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DISC COUNTERS

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ABSTRACT

Features and design criteria of DISC counters are discussed. Examples of designs extrapolated for the new energy range are given.

Cerenkov counters of the DISC type are essentially differential counters designed to accept the widest spectral range of the Cerenkov light afforded by the spectral response of the photomultipliers and spectral transmission of the optical elements of the counter, such as mirror, window, etc.

The associated increase in the number of photoelectrons leads to a counter design of acceptable length--less than 10 meters--for the 100 to 500 GeV/c range, and to the possibility of a multiple-coincidence definition of the Cerenkov angle, and of the angle of the particle with the beam axis.

These two properties mean high rejection of unwanted particles even if they are very close in velocity, and self-collimation of the counter. If particles make an angle larger than $\delta\theta/2$ - ($\delta\theta$ is the angular width of the diaphragm), they are not counted in this arrangement irrespective of their velocity.

These features can be obtained together with a good velocity resolution only if the dispersion of the Cerenkov angle versus wavelength, caused by the variation of the refractive index n (λ), is corrected. The chromatic error is given by

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{\theta}{2\nu}\left(1 + \frac{1}{\gamma^2\theta^2}\right), \quad \nu = \frac{n_{\lambda 1} - n_{\lambda 3}}{n_{\lambda 2} - 1}, \tag{1}$$

where θ is the Çerenkov angle. ν has the same definition as the usual Abbe number which characterizes the dispersion of glasses, but for wavelengths appropriate to the counter spectral range. These wavelengths are

$$\lambda_1 = 2800\ \text{Å}$$
$$\lambda_2 = 3500\ \text{Å}$$
$$\lambda_3 = 4400\ \text{Å}.$$

The formula (1) shows that the chromatic error is a function of γ. The chromatic correction provided by the optics therefore has to be variable with β.

The definition of the Cerenkov angle of a given particle is made by the requirement that the focused circular line of discrete photons is entirely contained between the inner and outer diameter of an annular diaphragm of angular aperture $\Delta\theta$.

If we were to collect all the light crossing the annular diaphragm on a single phototube, a mismatch in velocity and angle of the particle with the counter axis would still be feasible. This can be corrected, but only to a limited extent, if an anticoincidence is made with the light exceeding the limits of the diaphragm. This method can lead to high rates in the anticoincidence channel and is of limited effectiveness due to the finite threshold of detection of the anticoincidence. A threshold requirement on the pulse can also be used for improving the selection of the counter.

In DISC-type counters the selection of the events is made by the requirement of simultaneous detection of light in q sectors of the annular diaphragm.

Simple geometrical arguments show that for $q = 2$ or 3, a circle of light exceeding the limits of the annular diaphragm can still cover all the sectors, and the mass resolution deteriorates.*

The velocity resolution of a counter has not much significance at half height as it is usually quoted. The right criterion is how large is the contamination of the tails of a prominent adjacent peak. Straight slopes are of much more importance in this respect for particle selection than the width of the peak.

For $q = 4$ (fourfold coincidence), the rejection is still marginal. Experiments with actual counters show that excellent rejection (peak-to-valley ratio greater than 10^5 with straight slope) can be obtained with $q = 6$ or more.[1-4] Explicitly, the width of the curve doubles at a level where the efficiency is reduced to 10^{-4} to 10^{-5} of the peak counting rate.

Above $q = 6$, combinations like $q = 8$ or 9 still afford greater rejection far from the peak, but this is mostly a reduction of accidental coincidence between the singles rate in each phototube. Aberrations or fuzziness of the light circle produce effects similar to a low order of coincidence, with a difference however. We shall call $\delta\theta$ the sum of the angular errors. A low order of coincidence increases the width of a peak at all heights. Lack of sharpness of the focused light leads to a slightly improved resolution near the peak of the curve and to a widening of its base.

The reduced width at the top of the curve is detrimental to the efficiency of counting particles with a given velocity range, i.e., over a given momentum band.

The optical correction of the focusing system of a DISC is therefore mostly needed to improve the acceptance of the counter and to bring its acceptance closer to a theoretical rectangular window in velocity.

*See, for example, Yu. P. Gorin et al., IHEP 70-48 - Note added in proofs.

These different considerations are the starting points in the design of DISC counters. They have been used in practice and will now be discussed in more detail.

Detection Efficiency of DISC Counters

This efficiency refers only to the electronic efficiency of detection of an event and not to the loss that might also be incurred from a mismatch between the acceptance of the DISC and the beam properties.

