A SEARCH FOR HEAVY MUONS BY WIDE-ANGLE μ BREMSSTRAHLUNG AT NAL

K. W. Chen
Princeton University

ABSTRACT

It is shown that a test of QED in the muon propagator can be performed at NAL with a sensitivity substantially higher than the g-2 experiment. Utilizing a highly collimated μ beam of high intensity ($\sim 10^8$/pulse) and low divergence (< 2 mrad), heavy muon mass up to 8 GeV/c^2 can be searched for. In the more conventional description, an upper limit of about 1.0×10^{-15} cm or better on Λ^{-1} could be assigned.

The contributions to wide-angle bremsstrahlung from nuclear Compton scattering and inelastic bremsstrahlung need to be carefully investigated but are expected to contribute insignificantly if sufficient care is taken to choose the proper kinematical configuration of the outgoing muon and the gamma ray.

I. INTRODUCTION

This note is an attempt to consider the feasibility and desirability of studying wide-angle μ bremsstrahlung using the high-intensity μ beam available at NAL. It is intended to supplement other μ beam usage considerations made in several other summer reports, (e.g. L. Hand SS-151, H. Anderson SS-109, L. M. Lederman, B.2-68-74, M. L. Perl, B. 2-68-47, and M. Tannenbaum B.2-68-32).

II. KINEMATICS

The Feymann diagrams for muon bremsstrahlung are shown in Fig. 1. These diagrams are particularly interesting as they are sensitive to an anomalous propagator or coupling of both the time-like and space-like virtual muons. Defining kinematic quantities involved here:

Q = four-momentum transfer to target nucleus

$q^2 = m'^2$ = invariant mass squared of the propagator muon

p_1, E_1 = initial muon momentum, energy

p_2, E_2 = final muon momentum, energy

k = final photon energy.

It is immediately seen that in the laboratory frame

$$m'^2 = p_2 k \sin^2 \theta/2, \tag{1}$$

where θ is the angle between the final muon and the photon. For target mass and incident energies large compared to m', the momentum transfer to the nucleus Q is given by

$$Q^2 = \left[\left(m'^2 - m_\mu^2\right)/2\,E_1\right]^2 + Q_T^2, \tag{2}$$

where Q_T is the component of the m' system transverse to the incident muon. In wide-angle bremsstrahlung, Q^2 takes on a minimum value when $Q_T^2 = 0$. Thus

$$Q_{min}^2 = \left[\left(m'^2 - m_\mu^2\right)/2\,E_1\right]^2. \tag{3}$$

Typically, at 100-GeV muon incident energy $Q_{min}^2 = (0.05\ \mathrm{GeV/c})^2$ for a m' mass of 3 GeV/c^2. Heavy targets can be used for coherent production of m'. For higher m' masses, at a fixed μ energy, it would be necessary to choose a lower Z target (such as carbon) to preserve coherence. We shall come back to the actual choice of target in Section IV-A.

If nuclear recoil is ignored, the mass of the virtual muon is given by[1]

$$\left(m'^2 - m_\mu^2\right) = \left[P_T^2 + \left(m_\mu^2 k^2/E_1^2\right)\right] \cdot E_1^2/k\left(E_1 - k\right) \simeq p_2 k\theta^2, \tag{4}$$

(for space-like m')

$$-\left(m'^2 - m_\mu^2\right) = \left[P_T^2 + \left(m_\mu^2 k^2/E_1^2\right)\right] \cdot \left(E_1/k\right) \simeq p_1' \; k\theta^2, \tag{5}$$

(for time-like m')

where P_T is the transverse momentum of k or p_2 with respect to p. It is readily seen from (4) and (5) that the space-like propagator will have a considerably lower m'^2 for a given P_T, thus giving rise to a larger fraction of space-like events. The transverse momentum is limited by the requirement of minimizing pion-induced background and eliminating μ-e background In Section IV-C we will discuss the mass resolution problem due to this requirement.

III. SENSITIVITY TO QED BREAKDOWN

Two recent experiments[2] have been performed at low energy to measure the bremsstrahlung yield with incident electrons or muons. The Cornell experiment measured the $e^- + C \rightarrow e^- + C + \gamma$ reaction using incident electron energies up to 9.5 GeV. The data confirm the predictions of QED for time-like electron four-momenta up to 1 GeV/c. Figure 2 shows the ratio, R, of experimental cross sections to the theoretical cross sections as a function of m'. No evidence of a heavy electron is found. At 95% confidence level, the cut-off parameter, Λ^{-1}, is given by

$$\Lambda^{-1} = \left[\frac{1}{2} \frac{dR}{dm'^2}\right]^{1/2} < 9 \times 10^{-15} \text{ cm (for electrons).}$$

