FACILITY FOR TWO-BODY ELASTIC SCATTERING REACTIONS

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ABSTRACT

It is shown that a large-angle, low-momentum, non-focusing spectrometer of ~ 0.5 steradians, in coincidence with a small-angle, strong-focusing spectrometer, provides a setup which should be able to utilize the full beam intensities available at NAL. With such a facility, measurements could be made with both unpolarized and polarized targets up to about 130 BeV for the u channel and up to about 175 BeV for the t channel. Merits and demerits of strong focusing versus non-focusing high-energy spectrometers are discussed for this application. It is believed that on grounds of physics, engineering, and economy, a strong-focusing spectrometer is clearly superior to a non-focusing spectrometer for high momentum measurements. Preliminary design parameters are given for such a device.

I. DESIGN OF TWO-ARM SPECTROMETERS

A considerable number of Summer Study reports[1-7] have by now appeared on two-body scattering reactions in the u and t channel of the form

$$X + p \rightarrow X + p$$

where X is $\pi^{\pm}$, $K^{\pm}$, p, or $\bar{p}$.

All authors have proposed coincident detection of the two outgoing particles, although in principle enough precision could be obtained with one spectrometer arm to measure many, if not all, of the reactions.

The typical apparatus is shown in Fig. 1 and consists of a low-energy detector system A subtending a large solid angle and a small-angle, high-energy detector system B.

The low-energy detector, A, should work over the range of 50° to 180° in the laboratory and should measure momenta in the range of 500 MeV/c to 3.0 BeV/c. Such devices have been built for AGS and CERN experiments and typically subtend solid angles of the order of 0.5 steradians. Focusing devices do not cover solid angles much in excess of 20 millisteradians, and we do not believe that they are in any sense competitive with the presently used non-focusing 0.5 steradian "spectrometers." The attainment of larger solid angle acceptances, while possible in principle, would greatly increase complexity, and we believe that a 0.5 steradian acceptance is a reasonable objective for these experiments.

Once we have set the acceptance for the low-energy "spectrometer" A at 0.5 steradians, kinematics determine the solid angles required for spectrometer B. At 75 BeV the solid angle necessary for B to collect all particles in coincidence with A is of the order of 10^{-4} that of A, namely 50 μsr. At 200 BeV the required solid angle is down to about 5 μsr.

Focusing

Both focusing and non-focusing spectrometer systems can be designed to meet these requirements. In AGS and CERN experiments non-focusing wire-chamber magnet systems have been used; these instruments have functioned well. In comparable experiments at SLAC, focusing spectrometers have functioned excellently. There are, however, important, and we believe, decisive differences between focusing and non-focusing systems.

1. Non-focusing systems require detection devices near the target. Users of AGS non-focusing systems have stated that the problems of halo and beam interferences with their downstream detectors have necessitated holding input beams to a level of 10^6 particles per burst. This is to be contrasted with SLAC experience with focusing systems, where the first detecting devices can be placed far downstream. SLAC spectrometers have operated at 0° to the beam line at instantaneous rates of 3×10^{15} photons per sec, or, in vector-dominance equivalence terms, at instantaneous rates of 10^{13} "ρ^o-mesons" per sec, or 10^{14} electrons per sec.

As we show later, it should be possible to use our proposed strong-focusing setup in conjunction with the beams of 10^8-10^9 π's per burst that will be available at NAL. These intensities will be important for u-channel processes and for measurements with polarized targets.

2. Non-focusing spectrometer systems of the required solid-angle acceptance are not matched to DISC Cerenkov counters, the range of beam angles being too large. Our proposed focusing design incorporates a 3:1 demagnification of acceptance angles and is a good match to DISC Cerenkov counters. This discrimination is particularly necessary for u channel processes. Non-focusing systems would have to rely on threshold anti-coincidence counters (whose size at 150 BeV would be ~ 50 meters long).

3. In a well-designed focusing system, the number of decoding elements is small, and the size of the decoding elements is also small. Therefore, if required, scintillation hodoscopes can be used. This again permits use of high-intensity primary beams.