Let us suppose that the average number of photoelectrons detectable by all the phototubes is $\bar{N}$, and that the electronics can detect single photoelectrons.

The efficiency for the q-fold coincidence arrangement is

$$\epsilon_q = \left(1 - e^{-\frac{\bar{N}}{q}}\right)^q .$$

Table I. Efficiencies for Different Values of q and for $\bar{N}$ = 12, 24, and 48.

	$\bar{N}$ = 12	$\bar{N}$ = 24	$\bar{N}$ = 48
E_3	94.60	99.90	99.999
E_4	81.52	99.01	99.997
E_6	41.79	85.50	99.80
E_8	13.27	66.46	98.54

The table shows that $\bar{N}$ = 12 is definitely too small to give reasonable efficiencies in six-or eight-fold coincidences. On the other side $\bar{N}$ = 48 is not enough of an improvement over $\bar{N}$ = 24 that the associated increase in the cost of the counter could justify this choice. Moreover, the efficiencies quoted in the table are not necessarily the real ones.

A less stringent requirement is to ask, for example, for at least 7 pulses in coincidence out of 8 photomultipliers instead of an eight-fold coincidence.

The rejection is not sensibly affected by this, and the efficiency is sensibly increased.

$$E_8 + E_{7 \text{ out of } 8} = \left(1 - e^{-\frac{\bar{N}}{8}}\right)^8 + 8e^{-\frac{\bar{N}}{8}} \left(1 - e^{-\frac{\bar{N}}{8}}\right)^7 .$$

For $\bar{N}$ = 24 this efficiency is = 94.4%.

We shall assume that $\bar{N}$ = 24 detected photoelectrons is an acceptable design figure.

The number of photoelectrons detected is $\bar{N} = AL \sin^2\theta$; the value of A has been calculated and observed to be 65 for 56 UVP of 50 mA/W at 4400 Å, made before the manufacturer had improved the collection efficiency of the photoelectrons by the first dynode, which did show a drop of about 30% near 3000 Å.[5]

It can be assumed that for the new 56 DUVP with bialkali photocathode that the value of A is at least 80 to 100.*

The length of the counter follows:

$$L = \frac{\bar{N}}{A\theta^2},$$

$$L\ \mathrm{cm} \simeq \frac{0.24}{\theta^2}\ \text{to}\ \frac{0.3}{\theta^2}.$$

Velocity Resolution of the Counter

The velocity resolution is

$$\frac{\Delta\beta}{\beta} = \mathrm{tg}\theta\ \Delta\theta \simeq \frac{1}{2p^2}\left(M_i^2 - M_0^2\right),$$

where $\Delta\theta$ is the total range of Cerenkov angle which is accepted by the counter. In absence of optical aberrations and multiple scattering, $\Delta\theta$ is equal to the angular opening of the diaphragm,

$$\Delta\theta \simeq \frac{\Delta\rho}{F},$$

where ρ is the radius of the Cerenkov ring image and F is the focal length of the mirror. The velocity resolution of a DISC counter does not depend on the beam divergence. The lower limit of $\Delta\beta/\beta$ is set by the angular spread of the light on the diaphragm and is equal to

$$\frac{\Delta\beta}{\beta_{min}} = \mathrm{tg}\theta\ \delta\theta,$$

where $\delta\theta$ is the composite sum of the angular aberrations and multiple scattering.

A reduction of the width of the diaphragm below this limit does not improve the resolution but only reduces the electronic efficiency because only part of the light is collected on the phototubes. A diaphragm set exactly as wide as the light is spread will effectively provide $\Delta\beta/\beta_{min}$ in β or in angular acceptance.

Efficient working conditions are obtained when the angular aperture $\Delta\theta$ of the diaphragm is a few times larger than $\delta\theta$. Satisfactory conditions seem to be realized for

$$\Delta\theta = 2\ \text{to}\ 3\ \delta\theta,$$

and when this criterion has been met a k-peak to π-k valley ratio of the order of 10 in the separation of kaons from pions has been obtained, leading to a contamination of less than 10^{-3} of the kaons by pions. In practice, the maximum at which the DISC separates two particles with similar performances is

*Values of A equal to 82 and 84 at full aperture of the diaphragm can be deduced from the efficiency quoted in IHEP 70-48 preprint for XP1023 phototubes. (Note added in proofs.)

$$p^2 \max \approx \frac{M_i^2 - M_0^2}{4\ \mathrm{tg}\,\theta\ \delta\theta}.$$

Table II. Value of $M_i^2 - M_0^2$ for Successive Doublets in GeV^2.

	e – μ	μ – π	π – K	K – p	p – Σ	Σ – Ξ	Ξ – Ω	Ω – D
$M_{i+1}^2 - M_i^2$ =	0.01115	0.0083	0.224	0.636	0.553	0.312	1.050	1.109

The value of $\Delta\beta/\beta_{min} = \mathrm{tg}\,\theta\ \delta\theta$ which depends only on the counter design, and its state of correction, can be considered as the factor of merit of the DISC.

