POLARIZED TARGETS AT NAL<br>E. Berger and A. Yokosawa<br>Argonne National Laboratory<br>G. Burleson<br>Northwestern University<br>P. M. Patel<br>Mc Gill University<br>F. M. Pipkin<br>Harvard University<br>and<br>K. P. Pretzl<br>National Accelerator Laboratory


#### Abstract

The conclusions of the group studying the use of polarized targets at NAL are summarized. The report discusses the reasons for interest in polarized targets, the experiments with polarized targets, and the recommended target for NAL. This tar get uses a $50-\mathrm{kG}$ magnet and a temperature of $1^{\circ} \mathrm{K}$. The target is 13 cm long and has a cross section which is $2.5 \mathrm{~cm} \times 2.5 \mathrm{~cm}$. The polarization would be $70 \%$ and the reversal time 3.5 minutes.


## I. INTRODUCTION

As part of the 1970 Summer Study, a group was formed to consider the use of polarized targets at NAL. This report summarizes the conclusions of this group.

## II. REASONS FOR INTEREST IN POLARIZED TARGETS

Scattering experiments give a clue to the interaction between elementary particles, and an important method for studying this interaction is to measure the differential cross section for elastic scattering as a function of energy and momentum transfer. Measurements of only the differential cross section do not, however, completely determine the amplitude.

For example, the amplitude for scattering of a spin-0 particle from a spin-1/2 particle can be written in the form

$$
\mathrm{T}=\mathrm{f}+\mathrm{ig} \vec{\sigma} \cdot \overrightarrow{\mathrm{n}} .
$$

Here $\vec{n}$ is a unit vector normal to the scattering plane. If the target is polarized the differential cross section is

$$
\frac{\mathrm{d} \sigma}{\mathrm{dt}} \propto|\mathrm{f}|^{2}+|\mathrm{g}|^{2}+2\left[\operatorname{Im}\left(\mathrm{fg}^{*}\right)\right] \overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{n}}
$$

where $\vec{p}$ is the target polarization. The scattering from a polarized target depends on the interference between the flip and nonflip amplitudes, and it can be used to detect a small flip amplitude in the presence of a large nonflip amplitude. The scattering also depends on the relative phase of the amplitudes and can be used to detect a small real amplitude in the presence of a large imaginary amplitude. It in general requires a measurement of the differential cross section and two polarization experiments to completely determine the amplitude.

The real importance of polarization measurements lies in the fact that they give a good way to see if the amplitude is becoming simpler at high energy. Large polarizations in the diffraction region would indicate an important deviation from our present ideas and show that they are wrong. Present experiments and the current models predict large polarizations in the nondiffractive region.

Appendix A gives a theoretical discussion of the significance of polarization experiments and their role in distinguishing alternate models which fit the differential cross sections.

## III. EXPERIMENTS WITH POLARIZED TARGETS

As of July 20, 1970, there is only one proposal (Proposal 61) for the use of a polarized target at NAL. This is a proposal for an experiment to study $\pi^{+}, \pi^{-}$, and $p$ scattering from polarized protons at incident momenta of 50,100 , and $150 \mathrm{GeV} / \mathrm{c}$. At each energy the experiment covers the momentum transfer range from $-\mathrm{t}=0.15$ to $1.5(\mathrm{GeV} / \mathrm{c})^{2}$. The estimated precision in the polarization measurement for $-\mathrm{t}<1.0$ $(\mathrm{GeV} / \mathrm{c})^{2}$ is $\Delta \mathrm{P}=0.005$.

The apparatus consists of two spectrometers, one for the forward particle and one for the recoil particle. The spectrometer magnet for the recoil arm bends vertically so as not to confuse the measurement of the recoil momentum and the recoil angle. Wire proportional chambers are used to define the trajectories of each of the particles. The target is $12.5-\mathrm{cm}$ long and $2.5-\mathrm{cm}$ square; the target polarization is $70 \%$. It is proposed not to use counters in the beam so that the apparatus will tolerate $10^{8} \mathrm{p}$ /pulse when $\pi^{+}$scattering is being measured. Cerenkov counters in the forward arm are used to identify the scattered particle.

The apparatus used for the polarization experiment is very similar to that used in the elastic-scattering experiments, and the two measurements would be made with
the same apparatus. In a sense the polarization measurements are simpler than the elastic-scattering measurements; in another sense they are more difficult. The polarization measurements are simpler in that they are made by just reversing the polarization and holding everything else constant; this makes the results less sensitive to long-term drifts and not dependent on the absolute calibration of the apparatus. They are more difficult in that the backgrounds are larger and the measurements on the recoil and the scattered particle must be sufficiently precise to separate the elastic scattering from the quasielastic scattering. It is more proper to say that the elastic scattering can be done with the same apparatus as the polarized target meas urements rather than vice versa.

The group also looked into other polarization experiments that could and should be done at NAL. The following experiments were considered:

1. Forward polarization measurements at 20 to $60 \mathrm{GeV} / \mathrm{c}$ in all the elastic processes. One of the most interesting programs at NAL would be to make a systematic study of the scattering from a polarized target in the energy range below 60 GeV . Such a program could be carried out in the 15 mrad beam, and it could give data for $\pi^{ \pm}, \mathrm{K}^{ \pm}, \mathrm{p}$, and $\overline{\mathrm{p}}$ out to momentum transfers of $2.0(\mathrm{GeV} / \mathrm{c})^{2}$, and possibly higher. The apparatus required has been investigated and a more complete description of this experiment is given in Appendix B.
2. Polarization in backward scattering. One of the interesting phenomena thathave been studied at low energy is the dependence of the cross section for backward scattering on the polarization of the target. The polarization is large, and it is a strong function of $s$ and $u$. It is important to see if this behavior persists at high energy. The feasibility of extending these measurements to higher energy has been investigated, and it appears feasible to do so at NAL (see Appendix C).
3. Study of polarization in the charge-exchange reaction $\pi^{-}+p \rightarrow \pi^{\circ}+n$. In the simplest form of the theory only rho exchange can contribute. A reaction such as this with only one amplitude should display no polarization. There is, however, at 12 $\mathrm{GeV} / \mathrm{c}$ a substantial dependence of the scattering on the polarization of the target. It is important to extend this reaction to higher energy and study how the polarization changes with energy. This experiment has been investigated, and it is described more completely in Appendix D.
4. Measurement of the $R$ parameter in elastic pion-proton scattering. The purpose of this experiment is to study helicity conservation in pion-nucleon scattering and in particular to test the hypothesis that helicity is conserved in diffraction scattering. This experiment requires a configuration for the target such that the direction of polarization lies in the scattering plane and is perpendicular to the momentum of the
incident pion. The experiment is feasible, and it could be used to test helicity conservation. In conjunction with the results from a standard polarization experiment, it would give a complete determination of the amplitude (see Appendix E).
5. Inelastic muon scattering from a polarized target. Another very interesting experiment is the measurement of the dependence on target polarization when longitudinally polarized muons are scattered from a polarized target. This experiment requires a target which is polarized parallel to the direction of the incident muon. The experiment gives a unique test of some aspects of the parton model.

## IV. RECOMMENDED TARGET FOR NAL

In order to make precise measurements ( $\Delta \mathrm{p} \leq 0.01$ ) of the polarization, it will require good control of the systematic errors. The best way to reduce the systematic errors is to reverse frequently the polarization of the target. Because of the small cross sections, it is advantageous to have a large target polarization, and, also to have as large a target as is possible.

The best way to achieve these objectives is to use a target at $1^{\circ} \mathrm{K}$ in a field of 50 kG . This would give a polarization of $70 \%$ and a reversal time of 3.5 minutes. This target should have a length of 13 cm and a cross section 2.5 cm by 2.5 cm .

This target would require a superconducting magnet. A preliminary design for the magnet has been made at Argonne National Laboratory. This magnet would give an available $\phi$ angle of $\pm 12.5^{\circ}$ for the recoil proton. Dewars could be built so that the axis of polarization could be either perpendicular to or in the scattering plane. This would make it possible to use the target for either standard polarization experiments or for measurements of the R parameter.

The estimated cost of the entire target system including the superconducting magnet, dewars, and microwave apparatus is $\$ 220,000$. We recommend that NAL procure such a target. Further details are given in Appendix F.

The muon experiment requires a somewhat larger target with the axis of polarization along the direction of the incident beam. The diameter of the target should be 4 to 6 in . and the length roughly 1 ft . A superconducting solenoid could be used to supply the magnetic field. Since there is at the present time no proposal to do this experiment, no action concerning this target is recommended.

## APPENDIX A. POLARIZATION--THEORETICAL COMMENTS

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Polarization data supply important information concerning both the relative phase and the spin-flip/spin-nonflip structure of scattering amplitudes. This information complements that obtained from $\mathrm{d} \sigma / \mathrm{dt}$ measurements, and, moreover, can often serve as a more decisive test of models. A distinct advantage of polarization is that it is sensitive to relatively small effects in the overall high-energy amplitude. Precisely because of this, one may pinpoint unexpected developments in amplitudes better with polarization data than with $\mathrm{d} \sigma / \mathrm{dt}$ measurements. No doubt $\sigma_{\text {tot }}$ and $\mathrm{d} \sigma / \mathrm{dt}$ results at NAL will produce many new theoretical speculations; it is important to confront these theories with as complete a set of measurements as possible. Therefore, it is advantageous to obtain polarization data concurrently with $\mathrm{d} \sigma / \mathrm{dt}$ results, or as soon as possible thereafter.