4. A well-designed focusing system costs less than the comparable non-focusing system. (This is a weaker argument than the preceding, as the focusing magnet optics system will take only a fraction of the overall facility cost. We estimate below that magnets and quadrupoles for our proposed focusing systems will cost of the order of \$250 K.)

As second-order aberrations decrease in proportion to the acceptance, we believe that the attainment of appropriate precision for the focusing spectrometer will be relatively easy, and much easier than for the larger acceptance devices at SLAC.

Proposal

With these considerations in mind, we propose a scattering facility consisting of a half-steradian large-angle rotatable non-focusing spectrometer system and a forward rotatable spectrometer whose components could be used as a 100-μsr spectrometer at 75 BeV, or as a 15-μsr system at 200 BeV. This device would not only be useful for the large range of two-body reactions but could very quickly, at switch-on, make the initial high-energy beam surveys required for NAL operations.

The various parameters and design considerations are outlined in the subsequent sections.

II. KINEMATICS AND REQUIRED MEASUREMENT PRECISION

Ideally, each spectrometer arm should be sufficiently precise to exclude, on a missing-mass basis, the presence of an additional π meson. This is marginally achieved in practice. If a coincidence is required between the two spectrometers very high rejection is obtained without stringent precision requirements on the individual arms.

Consider background from the reaction:

$$X + p \rightarrow X + p + \pi. \qquad (1)$$

Figure 2 shows diagrammatically the phase-space plot for $MM^2_{p\pi}$ versus $MM^2_{X\pi}$. The coincidence requirement accepts all the two-body elastic-scattering process but accepts only a very minute fraction of the phase space for the three-body reactions. Suppose, for example the large-angle spectrometer defines a missing mass band for the recoiling boson of $\Delta MM^2_{X\pi}$ which includes the process $X + p \rightarrow \rho + p$; then the small-angle spectrometer accepts only a fraction $\sim \Delta MM^2_{p\pi}/s$, where s is then the center-of-mass energy squared and $\Delta MM^2_{p\pi}$ is the missing mass band defined by the spectrometer. Similarly, the fraction of an isobar accepted by the coincidence requirement is $\Delta MM^2_{X\pi}/s$.

Thus, at NAL energies of about 150 BeV, with $s \approx 300\ (\mathrm{BeV})^2$, the missing mass need only be defined to about $3\ (\mathrm{BeV})^2$ to give rejection ratios of 100:1 against unwanted processes. Over and above this, the requirement of coplanarity gives an additional rejection, most important for the slow recoiling resonances.

Generally, the spectrometers and input beam characteristics will be such as to define the missing mass bands to the ~ 0.1 BeV^2. However, it is clear that this value is extremely conservative and that it should be possible to run with substantially less resolution.

Table I shows representative values for πp scattering angles for the π and proton in the t channel at various values of t. In addition, the factor $dt/d\Omega_{\mathrm{lab}}$, the "sensitivity" factors $(\partial MM^2/\partial\theta)_p$, and $p(\partial MM^2/\partial p)_\theta$, and the ratio of the large to small angle spectrometer apertures for the horizontal and vertical dimensions are given. Table II gives the same quantities for the u channel as a function of u and primary momentum.

Table I. Some Kinematics for $\pi p \to \pi p$ Elastic Scattering in the t-Channel.

The spectrometer angle θ, $dt/d\Omega_{\mathrm{lab}}$, the sensitivity factors $(\partial MM^2/\partial\theta)_p$, and $p(\partial MM^2/\partial p)_\theta$ are tabulated for input beam momenta of 75 BeV/c and 150 BeV/c versus u for both the pion and proton. HMAG is the ratio of $(d\theta_p/d\theta_\pi)$ for the two lab production angles and VMAG is the ratio of the corresponding $(d\phi_p/d\phi_\pi)$ for the lab azimuthal angles. These factors are required to calculate the dimensions of one spectrometer matched to the other spectrometer.