The Harvard experiment measures $\mu + C \rightarrow \mu + C + \gamma$, yielding a result shown also in Fig. 2. Using the AGS muon beams of 9 - 13 GeV/c, values of m' up to 650 MeV/c^2 were searched for. No heavy muon was found. A fit of R is given by

$$R = 0.93\ [(1.0 \pm 0.045) - (0.09 \pm 1.37) \times 10^{-12}\ (\text{MeV/c})^{-4} (m')^4]. \tag{6}$$

This corresponds to a $\Lambda > 0.7$ GeV or

$$\Lambda^{-1} < 1.4 \times 10^{-14} \text{ cm (for muons).}$$

Neither of these results is as sensitive as the g-2 result, which assigns an upper limit to Λ^{-1} of 8×10^{-15} cm.

At NAL, utilizing a 100 GeV/c μ beam, assuming an accuracy of 10% measurement of R up to m' = 10 GeV/c^2, the limit to Λ^{-1} is

$$\begin{aligned} \Lambda^{-1} = \left[\frac{1}{2} \left(\frac{0.1}{100}\right)\right]^{1/2} &< 0.22 \times 10^{-2}\ (\text{GeV/c})^{-1} \\ &\approx 4 \times 10^{-16} \text{ cm.} \end{aligned} \tag{7}$$

This is many times more sensitive than the g-2 experiment for high momentum transfer. It also has the advantage that, if the mass measurement is made sensitive enough, detailed nature of the breakdown can be looked for. If the heavy muon should exist it would show up as a peak in the invariant mass distribution of the final muon and gamma at $m' = \Lambda$ and this distribution should behave strikingly differently from the usual $1/m'^2 - m_\mu^2$ bremsstrahlung spectrum. Figure 3 shows the usual bremsstrahlung mass distirbution in the Harvard experiment at lower values of m'.

IV. EXPERIMENTAL CONSIDERATIONS

We want to consider an experiment with the following requirements:

1. Highest mass sensitivity, m' up to 10 GeV/c^2.

2. Sufficiently good mass resolution to distinguish various different types of possible QED breakdown.

A. Cross Sections and Target Material

The bremsstrahlung cross section as a function of the relevant kinematical variables is given by[2]

$$\frac{d\sigma}{dp_2 d\Omega_2 d\Omega_k} = \frac{8Z^2\alpha^2}{\pi^2} \frac{p_2^2\left(p_1^2 + p_2^2\right)}{k\, p_1^4\, \theta_2^2\, \theta_\gamma^2 \left(\theta_2 + \theta_\gamma\right)^2} \frac{r^2}{\left(1 + r^2\right)}, \tag{8}$$

where $r = Q_\perp/Q_\parallel$ is the ratio transverse and longitudinal momentum transferred to the nucleus. The total cross section for $k > 0.1\ E_1$ is $\approx 0.4\ \mu b$. The energy and μ' mass dependence is simply

$$d\sigma \sim \frac{1}{Q^4} \cdot \frac{1}{\left(m'^2 - m_\mu^2\right)^2}. \tag{9}$$

Thus for a fixed set of conditions the most favored events will be those with the lowest Q^2 or $m'^2 - m_\mu^2$. Also for a given Q^2 and m'^2 at a higher E_1 (say 100 GeV/c) one has an average of 3-4 times more signal. However, in our estimate of the rates, we will not assume this gain, thus providing substantial margin of signal-to-noise ratio.

At AGS, an average flux of 10^4/pulse and a median muon energy of 10 GeV/c, the signal is observed to be 2 events per 10^6 muons in the m' range from 220 to 700 MeV/c^2. We assume the availability of the following μ beam at NAL

100 ± 5 GeV/c

< 2 mrad divergence

$\pi/\mu = 10^{-6}$

2.5 cm diameter

$> 10^8\ \mu$/pulse.

We choose the target thickness of 1 radiation length so that bremsstrahlung photons would emerge with high probability (~ 69%) without showering. The effective rate depends on the factor

$$\frac{Z^2 \cdot \rho L}{A},$$

where Z is the charge, A is atomic number, ρ is the density, and L the thickness. Table I shows the rate dependence of the target material at 1 radiation length.

Table I. $Z^2 \rho L/A$ as a Function of Target Material (for 1 r. ℓ.).

Target	Z	A	$\underline{X}_0$ (g/cm^2)	$\frac{Z^2 \rho L}{A}$	Target Length (cm)
H_2	1	1	58	58	820
Be	4	9	62	110	33
C	6	12	42	126	27
Pb	82	207	5.8	190	0.5

It is clear that by using a heavy target one gains only about ~ 2-4 times the rate of hydrogen. This is not the Z^2 dependence customarily assumed since the shower probability limits the thickness allowed to ~ 1 r. ℓ.

