QED EXPERIMENTS WITH MUONS

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ABSTRACT

Muon experiments primarily studying quantum electrodynamics (QED) at small distances or μ-e differences are considered. Several of these experiments appear attractive at NAL without regard to the storage-ring considerations. The quality of muon beam required and sensitivity of the experiments to QED breakdown are presented.

I. INTRODUCTION

The agreement of experimental data in purely electromagnetic processes at high energy with Bethe-Heitler cross sections has usually been expressed in terms of a "radius," or a "cut-off" parameter, down to which the theory is said to be valid.[1] Low suggested a more natural way of describing a breakdown of QED in terms of a coupling of "heavy" leptons to other elementary particles.[2] The continued experimental confirmation of QED predictions is equivalently expressed by the upper limits to the coupling strength and lower limits to the mass of a hypothetical particle coupled to the leptons and photons. From this point of view, an experimental search for "heavy" leptons would provide not just a direct check of QED but might have the additional advantage of isolating the electrodynamic system from the nuclear target, without the use of as yet non-existent storage rings. Also there is no reason to believe that electrons and muons are unique in that they are not coupled to a large family of elementary particles.

In the following we briefly describe the current status of tests of QED. The final results[3] of the CERN g-2 experiment are

$$1/2\,(g-2)_{\mu^-} - 1/2\,(g-2)_{\mu^+} = (50 \pm 75) \times 10^{-8},$$

and

$$1/2\,(g-2)_{\mu} = (116,\ 616 \pm 31) \times 10^{-8}.$$

The theoretical value of the anamolous moment of the muon is given by

$$1/2\,(g-2)_{\mu} = 116,\ 560 \times 10^{-8}.$$

The difference between the experimental and theoretical value is given by

$$\delta = +(480 \pm 270)\ \text{ppm}. \tag{1}$$

Taken at face value, this is a 1.8 standard deviation effect. If a heavy photon exists with a coupling $g^2 = \alpha$, this means $\alpha/M^2 = (4.80 \pm 2.7) \times 10^{-4}$; or $1/M^2 = 0.067 \pm 0.037\ \text{GeV}^2$. Thus $M \geq 3.9$ GeV.

A more conservative point of view would be to assign a cut-off parameter or a fictitious "radius," Λ^{-1}, down to which the theory is found to hold, casting the entire uncertainty in the (g-2) experiment. A high momentum transfer cut-off of $\Lambda \sim 5$ GeV/c or

$$\Lambda^{-1} = 4 \times 10^{-15}\ \text{cm},$$

would reduce the theoretical result by 270 ppm. In spite of the extreme accuracy attainable in this experiment it can be argued that g-2 is a test of QED at low q^2. The g-2 experiment studies the electromagnetic process to a value of four-momentum transfers of the order of the ρ mass while high-energy experiments reach momentum transfers substantially higher. Possibilities cannot be overlooked that certain breakdowns of QED, if any exist, may manifest themselves with powers higher than the square of momentum transfer. No firm conclusions can be drawn even when one has a negative result at lower momentum transfers with high precision. Thus, the high-energy experiments serve as a complement to the g-2 experiment.

Several high-energy experiments, such as μp elastic scattering vs ep scattering (μ-e universality), wide-angle bremsstrahlung, μ-trident production, electron-electron scattering,[4] have been performed in the past decade. These experiments generally yielded results of either comparable or somewhat less sensitivity than the g-2 experiment.

II. POSSIBLE NAL EXPERIMENTS

We evaluate four NAL high-energy experiments capable of testing QED at small distances, using the proposed high intensity muon beam. As will be shown, most of them can significantly extend the range of validity of the theory for the high momentum transfer electromagnetic processes. This work could substantially broaden the muon physics program.

A. μ-e Scattering

The maximum attainable momentum transfer, q, for μ-e scattering at 150 GeV/c is only 0.33 GeV/c. Let the photon propagator be modified in the form,

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2} \simeq + \frac{1}{q^2}\left(1 + \frac{q^2}{\Lambda^2}\right), \tag{2}$$

for

$$q^2 << \Lambda^2.$$

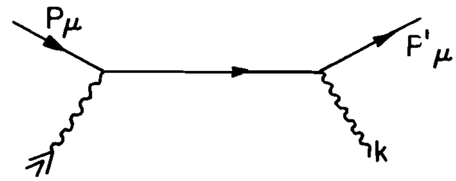
We expect a maximum deviation of 0.6% from the Bethe-Heitler cross section for $\Lambda \simeq 5$ GeV/c. Assuming that one is interested in a 3-standard deviation effect, $\Delta\sigma/\sigma = 0.2\%$ is needed for μ-e scattering measurement at 150 GeV/c. To achieve this, one is not limited by statistics since the cross section is ample ($\sigma \simeq 0.5$ μb for $E'_\mu > 50$ GeV). At a flux of 5×10^6 μ/pulse 10^6 events can be collected in one day. It is unlikely that systematic errors (i.e. normalization, efficiency, etc.) can be reduced below ~ 1%. Thus μ-e scattering does not appear attractive as far as testing of QED is concerned. Since μ-e scattering occurs mostly as background to other muon experiments, a measurement of the cross section can be done along with other experiments. Also searches of rare processes such as $\mu^+ + e^- \rightarrow \mu^- + e^+$ can be performed together with μ-e scattering. The point is that μ-e scattering alone does not appear attractive.