A. Beam Momentum 75 GeV/c

	θ degrees	$dt/d\Omega_{\mathrm{lab}}$ $\mathrm{BeV}^2/\mathrm{sr}$	$(\partial MM^2/\partial\theta)_p$ $\mathrm{BeV}^2/\mathrm{sr}$	$p(\partial MM^2/\partial p)_\theta$ $\mathrm{BeV}^2/\mathrm{sr}$
	1. t = -0.1	VMAG = 232.4	HMAG = 33.12	
π	0.241	1788	47.4	-140.7
p	78.6	0.232	55.8	-10.46
	2. t = -0.5	VMAG = 98.4	HMAG = 33.7	
π	0.541	1778	105.8	-140.7
p	68.3	0.535	109.9	-33.4
	3. t = -1.0	VMAG = 65.4	HMAG = 30.4	
π	0.766	1765	149.4	-140.7
p	61.1	0.885	152.3	-52.9
	4. t = -1.5	VMAG = 50.5	HMAG = 27.5	
π	0.940	1752	182.7	-140.7
p	56.0	1.260	185.0	-66.5

B. Beam Momentum 150 GeV/c

	θ degrees	$dt/d\Omega_{lab}$ BeV^2/sr	$(\partial MM^2/\partial\theta)_p$ BeV^2/sr	$p(\partial MM^2/\partial p)_\theta$ BeV^2/sr
	1. t = -0.1	VMAG = 466.1	HMAG = 65.5	
π	0.120	7158	94.6	-281
p	78.7	0.234	111.8	-20.76
	2. t = -0.5	VMAG = 197.2	HMAG = 67.0	
π	0.270	7137	211.8	-281
p	68.5	0.539	220.0	-66.4
	3. t = -1.0	VMAG = 131.3	HMAG = 60.7	
π	0.382	7112	299	-281
p	61.3	0.891	305	-105.1
	4. t = -1.5	VMAG = 101.6	HMAG = 54.9	
π	0.469	7087	366	-281
p	56.3	1.268	371	-132.1
	5. t = -1.9	VMAG = 86.8	HMAG = 51.0	
π	0.528	7066	412	-281
p	53.1	1.59	416	-148.4

III. INPUT BEAM REQUIREMENTS

The coincidence requirement between the large- and small-angle spectrometers permits a great deal of latitude on the input beam characteristics.

As an extreme example, consider "u" channel π-p scattering at 150 BeV. Suppose to obtain the beam intensity necessary to get reasonable counting rates, we used a 5 μsr acceptance beam with a 2% $\Delta p/p$ spread. Table II shows that the small-angle spectrometer would determine the "missing mass squared" to ~ 3 BeV^2 and the large-angle spectrometer would determine the missing mass squared to ~ 1 BeV^2.

As discussed in Sec. II above, coincidences between the two spectrometers would give rejection ratios against boson resonances of the order of 300:1 and against baryon resonances of the order of 1000: 1 when the coplanarity criterion is included; u values or t values are very well determined by the large-angle spectrometer. The input beam characteristics can be checked by measuring the input beam with the small-angle spectrometer at zero degrees.

In practice, we would expect to put a small scintillator hodoscope at the momentum crossover of the input beam and to know the input beam energy to a fraction of a percent.

The most suitable beam for this kind of work would be a compromise between

Table II. Some Kinematics for $\pi p \rightarrow \pi p$ Elastic Scattering in the u Channel.

Symbols used are the same as for Table I.

A. Beam Momentum 75 GeV/c

	θ degrees	$dt/d\Omega_{lab}$ BeV^2/sr	$(\partial MM^2/\partial\theta)_p$ BeV^2/sr	$p(\partial MM^2/\partial p)_\theta$ BeV^2/sr
	1. u = -0.1	VMAG = 146.4	HMAG = 146.2	
π	140.8	0.0845	48.6	-135.2
p	0.246	1810	48.7	-140.7
	2. u = -0.5	VMAG = 102.5	HMAG = 101.5	
π	104.4	0.172	106.4	-136.8
p	0.540	1800	106.4	-140.7
	3. u = -1.0	VMAG = 74.7	HMAG = 73.9	
π	85.0	0.323	149.8	-137.8
p	0.764	1787	149.8	-140.7
	4. u = -1.5	VMAG = 58.7	HMAG = 58.1	
π	73.7	0.519	183.0	-138.4
p	0.93	1775	183.0	-140.7
	5. u = -1.9	VMAG = 50.0	HMAG = 49.6	
π	67.3	0.709	205.6	-138.7
p	1.055	1765	205.6	-140.7

