaction:

$$\gamma + \text{nucleus} \rightarrow \begin{array}{l} \left. \begin{array}{l} \text{---} e^+ + \nu_e \\ | \\ w^+ \end{array} \right\} + \begin{array}{l} w^- \\ | \\ \mu^- + \bar{\nu}_\mu \end{array} + \text{anything.} \end{array}$$

A simple estimate⁷ of the counting rates shows that the experiment can test the existence of w^\pm and w^\pm up to mass of 7 GeV using a photon beam derived from a 500 GeV proton beam.

D. Inelastic μ (or e) + p Scattering

The most fundamental question in strong-interaction physics is whether the local field theory is really necessary to describe the hadronic system. For example, according to Chew,¹⁰ a hadron is a composite particle of composite particles observed in the laboratory, and hence it is not necessary to talk about a bare field, dressing it up by turning on the strong interactions, etc. It is true that the local field theory gives us the neatest way to derive CPT theorem, spin and statistics, substitution rules, etc., but it is also beset with difficulties such as the ultraviolet divergences and infinite renormalization constants. However, the same difficulties arise in quantum electrodynamics and yet experimentally it has been shown that the locality is good up to $\sim 4 \times 10^{-15}$ cm in quantum electrodynamics. It is interesting to see whether any experiment can be devised so that one can make a similar statement for hadrons. Large momentum transfer events in $\mu + \text{nucleon}$ inelastic scatterings at NAL offer the best hope of being able to tell us something about this question. When a μ is scattered at, say, $q^2 = 25 \text{ GeV}^2$, one would like to know what really kicked the μ so hard. The best way to find the answer is to see what comes out from the target. Maybe one will see surprises such as a quark or a monopole getting knocked out. Maybe less exciting, but of equal physical significance, the multiplicity, species of particles emitted, and the momentum distribution of particles become functions of mission mass $m_f^2 = q^2 + 2m_p \nu + m_p^2$ only but is independent of q^2 after q^2 reaches a certain value. This will mean that small q^2 events are mainly scattering from the cloud, but large q^2 events represent photons hitting something very tightly bound inside the proton resulting in a formation of a highly excited state whose mode of decay is independent of its mode of excitation, similar to the formation and decay of a compound nucleus in nuclear physics. If this picture is right, then the angular distribution of the secondaries

should be isotropic in the rest frame of the final hadron system. Another possibility is Feynman's idea¹¹ of parton model. In this model a proton is accompanied by (or consists of) a beam of virtual field quanta (called partons) which have a momentum distribution of $1/p_i$ when viewed in a frame where the target proton is moving at infinite momentum, but the photon has a zero energy. $1/p_i$ momentum distribution of the virtual field quanta comes from the analogy to the Weissacher-William's photons for a charged particle, and it also explains the flat part of νw_2 . The virtual photons interact with the partons incoherently. The energy-momentum conservation requires then that the virtual photon characterized by q^2 and ν can hit only those partons which have momentum p_i , $p_i/p = q^2/2m_p \nu$, where p is the momentum of the proton. There are many variations and versions to Feynman's parton idea. In Drell, Levy, and Yan's version,¹² the bare proton is the parton. In Bjorken's version,¹³ quarks and anti-quarks are partons. In Feynman's original version, charged vector mesons are the partons. If partons are scattered incoherently, we expect that there will be a jet of particles coming out from the target. Since nature seems to prefer exchange of zero quantum numbers at high energy, we expect this jet to have a large component which has the same quantum number as that of a parton. It is hoped that by observing the final states of hadrons in the μ inelastic scatterings one can make the parton idea of Feynman from a science fiction into science.

We have emphasized the importance of the measurements of hadronic final states in the muon scattering. This is meant to overcome some of the impression of experimentalists that theorists are interested only in $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$. W_1 and W_2 essentially represent the total cross sections of virtual $\gamma + p$ interactions as functions of ν and q^2 . The reason why most of the theoretical papers are concerned with W_1 and W_2 is that they are the only information available at this moment. The reason why there are so many papers on W_1 and W_2 is that there is a lot of freedom available to construct different theories to explain the behavior of total cross sections in the absence of any knowledge about partial cross sections. The true understanding of what is really going on can be obtained only after the measurement of the final states of hadrons.