For the pion-kaon separation we found

p =	200	300	400	500 GeV/c.
$\frac{\Delta\beta}{\beta_{min}}$ =	1.4×10^{-6}	6.2×10^{-7}	3.5×10^{-7}	2.2×10^{-7}

Only the simultaneous reduction of the Cerenkov angle and the sum of angular error $\delta\theta$ can realize such resolution.

Evaluation of $\delta\theta$

There are at least six errors contributing to $\delta\theta$:

1. The chromatic aberration due to the dispersion of the gas

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{\theta}{2\nu}\left(1 + \frac{1}{\gamma^2\theta^2}\right).$$

2. The secondary spectrum: The optical corrector can only fold the primary spectrum, exactly like, in optics, an achromatic doublet does. The resulting folded spectrum has a $\delta\theta$ of the order of 1/15 of the primary spectrum depending slightly on the particular optic.

3. Longitudinal chromatic aberration: The chromatic corrector introduces in general a variation in focal length versus wavelength. This can be corrected for in the design of the optics, but residuals of the order of the secondary spectrum are not uncommon. In fact, there is a relation between the primary spectrum corrected by the chromatic corrector and the longitudinal chromatic aberration.

The ratio between the angular error produced by the longitudinal chromatic aberration AELC and the amount of angular chromatic correction brought by the corrector ACCC is

$$\frac{AELC}{ACCC} = \frac{xF\theta}{h}$$

for the thin-lens approximation and does not depend on the type of corrector; single lens, or triplet of null power. In this formula x is the distance of the corrector from the diaphragm in unit of F, and h is the height at which the main ray hits the corrector.

This formula emphasizes a chromatic corrector positioned close to the diaphragm and also of strong chromatic power. The ACCC for a corrector of total curvature

$$C_T = \frac{1}{R_1} - \frac{1}{R_2}$$

is given by

$$ACCC = x\,h\,C\,\Delta n^* = \frac{\Delta\theta}{\Delta\lambda} = \frac{\theta}{2\nu}\left(1 + \frac{1}{\gamma^2\theta^2}\right).$$

Δn^* is for a single lens equal to Δn of the substance for the two extreme wavelengths. If the corrector is a triplet of null deviation--the preferred design--made with a central lens of NaCl enclosed in two fused-silica lens, C represents the total curvature of the NaCl lens, the essential element of the corrector, and $\Delta n^* = 0.0284$. This value is nearly equal to that for a fused quartz lens = $\Delta n_{SiO_2} = 0.0280$.

The total curvature of the central lens of NaCl is nearly equal to the curvature of a single fused-silica lens providing the same chromatic correction with similar residuals. Such a single lens made as a corrector would have very strong total curvature, would be strongly converging, and would reduce considerably the equivalent focal length of the optical system, geometrical aberrations would also be introduced. The diaphragm would have to be reduced in proportion. This is clearly not a workable solution; moreover, the amount of chromatic correction would be fixed. The dispersion of the gas used depends on its nature and its purity, and the spectral response of the phototubes might not be known before operating the counter.

Further, the chromatic aberrations vary with γ. These facts make the feature of variable chromatic correction a necessity. A triplet of SiO_2, NaCl, SiO_2 of zero power for the main wavelength has been found to be free of the previous defects and leaves some possibilities for correcting the AELC and coma of the mirror.

This triplet is moved along the axis of the counter to adjust the amount of chromatic correction. It can be made such that its movement will not affect the value of the Cerenkov angle for light of wavelength $\lambda_2 = 3500$ Å.

In the new design all the surfaces are spherical, and no axiconic surfaces (which are difficult to produce) are needed anymore, as in previous designs.[6]

4. COMA: The primary mirror receives rays parallel to a direction at an angle $\pm\theta$ with the axis of the counter. It is not possible to find a meridian curve for this mirror which can be aberration-free. A solution for the angle $+\theta$ is a parabola which is not the same as the parabola needed for the angle $-\theta$. Some improvement might be obtained by figuring the mirror, but the added cost and complexity is not worth the result. The simplest mirror, and the most accurate, is the spherical mirror which is used in all the new designs. The geometrical error is coma and is

$$\delta\theta_{\text{coma}} \approx \theta^3 .$$

The formula is exact for a small beam size in comparison to the mirror diameter. The coma is strongly dependent on the Cerenkov angle.