The bremsstrahlung cross section has been calculated at $E_\mu \simeq 12$ GeV by Liberman[2] as a function of m'. Table II gives the values:

Table II. Total μ Bremsstrahlung Cross Section for Carbon at E_μ = 11.5 GeV as a function of m'.

m' (MeV/c^2)	σ_T ($\times 10^{-2}$ μb)
200-300	16.39
300-400	10.35
400-500	3.427
500-600	1.36
600-700	0.56

At 100 GeV we expect an increase of a factor of 3 from these values.

Assuming the above-mentioned beam parameters we can estimate the minimum cross section we are sensitive to. We want a minimum of 10 evnets/(10% Δm/m) in 100 hours. This gives for a carbon target

$$\sigma_{min} = \frac{10}{10^8 \cdot 10^5 \cdot \left(\frac{\rho L}{A}\right) N_0 \eta} = \frac{10}{10^{13} \cdot 6 \cdot 10^{23} \left(\frac{5.5}{12}\right) \cdot \eta} \approx 4 \times 10^{-36} \, \eta ,$$

where η is efficiency for detection.

Assuming a $(1/m')^4$ dependence we expect to be sensitive to a cross section for m' up to m' $\simeq$ 8 GeV/c^2 if η = 1/4.

The use of lead allows a gain of ~ 50% in rate but the coherence of the reaction is no longer guaranteed.

B. Hydrogen Target

Lai[13] suggested the possibility of utilizing the missing mass technique to search for heavy muon, X, in the reaction,

$$\mu + p \to p + X \qquad X \to \mu\,\gamma, \tag{11}$$

where X is the heavy particle with decay lifetime of the order of 10^{-21} sec. We show below that the missing mass technique can be used only for large m' search and is hopeless for the low-mass region.

Figure 4 shows the kinematics for reaction (11) at 100 GeV/c. One envisages measuring protons with momentum typically between 200 to 800 MeV/c for M_x ranges from 1 GeV/c^2 to 10 GeV/c^2.

The minimum longitudinal momentum imparted to the nucleon at a muon energy of 100 GeV/c is 200 MeV/c for m' = 20 GeV/c^2 and is only 100 MeV/c for m' = 5GeV/c^2. The typical error in longitudinal momentum with a hydrogen target is 10 MeV/c mainly due to the multiple scattering of the proton. Thus we would expect $\gtrsim$ 50% in mass resolution at 2 GeV/c^2 and ~ 10% at m' = 5 GeV/c^2.

Although the resolution gets better at higher virtual muon mass, the signal rate decreases as $(1/m')^4$, the yield at > 5 GeV would exclude the use of hydrogen target even at 5×10^8 μ/pulse unless a target length > 10 meters can be tolerated.

C. Mass Resolution for a High-Z Target

When the outgoing muon and gamma are detected in coincidence the mass resolution is primarily dependent on the p_γ and $\theta_{\mu\gamma}$ measurements of the angle between the muon and γ, we write

$$\begin{aligned} m'^2 &= m_\mu^2 + m_\gamma^2 + p_\mu p_\gamma \theta_{\gamma\mu}^2 \\ &\simeq p_\mu p_\gamma \theta_{\gamma\mu}^2 \\ \Delta m' &\sim p_\gamma \theta_{\mu\gamma}^2 \Delta p_\mu + \Delta p_\gamma p_\mu \theta_{\mu\gamma}^2 + 2\theta_{\mu\gamma} p_\mu p_\gamma \Delta\theta_{\mu\gamma}. \end{aligned} \tag{12}$$

We attempt a few sample numbers:

The vector p_γ is well determined if $\Delta p_\gamma / p_\gamma = 0.1$ and $p_\gamma \Delta\theta_\gamma = 0.3$ with a 2 in. target. $\Delta p_\mu / p_\mu = 0.05$ and $p_\mu \simeq p_\gamma \simeq 50$ GeV for the worst case,

$$\left\langle\frac{\Delta m'}{m'}\right\rangle = \sqrt{\left(\frac{\Delta p_\mu}{p_\mu}\right)^2 + \left(\frac{\Delta p_\gamma}{p_\gamma}\right)^2 + 2\left(\frac{\Delta\theta_{\mu\gamma}}{\theta_{\mu\gamma}}\right)^2} \simeq \sqrt{2}\left(\frac{\Delta\theta_{\mu\gamma}}{\theta_{\mu\gamma}}\right) .$$

Thus at the worst case without incident μ energy and angle measurement,

$$\langle \Delta m'/m' \rangle \simeq 10\% .$$

to improve the mass resolution, we see immediately that the important thing is to keep the beam spot down to a minimum. Thus, we will impose a severe requirement on the spot size of the beam. To increase the mass resolution to 5%, we need to bring the μ beam down to ~ 1 in. in the cross section.