It is easy to say that the measurements of hadron final states are important, but it is not easy to state precisely what features of the final states should be measured so that one can understand the nature better. The difficulty arises from the fact that in general the final hadron states are expected to consist of many particles; one-or two-particle final states are relatively rare. For example, when there are 6 particles in the final states we have 18 degrees of freedom from the momenta of the particles alone (besides momenta, we have to know their masses, spins, charges, etc.). Hence the analysis and presentation of the data depend very critically upon one's prejudice about what features of the data are physically significant.

Personally, I would like to have the following questions answered:

1. When $-q^2$ is large, say $-q^2 > 5 \text{ GeV}^2$, are the composition of the final hadron states, multiplicity, and momentum distributions drastically different from the hadron final states at $q^2 = 0$? In the parton model, one assumes that partons are scattered incoherently when $-q^2$ is large. Hence if the nature of the hadron final states is quite insensitive to q^2 , one would wonder the relevance of the parton picture in describing the deep inelastic μ -p scatterings.

2. There is a possibility of rare but interesting events associated with large $-q^2$ scatterings. Quarks, magnetic monopoles¹⁴, or dyons¹⁵ which are normally tightly bound may get knocked out by a very localized strong impact in the high ($-q^2$) muon scatterings.

3. Instead of the revolutionary parton picture, a more conventional picture may prevail, namely, even when $-q^2$ is large, as long as $m\nu \gg -q^2$, multiperipheral mechanism and diffraction dissociation mechanism may dominate the cross section. The diffraction-dissociation mechanism manifests itself experimentally as having a high-energy secondary which has the quantum number of a photon. It would be interesting to know what fraction of the total cross section is due to the diffraction dissociation. However, this is probably too much to ask because at this moment. Even for much investigated processes such as pp, π p, or γ p, we do not know the answer to this question. It is rather difficult to pin down what are the experimental manifestations of the multiperipheral model. The reason is that in any Feynman-diagram calculation the detailed predictions of energy and angular distributions depend very critically upon the spins and the types of couplings between the particles, and we do not know what are the particles (π , ρ , ω , proton, Pomeron, etc.) which occupy the legs and propagators of the multiperipheral diagrams. One thing which can be checked easily is whether the baryon exchange is important in the multiperipheral model. If baryon exchange is important, as in the model of Drell, Levy, and Yan,¹² one would expect most of the energy and momentum of the photon to go to a baryon instead of mesons.

4. A simple kinematical consideration shows that even when $-q^2$ is large, as long as $m_p \nu \gg -q^2$ peripheral or diffraction-dissociation types of mechanisms can be significant. However, when $m\nu$ is not much larger than $-q^2$, we expect that these mechanism no longer dominate the total cross section. The rising part of νW_2 in the SLAC data as a function of $-q^2/m_p \nu$ corresponds to this kinematical region. In my opinion, this region is the most interesting because it represents the central collision. If a proton is a loosely bound system, we expect most of the energy and momentum of the photon to go to a small number of particles and the angular distribution in the rest frame of the final hadron system to be highly unisotropic. On the other hand, if the proton is a tightly bound system, then we expect the energy and momentum of the photon to be

evenly distributed among the decay product and the angular distribution in the rest frame of the final hadron system to be almost isotropic.

E. Production of ρ by a Virtual γ from Proton

The parton idea emphasizes the investigation of the virtual field content of a proton by a virtual γ . One unsatisfactory feature of this idea is that it completely ignores the fact that a virtual photon also dissociates itself into hadrons and has its own virtual field content. If a photon turns into hadrons before it hits the target, then the simple picture of partons given in the previous section no longer holds. One is thus interested in what is the probability of a photon turning into hadrons before it interacts with a proton. However, we know such a question is meaningless because the concept that a photon turns into something before it hits the target is not Lorentz invariant. To illustrate this point, let us consider two old-fashioned perturbation theory diagrams:



The sum of two diagrams is Lorentz invariant, but which diagram is more important depends upon the choice of coordinate system. However, one can test the idea whether partons interact incoherently with a hard photon. A photon turning into "a" vector meson and then scattering from the target has no analogue in the parton picture. If such a process is important, then it is a mistake to use the parton picture alone to describe the total cross sections.

