MEASUREMENT OF π^o MOMENTUM WITH METAL PLATE ARRAYS IN THE 25-FT BUBBLE CHAMBER

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ABSTRACT

Measurement of π^o momentum with metal plate arrays in the 25-ft bubble chamber is considered. If both γ's from π^o decay convert in the array, a momentum measurement on each pair in the first gap following conversion, combined with the measured photon angles, permits a one-constraint fit to $\pi^o \rightarrow 2\gamma$. For π's in the momentum range 1-10 GeV/c, a 2-meter long array of 9 plates, each 0.22 radiation lengths thick, gives reasonable errors. The median errors in momentum and angle for a 5 GeV/c π^o are $\Delta p_\pi / p_\pi$ = 7% and $\Delta \zeta_\pi$ = 0.2 deg, assuming a setting error of 500μ and 40 kG field. Several possible arrangments of the arrays in the chamber are considered.

I. POSSIBLE CONFIGURATIONS FOR A PLATE ASSEMBLY

The array must be viewed by at least two cameras whose stereo axis is parallel to the horizontal axis of the plates as in Figs. 1a and 3a. The plates must be tilted from the vertical direction as in Figs. 1b and 3b, so that the incident light is reflected by the Scotchlite.

Some possible configurations are discussed below:

1. The plates are mounted in the third spherical section of the chamber. This arrangement necessitates two camera ports in the third section (quite feasible according to W. Fowler). The assembly is 2 meters in length parallel to the beam and subtends an area 2.5 × 2.5 sq meters perpendicular to the beam direction.

The front plate is coated with Scotchlite. As seen in Fig. 1b, the maximum angle of incidence for cameras 2 and 3 is 63°. Chamber illumination is diminished when this angle is greater than 60°; clearly this angle can be reduced by moving the array forward into the middle section.

The array subtends an angle of about 35° in the horizontal and vertical planes at a typical interaction vertex and should suffice for a wide range of strong-interaction studies.

2. In this V-shaped array, (Fig. 2), a much wider range of π^0 emission angles is encompassed, and no additional camera ports are required. Also, this arrangement could probably be made more rigid than either A or C. It does, however, occupy a considerable volume of the chamber.

3. The two 1-meter arrays in Fig. 3 are designed to detect π^0's emitted at about 90°. They are viewed by cameras 2, 6 and 3, 5 respectively. The inner sides of the array are coated with Scotchlite.

This arrangement would probably be used in conjunction with 1.

II. DESIGN OF THE PLATE SYSTEM

We refer to the study by Trilling in SLAC 5, (Appendix 3).[1] We define the following parameters for a parallel plate array:

- t - thickness of each plate
- d - separation of the plates
- D - length in meters of the array
- L_R - radiation length of plate material
- n - number of radiation lengths in array
- ϵ - setting error in bubble chamber in microns
- H - magnetic field in kG.

When a photon converts in a plate, its momentum is estimated from the measured curvature of the electron-positron pair in the following gap. Trilling estimates the uncertainty in photon momentum as

$$\frac{\Delta p}{p} = \left[\left(\frac{0.6t}{L_R} \right)^2 + \left(\frac{2.0 \times 10^{-4}\, \epsilon p}{d^2 H} \right)^2 \right]^{1/2} . \qquad (1)$$

This formula takes into account:

1. setting error,
2. uncertainty as to where in the plate the conversion of the photon occurred, and
3. the spread in momentum due to fluctuations in the radiative energy loss of the electron and positron pair as they emerge from the plate.

The error due to multiple scattering in hydrogen can be neglected for $p_\gamma > 400$ MeV/c.

Trilling has shown that, for an array characterized by a given n and D, the value of t which minimizes $\Delta p/p$ for a photon of momentum p (GeV/c) is:

$$t/L_R = 7.4 \times 10^{-2} \left(\frac{\epsilon p}{H}\right)^{1/3} \left(\frac{n}{D}\right)^{2/3}, \tag{2}$$

and

$$d = \frac{Dt}{nL_R},$$

assuming d >> t. The corresponding value of $\Delta p/p$ is:

$$\left(\frac{\Delta p}{p}\right)_0 = 5.7 \times 10^{-2} \left(\frac{\epsilon p}{H}\right)^{1/3} \left(\frac{n}{D}\right)^{2/3}. \tag{3}$$

In Table I we list t, and $(\Delta p/p)_0$ for a 2 meter array, optimized for a series of photon momenta between 1 and 50 GeV/c. The quantities are calculated for 2 and 4 radiation lengths, with $\epsilon = 500\mu$ and H = 40 kG. The $(\Delta p/p)_0$ values for n = 4 are valid for any array with n/D = 2, and those for n = 2 for any array with n/D = 1.