V. BACKGROUNDS

A. Nuclear Compton Process

Figure 5a shows the process of nuclear Compton scattering. We could make an estimate of the size of the term by calculating[4]

$$\frac{d\sigma}{dE'_\mu} = \frac{Re}{k}\,\sigma^{tot}_{\text{Compton effect}}(k), \tag{14}$$

where $k = E_\mu - E'_\mu$, and Re is the "equivalent radiator" and contains the muon part of muon-nuclear interaction. Re is obtained by integrating $d^2\sigma/dE'_\mu dQ^2$, the inelastic cross section at Q^2 and yielding E'_μ. If the experiment accepts only large Q^2, the $1/Q^2$ dependence depresses Re. For a typical value of Q^2, Re $\lesssim 10^{-6}$.

The total cross section for the Compton effect (real photons) can be estimated using Sakurai's vector-dominance model (VDM):

$$A(\gamma + A \to \gamma + A) = \Sigma_{\rho,\omega,\phi}\,\frac{e}{g_V}\,A(\gamma + A \to V + A). \tag{15}$$

This yields

$$\sigma_{\text{Compton}}(k) = \left| \Sigma \left[\alpha \cdot \left(\frac{4\pi}{g_V^2}\right) \cdot \sigma_{tot(\gamma + A \to V + A)}(k) \right]^{1/2} \right|^2 . \tag{16}$$

Using the 5-GeV cross sections, it is estimated that $\sigma_{tot}(k) \simeq 0.6\,\mu b$. The cross section for total nuclear Compton scattering is then at best $\sigma \lesssim 10^{-35}\,cm^2$. We expect therefore that nuclear Compton scattering is of importance, but its contribution can be minimized by properly choosing the kinematics. It is noted that Re is heavily dominated by the $1/Q^2$ dependence of the photon propagator. For example σ is of the order

10^{-37} cm^2 at high value of Q^2 by choosing proper kinematics. The interference of the Compton term with the Bethe-Heitler term is odd under a change of the lepton charge. This allows a direct measurement by determining the bremsstrahlung yields from $\mu^+ + Z \rightarrow \mu^+ + \gamma Z$ and $\mu^- + Z \rightarrow \mu^- + \gamma + Z$. The ratio

$$R = \frac{\sigma(\mu^+) - \sigma(\mu^-)}{\sigma(\mu^+) + \sigma(\mu^-)} \,, \tag{17}$$

is a measure of the magnitude of the interference of bremsstrahlung and Compton scattering.

To be sure of the fact that virtual nuclear Compton scattering is small, improved calculation is needed. Presumably one could calculate the ρZ total cross section via the Drell-Trefil model and derive the forward scattering Compton amplitude via the optical theorem.

B. μ-e Scattering

μ-e scattering is a serious background if the triggering does not contain sufficient kinematic constraint. The maximum energy of the scattered μ in μ-e collision is given by

$$E'_{max} = E \left\{ 1 - \left[1 + \left(\frac{m_\mu^2}{m_e^2} \right) \left(\frac{m_e}{2E} \right) \right]^{-1} \right\} . \tag{18}$$

At 100 GeV/c, $E'_{max} = 0.9\,E$. Thus kinematics alone would not help to reject μ-e unless one restricts to final muon energies > 0.9 E. To keep the trigger rate low, anticounters in front of the γ-ray chamber would work at the level of random coincidence between μ from μ-e scattering and pion decay and halo in the beam. Thus keeping the spot size small is a gain here. Similarly μ-p scattering is suppressed even further.

C. Pion Contamination in Beam

For purposes of estimating the upper limit of the pion contamination allowed in the μ beam, the serious backgrounds due to the presence of pion contamination in the beam are mainly

$$\pi Z \rightarrow n\pi \text{ (where } n = \text{multiplicity)} + Z'$$
$$\quad \hookrightarrow \begin{cases} \mu \\ \gamma \end{cases} .$$

We can estimate the effective cross section for this process provided that we know the transverse momentum distribution of the final states.