There are two main guides for choosing specific high-energy polarization experiments. One is entirely model independent and the other relies on model considerations. First, low-energy data display characteristic features as a function of both energy and momentum transfer. Examples are the approximate mirror symmetry of $\pi^{ \pm}$p polarization (see Fig. 1, Appendix B); the fact that polarization falls $\approx$ as $s^{-\frac{1}{2}} \mathrm{in}$ all elastic reactions; the fact that $K^{+} p$ polarization is much larger than $K^{-} p$. All of these features are "explained" by present theories; however, independent of specific models, it is interesting to determine whether these characteristic features persist at very high energies. The issue at stake is whether existing polarization data reflect asymptotic behavior or are merely low-energy phenomena. For these reasons, it is essential to design measurements which cover a wide enough range of $s$ and $t$ and which are of sufficient accuracy that physics information is obtained without recourse to detailed (parameter sensitive) fits to data.

An example may help to make this argument more transparent; let us examine the mirror symmetry of $\pi^{ \pm} p$ polarization. For both charge states, values of low energy polarization approach zero near $t=-0.6(\mathrm{GeV} / \mathrm{c})^{2}$ and are nonzero away from that point. High -energy studies of $\pi^{ \pm} p$ polarization should be designed to encompass fully the $0<|t|<1(\mathrm{GeV} / \mathrm{c})^{2}$ region, in order to detexmine experimentally the energy dependence of the $t=-0.6(\mathrm{GeV} / \mathrm{c})^{2}$ phenomenon. It would be unfortunate, for example, to make measurements to $t=-0.5(\mathrm{GeV} / \mathrm{c})^{2}$ and to have to rely on theoretical fits to determine whether polarization changes sign near $t=-0.6(\mathrm{GeV} / \mathrm{c})^{2}$. Similarly, when measurements are made at large $t$, attempts should be made to encompass fully the
regions in which structure has been observed experimentally at low energy [ $|\mathrm{t}|$ from 2.0 to $\left.3.0(\mathrm{GeV} / \mathrm{c})^{2}\right]$ and expected theoretically $\left[|\mathrm{t}| \approx 2.5(\mathrm{GeV} / \mathrm{c})^{2}\right]$.

For energy dependence, the accuracy required is easily specified in terms of exponent $n$ in the equation

$$
\begin{equation*}
\mathrm{P}=\mathrm{bs}{ }^{-\mathrm{n}} . \tag{1}
\end{equation*}
$$

Over the energy range 50 to $150 \mathrm{GeV} / \mathrm{c}$, if n were known to $\pm 0.1$, it would be possible to state fairly unambiguously whether Regge poles, cuts, or other phenomena govern polarization. This deduction could be made readily, without recourse to detailed fits.

Specific models suggest further guides to choice of experiments. From the standpoint of Kegge exchange models, forward $\pi^{-} p \rightarrow \pi^{\circ} n$ and backward $\pi^{-} p$ elastic are the most valuable experiments because they provide the most directly interpretable results. Lach process has only one exchanged Regge pole. Forward elastic scattering is generally complicated by the presence of many exchanges. However, polarization data on forward elastic provides crucial information concerning the Pomeron and its coupling properties. In the following pages, we discuss forward elastic reactions and then turn to exchange processes.

> Forward-Angle Elastic Scattering

Available data are reasonably well understood to values of $t \simeq-0.6(\mathrm{GeV} / \mathrm{c})^{2}$. In Regge-pole language, the interference of Pomeranchuk amplitudes with $\rho$ and $A_{2}$ exchange terms gives rise to polarization which falls with $s$ approximately as $s^{-\frac{1}{2}}$. This is also true in models with sizeable absorptive-cut corrections. Large $K^{+} p$ polarization is expected because the secondary poles $\left(\rho, A_{2}, P^{\prime}, \omega\right)$ add to give a real amplitude ; this interferes with a predominantly imaginary Pomeranchuk term. How ever, $\mathrm{K}^{-}$p polarization is smaller because, in this case, secondary poles add to an imaginary result.

In Regge-pole theory, simple assumptions also give a correct description of the t dependence of polarization for all elastic-scattering data. As an example, we discuss $\pi^{ \pm} p$ scattering briefly. First, it is well documented that the s-channel spin-flip amplitude, $\mathrm{H}_{+-}$, is dominated by $\rho$ exchange. Then, if we assume that the s-channel non-flip amplitude, $H_{++}$, is purely imaginary (as would be the case for a diffractive Pomeron with zero slope), we derive ${ }^{1}$

$$
\begin{equation*}
\operatorname{Pol}\left(\pi^{ \pm} p\right) \propto \pm\left(1-\cos \pi \alpha_{\rho}\right) \tag{2}
\end{equation*}
$$

Notice that this equation gives both the famous mirror symmetry of $\pi^{ \pm} p$ polarization, as well as the correct behavior $\left[\infty(t+0.6)^{2}\right]$ near $-t=0.6(\mathrm{GeV} / \mathrm{c})^{2}$. The above argu ment is extended trivially to other elastic processes, giving Pol $\propto \cos \pi \alpha_{\rho}$ in $K^{-} p$ and $\bar{p} p$; no structure (as a function of $t$ ) is predicted in $K^{+} p$ and pp scattering. These results are all in good agreement with accurate data near $5 \mathrm{GeV} / \mathrm{c}$.

Actually, everything is far from being as consistent as the above paragraph would suggest. There are both experimental and theoretical loopholes. On the theoretical side, we have treated the spin-non-flip amplitude naively and have totally ignored (absorption) cuts. Indeed, if the Pomeron has canonical slope, $\alpha^{\prime}{ }_{P} \approx 1$, as is suggested by shrinkage observed in Serpukhov pp elastic-scattering data, then (a) there should be a sizeable real part in $\mathrm{H}_{++}$; and ( $b$ ) in all reactions, one would expect structure in Pol near $-t \simeq 1.0(\mathrm{GeV} / \mathrm{c})^{2}$, where $\alpha_{\mathrm{P}}(\mathrm{t})=0$. Moreover, the $\mathrm{P}^{\prime}$ is known to contribute significantly to $\mathrm{H}_{++}$, and its contribution is not purely imaginary [its phase is given by the signature factor $\left.1+\mathrm{e}^{-i \pi \alpha^{\prime}} ; \alpha_{P^{\prime}}(t) \simeq 0.5+t\right]$. Finally, in a model with strong cut contributions, the entire discussion is modified. In particular, the SCRAM (Michigan) model ${ }^{2}$ prefers a single zero near $t=-0.6(\mathrm{GeV} / \mathrm{c})^{2}$ in the effective $\rho$ residue rather than the quadratic zero natural in Regge-pole theory. Therefore, in SCRAM $\pi^{ \pm} p$ polarization would be expected to change sign at $t=-0.6$, rather than obey the mirror symmetry observed at low energy. At present, it is not fully understood whether the mirror-symmetry characteristic is really an asymptotic property (such as suggested by Regge-pole theory) or merely a nonasymptotic feature, as indicated by SCRAM.

In addition to these theoretical problems, there are also uncertainties of an experimental nature. For example, very low energy $\pi^{ \pm} p$ data indicate that polarization changes sign at $|t| \approx 2.0$ to $3.0(\mathrm{GeV} / \mathrm{c})^{2}$. This result is definitely unexpected. The phenomenon is fitted by Barger and Phillips, ${ }^{3}$ who introduce unconventional cyclic residues to accomplish the task. It is essential to determine whether this sign change at large $|t|$ is merely a very low energy effect or whether it persists at high energy. Another indication of difficulty comes from polarization data near $10 \mathrm{GeV} / \mathrm{c}$ (see Figs. 1-3 in Appendix B). There is a hint of a zero in all reactions near $\mathrm{t}=-1.0(\mathrm{GeV} / \mathrm{c})^{2}$. Is this an indication of the Pomeron effect we mentioned above? Unfortunately, available high-energy data are as yet too poor for us to be sure that an effect is really present. Finally, let us return to the crucial matter of energy dependence. A brief glance at the data suffices to show that energy dependence $\propto s^{-\frac{1}{2}}$ is hardly established in the canonical "high-energy domain" ( $p_{l a b} \geq 5 \mathrm{GeV} / \mathrm{c}$ ). The range over which measurements have been made is too small and the statistical errors too large for anyone to be comfortable with results so far.

In summary, then, although simple assumptions do explain the qualitative behavior of low-energy elastic polarization, the picture is very much more complicated when one probes beneath the surface. It is obvious that there is very valuable information to be extracted from early measurements of elastic polarization at NAL. In addition to general suggestions listed earlier, we now cite some specific measurements and what will be learned.