Table I. Momentum Accuracy of Gamma Rays.

D = 2 m H = 40 kG $\epsilon = 500\mu$

p_γ (GeV/c)	t/L_R		No. of Plates		$(\Delta p/p)_0$, percent	
	n =		n =		n =	
0.5	0.13	0.22	15	18	10	17
1.0	0.17	0.26	12	15	13	21
2.5	0.22	0.36	9	11	18	28
5.0	0.29	0.45	7	9	22	35
10.0	0.33	0.57	6	7	28	45
20.0	0.50	0.67	4	6	35	56
50.0	0.67	1.00	3	4	48	76

III. π^0 MOMENTUM AND ANGLE ERRORS AFTER CONSTRAINT

If the error in photon direction is assumed negligible the error in π^0 momentum and direction after constraint is:[1]

$$\frac{\Delta p_\pi}{E_\pi} \simeq \frac{\Delta p_\pi}{p_\pi} = \cos\theta^* \left[\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2\right]^{-1/2}, \tag{4}$$

and

$$\Delta\zeta = \frac{m_\pi}{p_\pi} \sin\theta^* \left[\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2\right]^{-1/2} , \tag{5}$$

where θ^* is the π^o decay angle in its rest frame, m is the π^o mass, and p_1, p_2 are the photon momenta measured in the laboratory. The momentum uncertainty goes to zero for $\theta^* = 90°$ because this decay configuration corresponds to equal photon momenta and minimum opening angle between the two photons in the laboratory. Then the constraint equation $m_\pi = 2^{-1/2}E_\pi(1 - \cos\theta_{min})^{1/2}$ implies that an exact measurement of θ_{min} gives an exact value of p_π.

The majority of π^o's in both hadronic and neutrino interactions are expected to be in the momentum range 1-10 GeV/c. We consider now how to determine the parameters of the array for this range of momenta, assuming that the length D of the array is fixed at 2 meters, H = 40 kG, and $\epsilon = 500\mu$. We consider only the arrays in Figs. 1 and 2. The arrays in Fig. 3 should be designed for a lower range of π^o momenta.

Figures 4a and 4b show the median values of $\Delta p_\pi p_\pi$ and $\Delta\zeta$ for $p_\pi = 2$, 5 and 10 GeV/c, optimized for photon momenta from 1 to 20 GeV/c. (The distribution in $\cos\theta^*$ is flat, so that the median values of $\cos\theta^*$ and $\sin\theta^*$ are 0.5 and 0.87 respectively.) It seems reasonable to optimize the array for $p_\gamma = 2.5$ GeV/c.

Figure 5 shows the median errors, $(\Delta p/p)_M$ and $(\Delta\zeta)_M$, for a 5 GeV/c π^o in the $p_\gamma = 2.5$ GeV/c optimized array, plotted against n, the number of radiation lengths. For n = 2, we have $(\Delta p/p)_M = 6.3\%$ and 79% conversion efficiency; for n = 4 the conversion efficiency is 95% and $(\Delta p/p)_M = 10\%$. The choice of n = 2 represents a good compromise between the demand for high conversion efficiency, low errors, and low rate of background interactions in the plates.

From Table I we then find that the array should have 9 plates of thickness 0.22 radiation lengths.

It can easily be verified, either by calculation or by inspection of Eq. (1), that in an array optimized for $p_\gamma \approx p_\pi/2$ the quantity

$$\left[\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2\right]^{1/2} ,$$

is essentially independent of $\cos\theta^*$ and $\approx \sqrt{2}\ (p_\gamma/\Delta p_\gamma)_0$ where $(p_\gamma/\Delta p_\gamma)_0$ is the optimized error in Eq. (3). Then

$$\left(\frac{\Delta p_\pi}{p_\pi}\right)_M = 2^{-3/2}\left(\frac{\Delta p_\gamma}{p_\gamma}\right)_0 , \tag{6}$$

and

$$\Delta\zeta_M = 0.86\ m_\pi \left(\frac{\Delta p_\gamma}{p_\gamma}\right)_0 \Big/ \sqrt{2}\ p_\pi , \tag{7}$$

showing that the constrained value of $(\Delta p_\pi/p_\pi)_M$ is, on the average, one-third of the unconstrained error. The unconstrained error is $1/\sqrt{2}\ (\Delta p_\gamma/p_\gamma)_0$ when the π^0 decays symmetrically and $\sqrt{2}\ (\Delta p_\gamma/p_\gamma)_0$ when the π^0 decays fore-aft.