1. The total cross section of pions on Pb is 500 mb at 50 GeV[5]. It is nearly constant at higher energy.

2. The multiplicity at ~ 100 GeV/c pion energy is ~ 5.

3. Decay probability of 20 GeV/c pion into μ within 10 m is ~ 1%.

4. S = the suppression factor of π to have transverse momentum $p_\perp$: $S \sim e - (p_\perp/0.2)^2$.

5. $R = \pi/\mu$ in μ beam.

These are related to background cross section

$$"\sigma" = (\pi \to n\pi, \pi \to \mu, \gamma)/\mu = R(\pi/\mu) \times 5 \times 10^{-25}\,\mathrm{cm}^2 \times 5 \times 0.01 \times S = R \times S \cdot 0.25 \times 10^{-25}\,\mathrm{cm}^2.$$

When $"\sigma" = 2.5 \times 10^{-37}\,\mathrm{cm}^2$, the background is small; we select $p_\perp > 2$ GeV so that $S \simeq 10^{-5}$.

$$R \times S = 10^{-11}.$$

This states that $R < 1 \times 10^{-6}$ would be adequate to guarantee negligible pion background for events with $p_\perp > 2$ GeV/c.

D. Inelastic Bremsstrahlung

The important inelastic process for bremsstrahlung with a nuclear target are:

1. excitation of the nucleus
2. quasi-elastic scattering with a single nucleon
3. pion production (most likely through a resonant mechanism).

The contribution from (1) and (2) depends on the momentum transfer to the nucleus, Q. If the outgoing muon and gamma-ray energy and angle are both measured, it is possible to keep Q as small as possible. Here lies the advantage of being at high incident energy. We expect $Q \leq 150$ MeV/c for a m' up to 5 GeV. The relative amount of elastic and inelastic scattering in the region Q_1 to Q_2 is (above Q ~ 50 MeV/c)

$$\int_{Q_1}^{Q_2} \left[F(Q^2) \right] \frac{dQ}{Q} , \tag{20}$$

where $F(Q^2)$ is the form factor of the target nucleus.

Typically if $Q < 150$ MeV/c and with a carbon target inelastic contribution is estimated to be no more than 3%.

VI. APPARATUS

We require that

1. Energy of gamma ray be measured to 10% for 10 GeV < k < 100 GeV,
2. Angle of gamma ray be known to 2 mrad,
3. Energy of scattered muon be known to 5%,

4. Angle of scattered muon be known to better than 1 mrad,
5. Accept only events with $p_{\perp} > 2$ GeV/c.

Figure 6 shows an apparatus capable of the fulfilling of the above requirements. Collimated to 1 in. in cross section, the μ beam (~10 /pulse) is incident on a high-Z target (Pb or carbon). Outgoing muon directions are measured before and after the 2.5 m magnet (at 18 kG). The apparatus is capable of "seeing" a muon and a gamma ray in either configuration. The gamma ray is detected in a shower-chamber counter hodoscope combination. With 15 r. ℓ. of Pb, the energy resolution is expected to be < 10% and the vertex of the shower is known better than 0.5 cm. The outgoing muon is detected with a hodoscope and allowed to traverse through 200 in. of Fe. The energy loss is 10 GeV and is corrected for. This serves to reject pions and low-energy muons from electromagnetic and decay processes. Each hodoscope covers angular range between 30 mrad and 300 mrad. Approximately equal θ_{μ} and θ_{γ} can be selected in the hodoscope matrix by triggering on only diagonal or near-diagonal combinations.

Right Hodoscope - RH = (1, 2 . . . , N)
Left Hodoscope - LH = (1, 2 . . . , N).

The accepted triggers are 11, 22, . . . NN or 12, 23, . . . (N-1) N, etc.

Anti-counter A_1 is large enough to exclude all outgoing particles which are not in the forward direction. It is capable of converting γ rays and some neutrons.

The μ beam is monitored by placing counters in the beam. For relative measurements, side counters will be used.

REFERENCES

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[2] R. Siemann et al., Phys. Rev. Letters 22, 421 (1969); A. D. Liberman et al., Phys. Rev. Letters 22, 663 (1969).

[3] K. W. Lai, Some Speculative μ Experiments, National Accelerator Laboratory 1968 Summer Study Report B.2-68-28, Vol. II, p. 43.

[4] We thank J. D. Bjorken for a conversation on this point.

[5] G. Cocconi, private communication.

(a)

(b)

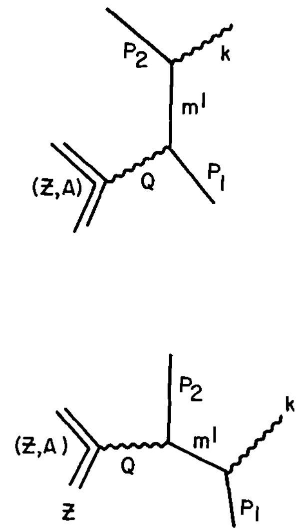

Fig. 1. Feymann graphs for μ bremsstrahlung for a) time-like and b) space-like virtual photons.