For helium, a very low dispersion gas, the chromatic error for $\beta = 1$ and the coma error are equivalent for a Cerenkov angle of 100 mrad.

Coma affects the uniformity of the light collection of the counter, depending on the distance of the particle to the beam axis.

Correction of coma, at least for $\theta < 50$ mrad, can be accomplished with a fixed positive silica lens which serves the purpose at the same time of refracting the main ray parallel to the axis of the counter, so its intercept with the chromatic corrector is always at the same height from the axis regardless of the corrector's position.

A substantial reduction in the thickness and size of the corrector is provided in this way.

The total curvature of the lens needed is

$$C_T = C_1 - C_2 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2(n-1)\,L\left(1-\frac{x}{2}\right)} .$$

The correction of the coma depends on the bending parameter of the lens

$$\beta = \frac{C_1+C_2}{C_1-C_2},$$

and on its position x, and they have to be determined by optical-ray tracing. The chromatic correction of this lens is

$$\text{ACCC} = \frac{x\theta}{2\nu_{SiO_2}},$$

and is always less than the minimum needed to chromatically correct the counter. Nevertheless, it helps to reduce the chromatic power of the corrector.

For small Cerenkov angles $\theta \leq 20$ mrad this lens can be omitted.

For large Cerenkov angles $\theta \geq 100$ mrad, the amount of coma is such that correction is obtained only with a rear surface mirror of the MANGIN type. Such a design of only 30 cm in length is being built for charged hyperon separation at 20 GeV/c.

5. Light diffraction is $\delta\theta \approx \frac{0.4\lambda}{L\theta} = \frac{0.4\lambda A\theta}{\bar{N}}$.

This effect is in general negligible.

6. Multiple scattering: The multiple scattering angle is given by the formula

$$<\Delta\theta^2>^{1/2} = \frac{21\ 10^{-3}}{p\beta}\left[\frac{\bar{N}}{2A}\ \frac{\rho_0}{(N_0-1)x_0}\ \left(1+\frac{1}{\gamma^2\theta^2}\right)\right]^{1/2}.$$

For $\gamma \rightarrow \infty$

$$<\Delta\theta^2>^{1/2} = \frac{15\ 10^{-3}}{p\ \mathrm{GeV/c}}\left(\frac{\bar{N}}{A}\right)^{1/2}\left[\frac{\rho_0}{(N_0-1)x_0}\right]^{1/2}.$$

The nature of the gas enters only through the term

$$\left[\frac{\rho_0}{(N_0-1)x_0}\right]^{1/2}.$$

Table III gives the values of this coefficient.

Table III. The Multiple Scattering Coefficient for Various Gases.

Gas	$\left[\frac{\rho_0}{(N_0-1)x_0}\right]^{1/2}$
He	2.43×10^{-1}
H_2	1.03×10^{-1}
Air	$3.4\ \times10^{-1}$
CH_4	1.84×10^{-1}
CO_2	3.44×10^{-1}
SF_6	5.49×10^{-1}
Freon 13	4.57×10^{-1}

The multiple scattering is independent of the Cerenkov angle. The multiple scattering in a differential counter is not much greater than in a threshold counter, as it is proportional to $\bar{N}^{\frac{1}{2}}$.

The evaluation of these errors by computation and ray tracing gives a value of $\delta\theta$ by proper summation.

In order to check experimentally the procedure for deducing the $\Delta\beta/\beta_{\lim}$ the spread of light has been verified to be properly accounted for by the computation, by measurement of the light received by the phototube versus the opening of the diaphragm. From this experiment $\Delta\beta/\beta_{lim}$ is deduced and is also compared to the limiting value of $\Delta\beta/\beta$ as observed on a pressure curve.

This procedure gives consistent results to 10% accuracy and supports the view that performance can be safely predicted from a knowledge of $\delta\theta$.

Range of Momentum for a DISC

The range in γ is limited on the low-momentum side by the maximum amount of chromatic correction afforded by the chromatic corrector. If k is the ratio of the maximum chromatic correction to the correction needed for $\beta = 1$ particles,

$$k = \frac{1}{\gamma_{min}^2 \theta^2} \qquad \gamma^2 \geq \frac{1}{k\theta^2}.$$

The value of k is also the ratio of the position of the corrector $= x_{\beta\ min}/x_{\beta\ =\ 1}$.