B. Beam Momentum 150 GeV/c

	θ degrees	$dt/d\Omega_{lab}$ BeV^2/sr	$(\partial MM^2/\partial\theta)_p$ BeV^2/sr	$p(\partial MM^2/\partial p)_\theta$ BeV^2/sr
	1. u = -0.1	VMAG = 292	HMAG = 291	
π	141.45	0.0845	96.0	-270.5
p	0.122	7202	96.2	-281.5
	2. u = -0.5	VMAG = 205	HMAG = 202	
π	104.8	0.1727	212	-273.6
p	0.270	7182	212	-281.4
	3. u = -1.0	VMAG = 149.5	HMAG = 147.8	
π	85.4	0.323	299.8	-275.6
p	0.382	7157	299.9	-281.4
	4. u = -1.5	VMAG = 117.7	HMAG = 116.5	
π	74.0	0.519	366	-276.8
p	0.468	7131	366	-281.4

the "high-intensity" Barish[8] beam and the high-quality Reeder[9] beam. The most desirable compromise would be in the range of 4-5 μsr and 2-3% $\Delta p/p$. This would provide adequate precision, particularly if hodoscoped with scintillators, and would permit the NAL potential for high-intensity beams to be utilized.

IV. LARGE-ANGLE DETECTOR

We would propose for the detectors conventional wire-chamber or Charpak proportional-chamber arrays. Because of interferences at the larger angles of such a device with the beam, we would propose using a C magnet, although H magnets have been used successfully in previous experiments. For trigger purposes, a crude scintillator hodoscope would be added.

Rough design specifications and costs are given below:

Design Specifications	
Solid angle acceptance	1/2 steradian
Precision of angle measurement* for ± 0.1 GeV^2 in MM^2	± 1 mrad
Precision of momentum measurement** for 0.2 GeV^2 in MM^2	$\pm 0.15\%$
C-Magnet aperture	1 m $\times$ 2 m
C-Magnet depth	1 m
Maximum field	12 kG
Weight of magnet is ~150 tons	

Estimated Costs	
Magnet and power supplies	\$250 K
Wire planes and electronics	\$250 K
Rotatable carriage	\$ 50 K
TOTAL	\$550 K

V. SMALL-ANGLE SPECTROMETER

Figures 4a, b, and c show the schematic layout of the spectrometer. The first focusing element is a quadrupole triplet. (A doublet would not provide the right aspect ratio to match the aperture of the bend magnet.) The triplet provides a parallel beam in the bend plane and in the vertical plane focuses rays emerging from the target to a point downstream. Deflection is provided by a ten-meter bend magnet which is followed by a single quad which focuses the beam horizontally to the momentum plane and leaves the vertical beam unchanged.

The quad bores are 6 inches for the first quad set and 8 inches for the single quad. Momentum is measured by scintillators or wire chambers at the momentum focus. Vertical angles at the exit of the single quad and horizontal angles are

*For t-channel processes, the angle is important. The momentum needs to be much less precisely determined.

**For u-channel processes, the momentum information is the most important.

measured by combining the information from the momentum plane and horizontal angle plane. Again, the use of wire chambers, Charpak planes, or scintillators is according to taste. Scintillators are by no means precluded, as only about twenty elements are required at each of the three detection planes. If long targets are used, the depth of field and target angle corrections will be of the order of a percent. However, the production point in the target is known from the large-angle spectrometer, and the corrections, if necessary, can be easily made.

A large number of elegant tricks are possible with such a setup. For instance, a field lens placed at the momentum focus would provide another focal plane downstream for the production angles. Additional planes could be used to provide redundancy. At this stage, however, the simple design of Fig. 4 seems entirely adequate.