1. Assuming elastic Pol of about $20 \%$ at $10 \mathrm{GeV} / \mathrm{c}$ and using the $\approx \mathrm{s}^{-\frac{1}{2}}$ prediction of current theories, we expect Pol at $100 \mathrm{GeV} / \mathrm{c}$ to be no bigger than $20 \times(10 / 100)^{\frac{1}{2}}$ $\approx 7 \%$. It is worth emphasizing strongly here that if one measures such small Pol in pp or $\pi^{ \pm} p$ at $100 \mathrm{GeV} / \mathrm{c}$, this will be strong evidence that the Pomeron is an ordinary Regge pole and that existing theories are fairly adequate. This will certainly be a nontrivial result because low -energy data really cannot determine precise properties of the Pomeron: there are too many other nonasymptotic effects also present. On the other hand, if one measures sizeable nonzero polarization at $100 \mathrm{GeV} / \mathrm{c}$, it will be a good indication that exciting, significantly new physical effects are being observed.
2. In essentially all theories, the Pomeron is an isospin object in the exchange channel. We can isolate polarization associated with $I_{t}=0$ by adding the $\pi^{+} p$ and $\pi^{-} p$ polarizations. Available data suggest that this sum goes to zero faster than $s^{-1}$. On the other hand, it is not inconsistent with present data to have small $\mathrm{P}^{\prime} \times$ Pomeron or Pomeron $\times$ Pomeron contributions to polarization at the $5 \%$ level. The $P^{\prime} \times P$ and $P \times P$ contributions to polarization have energy dependences of $s^{-\frac{1}{2}}$ and constant, respectively. Precise measurements of energy dependence of (the sum of) $\pi^{ \pm} p$ polarizations in the energy range 20 to $150 \mathrm{GeV} / \mathrm{c}$ would allow us to identify and separate possible $P^{\prime} \times P$ and $P \times P$ contributions. A nonzero $P \times P$ contribution would, of course, require a new view of the Pomeron. At high energy, the most striking aspect of $P \times P$ dominance of polarization is the prediction of equal magnitude and sign of polarization for $\pi^{+} p$ and $\pi^{-} p$. This is in sharp contrast to present low energy results. Experimentally, therefore, it is important to look for systematic changes in $\pi^{ \pm} p$ polarization as a function of energy, at fixed $t$.
3. We have discussed the relevance of structure in polarization distributions near $t=-0.6$ and $-1.0(\mathrm{GeV} / \mathrm{c})^{2}$, as well as at larger $t$ values. From such measure ments we stand to learn a great deal about the properties of amplitudes which cannot be obtained simply from $d \sigma / d t$ results. In Regge-pole theory, these observations become statements about the structure of residue functions, presence or absence of wrong-signature -nonsense-zeroes, and the like; in absorptive cut models, they provide information about pole-cut interference. Experiments are necessary with all particle types: $\pi^{ \pm} p, K^{ \pm} p, p p, n p, \bar{p} p$.

Consequently, two groups of elastic-scattering experiments may be identified. One set of measurements on pp and $\pi^{ \pm} p$ at lab momenta $\geq 50 \mathrm{GeV} / \mathrm{c}$, for $|\mathrm{t}|<1.0$ ( $\mathrm{GeV} / \mathrm{c})^{2}$, should provide essential insight into the nature of diffractive (Pomeron) amplitudes. A second set of experiments, at energies of 20 to $60 \mathrm{GeV} / \mathrm{c}$, could be directed primarily at verifying features of secondary trajectories and of absorptivecut models. These measurements should cover a wide range of $t$ values and include measurements with all particle types. (See Appendix B.)

## Exchange Processes

The present status of polarization in exchange (i.e., non-Pomeron) processes may be easily summarized. Simply, there is very little high-energy data above $5 \mathrm{GeV} / \mathrm{c}$, and although ruling out very simple models, the present results are consis tent with a veritable horde of more complicated theoretical speculations.

The important data on $\pi N$ CEX rule out the simple one- $\rho$ Regge -pole exchange model which predicts zero polarization. Unfortunately, various other models (e.g., absorption, subsidiary poles, p-dipole or colliding poles) fit the present data. However, they predict polarization whose energy dependence varies from model to model between $s^{-\frac{1}{2}}$ and essentially constant. Given that the polarization is some $10 \%$ at present energies, it is clearly important to design higher-energy experiments to dis tinguish these predicted s dependences. Further, for $\pi N C E X$, these models obtain the dip in $\mathrm{d} \sigma / \mathrm{dt}$ at $\mathrm{t} \approx-0.6(\mathrm{GeV} / \mathrm{c})^{2}$ by quite different mechanisms; correspondingly, the polarization at and above $|t|=0.4(\mathrm{GeV} / \mathrm{c})^{2}$ is usually predicted to be large and varies in sign from model to model. Future experiments should investigate this region. (See Appendix D.)

Data on the other one -pole exchange processes, backward $\pi^{-} p \rightarrow p \pi^{-}$and $K^{-} p \rightarrow \Sigma^{+} \pi^{-}$, are similarly important. Based on fits ${ }^{4}$ to the available d $\sigma / \mathrm{dt}$ data, we give the predictions of four distinct models for $\pi^{-} p \rightarrow p \pi^{-}$in Fig. 1. In this case, all the models give essentially energy-independent polarization. In fact, polarization in all exchange reactions is nearly always (predicted to be) both large and energy independent. A relevant example is shown in Fig. 2; our predictions for backward $\pi^{+} \mathrm{p} \rightarrow \mathrm{p} \mathrm{\pi}^{+}$show very significant differences between the available models. ${ }^{4}$ (See Appendix C.)

Good polarization data also exist for the hypercharge-exchange reactions, i.e., $\pi \mathrm{N} \rightarrow \mathrm{K}(\Lambda, \Sigma)$ and $\overline{\mathrm{K}} \mathrm{N} \rightarrow \pi(\Lambda, \Sigma)$--especially for $\pi^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \Sigma^{+}$. The duality diagram approach (perfect exchange degeneracy) predicts zero polarization, in disagreement with present data. Crucial tests of other models require data on (1) $\pi N \rightarrow N \Lambda$, which is predicted to have equal and opposite polarization to the $\Sigma$ data; and (2) the linereversed process $\overline{\mathrm{K}} N \rightarrow \pi(\Sigma, \Lambda)$. Here, a simple pole model fits current d $\sigma / \mathrm{dt}$ data, but its prediction of the opposite sign for polarization in line-reversed reactions remains untested. Again, all the standard models (poles with broken exchange degeneracy, absorption, etc.) predict little energy dependence ( $n \leq 0.25$ ) in the polarization of any hypercharge reaction [ $n$ is defined in Eq. (1)].

Although polarization data give stringent tests for models, $\mathrm{d} \sigma / \mathrm{dt}$ and polarization do not form a complete set of measurements. Both can be fitted by theories with quite different amplitudes. Experimentally, however, all amplitudes can be
determined, up to a common phase, by measurements of the spin-rotation parameters $R$ and $A$ (in addition to $d \sigma / d t$ and $P$ ). ${ }^{5}$ Determination of $R$ and $A$ is vital in both elastic and exchange reactions. In Appendix E, Pipkin describes R and A studies for elastic processes. Here, we concentrate on exchange reactions, of which the most accessible experimentally is $\pi^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \mathrm{\Sigma}^{+}$. If this reaction is done with polarized protons, one need only observe the weak decay of $\Sigma$ to determine $R$ and $A$. In contrast to elastic processes, difficult rescattering measurements are not required.

Typical of the predicted differences ${ }^{5}$ between models are the results shown in Fig. 3. We emphasize that the pronounced structure shown in the SCRAM curves for $R$ and $A$ at $t$ or $u$ of -0.2 and $-0.6(\mathrm{GeV} / \mathrm{c})^{2}$ should occur in all reactions and is the most distinctive prediction of this model. Absence of such structure in nature would rule out strong-cut models. ${ }^{5}$

In elastic reactions, we are searching for small effects in relatively wellunderstood phenomena. Thus, it is desirable to measure over as wide an energy range as possible. The theoretical understanding of exchange processes is still poor, and we are looking for large effects. It appears quite sufficient (as is perhaps only technically feasible) to measure up to $60 \mathrm{GeV} / \mathrm{c}$. The first experiments should cover the $t$ or $u$ range of 0 to $1.0(\mathrm{GeV} / \mathrm{c})^{2}$ and aim for an accuracy sufficient to determine n in Eq. (1) to $\pm 0.1$.

## REFERENCES

${ }^{1}$ To obtain this result, note that $\mathrm{Pol} \propto \operatorname{Im}\left(\mathrm{H}_{++}{ }^{*} \mathrm{H}_{+-}\right) /(\mathrm{d} \sigma / \mathrm{dt})$. Our assumptions are that $H_{++} \propto i$ and $H_{+-} \propto 1-\mathrm{e}^{-\mathrm{i} \pi \alpha} \rho$. Thus, Eq. (2) follows. See also the review by G. Bellettini, Vth Rencontre de Moriond, March, 1970. (In our expressions for Pol, we express only the dynamical t dependence; the kinematic factor $\sin \theta$ is, of course, also present always.)
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Fig. 1. Polarization in $\pi^{-} p \rightarrow p \pi^{-}$from four different models.


Fig. 2. Polarization in backward $\pi^{+} p \rightarrow p \pi^{+}$from four different models


Fig. 3. Polarization parameters as functions of $t$ and $u$.