On the high-momentum side the counter is limited by its usefulness to just separate two particles

$$\gamma^2 \leq \frac{2\Delta M}{M tg\theta\delta\theta}.$$

The practical range of the counter is

$$\frac{1}{k\theta^2} \leq \gamma^2 \leq \frac{2\Delta M}{M tg\theta\delta\theta}.$$

The value of k is from 1 to 4.

The interest afforded by an extended range at low values of γ comes from the usefulness of the detection of heavy particles like $\overline{p}$, d, for beam tuning, even if the experiment does not use them. A DISC counter might count these rare particles in a beam which would otherwise be too intense to be counted.

Momentum determination of the beam can be obtained with heavy particles. It is quite possible to measure the position of a peak in a mass spectrum to a precision of 1/20 of its width.

The momentum precision is of the order of

$$\frac{\Delta p}{p} = \gamma^2 \frac{\Delta\beta}{\beta} \times \frac{1}{20},$$

and can be as low as $\Delta p/p = 10^{-4}$ for 100 GeV $\overline{p}$ with a counter of $\Delta\beta/\beta_{lim} = 10^{-6}$. A corresponding precision has to be obtained for the measurement of the index of refraction

$$\Delta(n-1) \leq \frac{1}{20} \frac{\Delta\beta}{\beta_{lim}}.$$

The momentum bite of the beam can be experimentally observed for heavy particles which show widening of the peak. Comparison between mass spectra for pions and heavy particles might provide a way of measuring this momentum bite.

The flexibility of the DISC counter to count indifferently particles over the full spectrum of masses is an important feature which permits many checks to be made.

Matching of a DISC and a Beam

The DISC counter imposes conditions on the particles to be counted. Their divergence with the axis and their velocity have to be inside limits. Figure 1 gives the acceptance diagram of a DISC in the plane of coordinates x', $\Delta p/p$. It should be pointed out that beam divergence might reduce efficiency but does not affect velocity selection in a self-collimating counter.

The design of an efficient beam (or spectrometer) requires that the DISC be considered as a beam-transport element having properties of its own. The acceptance of a DISC has limits in angle x', and $\Delta p/p$ but practically not in x, the distance of the particle to the axis of the beam, because the useful diameter of such a counter can be made as large as the beam size without much constraint.

The beam divergence is $\delta\alpha = w/F_L$ where w is the target width and F_L the focal length of quadrupole system making the beam parallel. It can be shown that the length of quadrupole needed is independent of the momentum when the lens aperture is matched so as to have constant capture efficiency. The target width equal to the incident proton beam size decreases essentially like $1/p$. The beam divergence varies therefore like $1/p$ when secondary beams are produced with accelerators of increasing energy.

The velocity resolution needed, $\Delta\beta/\beta = tg\theta\,\delta\theta \approx tg\theta\,\delta\alpha$ for efficient match, decreases like $1/p^2$. The Cerenkov angle has to be scaled down like $1/p$ to achieve a match between DISC and beam equivalent to the one obtained at lower energies (CERN and Serpukhov). The length of the counter therefore scale up like p^2, but mass separation and beam matching stay the same. Differential counters can therefore be used over a large span of accelerator energies.

Measurement of the Index of Refraction

A precise measurement of the index of refraction is required for operating a DISC. The knowledge of the index allows one to know the momentum of particles in a way independent of the magnetic-rigidity determination. Practically an absolute interferometric measurement with a digital readout is very desirable. An interferometer one meter long has been built at CERN. The readout provides four pulses per fringe or a phase difference of 2π. The monochromatic light of a small-power plug-in laser is used.

The number of pulses counted in a forward-backward scaler is

$$N_{\frac{1}{4}F} = \frac{4\,L_{tube}}{\lambda\,laser}\left(\frac{n_{\lambda\,laser}-1}{n_{3500}-1}\right)\frac{1}{\cos\theta}\left(1-\cos\theta + (1-\beta) + (1-\beta)^2 + \ldots\right).$$

Interpolation is possible with an oscilloscope display of the phase difference. Temperature differences between DISC and refractometer have to be avoided. The refractometer should be integrated into the counter body.

Absolute temperature regulation of the counter is not required, as the index of refraction of gas does not depend to first order on the temperature at constant volume.