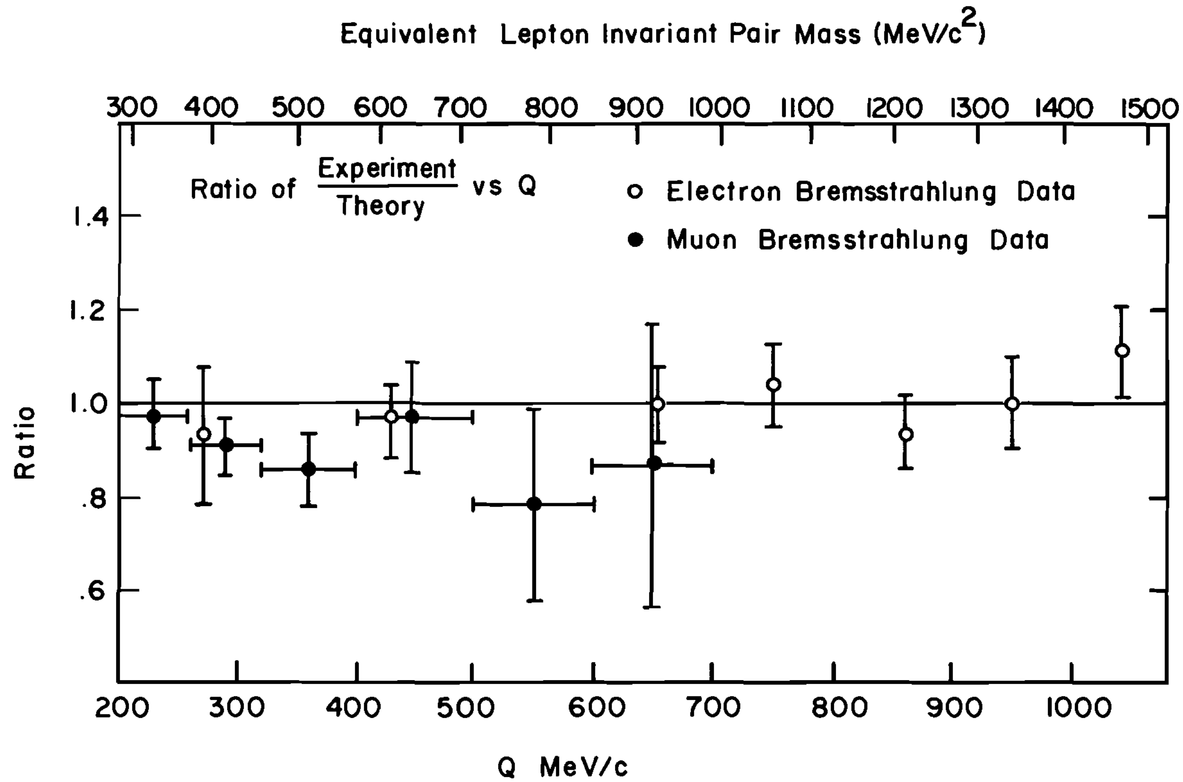

Fig. 2. Experimental values of the ratio, R, as a function of m' for the two bremsstrahlung experiments. The μ bremsstrahlung result extends up to m' = 600 MeV/c.