The magnets and detectors would mount on a 100 meter long rotatable carriage (or carriages). The same magnets and quads could be arranged to provide a 100-μsr spectrometer up to 75 GeV/c or a 15-μsr spectrometer good to 200 GeV/c.

The overall alignment problems (~ 1/2 mm tolerance) are comparable to those of the SLAC spectrometers. At any time, the high fluxes provided by elastic πp scattering could be used to check the alignment and calibration. The alignment could also be checked with a simple optical system attached to the carriage, or by a stretched wire.

As this is a horizontal bend system without the thousand-ton shield caves of the SLAC vertical bend systems, we do not believe that carriages will present any insurmountable engineering problems.

Second-order aberrations are proportional to the aperture. In general, well-constructed symmetric quads are free of second-order aberrations, and we would expect small-aperture spectrometers either not to need appreciable corrections, or to be easily correctable.

We give the parameters for two typical configurations. We have used the results of E. J. N. Wilson[10] to obtain these estimates.

200-BeV System

Ω = 15 μsr		
Drift	15 meters	
Q_1	3 meters	15 cm bore quad (10 kG)
Q_2	6 meters	15 cm bore quad (10 kG)
Q_3	3 meters	15 cm bore quad (10 kG)
Drift	1 meter	
Bend	10 meters	20 cm × 10 cm, bend 30 mrad (20 kG)
Drift	1 meter	
Q_4	3 meters	18 cm bore quad (10 kG)

200-BeV System (Continued)

Drift to momentum plane	35 meters
	77 meters (Total)

Vertical dispersion	~ 2 cm per mrad
Momentum dispersion	0.9 cm per 1% $\Delta p/p$
Precision	±0.2% for 1 mm object

Angular demagnification is 3:1 in horizontal dimension and 2:1 in vertical dimension permitting a good match to a DISC Cerenkov counter.

75-BeV System

Ω = 100 sr	
Drift	6 meters
Q_1	3 meters, 15 cm bore quad (10 kG)
Q_2	6 meters, 15 cm bore quad (10 kG)
Q_3	3 meters, 15 cm bore quad (10 kG)
Drift	1 meter
Bend	10 meters 80 mrad (20 kG)
Drift	1 meter
Q_4	3 meters, 18 cm bore quad (10 kG)
Drift	22 meters
	55 meters (Total)

Momentum dispersion	~1.2 cm per 1% $\Delta p/p$
Vertical angle dispersion	~ 2 cm per mrad
Precision	±0.2% for 1 mm object

Small-Angle Spectrometer Components

Item	Comments		Cost*
Bend M1	10 meters long 10 cm vertical by 20 cm horizontal (10 tons)		$50 K
Q_1	3 meters	15 cm bore	$20 K
Q_2	6 meters	15 cm bore	$40 K
Q_3	3 meters	15 cm bore	$20 K
Q_4	3 meters	20 cm bore	$30 K
		Subtotal	$160 K
		Electronics Counter	$250 K
		Carriage	$140 K
		Total	$550 K

*Based on Frauenfelder and Wenzel, Summer Study 1968, Vol. II, p. 291.

VI. CROSS SECTIONS AND RATES

We believe that it should be possible to study diffractive processes up to energies of about 175 BeV. Both the cross sections and acceptance of our proposed coincidence devices remain roughly constant. At beam intensities as low as 10^6 particles per burst and with a 1-meter liquid hydrogen target, we can measure $d\sigma/dt$ as low as 10^{-2} $\mu b/(BeV/c)^2$ at a rate of ten events per hour. Forward-diffraction cross sections should be of the order of 3×10^4 $\mu b/(BeV/c)^2$. Accordingly, all the forward diffractive processes should be easily measurable up to energies close to 200 BeV and over many decades.

u-channel processes such as $\pi^- p$ have cross sections around u = 0 of the order of:

$$d\sigma/dt = 300/E^2 \mu b/(BeV/c)^2.$$

The acceptance in t remains roughly constant. At 130 BeV, the forward $d\sigma/dt$ would be approximately 1.7×10^{-2} $\mu b/(BeV/c)^2$ close to the limits of measurability, with beams of 10^6 pions per burst. If the beam is opened up to accept the order of 10^8 pions per burst, the cross section could be followed for two decades. As we have shown previously, it should be perfectly possible to operate with wide-open input beams at this level.