The number of fringes observed between two peaks of masses M_0 and M_1 is

$$N_{\frac{1}{4}} = \frac{2L}{\lambda}\left(\frac{n_\lambda - 1}{n_{3500} - 1}\right)\left(\frac{M_1^2 - M_0^2}{p^2}\right).$$

For a one-meter long refractometer $N_{\frac{1}{4}} = 20$ between π and k at 200 GeV/c.

DISC Examples

The parameters of the DISC counters of type A, B, and C of Ref. 7 have been worked out in more detail.

Type A is now a revised version of the 2-meter counter with θ - 45 milliradian used at CERN and Serpukhov. Parameters of type B have been chosen such that it can be considered a general-purpose counter for CERN II or NAL accelerators. The gas pressure in this counter is always below 1 atmosphere; since both SF_6 and Freon 13 are good insulators, the phototubes can be situated inside the vessel, alleviating the need for optical windows. Counter B is 5 meters long with a Cerenkov angle = 25 mrad.

The type C has the highest resolution but somewhat less flexibility; its range does not go below $\gamma = 50$. It uses helium under pressure.

This counter is 8 meters long, the Cerenkov angle 20 mrad, and the resolution should be enough to just separate pions from kaons at 500 GeV/c.

Table IV gives the parameters of these counters.

Optical Parameters

Figures 2, 3, and 4 indicate the position of the edges of the optical elements from an origin close to the diaphragm and their radius of curvature.

The tables give the value of the computed residual angular error $\delta\theta$ versus β and the value of Δ (mm) which fixes the position of the chromatic corrector for SF_6 or He.

Type A

β	$\delta\theta$	Δ
0.997	4.8×10^{-4}	207
0.9985	2.2×10^{-4}	124
1.000	0.8×10^{-4}	40

Type B

β	$\delta\theta$	Δ
0.9992	1.7×10^{-4}	310
0.9994	1.2×10^{-4}	250
0.9996	8.7×10^{-5}	190
0.9998	5.6×10^{-5}	130
1.0000	3.4×10^{-5}	70

Type C

β	$\delta\theta$	Δ
0.9998	4.4×10^{-5}	334
0.9999	3.1×10^{-5}	245
1.0000	1.9×10^{-5}	155

REFERENCES

[1] J. V. Allaby et al., The First Experiment Using the CERN Slow Extracted Proton Beam; Particle Production in 19.4 GeV/c Proton-Proton Interactions, CERN Report NP/Int. 66-2, June 22, 1966.

[2] Yu. B. Bushnin et al., Phys. Letters 29B, 48 (1969).

[3] F. Binon et al., Phys. Letters 30B, 506 (1969).

[4] F. Binon et al., Phys. Letters 30B, 510 (1969).

[5] P. Duteil et al., Efficacite de Detection de Divers Phototubes pour la Lumiere Cerenkov, CERN Report 68-14, April 24, 1968.

[6] P. Duteil et al., Rev. Sci. Instr. 35, 1523 (1964).

Table IV. Specifications of DISC A, B, and C.

Type	mrad	L_m	$\delta\theta$ radius	$\Delta\beta/\beta_{lim} = tg\theta\delta\theta$	γ_{min}	πk Separation p_{max}[a]	p_{lim}[b]	Gas	Pressure Range
A	45	2	8×10^{-5}	3.6×10^{-6}	11	130 GeV/c	176 GeV/c	SF_6 or FR13	1.6 to 6.4 atm
B	25	5	3.4×10^{-5}	8.5×10^{-7}	25	250 GeV/c	350 GeV/c	SF_6 or FR13	0.4 to 1.0
C	20	8	1.9×10^{-5}	3.8×10^{-7}	50	380 GeV/c	540 GeV/c	He	5.7 to 11.4

[a] p_{max} is the maximum momentum for separation of π from k with $\Delta\theta = 2\delta\theta$.

[b] p_{lim} is the maximum theoretical momentum for separation of π from k.