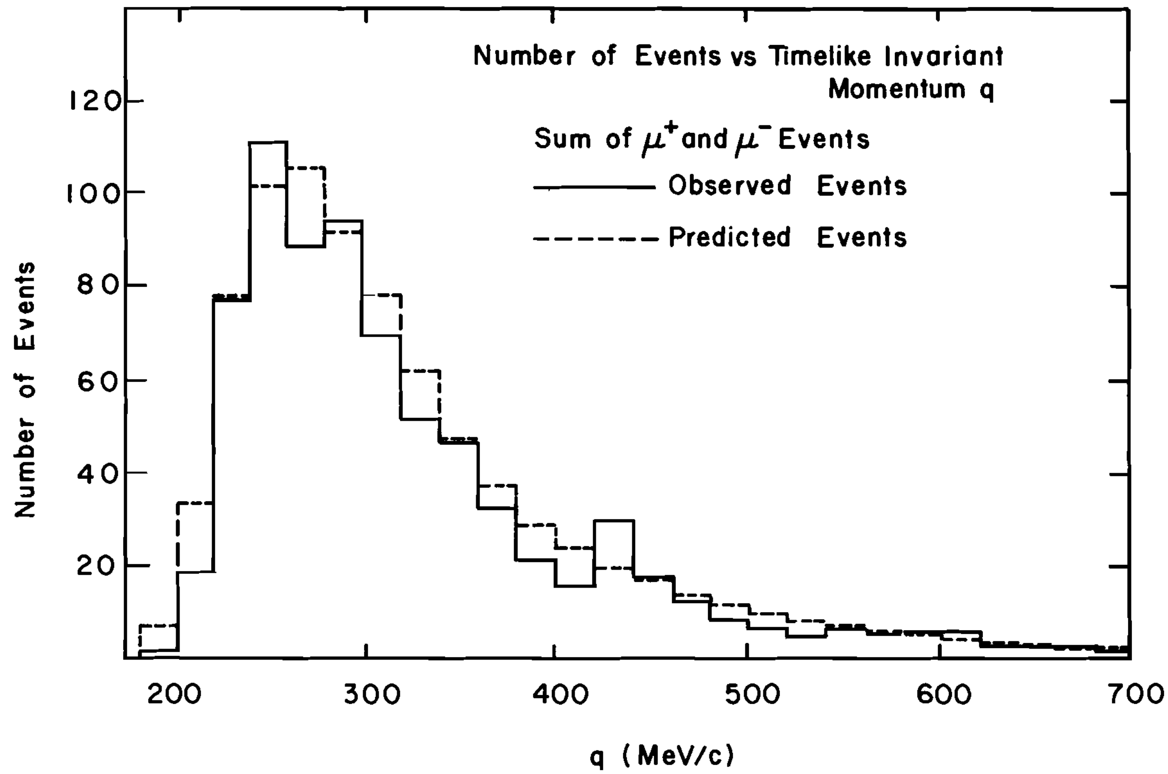

Fig. 3. Event distribution as a function of virtual muon mass in the Harvard experiment.

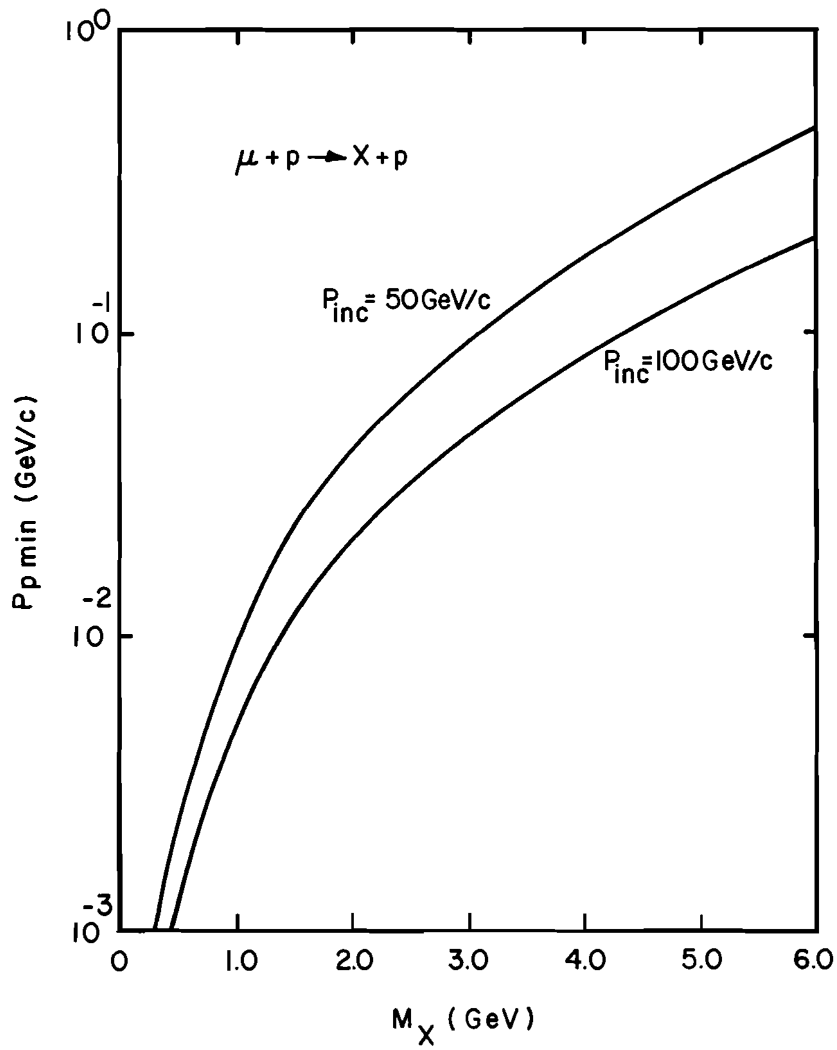

Fig. 4. Kinematics for $\mu + p \to p + X$ as a function of mass of X.