# APPENDIX B. FORWARD POLARIZATION MEASUREMENTS AT $20 \mathrm{TO} 60 \mathrm{GeV} / \mathrm{c}$ IN ALL THE ELASTIC PROCESSES 

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The $P$ parameter in elastic scattering has been measured up to $14 \mathrm{GeV} / \mathrm{c}$ for $\pi p$ and Kp and $17 \mathrm{GeV} / \mathrm{c}$ for pp in the range $0.1 \leq|\mathrm{t}| \leq 1.00(\mathrm{GeV} / \mathrm{c})^{2}$. The results have revealed very interesting phenomena such as a mirror symmetry with respect to $|t|$ in $\pi^{ \pm} p$ polarization, as shown in Fig. 1. Both the $K^{+} p$ and $K^{-} p$ polarization are positive dominant, as shown in Fig. 2, and the pp polarization has a fairly constant t dependence, as shown in Fig. 3. In general, the energy dependences of all the processes are not clear because of the available data lying in a small energy range. In contrast to interesting variations in P parameter, the differential cross sections show a similar $t$ dependence in the diffractive region from one process to another. It is extremely interesting to compare polarization phenomena of all the processes at higher energies. The medium-energy beam ( 15 mrad ) will provide excellent intensities of $\pi^{ \pm}, K^{ \pm}, p$ and $\bar{p}$ particles in the energy region of 20 to $60 \mathrm{GeV} / \mathrm{c}$. In this energy region, the cross sections are high enough that we should be able to cover the nondiffractive region, $0.6<|t|<2.0$, where currently available data indicate relatively high polarization. The medium-energy beam offers a reasonable ratio of $\pi$ to $p$ as well as $K$, as shown in Table I, while the $p$ to $\pi$ ratio goes as high as 100 at

Table I. Intensities in the Medium-Energy Beam $/ 10^{13}$ at $200 \mathrm{GeV} / \mathrm{c}$.

| $\begin{gathered} \mathrm{p} \\ (\mathrm{GeV} / \mathrm{c}) \end{gathered}$ | $\pi^{+}$ | $\mathrm{K}^{+}$ | p | $\pi^{-}$ | $\mathrm{K}^{-}$ | $\bar{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | $\overline{2 \times 10^{8}}$ | $\overline{3 \times 10^{7}}$ | $2 \times 10^{7}$ | $2 \times 10^{8}$ | $4 \times 10^{7}$ | $5 \times 10^{6}$ |
| 40 | $10^{8}$ | $2 \times 10^{7}$ | $3 \times 10^{7}$ | $8 \times 10^{7}$ | $10^{7}$ | $2 \times 10^{6}$ |
| 60 | $2 \times 10^{7}$ | $2 \times 10^{7}$ | $8 \times 10^{6}$ | $10^{7}$ | $10^{6}$ | $10^{5}$ |

higher energies. We should be able to obtain a polarization accuracy of $\Delta P=0.01$ to 0.10 in the $|t| r e g i o n ~ o f ~ 0.15$ to 2.00 at 20,40 , and $60 \mathrm{GeV} / \mathrm{c}$ in all the six processes within one month of machine time. The rates for such an experiment are shown in Table II. An experimental arrangement suitable for this experiment is similar to that described in Proposal 61 (ANL, Harvard, LRL, NU, Wyoming, Yale, and NAL) in which angles and momenta of scattered recoil particles are measured. The forward arm requires one BM-109 (ANL) magnet or equivalent and the recoil arm needs one SCM-105 (ANL) magnet or equivalent for momentum determination. A sketch of setup is shown in Fig. 4. However, a much simpler setup has been used at CERN in meas urements up to $17 \mathrm{GeV} / \mathrm{c}$, in which both the inelastic and quasi-elastic background

Table II. The Rates for P Measurements from 20 to $60 \mathrm{GeV} / \mathrm{c}$.
Accuracy: $\quad \Delta P=0.01$ to 0.10 depending upon the $t$ region
|t| range: Forward (F), 0.15-0.60
Larger momenta transfer (L), 0.60-2.00
I. Positive Run

| Reaction | $(\mathrm{GeV} / \mathrm{c})$ | Incident particle/pulse | \|t|range | Shifts |
| :---: | :---: | :---: | :---: | :---: |
| $\pi^{+} p$ | 20 | $2 \times 10^{6}$ pions | $F$ | 2 |
|  |  | $10^{8}$ | L | 2 |
|  | 40 | $3 \times 10^{6}$ | F | 2 |
|  |  | $10^{8}$ | L | 3 |
|  | 60 | $3 \times 10^{6}$ | F | 2 |
|  |  | $2 \times 10^{7}$ | 0.60-1.50 | 4 |
| $\mathrm{K}^{+} \mathrm{p}$ | 20 | $2 \times 10^{6}$ kaons | F | 2 |
|  |  | $5 \times 10^{7}$ | L | 4 |
|  | 40 | $3 \times 10^{6}$ | F | 2 |
|  |  | $10^{7}$ | 0.60-1.50 | 6 |
|  | 60 | $3 \times 10^{6}$ | F | 2 |
|  |  | $3 \times 10^{6}$ | 0.60-1.00 | 6 |
| pp | 20 | Simultaneously run |  |  |
|  | 40 | Simultaneously run | $\mathrm{K}^{+} \mathrm{p}$ |  |
|  | 60 | Simultaneously run | ${ }^{+} \mathrm{p}$ |  |

Total 37
II. Negative Run

| $2 \times 10^{6}$ pions | F | 2 |
| :---: | :---: | :---: |
| $10^{8}$ | L | 2 |
| $3 \times 10^{6}$ | F | 2 |
| $10^{8}$ | L | 2 |
| $3 \times 10^{6}$ | F | 3 |
| $10^{7}$ | $0.60-1.00$ | 2 |
| $2 \times 10^{6}$ kaons | F | 4 |
| $5 \times 10^{7}$ | L | 2 |
| $3 \times 10^{6}$ | F | 4 |
| $10^{7}$ | $0.60-1.50$ | 2 |
| $10^{6}$ | F | 6 |
| $2 \times 10^{6}$ anti p | F | 6 |
| $5 \times 10^{6}$ | $0.60-1.50$ | 6 |
| $2 \times 10^{6}$ | F | 2 |
| $2 \times 10^{6}$ | $0.60-1.00$ | 8 |
|  |  | Total |
|  | Grand Total | 56 |

were suppressed by angular measurements in both arms, with a determination of the angle of the incoming particle. We may use this simpler system for $\pi^{ \pm} p$ and $p p$ measurements. For the rest of the process in which we require maximum beam intensity, we cannot put counters in the beam so that momentum measurements in both arms will compensate for this.

Finally, we compare this experiment at 20 to $60 \mathrm{GeV}^{-} / \mathrm{c}$ with competing or planned experiments. A group at Serpukhov is preparing a polarization measurement in $\pi^{-} p$ forward elastic scattering at 40 and $60 \mathrm{GeV} / \mathrm{c}$ and will be ready by the end of 1972. However, their beam intensity will be $10^{5} \pi^{-} /$pulse at $40 \mathrm{GeV} / \mathrm{c}$ and $10^{4}$ $\pi^{-} / p u l s e$ at $60 \mathrm{GeV} / \mathrm{c}$, and these numbers are considerably lower than expected NAL intensity as shown in Table I. There is no planned experiment on $\pi^{+} p$, Kp, and $\bar{p} p$ in this energy region.


Fig．1．$\pi^{ \pm} p$ polarization data



Fig. 3. pp polarization data.


# APPENDIX C. POLARIZATION MEASUREMENTS IN BACKWARD ELASTIC MESON-NUCLEON SCATTERING 

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Although there exist a number of good measurements on meson nucleon elastic backward scattering up to $16 \mathrm{GeV} / \mathrm{c}$ in the case of $\pi^{ \pm} \mathrm{p}$ scattering and $7 \mathrm{GeV} / \mathrm{c}$ in the case of $\mathrm{K}^{+} \mathrm{p}$ scattering, very little is known about their polarization parameters at momenta above $3 \mathrm{GeV} / \mathrm{c}$. The interpretation of meson-nucleon backward scattering at high energies in terms of baryon-exchange models has been moderately successful up to $16 \mathrm{GeV} / \mathrm{c}$. However, the predictions of the polarization parameter within the framework of those models seem to differ very violently from one to another. ${ }^{1}$ Extensive experimental study of the polarization up to NAL energies should be very valuable for the understanding of the exchange mechanism which takes place in this channel

## A. Rates

We know from existing data that the differential cross section for $\pi^{ \pm} p$ backward scattering at fixed $u=0$ varies as $d \sigma / d u \sim s^{-2}$ while the differential cross section for $\mathrm{K}^{+} \mathrm{p}$ and $\mathrm{K}^{-} \mathrm{p}$ vary as $\sim \mathrm{s}^{-4}$ and $-\mathrm{s}^{-10}$ respectively. The angular distributions show, in the case of $\pi^{-} p$ and $K^{+} p$, a rather smooth exponential behavior like $d \sigma / d u \sim e^{4 u}$, with very little shrinkage. In the case of $\pi^{+} p$ backward scattering, considerable structure has been observed which persists up to $16 \mathrm{GeV} / \mathrm{c}$. In $\mathrm{K}^{-} \mathrm{p}$ backward scattering no backward peaks have been observed so far.

We extrapolate to higher energies the differential cross sections from the results of existing $\pi^{-} p$ backward-scattering data as well as $K^{+} p$ backward-scattering data with the formula

$$
\begin{aligned}
& \left.\frac{d \sigma}{d u}\right|^{\pi-p}=2 \times 10^{5} \frac{1}{p^{2}} \exp (4 u) \frac{n b a r n}{(\mathrm{GeV} / \mathrm{c})^{2}} \\
& \left.\frac{d \sigma}{d u}\right|^{K^{+} p}=7.8 \times 10^{6} \frac{1}{p^{4}} \exp (4 u) \frac{n b a r n}{(\mathrm{GeV} / \mathrm{c})^{2}} .
\end{aligned}
$$

We take a $12.5-\mathrm{cm}$ effective liquid hydrogen target and an azimuthal angle $\Delta \phi=\left(25^{\circ}\right)$.