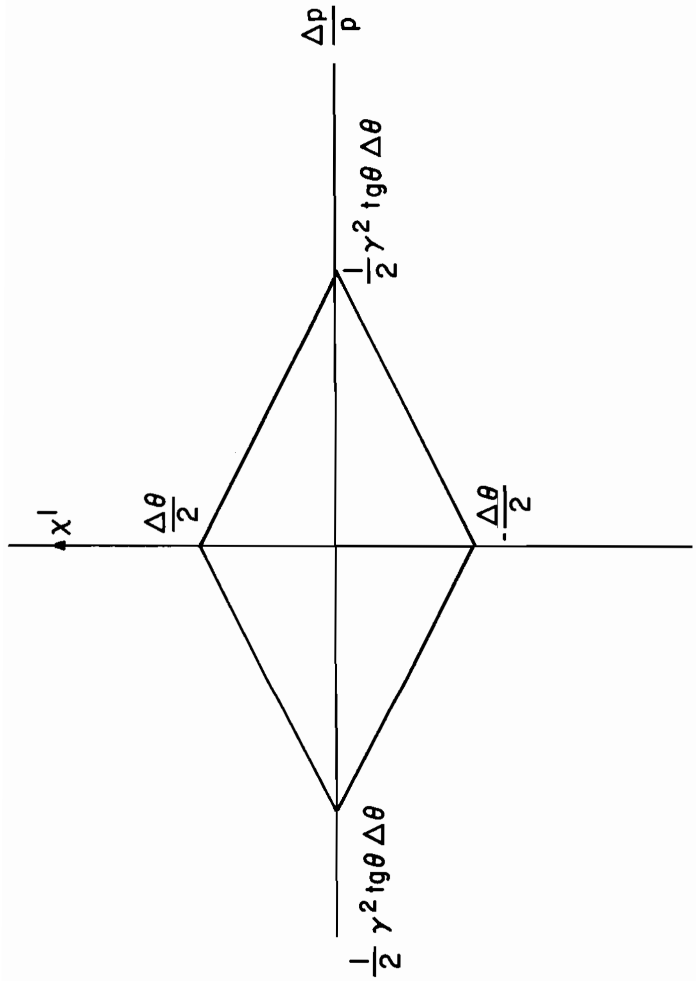

Fig. 1. Acceptance of a DISC in the plane, x', $\Delta p/p$.

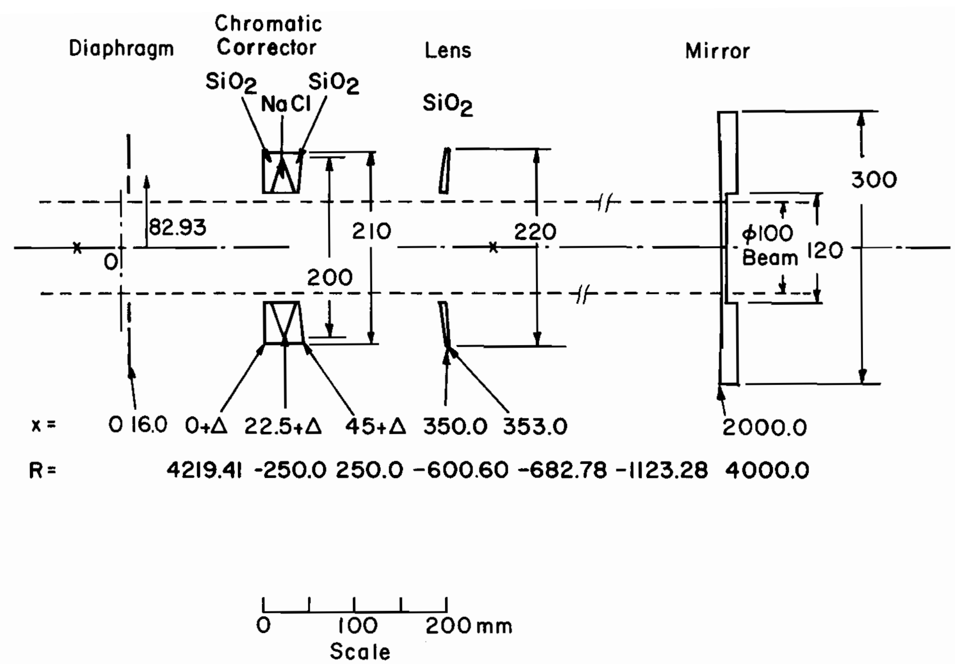

Fig. 2. DISC A. θ = 45 mrad.