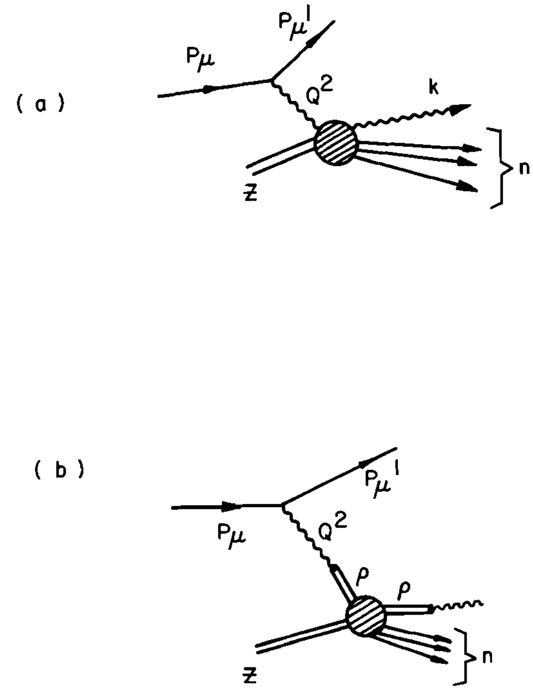

Fig. 5. (a) Nuclear Compton scattering Feymann diagram. (b) Vector (ρ) dominance diagram for virtual Compton scattering.

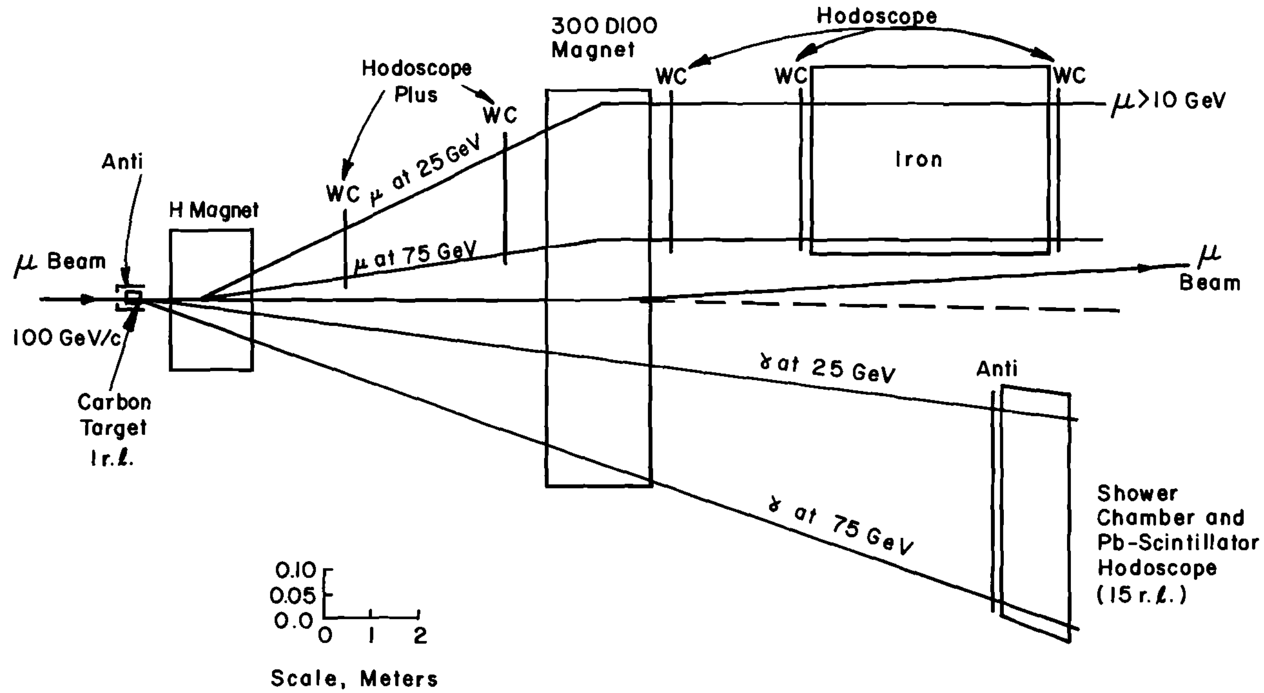

Fig. 6. Possible apparatus for detecting the muon and gamma ray in coincidence.