B. Wide-Angle Muon Bremsstrahlung

The presences of the virtual muon in the following process allows a test of QED in the muon propagator.

$$\begin{aligned} \mu + p &\rightarrow \mu + \gamma + p \\ &\rightarrow \mu' + p \\ &\qquad \llcorner\!\!\rightarrow \mu\ \gamma. \end{aligned} \tag{3}$$

The Feymann diagram goes as follows



The virtual mass of the muon is given by $m'^2 = p'_\mu k\, \theta^2_{\mu\gamma}$ where p'_μ, k, and $\theta_{\mu\gamma}$ are scattered muon energy, radiated photon energy, and the angle between the muon and photon. A measurement of these three quantitites determines the signature and the mass. A detailed consideration of a possible experiment is discussed in NAL summer study SS-68.[5]

The bremsstrahlung cross section is a complicated function of the kinematic variables:

$$\frac{d\sigma}{dk} \sim \frac{8}{3}\,\alpha\,\frac{Z^2 e^4}{m^2}\,\frac{dk}{k}\left(1 - \frac{k}{E} + \frac{3}{4}\frac{k^2}{E^2}\right)\left[2\,\ell n\,\frac{2E(E-k)}{mk} - 1\right],$$

where k is the radiated photon energy, E is the incident muon energy, and m is the muon mass. We show here a simple integration to get the estimated cross section from k = 0.1 E to E.

$$\int_{0.1E}^{E} \frac{d\sigma}{dk}\,dk \simeq \frac{8}{3}\,\alpha Z^2 \left(r_\mu\right)^2 \cdot 2\,\ell n\left(\frac{2E}{m}\right) \cdot 0(1) \tag{4}$$

$$\simeq 70\ Z^2 \times 10^{-32}\ \mathrm{cm}^2$$

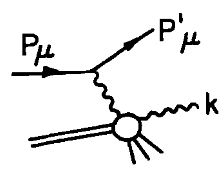
$$= 0.7\ \mu\mathrm{b}\ \text{for}\ Z = 1\ \text{at}\ 100\ \mathrm{GeV/c}.$$

It is necessary to use a thin target otherwise the outgoing photon will shower within 1 radiation length. With a 1-meter H_2 target (~ 0.1 r. ℓ.) and a large muon spectrometer, the rate is estimated to be $\sim 10^4$ events/100 hour for virtual muon mass (m') of ~ 4 GeV/c^2. At m' = 6 GeV/c one achieves a precision of 10% in cross section in a 500-hour run. If the cut-off parameter is defined by

$$\Lambda^{-1} = \left(\frac{1}{2}\,\frac{dR}{dm'^2}\right)^{-1/2},$$

where R is the ratio of experimental data to the Bethe-Heitler cross sections, m' is the muon mass, then $\Lambda^{-1} = 5 \times 10^{-16}$ cm or ~ 11 times g-2 results.

The uncertainty about the contribution from the nuclear Compton scattering

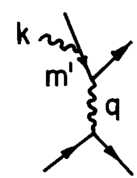


can be estimated with the vector-dominance model. A crude estimate indicates that the contribution is negligible amounting to less than a few percent. Furthermore, runs with μ^+ and μ^- incident beam allows a direct measure of the interference. A typical setup would require a muon spectrometer (25 GeV < E'_μ < 75 GeV) together with a gamma-ray shower-counter-spark chamber μ combination. The error on the mass resolution comes mainly from the angle measurements. Since one wants to measure θ_γ to 2 mrad, one needs a 1-in. beam spot and converting the γ's at 20 meters downstream from the target. Hodoscope coincidences allows selective triggers for a large range of masses simultaneously. Pion contamination-induced background is eliminated by requiring the muon or gamma to have a transverse momentum, $p_\perp > 1$ GeV/c. We estimate that a pion contamination of the order of 10^{-6} can be tolerated. Needless to stress here, one must use a muon beam of highest possible intensity since the cross section varies as $\sigma \sim (1/m')^4$. At a given incident muon energy and q^2 to the target nucleon, a factor of 16 increase in intensity doubles the range of m' explored.