For particles such as $\bar{p}$, $\bar{K}$, and an opened up beam with 10^8-10^9 π's per beam and 10^6-10^7 kaons and antiprotons could measure over two decades of cross section at energies up to 100 BeV.

For most of these processes, it should be entirely feasible to make polarized-target measurements up to 75 BeV.

APPENDIX I. SCALING LAWS FOR SPECTROMETERS AND BEAM TRANSPORT SYSTEMS

As we show below, it is possible to take a given existing transport system and scale its parameters. The quantities we shall be interested in are:

Solid angle	Ω
Length	L
Cost	C
Relative quad bores	B
Relative quad gradients	G
Momentum	p
Momentum resolution	R

The transfer matrices of the system are of the form

Drift
$$\begin{vmatrix} 1 & L \\ 0 & 1 \end{vmatrix} \begin{vmatrix} z \\ \Theta \end{vmatrix}$$

Focusing
$$\begin{vmatrix} \cos\left(\frac{z_0}{\lambda}\right) & \lambda \sin\left(\frac{z_0}{\lambda}\right) \\ \frac{1}{\lambda} \sin\left(\frac{z_0}{\lambda}\right) & \cos\left(\frac{z_0}{\lambda}\right) \end{vmatrix}$$

Defocusing
$$\begin{vmatrix} \cos h\left(\frac{z}{\lambda}\right) & \lambda \sin h\left(\frac{z}{\lambda}\right) \\ \frac{1}{\lambda} \sin h\left(\frac{z}{\lambda}\right) & \cos h\left(\frac{z}{\lambda}\right) \end{vmatrix}$$

where z is the displacement down the system. If natural units of z/λ are used and lengths are scaled as $\alpha\lambda$ (both L and z_0) the transfer matrices change identically and therefore the spectrometer scales with unchanged focusing properties; λ is given by:

$$\lambda \propto \sqrt{\frac{p}{G}} \quad . \tag{1}$$

In order to optimize the system, we let the relative gradients in the quads vary as:

$$G \propto p^n, \tag{2}$$

where n is an arbitrary parameter. Then the relative bore is given by the maximum field permitted at the pole tips, i.e.,

$$B \propto G^{-1} \propto p^{-n}. \tag{3}$$

The length L is proportional to λ, the scale factor, or

$$L \propto p^{(1 - n)/2}. \tag{4}$$

Assuming the cost C is proportional to the volume of magnetic field

$$C \propto L \cdot A^2 \propto p^{(1 - 5n)/2}. \tag{5}$$

The solid angle Ω accepted is

$$\Omega \propto A^2/L^2 \propto p^{-(1 + n)}. \tag{6}$$

Combining (5) and (6) gives the result

$$\Omega \propto C^{2/5} p^{-6/5}. \tag{7}$$

Combining (4) and (5) gives

$$L \propto C^{1/5} p^{2/5}. \tag{8}$$

The second-order circle of confusion in the momentum plane is proportional to the solid angle, and the momentum dispersion is proportional to L/p. Therefore, the resolution R is given by:

$$R \propto \frac{L}{p} \propto p^{3/5}/C^{1/5}, \tag{9}$$

i.e., the system improves at high momentum unless absurd sums of money are expended.

APPENDIX II. PARAMETERS FOR DISC CERENKOV COUNTER TO DISCRIMINATE PROTONS FROM PIONS AT 150 GeV

Length	15 m
Cerenkov angle	12 mrad
No. of photoelectrons	~10
Angular separation of pions from protons in counter	2 mrad
Radius	20 cm

APPENDIX III. SPECIAL PROBLEMS IN π^+p u-CHANNEL ELASTIC SCATTERING

At small $|u|$ values, the primary π beam and scattering protons are only marginally separated. At high $|u|$ values there are no unusual problems.

At the highest energies and for small $|u|$ values we would rely on the use of one or more DISC Cerenkov counters to identify the protons coming down the spectrometer arm and would use 200-megacycle scintillator hodoscopes (as in the SLAC spectrometers) to split the information if rates of the order of 10^8 π's per burst were being used.