Combining Eqs. (3), (6), and (7) we get

$$\left(\frac{p_\pi}{p_\pi}\right)_M = 2.0 \times 10^{-2} \left(\frac{\epsilon p_\gamma}{H}\right)^{1/3} \left(\frac{n}{D}\right)^{2/3} , \text{ and}$$

$$(\Delta\zeta)_M = 3.5 \times 10^{-2}\ \frac{m_\pi}{p_\pi} \left(\frac{\epsilon p_\gamma}{H}\right)^{1/3} \left(\frac{n}{D}\right)^{2/3} ,$$

showing that the momentum and angle errors vary as $n^{2/3}$ (Fig. 5) and as $(1/H)^{1/3}$.

It should be emphasized that the actual errors will probably exceed those calculated here. We have neglected the error in photon direction, which we estimate will not exceed 0.05 degrees. Also, the error calculations were made for a parallel plate array, not realizable in practice. More serious is the possibility of pressure waves between the plates, which would increase the effective value of the setting error ϵ. Such turbulence has been observed close to metal plates in some experiments.[2]

IV. COST AND MATERIAL

We consider a 2 meter long array consisting of 9 plates each 2.5 × 2.5 meters square and 0.22 radiation lengths thick. If aluminum is used, the plates are 2 cm thick and the weight of the array is 3 tons. Cost of the material is about \$10 K and fabrication about \$25 K (D. Jovanovic). Cost of assembling the arrays in the chamber will probably be ~ \$100 K.

The plates might also be constructed from copper, 3 mm thick, or a lead-aluminum sandwich, (1.3 mm of lead between 0.75 mm sheets of aluminum). These structures are superior insofar as the pair production vertex and hence the photon direction can be more precisely located.

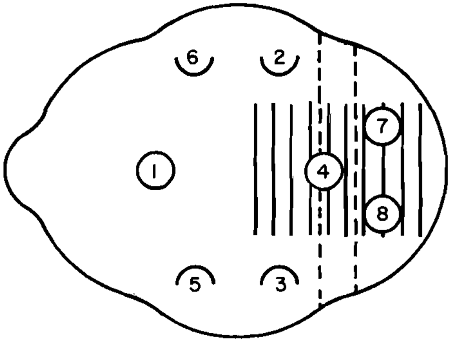
Cost of the two additional portholes for configuration A would be in the neighborhood of $100 K.

W. Fowler and F. R. Huson supplied much of the information in this note.

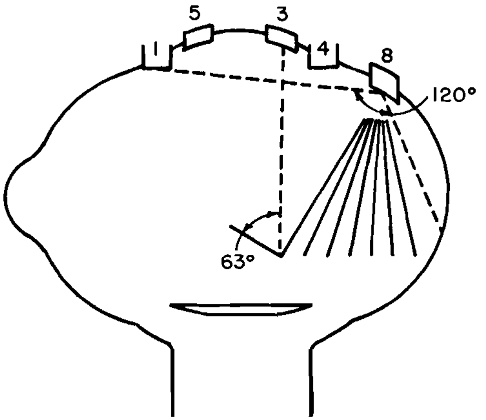
REFERENCES

[1] G. Trilling, The Use of Hydrogen Bubble Chambers at SLAC, SLAC Report No. 5, 1962.

[2] F. Huson and A. Erwin, private communication.



Top View



Side View

Fig. 1. Two-meter plate array in third spherical section of 25-foot chamber.

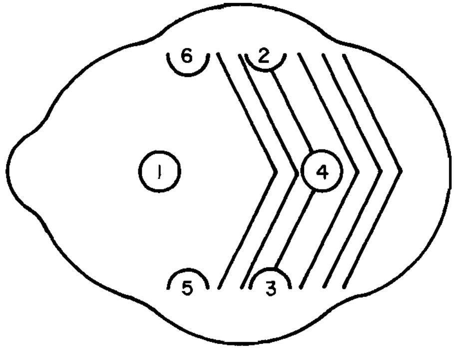


Fig. 2. V-shaped plate array viewed by cameras 2, 3, and 4.