By measuring the cross section

$$\text{virtual } \gamma + p \rightarrow \text{vector particles} + p,$$

one can evaluate what is the probability of a photon turning into vector mesons before it interacts with the target proton. Many assumptions¹⁶ are necessary in order to make this estimate, but it is hoped that this estimate can be made by the following measurements:

1. Are there any more neutral vector mesons other than ρ , ω , and ϕ ?
2. Ratio of longitudinally polarized vector mesons to transversally polarized vector mesons.
3. What is the q^2 dependence of the production cross section, especially near the forward direction?

REFERENCES

- ¹L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969).
- ²H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934).
- ³F. Low, Phys. Rev. Letters 14, 238 (1965).
- ⁴A. Barna et al., Phys. Rev. Letters 173, 1391 (1968).
- ⁵Y. S. Tsai and Van Whitis, Heavy Lepton Production Cross Sections, SLAC Computation Center Program Library (1966). It should be pointed out that when a pair of heavy particles, each with mass M , are photoproduced, the minimum momentum transfer to the target system is $q_{\min} = 2M^2/k$. For $M = 7$ GeV and $k = 200$ GeV, we have $q_{\min} \approx 0.5$ GeV. For this case, the coherent production from the nucleus is completely negligible. In the incoherent production, the target nucleon either remains as a nucleon or breaks up. Because of the mildness of the t dependence of W_1 and W_2 found in the SLAC deep-inelastic ep scatterings compared with the t dependence of the elastic form factors, it will grossly underestimate the production cross section if only the elastic form factors $G_e(t)$ and $G_m(t)$ are used in the calculation. The inclusion of the contribution from inelastic form factors W_1 and W_2 is trivial. SLAC experiments show $\nu W_2 \sim 0.3$ and $\sigma_l/\sigma_t \sim 0$ which implies $W_1 \approx [(\nu^2 - q^2)/-q^2] W_2 \gg W_2$ when $\nu^2 \gg -q^2$. Because of the behavior of W_1 and W_2 , Bethe-Heitler's formula, where the target is assumed to be a point Coulomb source, actually gives quite an adequate order-of-magnitude estimate of the cross section, due to inelastic W_1 and W_2 , provided z^2 is replaced by 0.3. The suppression factor due to the phase space can be estimated using the formula

$$I = \left[1 - 2 \left(M_1^2 + M_2^2 \right) s^{-1} + \left(M_1^2 - M_2^2 \right)^2 s^{-2} \right]^{1/2},$$
 where $M_1 =$ invariant mass of w^+ and w^- , $M_2 =$ invariant mass of hadron final state and $s =$ total $\gamma + p$ center-of-mass energy squared. The suppression factor I is about 0.5 when

$$s = 2M_p k + M_p^2 \approx 400 \text{ GeV}^2, \quad M_1 = 14 \text{ GeV} \text{ and } M_2 = 2 \text{ GeV}.$$
- ⁶S. M. Berman and Y. S. Tsai, Phys. Rev. Letters 11, 483 (1963).
- ⁷W. Y. Lee et al., Proposal to Search for Heavy Leptons and Intermediate Bosons from Photon-Nucleon and Photon-Nuclei Collisions, National Accelerator Laboratory Proposal 87, 1970.
- ⁸Y. S. Tsai and A. C. Hearn, Phys. Rev. B721, 140 (1965).
- ⁹T. Saltzman and G. Saltzman, Phys. Letters 15, 91 (1965).
- ¹⁰G. F. Chew, S Matrix Theory of Strong Interactions (W. A. Benjamin, Inc., New York, 1961).

- ¹¹R. Feynman, High Energy Collisions, edited by C. N. Yang et al., (Gordon and Breach, New York, 1969).
- ¹²S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. Letters 22, 744 (1969).
- ¹³J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
- ¹⁴P. A. M. Dirac, Phys. Rev. 74, 817 (1948); T. M. Yan, Thesis, Harvard University (1968).
- ¹⁵J. Schwinger, Science 165, 757 (1969); D. Zwanziger, Phys. Rev. 176, 1489 (1968).
- ¹⁶The assumptions necessary are
- a. Photon-vector meson coupling constants are given by $e^+ + e^-$ colliding-beam experiments.
 - b. Helicities are conserved in the interaction $v + p \rightarrow v + p$ near the forward direction, where v is a vector particle.
 - c. Forward amplitudes for $v + p \rightarrow v + p$ are imaginary.
 - d. The cross section $v + p \rightarrow v + p$ is independent of q^2 (i. e., no off-the-mass shell correction). The last assumption is the most dubious one.