There do not appear to be any insurmountable problems to obtaining u-channel cross sections out to 130 BeV in this reaction.

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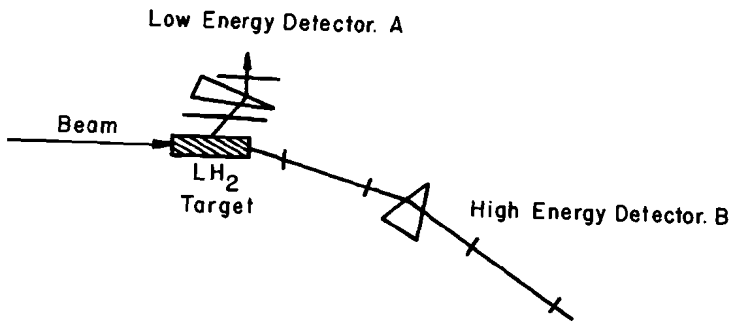

Fig. 1. Schematic of the detection system.

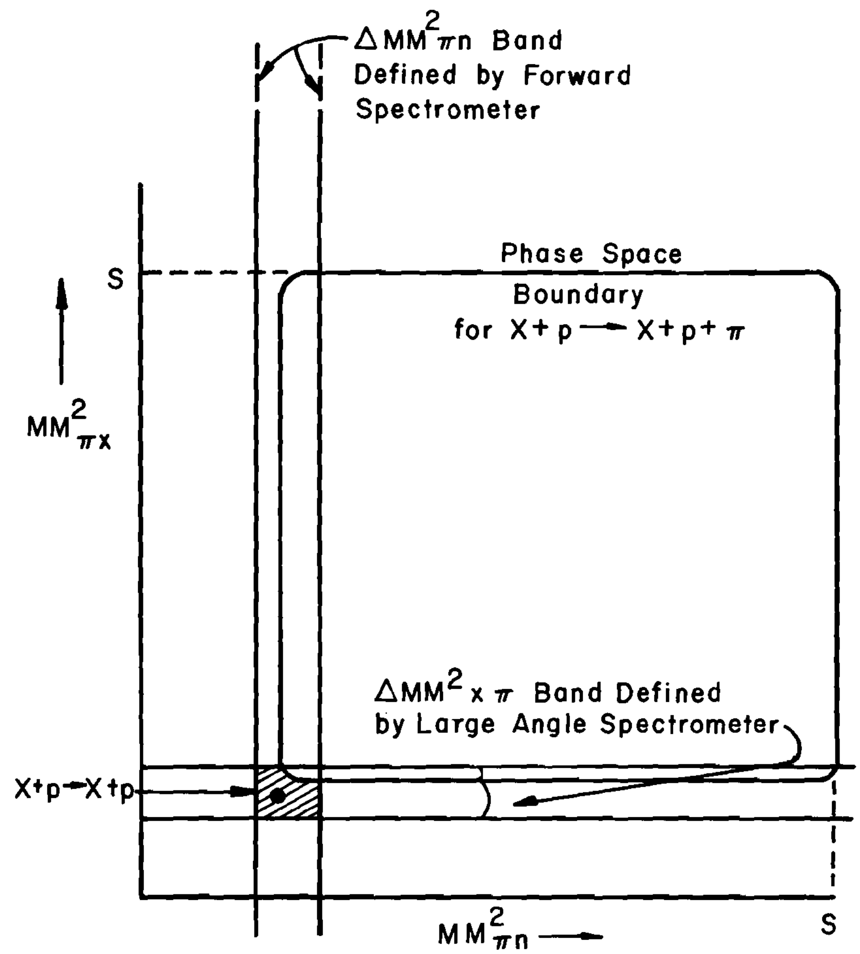

Fig. 2. Diagrammatic representation of the Dalitz plot and the spectrometer missing mass bands.