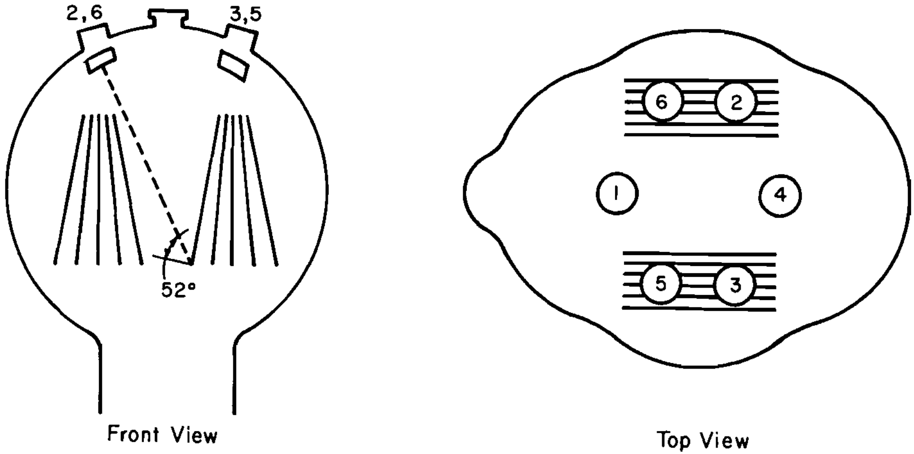


Fig. 3. One-meter arrays for detection of π^0's emitted near 90°.

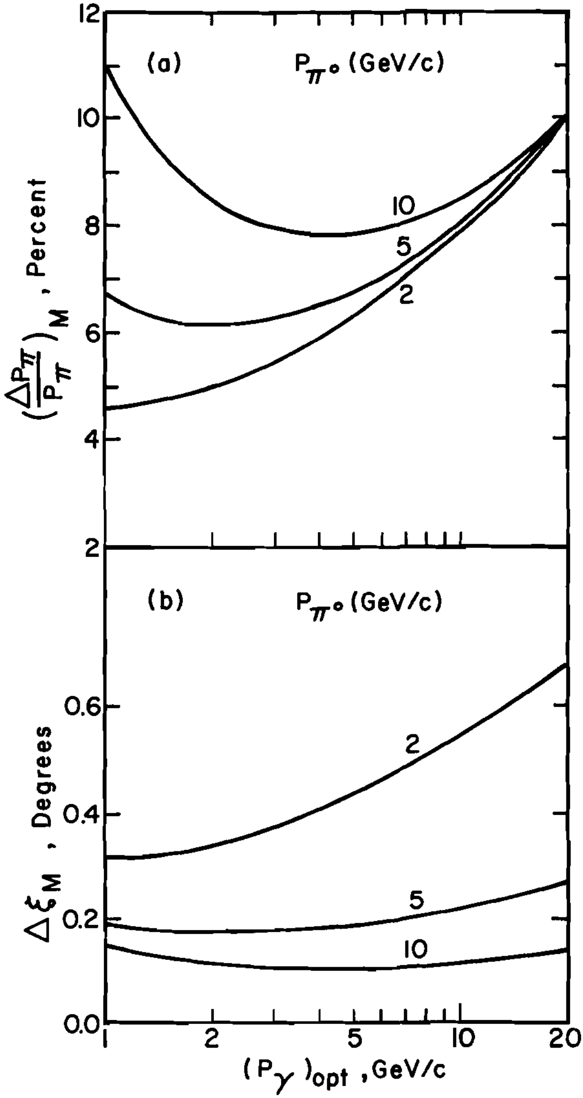


Fig. 4. (a) Median momentum error $(\Delta p_\pi/p_\pi)_M$, and (b) median angle error $\Delta\xi_M$ as a function of the photon momentum for which the plate array has been optimized. Curves are given for p_π = 2, 5, and 10 GeV/c. (The array is 2 meters long, 2 radiation lengths thick with ϵ = 500μ, H = 40 kG.)

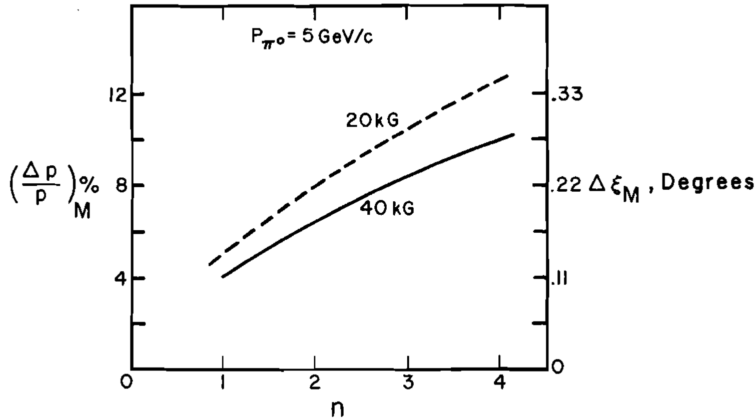


Fig. 5. Median momentum error $(\Delta p_\pi/p_\pi)_M$ for a 5 GeV/c π^0 as a function of n, the number of radiation lengths in the array. The array is 2 meters long, optimized for p_γ = 2.5 GeV/c with ϵ = 500μ.