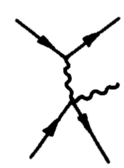
The high-luminosity storage rings ($10^{32}/\mathrm{cm}^2/\mathrm{sec}$) at CEA and SLAC could conceivably study the muon propagator via the reaction

$$\begin{array}{l} e^+ + e^- \rightarrow \mu' + \mu \\ \qquad\qquad\ \ \llcorner\!\!\rightarrow \mu\gamma . \end{array} \tag{5}$$

The Feymann diagram goes as



There are difficulties. The main difficulty lies in the fact that there are two propagators, $1/q^2$ and $1/m'^2$ in the amplitude competing to reduce the cross section. One runs out of signals to perform an experiment with precision. In addition one has the difficulty of understanding the radiation in a diagram such as



since one expects this amplitude to be just as probable as the μ bremsstrahlung.

C. Muon Tridents

M. Tannenbaum[5] pointed out the desirability of studying μ tridents at NAL. The total trident cross section is estimated to be 0.05 μb at 100 GeV/c. In contrast to the μ bremsstrahlung one could use a thick target for the Z^2 factor in coherent production of μ pairs. The total cross section is dominated by low mass states in the virtual gamma or muon as the cross section varies as

$$\sigma \sim \frac{Z^2 \alpha^4}{q^4} \cdot \frac{1}{m_\gamma^4} \cdot \frac{1}{(m')^4}, \tag{6}$$

where q, m_γ, and m' are 4-momentum transfer to the nucleus, virtual gamma ray, and the virtual muon. The most likely trident configurations are 1) virtual photons produced at large transverse momentum decaying into a lepton-antilepton pair with small transverse momentum, and 2) virtual photon production with small transverse momentum and decaying with large transverse momentum. Other configurations are possible but with smaller probability. All of these configurations are equivalent to a combination of muon-muon scattering, wide-angle photo-pair production, muon bremsstrahlung and muon-antimuon colliding beam experiments. The information content is rich, but it is a challenge to not only measure an effect, but also to pin down the proper channel that caused the effect.

The advantage of doing the μ trident at NAL is that one could probe the heavy photon propagator when it is coupled to muons <u>only</u>.[6] If the heavy photon can couple to electrons as well, the storage ring processes

$$e^+ + e^- \to \begin{cases} e^+ + e^- \\ \mu^+ + \mu^- \end{cases}, \tag{7}$$

can yield results up to $m_\gamma = 6$ GeV/c^2. This is hard to achieve with tridents due to the rapid decrease of cross section as m_γ increases. To have a rate of 10^2/100 hr and using a target(Pb) of thickness t = 500 g/cm^2, the value of the trident cross section required is

$$\sigma(\mathrm{coh}) = \frac{10^2}{f \cdot \left(\frac{6 \times 10^{23}}{82}\right) \cdot 10^5 \cdot (500)}, \tag{8}$$

where f = μ beam flux. At f = 10^8/pulse we get

$$\sigma = \frac{1}{4} \times 10^{-36}\ \mathrm{cm}^2 .$$

This cross section corresponds to a virtual photon mass of ~ 2 to 3 GeV/c^2. With a 10% experiment one assigns $\Lambda^{-1} \sim 1.5 \times 10^{-15}$ cm.

D. Comparison of ep and μp Scattering (μ-e Universality Tests)

Precision μp and ep Elastic Scattering

As is pointed out by R. Wilson[7] the limitation of rate in μp elastic scattering at NAL (1/10 of SLAC rate at $q^2 = 25\ (\mathrm{GeV}/c)^2$ cannot provide the detailed comparison of μp and ep elastic scattering at large q^2. The elastic scattering experiments can be used only to compare eγ and μγ vertices at low momentum transfers. AGS data in the momentum transfer range of $0.15 < q^2 < 0.9\ (\mathrm{GeV}/c)^2$ indicated that, with an average precision of 2 - 10% in measured cross section a cut-off parameter $\Lambda > 2.4$ GeV/c can be assigned.[8] Here Λ is defined by

$$\frac{1}{\Lambda^2} = \frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} , \tag{9}$$

where Λ_μ and Λ_e cut-off parameters of the muon and electron respectively. Hofstadter[9] estimated that with a beam of 10^7 μ/sec at 100 GeV/c, one has only 15 μp elastic scattering events per day at $q^2 = 10\ (\mathrm{GeV}/c)^2$ and 0.3/day at $20\ (\mathrm{GeV}/c)^2$. Scaling this to $q^2 = 4\ (\mathrm{GeV}/c)^2$ we could hope to get ~ 600 events/day at 10^7 μ/sec. Using a 10^8/sec beam we could hope to get ~ 6000 events/day. This is sufficient for 1% statistical precision.