We then calculate the minimum number of incident pions and kaons per pulse to be

$$
\begin{aligned}
& n_{\pi}=1.82 \times 10^{4} \times p^{2} \\
& n_{K}=4.6 \times 10^{2} \times p^{4}
\end{aligned}
$$

which is required to give us in 100 hours machine time 3000 valid events in the range of $0>u>-1.0(\mathrm{GeV} / \mathrm{C})^{2}$. Figures 1 and 2 show typical particle yields which would be avail able in the 15 mrad beam with a maximum momentum of $80 \mathrm{GeV} / \mathrm{c}$. For a $\pi^{-} p$ measure ment at $50 \mathrm{GeV} / \mathrm{c}$ we would need then a $\pi^{-}$flux of $5 \times 10^{7}$ particles per pulse and a $\mathrm{K}^{+} \mathrm{p}$ measurement at $25 \mathrm{GeV} / \mathrm{ca}^{\top}$ flux of $5 \times 10^{7}$ particles per pulse in order to match the requirements above. This would imply having no detectors in the incident beam.

With 3000 valid events and a $1: 1$ signal-to-noise ratio, we expect to determine the polarization parameter with a precision of $\Delta \mathrm{P}-0.07$ at $u=-0.1 \pm 0.05(\mathrm{GeV} / \mathrm{c})^{2}$ and $\Delta P \sim 0.2$ at $u=-0.9 \pm 0.1(\mathrm{GeV} / \mathrm{c})^{2}$. At lower energies these numbers will improve considerably since the cross sections are higher and therefore the number of valid events per running period larger.

## Experimental Layout

## Polarized Target

We propose to use the $2.5 \times 2.5 \times 13-\mathrm{cm}$ target in a 50 kG magnetic field as described in Appendix F. The azimuthal opening available is $25^{\circ}$. We have considered two options for the measurement of the proton and the pion parameters.

Option A: (i) Forward Spectrometer: This is almost identical to the forward spectrometer proposed in NAL Proposal 61 (Novey et al.) and to many of the spectrometers proposed in many of the elastic-scattering experiments. ${ }^{2}$ It consists of proportional wire chambers before and after two bending magnets of the BM109 type (recommended by the Magnets Summer-Study Group as standard laboratory items) and a threshold Cerenkov anticoincidence counter to reject pions and kaons. The intrinsic angular resolution, $\Delta \theta$, of the spectrometer is $\pm 0.10 \mathrm{mrad}$. The corresponding intrinsic resolution for the coplanarity angle $(\delta \phi)$ is $\approx \pm 20 \mathrm{mrad}$ at $-\mathrm{u}=0.05$ and $\pm 5$ $\operatorname{mrad}$ at $-\mathrm{u}=1.0$. The momentum resolution $(\Delta \mathrm{p} / \mathrm{p})$ will be $\pm 0.2 \%$ at $50 \mathrm{GeV} / \mathrm{c}$. For more details about the spectrometer, we refer the interested reader to Proposal 61.
(ii) Backward Spectrometer: The momentum range of the pions is approximately $450 \mathrm{MeV} / \mathrm{c}$ to $1 \mathrm{GeV} / \mathrm{c}$, while they emerge in the laboratory in the angular range of $152^{\circ}$ to $85^{\circ}$. These pions undergo in the $50-\mathrm{kG}$ superconducting magnet a bending angle ranging from $25^{\circ}$ to $10^{\circ}$ so that we get an effective focusing effect. For a detector placed two meters away from the center of the target, we need a horizontal aperture of $\sim 200 \mathrm{~cm}$ instead of 400 cm that would have been necessary without the
$50-\mathrm{kG}$ magnet. Hence, one magnet of the type SCM 105 (ANL) would be adequate for measurement of the pion momenta. For a field integral of $10 \mathrm{kG}-\mathrm{m}$, we get approximate parallelism between the outgoing pion trajectories. By providing a scintillation hodoscope we can utilize this effect to reject, in the trigger logic, quasi-elastic events either involving protons in the carbon target having large Fermi momenta or involving interaction of the pions with the residual nucleus. In other respects, the spectrometer design is quite conventional and resembles that in Proposal 61.

The intrinsic angular resolution of the spectrometer is $\pm 3 \mathrm{mrad}$. The corres ponding intrinsic resolution for the coplanarity angle $(\delta \phi)$ is $\approx \pm 6 \mathrm{mrad}$. The momentum resolution $(\Delta p / p)$ is $\pm 1.2 \%$ at pion momentum of $1 \mathrm{GeV} / \mathrm{c}$.
(iii) Resolution: Correlation of recoil and scattering angles: The horizontal beam divergence of $\pm 0.3$ mrad dominates the uncertainty in determining the proton recoil angle. This is effectively $\pm 0.3 \mathrm{mrad}$. This gives the equivalent cut of $\pm 25 \mathrm{mrad}$ on the pion side--so that, in principle, for this constraint, the pion angle does not have to be measured to better than this value. We still have a useful cut of $\sim 0.025$ on background. Coplanarity

Because the proton recoils at very small angles with respect to the beam, this is not a very useful constraint. Beam divergence again dominates. The values we estimate are

$$
\begin{array}{lll}
\Delta \phi= \pm 4^{\circ} & \text { at } & -\mathrm{u}=0.05(\mathrm{GeV} / \mathrm{c})^{2} \\
\Delta \phi= \pm 1^{\circ} & \text { at } & -\mathrm{u}=1.0(\mathrm{GeV} / \mathrm{c})^{2},
\end{array}
$$

so that we get a cut on background ranging from $\sim 0.3$ to 0.08 .

## Longitudinal Momentum Balance

This is dominated by the uncertainty in the momentum measurement of the fast proton and the $0.1 \%$ momentum uncertainty of the beam. It is approximately $\pm 0.23 \%$ or $\pm 115(\mathrm{MeV} / \mathrm{c})$. This is insufficient by itself to reject the process $\pi^{-} p \rightarrow p^{-} \pi^{-} \pi^{0}$ where the $\pi^{\circ}$ is produced almost at rest. However, this is a continuum background process and it is severely limited by phase-space considerations. A calculation based on extrapolation from bubble-chamber data at lower energies is in progress. Transverse Momentum Balance

The beam contributes $\pm 15 \mathrm{MeV} / \mathrm{c}$. The numbers for the proton side and the pion side, not including the beam contribution, are

$$
\begin{aligned}
&-\mathrm{u}=0.05(\mathrm{GeV} / \mathrm{c})^{2}: \quad\left(\Delta \mathrm{p}_{\perp}\right)_{\text {proton }} \approx \pm 18 \mathrm{MeV} / \mathrm{c} \rightarrow \pm 8 \% \\
&\left(\Delta \mathrm{p}_{\perp}\right)_{\text {pion }} \approx \pm 10 \mathrm{MeV} / \mathrm{c} \rightarrow \pm 4.5 \% \\
&-\mathrm{u}=1(\mathrm{GeV} / \mathrm{c})^{2}: \quad\left(\Delta \mathrm{p}_{\perp}\right)_{\text {proton }} \approx \pm 20 \mathrm{MeV} / \mathrm{c} \rightarrow \pm 2 \% \\
&\left(\Delta \mathrm{p}_{\perp}\right)_{\text {pion }} \approx \pm 4 \mathrm{MeV} / \mathrm{c} \rightarrow \pm 0.4 \% .
\end{aligned}
$$

It will be noticed that transverse momentum balance is our strongest constraint and ultimate justification for the magnetic analysis of the proton and pion momenta. It will also be seen that we require a divergence of the beam not worse than $\pm 0.3 \mathrm{mrad}$ with a $2-\mathrm{cm}$ diameter spot at the polarized target. Future improvements in proportional wire-chamber design, so as to handle approximately $10^{6}$ particles per second per wire with a $1 / 2-\mathrm{mm}$ spacing, may allow us to measure the angle of the incoming negative pions. We are not, however, counting on that.

Option B: Focusing Spectrometer: One proposal ${ }^{3}$ and one letter of intent have been submitted to study backward elastic scattering up to possibly $75 \mathrm{GeV} / \mathrm{c}$ with a focusing spectrometer. We evaluated this technique in doing a polarization experiment at backward angles. The solid-angle acceptance determined by the spectrometer is typically $55 \mu \mathrm{sr}$. The u range, which is covered by one spectrometer setting, ranges from $0>\mathrm{u}>-0.7(\mathrm{GeV} / \mathrm{c})^{2}$ at $50 \mathrm{GeV} / \mathrm{c}$ and $0>\mathrm{u}>-0.2(\mathrm{GeV} / \mathrm{c})^{2}$ at $20 \mathrm{GeV} / \mathrm{c}$. In order to measure polarization at larger negative $u$ values, the spectrometer has to be turned about its pivot or the incident beam should be able to hit the polarized target at different angles.

