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MUON PHYSICS

S. Derenzo
Lawrence Radiation Laboratory

and

M. Tannenbaum
Harvard University

ABSTRACT

Muon experiments may be divided into two broad categories. First, there are experiments studying the muon itself. These seek to understand the fundamental nature of the leptons: Why does the muon exist? If the muon is a heavy electron, why do the muon and electron have different neutrinos and separately conserved lepton numbers? What accounts for the observed lepton mass spectrum: ν_e, ν_μ, e, μ?

Second, there are experiments which use the virtual photon from muon inelastic scattering as a probe of hadronic structure. These experiments have been stimulated by the deep inelastic scattering results from SLAC.[1] In this report, we will discuss the hadronic probes and then compare the muon and neutrino inelastic experiments as to rates and other features.

Muons as Hadronic Probes

Muon beams have two unique features: (1) they interact via virtual photons with continuously variable invariant mass q^2 and energy ν, and (2) the final-state muon can be unambiguously identified by its penetration through meters of material. In addition, the momenta of both the incident and outgoing muon can be precisely measured to give the parameters of the virtual photon.

The inelastic μ scattering cross section is

$$\frac{d^2\sigma_\mu}{dq^2 d_\nu} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E}\left[W_2 \cos^2\frac{\theta}{2} + 2W_1 \sin^2\frac{\theta}{2}\right]$$

Experimental and theoretical results at SLAC seem to indicate that

1. The form factor W_2 for inelastic lepton scattering has a much weaker q^2 dependence than for elastic scattering or resonance production.

2. $\nu W_2(q^2, \nu)$, the inelastic proton structure function, is a universal function only of ν/q^2, a property called scaling.

3. $d^2\sigma$ is dominated by W_2.
4. For $\nu >> q^2$, νW_2 becomes a constant, 0.3.

The questions of interest at higher ν and q^2 are:

1. Does scaling continue to hold?
2. Does scaling hold in detail? (e.g., the effects of $\sigma_S/\sigma_T \neq 0$)
3. How do the final hadronic states depend on ν and q^2?

In particular, how much of inelastic scattering is due to rho-meson production?

4. What is the t distribution of recoil protons vs q^2, ν, missing mass?
5. What fraction of the time are strange particles produced?
6. What is the multiplicity of charged and neutral particles produced in the final state as a function of q^2 and ν?
7. What is the statistical distribution of p_L and $p_\perp$ for reaction secondaries?

<u>Relation to Neutrino Experiments</u>

The cross sections for muon and neutrino deeply inelastic scattering are nearly identical. We rewrite the muon cross section as

$$\frac{\partial^2\sigma_\mu}{\partial q^2 \partial\nu} = \frac{4\pi\alpha^2}{q^4} W_2(\nu, q^2)\left[1 - \frac{\nu}{E} + \frac{q^2}{4E^2}\left(2\frac{W_1}{W_2} - 1\right)\right] \quad (1)$$

and have a neutrino cross section of

$$\frac{\partial^2\sigma_\nu}{\partial q^2 \partial\nu} = \frac{G^2}{2\pi} W_2(\nu, q^2)\left[1 - \frac{\nu}{E} + \frac{q^2}{4E^2}\left(2\frac{W_1}{W_2} - 1\right) + \left(1 - \frac{\nu}{E} - \frac{q^2}{4E^2}\right)\frac{2E-\nu}{M}\frac{W_3}{W_2}\right] \quad (2)$$

W_2 and W_1 for the vector part of the neutrino cross sections are the same as the muon structure functions. However, the neutrino cross section also contains an axial-vector part as well as a vector-axial-vector interference, W_3. To account for this we double the neutrino cross section and ignore W_3. This may not be too bad an approximation since if the vector and axial parts were equal, the cross section would quadruple, but the form factor W_3 could be negative.[2] We take the neutrino cross section as

$$\frac{\partial^2\sigma_\nu}{\partial q^2 \partial\nu} \simeq \frac{G^2}{\pi} W_2(\nu, q^2)\left[1 - \frac{\nu}{E} + \frac{q^2}{4E^2}\left(2\frac{W_1}{W_2} - 1\right)\right]. \quad (3)$$