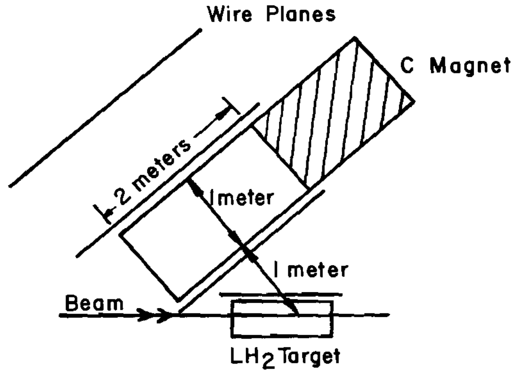

Fig. 3. Large-angle spectrometer setup.

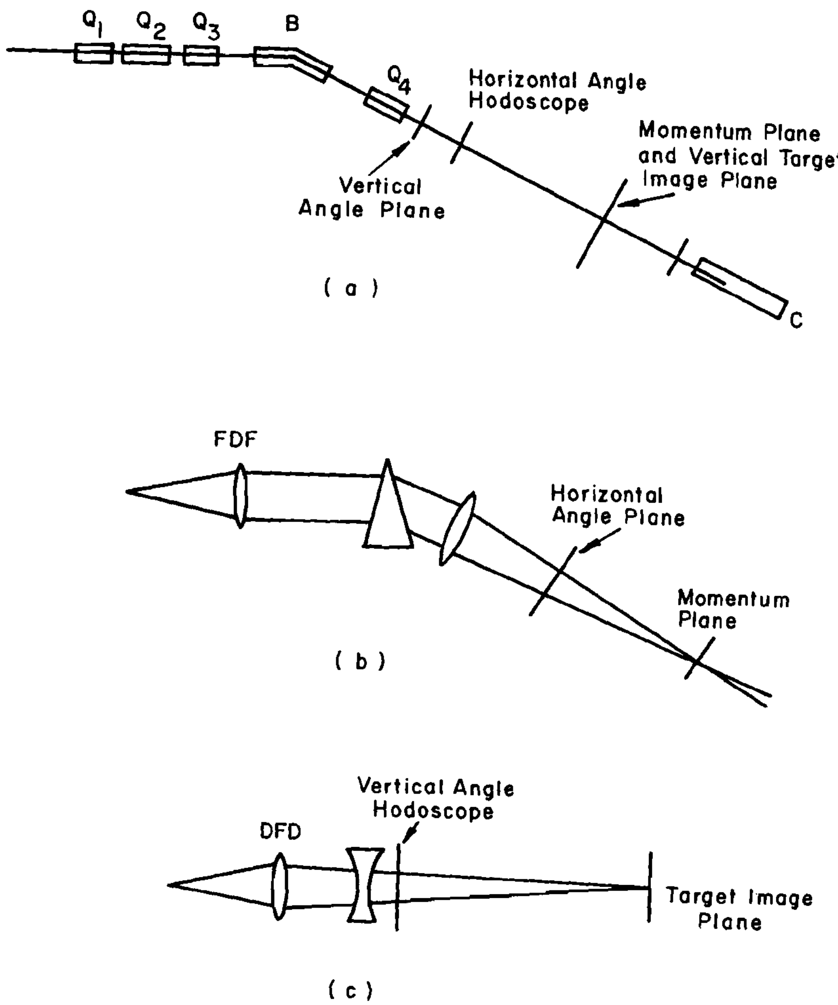

Fig. 4. The focusing spectrometer. (a) Beam layout. (b) Equivalent optics in horizontal plane. (c) Equivalent optics in vertical plane.

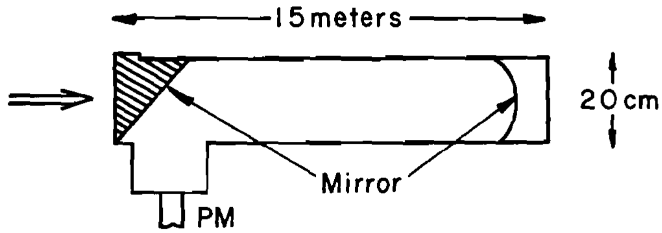

Fig. 5. Large DISC Cerenkov counter.