The scattering angle of the recoil proton at $50 \mathrm{GeV} / \mathrm{c}$ incident momentum is 4.5 mrad for a corresponding $u=-0.05(\mathrm{GeV} / \mathrm{c})^{2}$ and increases with increasing negative $u$ values. It can be measured by a proportional wire-chamber telescope in front of the focusing spectrometer or in the downstream end of the spectrometer. By using an inverse matrix transformation it is easy to project the angles measured at the downstream end of the spectrometer back to the front end. For reasons of reduced background, it is desirable to use this latter technique to determine the scattering angle. The accuracy with which the forward scattering angle can be determined depends largely on the incident-beam divergence, which can be as small as $\pm 0.3 \mathrm{mrad}$, as previously assumed.

The scattering angle of backward scattered mesons can be measured in a propor tional wire-chamber telescope, which should be brought as close as possible to the target in order to reduce decay losses from low -energy ( $0.5 \mathrm{GeV} / \mathrm{c}$ ) backward scattered particles (like K's). How close the telescope can be brought to the target depends largely on the background level at the target. The required resolution of the recoil proton has to be

$$
\frac{\Delta p}{p} \leq \frac{m_{\pi}}{2 p_{0}}
$$

or $\Delta p / p \leq 0.14 \%$ at $50 \mathrm{GeV} / \mathrm{c}$ and $\leq 0.35 \%$ at $20 \mathrm{GeV} / \mathrm{c}$ in order to reject the background from $\pi^{\circ}$ 's produced at rest. Lower precision like $\Delta p / p=0.2 \%$ still would lead to a powerful rejection of the background. Momentum resolutions in the incident beam are expected to be of this order. The azimuthal angle $\Delta \phi$ of the recoil particle has to be determined better than $\Delta \phi \leq m_{\pi} / p_{p}=2 \mathrm{mrad}$ at $50 \mathrm{GeV} / \mathrm{c}$ and $\Delta \phi \leq 5 \mathrm{mrad}$ at $20 \mathrm{GeV} / \mathrm{c}$,
which is well within the angular accuracy stated above. The scattering angle $\theta_{\pi}$ of the slow backward-scattered mesons ( $p_{\pi} \approx 0.6 \mathrm{GeV} / \mathrm{c}$ ) should be determined at least as good as $\Delta \theta_{\pi} \leq m_{\pi} / p_{\pi}$. With the proposed proportional wire-chamber telescope, a precision of a few milliradians in backward angles can easily be obtained. The coplanarity with $\Delta \phi-4^{\circ}$ at $u=-0.05(\mathrm{GeV} / \mathrm{c})^{2}$ is again not a very efficient rejection for inelastic events. At larger negative $u$ values $\Delta \phi$ becomes smaller and more sensitive to background rejection.

## REFERENCES



See Appendix A by E. L. Berger and G. C Fox
${ }^{2}$ T. B. Novey et al., A Proposal to Measure Polarization in $p p, \pi^{-} p$, and $\pi^{+} p$ Elastic Scattering at 50, 100, and $150 \mathrm{GeV} / \mathrm{c}$ at the National Accelerator Laboratory, National Accelerator Laboratory Proposal 61, 1970.
${ }^{3}$ R. L. Anderson et al., Proposal to Measure Two Body Flastic and Quasi-Elastic Scattering at High Energies, National Accelerator Laboratory Proposal 73, 1970. ${ }^{4}$ W. F. Baker et al., Backward Pion-Proton Elastic Scattering, National Accelerator I aboratory Letter of Intent, 1970; K. P. Pretzl, Design Criteria for a High Energy, High Resolution Focusing Spectrometer, National Accelerator Laboratory Internal Report TM-233, April, 1970.


Fig. 1. Yield curves based on Hagedorn-Ranft model for negative particles in the 15mrad beam.


Fig. 2. Yield curves based on a Hagedorn-Ranft model for positive particles in the $15-\mathrm{mrad}$ beam.

# APPENDIX D. MEASUREMENT OF POLARIZATION <br> IN THE REACTIONS $\pi p \rightarrow \pi^{-} n$ AND $\pi^{-} p \rightarrow \eta^{\circ} n$ <br> G. Burleson <br> Northwestern University 

There appear to be several feasible methods of measuring polarization in the reactions $\pi^{-} p \rightarrow \pi^{\circ}{ }_{n}$ and $\pi^{-} p \rightarrow \eta^{\circ}{ }_{n}$ at high energy. In ascending order of the number of kinematic variables detected by the apparatus, they can be listed as

1. Measurement of the angles of the decay $\gamma$ rays, together with a determination of which has the greater energy.
2. Same as (1), plus a measurement of their total energy.
3. Same as (1), plus a measurement of the energy of each of the $\gamma$ rays.
4. Same as (2) or (3), plus a measurement of the angles of the recoil neutron and its time -of-flight.

Method (1) has been used in a recent polarization measurement at CERN (O. Guisan et al., private communication), but the results are not available yet. A fairly large array of optical shower spark chambers was used. The energies of the y rays were measured, to a precision which was adequate to distinguish the higher energy one, by counting sparks. Because of the fairly large solid angle of the detectors, background due to neutral inelastic events, which would produce more than two y rays, should be largely eliminated. Background due to quasi-elastic events was eliminated by making separate runs with a background target, of the same composition as the polarized target, but without free protons. Since the results of this experiment should be available shortly, we will not attempt to discuss in detail the advantages and disadvantages of this method, except to note that the use of optical chambers requires bubble-chamber techniques of scanning and measuring film, which can be timeconsuming.

Method (2) is discussed in NAL Proposal 55 (A. V. Tollestrup, correspondent) which describes a method of measuring the differential cross sections. The setup consists of a 2 -foot long liquid-hydrogen target surrounded by anticoincidence counters to veto charged particles and $\gamma$ rays. The detector is in the forward direction and consists of a large shower counter which detects both $\gamma$ rays and measures this total energy with a resolution of $\Delta E / E<5 \%$. Toward the front of the counter (the front of the shower) there is a wire plane which measures the position of the $\gamma$ rays, and toward the center (the peak density of the shower) there is a proportional wire chamber, operated in the proportional mode, which measures their relative energies. Wire planes in the beam determine the incident $\pi^{-}$direction. The setup is rather compact;
at $100 \mathrm{GeV} / \mathrm{c}$ the distance between the target and detector is 10 m , and the area of the detector is about $50 \mathrm{~cm} \times 50 \mathrm{~cm}$. The experiment proposes to run between 20 and 200 $\mathrm{GeV} / \mathrm{c}$.

This setup has the advantage that it can be used with an on-line computer so that the results are immediately available. It depends rather heavily on a very efficient veto of inelastic final states with charged particles or with more than two y rays, such as the large cross-section diffraction process

$$
\pi^{-} p \rightarrow \pi^{-} \pi^{o} \pi^{o} p
$$

and

$$
\begin{array}{r}
\pi^{-} p \rightarrow \pi^{o} \Delta^{o} \\
\rightarrow \pi^{o} n .
\end{array}
$$

This limits the maximum beam intensity to about $10^{6} \pi /$ puise. (The proposal quotes $10^{5} \pi / p u l s e$, but Tollestrup agrees this is conservative.)

If this setup were successful in measuring differential cross section, it could be used to measure polarization as well. The principal modification would be the array of veto counters, which would have to be made larger to accommodate the polarized target. (This is potentially a serious problem.) Inelastic reactions would be suppressed by the same means, and the quasi-elastic reactions would be corrected for by a background target run.

This background target would have to be made very carefully so as to match the dimensions, density, and composition of the polarized target. To reduce systematic effects due to beam variations, these background runs would have to be made fairly often. Such frequent target changes would also pose problems, but we believe they can be solved.

To estimate rates, we note that the total number of counts required to give a polarization error $\Delta \mathrm{P}$ is given by

$$
N=\frac{\left(1+\left(P_{T}\right)^{2}\right)^{2(1+2 \beta)}}{P_{T}^{2}(\Delta P)^{2}}
$$

where $P_{T}$ is the fractional polarization of the target, and $\beta$ is the background-tosignal ratio. If we assume $\mathrm{P}_{\mathrm{T}}=0.7, \beta=4$ (due to the quasi-elastic events), $\mathrm{p}=0$, and require $\Delta P=0.025$, this gives $N \simeq 30,000$ counts. We use the cross sections given in Proposal 55 and assume the following:

$$
\begin{aligned}
\text { Target: } & 12.5-\mathrm{cm} \text { long }\left(6.5 \times 10^{23} \mathrm{p} / \mathrm{cm}^{2}\right) \\
& \Delta \phi=60^{\circ} \text { (total) } \\
\text { Beam: } & 10^{6} \pi / \text { pulse, } 900 \text { pulses } / \text { hour }
\end{aligned}
$$

This gives the following numbers of shifts:


With an equal amount of time for the background target runs, the experiment would require about 150 shifts. (We should note that the error $\Delta p=0.025$ refers to the total t range. If this were broken up into, say, 10 bins, it would give an average error of 0.08 per bin.)

Method (3) differs from this by wirtue of the detector being able to measure the energy of each $\gamma$ ray, rather than the energy of the sum. This has the advantage of giving a more positive identification of the event, since it can take advantage of the kinematics of two-body production and decay. (This corresponds to two additional constraints, which can be taken to be the mass of the two y-ray system and the relative division of the energies of the two $\gamma$ rays, which is predicted from angular measurements alone.)