A combined electron and muon vertex modification is of the form

$$G\,(q^2) = F\,(q^2)\left[1 + \frac{q^2}{\Lambda^2}\right]^{-1/2} .$$

If one attains a precision of $G^2\,(q^2) \sim 10\%$, Λ can be probed up to 10 GeV/c, provided that precise ep data is also available from SLAC. It is clear that μp elastic scattering can be fruitfully pursued at NAL for $q^2 < 4\ \mathrm{GeV}^2/c^2$. However, it is entirely possible that at this q^2, the AGS experiment[10] may be fully adequate.

The rapid decrease of μp elastic cross section as q^2 (1/10 × ep, and $1/q^8$ dependence) suggests that μp inelastic scattering is expected to be more fruitful to study QED at larger q^2.

Precision μp and ep Inelastic Scattering

Present SLAC inelastic data is confined to $q^2 < 2.5\ (\mathrm{GeV}/c)^2$ and $\nu = E_{\mathrm{incident}} - E \simeq 7$ GeV. It is likely that data from SLAC with $q^2 \simeq 10\ (\mathrm{GeV}/c)^2$ at $E = 20$ GeV will be available by NAL turn-on. Depending on the success of the new

AGS experiment,[10] an inelastic μp scattering experiment matching the SLAC data would be highly interesting. In contrast to the μp elastic scattering the inelastic form factors vary little at high q^2. The cross section is of the form

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(w_2 \cos^2 \theta/2 + 2\, w_1 \sin^2 \theta/2 \right) ,$$

where

$$w_2 = \frac{k}{4\alpha\pi^2} \; \frac{\sigma_L + \sigma_T}{1 + \nu^2/q^2}$$

$$\frac{w_1}{w_2} = \left(1 + \frac{\nu^2}{q^2} \right) \frac{\sigma_T}{\sigma_T + \sigma_L} .$$

Typical cross section at $E = 20$ GeV, $q^2 = 10\ (\mathrm{GeV/c})^2$, and $\nu = 7$ GeV is $2 \times 10^{-31}\ \mathrm{cm}^2$. This rate is ample with a $\Delta\Omega \approx 100$ msr detector, a 10^7 μ/pulse beam, and a liquid hydrogen target of ~ 1 m. For 1% statistics a 5-day run would be sufficient. The sensitivity to QED breakdown of the inelastic μp experiment at a given precision is 2-3 times as high as the elastic scattering due to the fact that higher q^2 could be reached. The rate factor helps to make the experiment simpler and less sensitive to backgrounds. We believe that a conventional setup using a medium intensity μ beam would be adequate.

At $q^2 > 10\ (\mathrm{BeV/c})^2$, SLAC would not be able to provide ep data with sufficient accuracy. The μp inelastic scattering as proposed by Hand[11] would not likely be useful for μ-e difference studies. Inelastic electron scattering experiments at NAL for $q^2 > 10\ (\mathrm{GeV/c})^2$ are much harder and suffer from 1000 times worse pion contamination in the beam.[11] The electron beam intensity is also at least 30 times lower than the muon beam intensity. Thus one cannot hope for high precision electron data at high q^2 (> 10 GeV/c) from NAL for a μe universality test.

III. CONCLUSIONS

1. μp elastic scattering up to 4 $(\mathrm{GeV/c})^2$ is worth studying for the purpose of comparing the data with SLAC ep data. Precise μp inelastic scattering up to $q^2 = 10\ (\mathrm{BeV/c})^2$ are desirable and extremely interesting. With a precision of ~ 3%, Λ up to 20 GeV/c can be studied. The μ beam intensity required of elastic scattering is > 10^8/pulse and ~ 10^7/sec for inelastic scattering. The beam quality is well described by Hand.[11]

2. Wide-angle μ bremsstrahlung offers a "unique" way of studying the virtual muon propagator. "Heavy" muon mass up to ~ 8 GeV/c^2 can be searched for yielding

a Λ^{-1} of 4×10^{-16} cm ($10 \times g$-2 sensitivity) with a 10% measurement. Pion contamination $\sim 10^{-6}$ is tolerable and a beam spot of < 1 in. is necessary for a mass resolution of ~ 10%.

3. μ trident at NAL suffers from a competition offered by the storage-ring experiments in probing the "heavy" photon propagator up to 6 GeV/c. Rate consideration prevents tests beyond the 6 GeV/c limit. The test of muon propagator is limited to ~ 3 GeV/c ($\Lambda^{-1} \sim 1.5 \times 10^{-15}$ cm) and is inferior to the μ bremsstrahlung. The uniqueness of μ trident is to probe heavy photons coupled only to muons. This is a difficult experiment and could possibly be designed in conjunction with the μ bremsstrahlung experiment.

4. μ-e scattering is not interesting for the purpose of studying QED since the momentum transfer involved will be small. It is likely to be a by-product of most other μ experiments.

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