From these equations it is apparent that the muon cross section is proportional to α^2 and drops like $1/q^4$ due to the photon propagator. On the other hand, the neutrino cross section is proportional to the much smaller G^2 $(G = 10^{-5}/M_p^2)$ but does not have a propagator since the interaction is believed to be local. Since the kinematic and structure factors are identical, they cancel out when finding the ratio of muon and neutrino cross sections

$$\frac{\partial^2\sigma_\mu}{\partial^2\sigma_\nu} = \frac{4\pi^2\alpha^2}{G^2}\,\frac{1}{q^4} \tag{4}$$

or

$$\frac{\partial^2\sigma_\nu}{\partial^2\sigma_\nu} = \left(\frac{64\ \mathrm{GeV}}{q}\right)^4 . \tag{5}$$

Calculation of Muon Counting Rates

The experimental results at SLAC seem to indicate that for $\nu >> q^2$ $\nu W_2 = 0.3$ for all values of ν and q^2 and also that $\sigma_1 << \sigma_T$. If σ_L can be ignored then the ratio of W_1/W_2 is simply

$$\frac{W_1(\nu, q^2)}{W_2(\nu, q^2)} = \frac{q^2 + \nu^2}{q^2} . \tag{6}$$

Substituting into Eq. (1) we get the expression

$$\frac{\partial^2\sigma_\mu}{\partial q^2\partial\nu} = \frac{4\pi\alpha^2}{q^4}\,\frac{0.3}{\nu}\left(1 - \frac{\nu}{E} + \frac{\nu^2}{2E^2} - \frac{q^2}{4E^2}\right) . \tag{7}$$

and if we assume that $\nu^2 >> q^2$

$$\frac{\partial^2\sigma_\mu}{\partial q^2\partial\nu} = \frac{1.2\pi\alpha^2}{q^4}\,\frac{1}{\nu}\left(1 - \frac{\nu}{E} + \frac{\nu^2}{2E^2}\right). \tag{8}$$

We take this as the expression for the muon inelastic cross section.

We can integrate this to find the ν distribution for all q^2

$$\frac{d\sigma_\mu}{d\nu} = \frac{1.2\pi\alpha^2}{q_{min}^2}\left(1 - \frac{q_{min}^2}{q_{max}^2}\right)\frac{1}{\nu}\left(1 - \frac{\nu}{E} + \frac{\nu^2}{2E^2}\right), \tag{9}$$

and the q^2 distribution integrated over all ν assuming that $\nu_{max} = E$

$$\frac{d\sigma_\mu}{dq^2} = \frac{1.2\pi\alpha^2}{q^4}\left[\ln\frac{E}{\nu_{min}} - \frac{3}{4} + \frac{\nu_{min}}{E}\left(1 - \frac{\nu_{min}}{4E}\right)\right], \tag{10}$$

and we can also find the total observable cross section

$$\sigma_{tot}(\nu_{min}, q_{min}^2) = \frac{1.2\pi\alpha^2}{q_{min}^2}\left[\ln\frac{E}{\nu_{min}} - \frac{3}{4} + \frac{\nu_{min}}{E}\left(1 - \frac{\nu_{min}}{4E}\right)\right]. \tag{11}$$

It is worthwhile to remember that if q^2 gets too large ν_{min} is no longer set by the acceptance but instead

$$\nu_{min} = \frac{q^2}{2M},$$

so that Eq. (10) has to be integrated carefully.