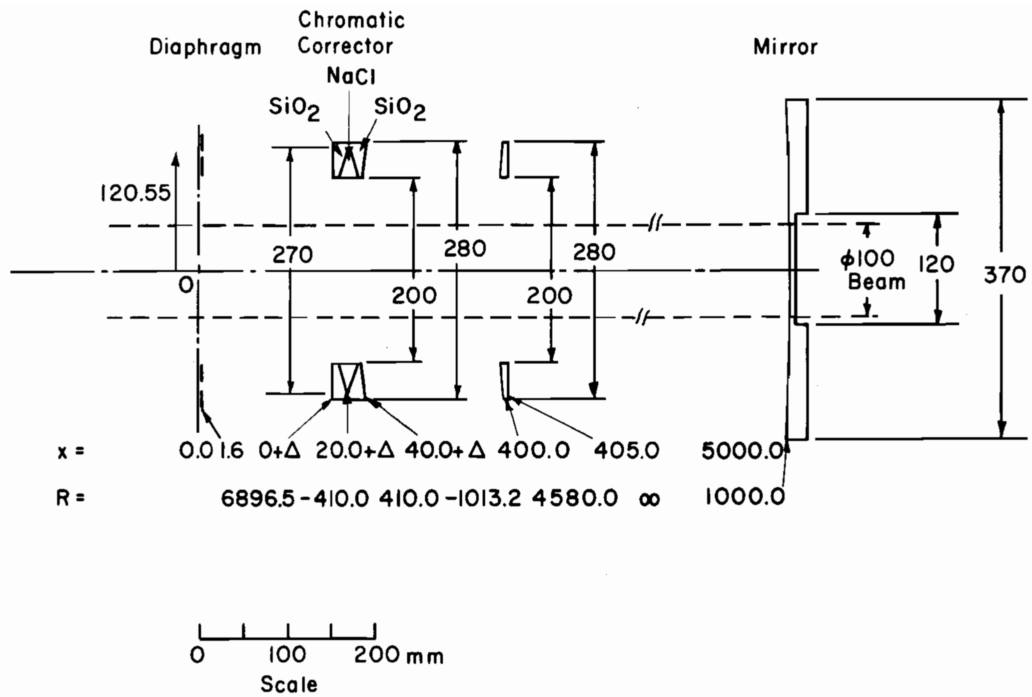

Fig. 3. DISC B. θ = 25 mrad.

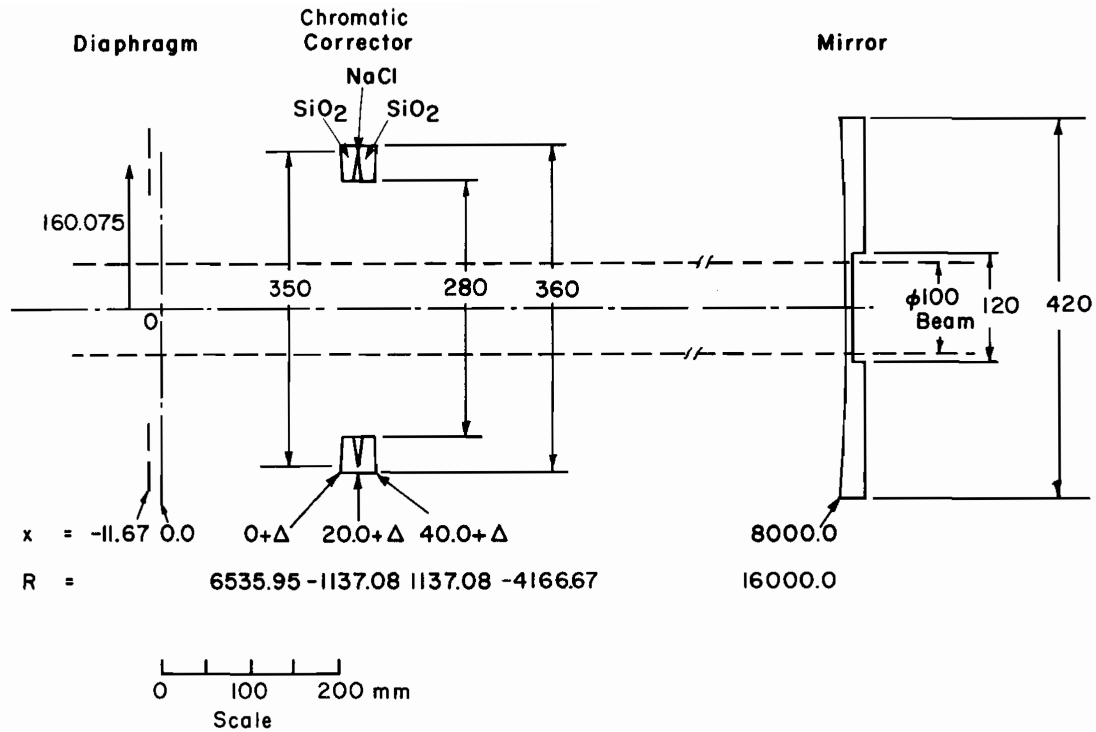

Fig. 4. DISC C. θ = 20 mrad.