For this method, the veto requirement is less stringent, so that it should be possible to run such an experiment simultaneously with a measurement of polarization in $\pi^{-} p$ elastic scattering, if the setup could be made compatible. (Such a dual setup is currently being planned for two experiments, E-278 and E-272, at ANL. ) Some anticoincidence counters would still be necessary, however, so the beam intensity would probably have to remain at $\leq 10^{6} \pi^{-}$/pulse.

The detection device would be a hodoscope of lead-glass Cerenkov counters, such as the one currently under construction at ANL and NAL. It will consist of an array of counters, each 2-1/2 in. square and of sufficient length to contain $200 \mathrm{GeV} / \mathrm{c}$ showers (about 27 radiation lengths). To determine the setup for these counters at NAL, we assume that the two showers would have to be separated at the hodoscope by at least two counters. At $50 \mathrm{GeV} / \mathrm{c}$, the minimum opening angle of a $\pi^{\circ}$ is 5.4 mrad ; if the distance between shower centers is taken to be 6 in., this gives a required distance of 100 feet. (This would scale linearly with beam momentum.) The number of counters currently under construction is about 150 . They could be arranged so as to give a coverage of $-t<1.0$, for $\Delta \phi-30^{\circ}$ (about half the effective solid angle assumed
in the rate calculations above), for the $\pi^{-} p \rightarrow \pi^{\circ} n$ reaction. For the $\pi^{-} p \rightarrow \eta^{0} n$ reaction the opening angle of the $\eta^{\circ}$ is about four times that of the $\pi^{\circ}$. Because of the limited solid angle of the hodoscopes, it would not be able to detect both reactions at once, so for the $\eta^{\circ}$ it would have to be moved closer to the target. This setup would then correspond to a factor of about 4 in running time over the one above, to secure the same polarization error. As was mentioned above, it could possibly be run simultaneously with elastic scattering whereas the previous one could not, because of its stronger veto requirement, and it might be particularly suitable for a series of low energy ( $p_{\pi} \leq 80 \mathrm{GeV} / \mathrm{c}$ ) measurements. (A background run would again be necessary.)

Method (4), which involves detection of the recoil neutron, could possibly also be run simultaneously with elastic scattering. Either type of detector could be used; probably the one with the larger solid angle would be better. Some typical parameters of an array of neutron counters are as follows:

Type: long scintillators, photomultiplier at each end; relative arrival time of light at each end meas ured to give position and time -of-flight
Distance from target: 4.5 m
Azimuthal angle covered: $30^{\circ}$, on each side of beam
Length corresponding to this angle: 2.4 m
Position resolution: $\pm 2.5 \mathrm{~cm}$
Angular resolution: $\pm 10 \mathrm{mrad}$
Time-of-flight: difference between neutron and $\beta=1$ particle: 3 to 22 nsec
Detection efficiency: - $20 \%$
Number required ( $0 \leq-t \leq 1.0$ ): about 36
To compare rates with those given above, we note the following factors:

1. No background run necessary: $\times 2.0$ in rates,
2. Neutron detection efficiency: $\times 0.2$
3. Signal-to-noise ratio: This is difficult to estimate without a detailed calculation, but we note that in a previous polarization experiment at CERN [Sondregger et al., Phys. Letters 23, 501 (1966)] which used a similar set up, the signal-to-noise ratio was about 10 to 1 . If we assume a pessimistic value of 1 to 1 , this would give: $\times 3.0$.
The product of these is $\times 1.2$ so that we have about the same results as above. Because this method gives such a positive identification of the event, beam counters should not be necessary, and the high-rate veto counters could be eliminated The beam intensity could then be increased provided that the neutron counters could oper ate in the resulting background.

# APPENDIX E. AN EXPERIMENT TO STUDY HELICITY CONSERVATION IN DIFFRACTION SCATTERING 

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#### Abstract

An experiment to study helicity conservation in elastic scattering is described. The experiment consists of a measurement of the $R$ parameter in $\pi p$ scattering. The target is polarized so that the polarization vector lies in the scattering plane and is perpendicular to the momentum of the incident pion. A carbon-plate spark chamber is used to determine the polarization of the recoil proton.


Despite the great deal of work that has been done, there is no good theoretical model for diffraction scattering. It has recently been suggested that s-channel helicity is conserved in diffraction scattering. ${ }^{1}$ This implies a special relationship between the t -channel flip and nonflip amplitudes and suggests that an s -channel description of the scattering is simpler and more fundamental than a $t$-channel description.

There is only one system for which there is good experimental information concerning the conservation of helicity in the $s$ channel; that is rho photoproduction. ${ }^{2,3}$ There is very good evidence from several experiments that $s$-channel helicity is conserved in rho photoproduction. There is no model for this behavior.

The purpose of this note is to outline an experiment through which helicity conservation in elastic scattering can be studied. The experiment uses a polarized target to study the reactions

$$
\begin{aligned}
& \pi^{+}+p \rightarrow \pi^{+}+p \\
& \pi^{-}+p \rightarrow \pi^{-}+p .
\end{aligned}
$$

The center-of-mass amplitude for pion-nucleon scattering can be written in the general form ${ }^{4}$

$$
T=x_{2}(f+i g \vec{\sigma} \cdot \vec{n}) x_{1}
$$

Here $x_{1}$ is the spinor for the incident nucleon, $x_{2}$ is the spinor for the scattered nucleon, $f$ is the non spin-flip amplitude, $g$ is the spin-flip amplitude, and $\vec{n}$ is a unit vector normal to the scattering plane. The vector $\vec{n}$ has the direction $\vec{q}_{1} \times \vec{q}_{2}$ where $\vec{q}_{1}$ is the momentum of the incident pion and $\vec{q}_{2}$ is the momentum of the scattered pion.

In terms of this amplitude the differential cross section for an unpolarized target is

$$
\frac{d \sigma}{d \Omega}=|f|^{2}+|g|^{2}
$$

If $s$-channel helicity is conserved, the amplitude can be written in the form

$$
T=A(s, t) x_{2}\left[\cos \left(\theta^{*} / 2\right)-i(\vec{\sigma} \cdot \vec{n}) \sin \left(\theta^{*} / 2\right)\right] x_{1}
$$

Here $A(s, t)$ is a general function of the energy and momentum transfer and $\theta^{*}$ is the scattering angle in the center-of-mass. Figure 1 shows diagramatically the notation used in this paper.

If the target polarization is $\mathrm{P}_{\mathrm{T}}$, then the scattering cross section in the center -of-mass is

$$
\frac{d \sigma}{d \Omega}='^{\prime} \mathbf{f}^{2}+' g!^{2}+2\left[\operatorname{Im}\left(f g^{*}\right)\right] \overrightarrow{\mathrm{n}} \cdot \overrightarrow{\mathrm{P}}_{\mathrm{T}}
$$

and the polarization of the recoil proton is

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega} \overrightarrow{\mathrm{P}}_{2}=2\left[\operatorname{Im}\left(\mathrm{fg}^{*}\right)\right] \overrightarrow{\mathrm{n}}-2\left[\operatorname{Re}\left(\mathrm{fg}^{*}\right)\right] & \overrightarrow{\mathrm{n}} \times \overrightarrow{\mathrm{P}}_{\mathrm{T}} \\
& +\left(|f|^{2}+|g|^{2}\right)\left(\vec{n} \cdot P_{1}\right) \overrightarrow{\mathrm{n}}-\left(|\hat{f}|^{2}-|\mathrm{g}|^{2}\left[\overrightarrow{\mathrm{n}} \times\left(\overrightarrow{\mathrm{n}} \times \overrightarrow{\mathrm{P}}_{\mathrm{T}}\right)\right]\right.
\end{aligned}
$$

It is conventional to speak of three polarization experiments. For the first experiment the target polarization is perpendicular to the scattering plane and the assymmetry is determined by reversing the polarization of the target. This experiment determines

$$
2 \operatorname{Im}\left(f g^{*}\right)
$$

For the other two measurements the target polarization lies in the scattering plane. For the first experiment with this configuration the target polarization lies along the direction of the incident particle and the component of the polarization of the recoil particle in the scattering plane perpendicular to the momentum of the recoil particle is measured. The ratio of the measured polarization to the initial polarization is called the A parameter. In terms of the scattering amplitudes in the center-of-mass system

$$
A=\frac{|f|^{2}-|g|^{2}}{|f|^{2}+|g|^{2}}
$$

For the second experiment, the polarization of the target is perpendicular to the direction of the incident particle and the component of the polarization of the recoil particle perpendicular to the momentum of the recoil particle is measured. This is called the $R$ parameter. In the center-of-mass

$$
R=-\frac{2 \operatorname{Re}\left(\mathrm{fg}^{*}\right)}{|\mathrm{f}|^{2}+|g|^{2}}
$$

In the laboratory coordinate system

$$
\begin{aligned}
& A=\frac{\left|f^{2}-|g|^{2}\right.}{|f|^{2}+|g|^{2}} \sin \left(\theta^{*}+\theta_{\mathrm{P}}^{\mathrm{L}}\right)+\frac{2 \operatorname{Re}\left(\mathrm{fg}{ }^{*}\right)}{|\mathrm{f}|^{2}+|\mathrm{g}|^{2}} \cos \left(\theta^{*}+\theta_{\mathrm{P}}^{\mathrm{L}}\right) \\
& \mathrm{R}=-\frac{|\mathrm{f}|^{2}-|\mathrm{g}|^{2}}{|\mathrm{f}|^{2}+|\mathrm{g}|^{2}} \cos \left(\theta^{*}+\theta_{\mathrm{P}}^{\mathrm{L}}\right)-\frac{2 \operatorname{Re}\left(\mathrm{fg} \mathrm{~g}^{*}\right)}{|\mathrm{f}|^{2}+|\mathrm{g}|^{2}} \sin \left(\theta^{*}+\theta_{\mathrm{P}}^{\mathrm{L}}\right)
\end{aligned}
$$