We can now calculate the counting rates for a typical muon experiment

$$E = 100 \text{ GeV}$$
$$q_{min}^2 = (E\theta_{min})^2$$
$$\theta_{min} = 7 \text{ mrad}$$
$$q_{min}^2 = 1/2$$
$$\nu_{min}/E = 1/2$$
$$\text{target length} = 2 \text{ m liquid hydrogen}$$
$$\text{flux} = 10^6 \text{ muons/pulse}$$
$$\Delta p/p = \pm 5\%$$
$$\text{flux and target length} = 8.4 \text{ events}/\mu b/\text{pulse}.$$

If we can obtain $\nu_{min}/E \sim 0.4$ to 0.5, then the bracketed factor in Eq. (11) is equal to about 1/2 so that we get

$$\sigma_{\mu\,tot}(q_{min}^2) \simeq \frac{1}{25q_{min}^2}\ \mu b\ GeV^2, \tag{12}$$

and the number of events per pulse is

$$N_\mu(q_{min}^2) \simeq \frac{1}{3q_{min}^2}, \tag{13}$$

where q_{min}^2 is in BeV^2.

The factor in parentheses in Eq. (8) doesn't vary from 1/2 by more than 20% for $\nu_{min}/E > 0.4$, so that we can also find a simple expression for the number of events per bin or Δq^2 and $\Delta\nu$ with all units in GeV

$$\Delta^2 N_\mu(q^2) \simeq \frac{1}{3}\ \frac{\Delta q^2}{q^4}\ \frac{\Delta\nu}{\nu}. \tag{14}$$

Hence, if we take as a standard bin:

$$\frac{\Delta\nu}{\nu} = 10\%$$

$$\Delta q^2 = 1\ \mathrm{GeV}^2,$$

we find the number of events per pulse in this bin

$$\Delta^2 N_\mu(q^2) = \frac{1}{30q^4} \tag{15}$$

This equation is plotted in Fig. 1.

Comparison of Muon and Neutrino Rates

We compare the counting rate for deeply inelastic scattering by muons to that for deeply inelastic neutrino scattering as proposed by Harvard, Penn, and Wisconsin (NAL Proposal 1). They use an 8 m long by 3 m diameter H_2 target. For comparison in the same range of ν available from 100-GeV muons, i.e.,

$$40 \le \nu \le 100.$$

We take the $40 < E_\nu \le 100$ GeV part of their spectrum and find 10^9 neutrinos per pulse shining on their hydrogen target. We throw in another factor of 2.5 out of generosity and take the ratio:

$$\frac{\text{Neutrino flux} \times \text{target length}}{\text{Muon flux} \times \text{target length}} = \frac{10^{10}}{10^6} = 10^4.$$

Using Eq. (5) for the cross-section ratio and plugging in the 10^4 factor of flux times target length, we find the event ratio between muon and neutrino experiments in a bin of

$$\Delta q^2 = 1\ \mathrm{BeV}^2$$

$$\frac{\Delta\nu}{\nu} = 10\% \tag{16}$$

is

$$\frac{\Delta^2 N_\nu}{\Delta^2 N_\mu} = \left(\frac{q}{6.4\ \mathrm{GeV}}\right)^4.$$

Thus, the muon experiment becomes equal in counting rate to the neutrino experiment at $q^2 = 41\ \mathrm{GeV}^2$.

We can then multiply by the muon counting rate $\Delta^2 N_\mu$ to find the neutrino counting rate in a bin with

$$\Delta q^2 = 1\ \mathrm{GeV}^2$$

$$\frac{\Delta\nu}{\nu} = 10\%$$

per 2.5×10^9 neutrinos/pulse on an 8 m H_2 target in the interval

$$40 \le E_\nu \le \infty$$

$$40 \leq \nu \leq 100.$$

Then, $$\Delta^2 N_\nu = 2 \times 10^{-5}/\text{pulse}.$$

Actually this counting rate is probably overestimated since each neutrino in the E_ν range quoted above can not supply the full range of $40 \leq \nu \leq 100$ GeV to the hadrons.

Assuming an experiment of 10^6 pulses there will be 20 events in each $\Delta q^2 = 1$ $\Delta\nu/\nu = 0.1$ internal for the neutrino experiment. Taking bins of $\Delta q^2 = 5$ BeV^2 gives 100 events per bin, and it may be that some crude estimates of νW_2 can be made in this q^2 region of $q^2 > 40$ GeV^2 by neutrinos.

In conclusion, it certainly looks that for $q^2 < 40$ GeV^2 the muon experiments are dominant. For $q^2 > 40$ GeV^2 the neutrinos should make some contribution in a limited ν range, the details of which depend on the magnitude and falloff of the neutrino flux at $E_\nu > 100$ GeV.

REFERENCE

[1] E. A. Paschos (The Rockefeller University), personal communication, 1970.

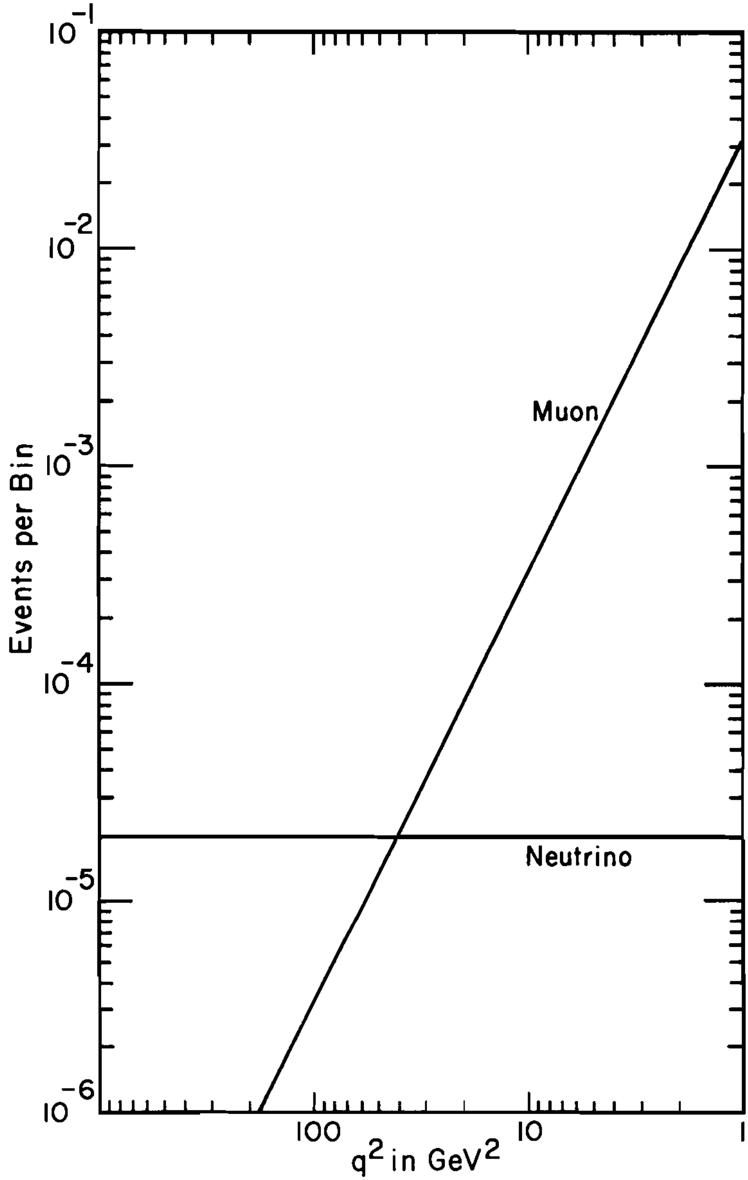

Fig. 1. Events per bin of $\Delta q^2 = 1\ \mathrm{BeV}^2$ $\Delta\nu/\nu = 0.1$ per 10^6 muons incident on 2 meters of H_2 or per 2.5×10^9 neutrinos incident on 8 meters of H_2 at 100 GeV.