Here $\theta^{*}$ is the scattering angle of the proton in the center-of-mass system and $\theta_{p}$ is the scattering angle of the proton in the laboratory system. Since

$$
\begin{aligned}
& \theta+\theta_{P}^{L} \approx \frac{\pi}{2} \\
& A=\frac{|f|^{2}-|g|^{2}}{|f|^{2}+|g|^{2}} \\
& R \simeq-\frac{2 \operatorname{Re}\left(f g^{*}\right)}{|f|^{2}+|g|^{2}}
\end{aligned}
$$

For the case in which s-channel helicity is conserved

$$
\begin{aligned}
& \mathrm{f}=\mathrm{A} \cos \left(\frac{\theta^{*}}{2}\right) \\
& \mathrm{g}=-\mathrm{A} \sin \left(\frac{\theta^{*}}{2}\right)
\end{aligned}
$$

As a result

$$
\begin{aligned}
& \mathrm{A}=\cos \theta^{*} \\
& \mathrm{R}=\sin \theta^{*}
\end{aligned}
$$

and there is no dependence of the scattering cross section on the polarization when the polarization is perpendicular to the scattering plane. Figure 2 shows a plot of the $R$ parameter as a function of momentum transfer for incident pion energies of 10 and 50 GeV . It is clear from this figure that a sensitive way to study helicity conservation is to measure the $R$ parameter as a function of momentum transfer.

Figure 3 shows a schematic view of the apparatus. The apparatus is similar to that used in conventional polarization experiments. The direction of polarization of the target lies in the scattering plane and is perpendicular to the incident pion beam. This arrangement can be best obtained by using a superconducting magnet and constructing the dewar so that the magnetic field is horizontal. The recoil particle would then come out through the center of the coils.

The apparatus differs from that used in a conventional experiment in the use of a carbon-plate spark chamber to measure the polarization of the recoil proton. The carbon-plate spark chamber would consist of wire planes plus carbon plates between the planes.

For the calculation of the rates we will assume the following parameters:
$\left.\begin{array}{rl}\text { Incident beam } & 5 \times 10^{6} \text { pions/pulse } \\ \text { Target length } & 12.5 \mathrm{~cm} \\ \text { Target polarization } & 70 \% \\ \text { Azimuthal acceptance } & \pm 10^{\circ} \\ \text { Beam repetition rate } & 10 \mathrm{pulse} / \mathrm{min} \\ \text { Quasi elastic triggering rate } & 4 \text { times elastic rate } \\ \text { Fraction of events analyzable } & 5 \% \\ \text { Analyzing efficiency of recoil chamber } \\ \text { Anamber (E) }\end{array}\right]$

These parameters are essentially the same as those in Proposal 61, and the reader is referred to that proposal for further details.

A simple calculation shows that the error in the R parameter is given by the expression

$$
\Delta R=\left(\frac{1}{N_{R}+\mathrm{N}_{L}}\right)^{\frac{1}{2}} \frac{1}{\mathrm{EP}_{0}}
$$

Here $E$ is the analyzing power of the carbon-plate spark chamber, $P_{0}$ is the initial polarization, $\mathrm{N}_{\mathrm{R}}$ is the number of useful scatters to the right in the carbon-plate spark chamber, and $N_{L}$ is the number of useful scatters to the left. With the assumed parameters it requires for a $\Delta R$ of 0.03

$$
\mathrm{N}_{\mathrm{R}}+\mathrm{N}_{\mathrm{L}}=1.2 \times 10^{4} \text { counts }
$$

Thus it would require a total of $2.5 \times 10^{5}$ events. With the assumed beam intensity and target it would require 50 shifts to obtain this many events in each of $0.1(\mathrm{GeV} / \mathrm{c})^{2}$ bins from 0.15 to $1.0(\mathrm{GeV} / \mathrm{c})^{2}$. This would give a very good test of helicity conser vation in diffraction scattering.

## REFERENCES

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${ }^{2}$ SLAC-Berkeley-Tufts Collaboration, Phys. Rev. Letters 24, 955 (1970).
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${ }^{4}$ L. Wolfenstein, Annual Reviews of Nuclear Science, E. Segre, Ed. (Annual Reviews Inc., 1956), Vol. 6.


0

d

Fig. 1. Notation used in Appendix E.


Fig．2．Polarization parameter $R$ as a function of momentum transfer


Fig. 3. Schematic view of apparatus.

## APPENDIX F. RECOMMENDED TARGET FOR NAL

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Most of the polarization measurements at NAI are involved with relatively small cross sections compared with experiments with current accelerators and some of the measurements will require good accuracy of $\Delta P \approx 0.01$. In order to minimize the systematic errors, the polarization of the target should be frequently reversed

Based upon the recent achievement at Saclay ${ }^{1}$ where they obtained relatively less build -up time (polarization time) with $50-\mathrm{kG}$ magnetic field, we looked into detailed design of a superconducting magnet suitable for the proposed target. We choose a length of 13 cm from both physics and technical point of view; attenuation of the beam ( $20 \%$ interaction length for 13 cm and $40 \%$ for 26 cm ) and background problems were considered. The cross-sectional area of the target is determined by the size of the beam and the allowed multiple scattering of the recoil particle, and a $2.5 \mathrm{~cm} \times 2.5 \mathrm{~cm}$ cross section appears to be appropriate.

The design specifications of the proposed target are shown in column 1 of Table I, together with a similar target with $25-\mathrm{kG}$ magnetic field and also a current target such as the one at ANL. Because of the target length proposed, the shape of $25-\mathrm{kG}$ and $50-\mathrm{kG}$ magnets will be elliptical. The access angle quoted in the table is the maximum value and, in general, we require much less access angle at the forward direction. The total cost shown in Table I includes magnet, cryostat, NMR system, microwave system, required control system, etc.

The $50-\mathrm{kG}$ target has several advantages over the $25-\mathrm{kG}$ target: (1) faster build -up time which minimizes the time for reversing polarization, (2) $50-\mathrm{kG}$ field gives better focusing for recoil particles and in return we minimize the size of the detectors that follow; otherwise, we may need a supplement bending magnet placed close to the target, (3) $\mathrm{He}^{4}$ cryostat offers a simpler operation than the $\mathrm{He}^{3}$ case.

On the other hand, the $25-\mathrm{kG}$ target requires less complicated microwave system and simpler magnet design and can be made longer. However, we conclude that the advantage of the $50-\mathrm{kG}$ magnet is more substantial. Figure 1 shows the sketch of superconducting magnets ${ }^{2}$ for $25-\mathrm{kG}$ (dotted line) and $50-\mathrm{kG}$ (solid lines) targets. Required correction coils are not shown in the figure. The field configurations are shown in Fig. 2 in which the solid curve is for $50-\mathrm{kG}$ and the dotted curve for $25-\mathrm{kG}$ magnet.

## REFERENCES

${ }^{1}$ A. Abragam (Saclay), personal communication, 1970.
${ }^{2}$ Provided by H. Desportes and C. Laverick, ANL.

Table 1. Design Specifications and Cost Estimates.

|  | 0-kG Target <br> $\mathrm{He}^{4}$ Temp. | 25-kG Target <br> $\mathrm{He}^{3}$ Temp. | Current Tar $\mathrm{He}^{4}$ Temp. | get (ANL) <br> $\mathrm{He}^{3}$ Temp |
| :---: | :---: | :---: | :---: | :---: |
| Length, cm | 13 | 13 to 26 | 5 | 5 |
| Gap, cm | 10 | 10 | 7.5 | 7.5 |
| Magnetic field, kG | 50 | 25 | 25 | 25 |
| Field uniformity, gauss | $\pm 10$ | $\pm 10$ | $\pm 2$ | $\pm 2$ |
| Temperature | $1.0^{\circ} \mathrm{K}$ | $0.5{ }^{\circ} \mathrm{K}$ | $1.0^{\circ} \mathrm{K}$ | $0.5{ }^{\circ} \mathrm{K}$ |
| Build-up time ( $70 \%$ ) minutes | ~ 3.5 | $\sim 10$ | 1 | -10 |
| Access angle | $\pm 12.5$ | $\pm 12.5$ | $\pm 25^{\circ}$ | $\pm 25^{\circ}$ |
| Target (hydrocarbon) polarization | 70\% | 70\% | 40-45\% | 70\% |
| Time required for completion | n 14 mo . | 12 mo . |  |  |
| Estimated cost of magnet | 100 K | $\begin{aligned} & 50 \mathrm{~K}(13 \mathrm{~cm}) \\ & 60 \mathrm{~K}(26 \mathrm{~cm}) \end{aligned}$ |  |  |
| Total cost of PP'T | 220 K | $\begin{aligned} & 170 \mathrm{~K}(13 \mathrm{~cm}) \\ & 180 \mathrm{~K}(26 \mathrm{~cm}) \end{aligned}$ |  |  |



Fig．1．Jayout of superconducting magnets．


Fig. 2. Field configurations.

