NEUTRINO EXPERIMENTS: TOTAL CROSS SECTION, INELASTIC CROSS SECTION, AND ADLER TEST

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ABSTRACT

The possibilities are examined of measuring the neutrino-nucleon total cross section, the inelastic cross section $d^2\sigma/dq^2 d\nu$ where q^2 = magnitude of the momentum transfer at the lepton vertex, $\nu = E_{\text{neutrino}} - E_{\text{final lepton}}$ and of performing the Adler test relating neutrino events with low q^2 to pion production data. It is concluded that the 25-ft bubble chamber is an excellent instrument to use for these measurements but must be used in a double chamber mode in order that gamma rays can be converted and the energy of individual neutrino events obtained. If this is done it would appear possible to obtain the total cross section out to the neighborhood of 100 BeV with perhaps 20% errors. This will allow one to set a "one way limit" on the mass of the intermediate boson of around 15 BeV. In the inelastic cross sections we can search to about $Q^2 = 15\ (\text{BeV}/c)^2$ for $40 < E_{\text{neutrino}} < 60$ BeV and in terms of $x = Q^2/2M\nu$ we can search down to $x \sim 0.005$ with decent statistics. The Adler test can be performed with about 10% statistics.

I. THE NEUTRINO-NUCLEON TOTAL CROSS-SECTION MEASUREMENT AS A FUNCTION OF ENERGY

At present the νN cross section is known to be rising linearly with ν lab energy beyond 5 BeV. It will be exceedingly interesting to see whether this rise persists up to 100 BeV, or whether it levels off or otherwise changes and hence shows structure in the 4-fermion interaction.

For instance, Bjorken has shown that a variation from a linear rise would probably correspond to a finite intermediate boson mass or a violation of the scale invariance of the inelastic form factors found at SLAC in electron scattering. We shall return to this later.

Table I shows event rates in the 25-foot bubble chamber calculated with the standard ν flux for this study and with two assumptions on the behavior of the cross section:

1. The cross section continues rising linearly up to 100 BeV [$\sigma = 0.8\,E\,(\mathrm{BeV}) \times 10^{-38}$].

2. The cross section rises as above to 10 BeV and then is flat at the 10 BeV value at higher energies.

As can be seen one can determine the cross section with a statistical error of 10% up to past 100 BeV under assumption (1) and to about 90 BeV under assumption (2). The major errors are, however, not statistical but systematic. The problem is to determine the energy of the neutrino. At 80 BeV a systematic error in the energy determination of 5% will cause a 40% change in the total cross section. This occurs because of the steeply falling flux distribution as a function of energy at the higher neutrino energies.

Table I.

Total cross section ν event rate per 10^6 pictures on one type of nucleon (n or p) as a function of energy. The 25-foot chamber estimate assumes a 21-foot fiducial length and the 14-foot assumes an 11-foot fiducial length. The standard ν flux is assumed.

Assumption 1: $\sigma_{tot} = 0.8\,E \times 10^{-38}\,\mathrm{cm}^2$.

Assumption 2: $\sigma_{tot} = 0.8\,E \times 10^{-38}\,\mathrm{cm}^2$, $E < 10$ BeV

$= 8 \times 10^{-38}\,\mathrm{cm}^2$, $E \geq 10$ BeV

	Assumption 1			Assumption 2		
E BeV	25-foot Hydrogen or Deuterium	25-foot Neon off 1 kind of nucleon only	14-foot Hydrogen or Deuterium	25-foot Hydrogen or Deuterium	25-foot Neon off 1 kind of nucleon only	14-foot Hydrogen or Deuterium
5-10	2.75×10^5	2.75×10^6	1.8×10^5	2.75×10^5	2.75×10^6	1.8×10^5
10-15	2.9	2.9	1.9	2.3	2.3	1.5
15-20	1.35	1.35	0.9	0.78	0.78	0.52
20-25	0.54	0.54	0.36	0.24	0.24	0.16
25-30	0.26	0.26	0.17	9.3×10^3	9.3×10^4	6.2×10^3
30-35	0.16	0.16	0.11	5.1	5.1	3.4
35-40	0.11	0.11	17.6×10^3	3.1	3.1	2.0
40-50	17.6×10^3	17.6×10^4	11.0	3.6	3.6	2.6
50-60	11.4	11.4	7.6	2.0	2.0	1.4
60-70	7.4	7.4	4.8	1.1	1.1	0.74
70-80	3.8	3.8	2.6	0.52	0.52	0.34
80-90	1.6	1.6	1.1	0.19	0.19	0.13
90-100	0.6×10^3	0.6×10^4	0.42×10^3	6.2×10^1	6.2×10^2	4.2×10^1
Total	8.4×10^5	8.4×10^6	5.7×10^5	6.4×10^5	6.4×10^6	4.3×10^5

In order to obtain total cross-section measurements, it would thus seem necessary to obtain an accurate measurement of the neutrino energy. One either must use a tagged neutrino beam and spark chambers or must measure (or estimate) the energy which goes into neutrals. However, Frisch, using reasonably optimistic assumptions, finds that with a beam of 10^7 π's and K's he can obtain a tagged neutrino beam of 110 ± 10 BeV/c which gives 2-1/2 events/day in a 10-meter long iron detector if the cross section keeps rising linearly. It would seem that techniques other than tagging might prove favorable.

In the 1968 CERN propane chamber neutrino experiment, it was found that the visible energy transferred to hadrons had to be multiplied by 1.34 to account for neutrals missed in that chamber (~20% gamma-ray detection efficiency). The 25-foot chamber filled with hydrogen is slightly smaller than the CERN chamber in interaction lengths and in radiation lengths. Also at NAL energies the fraction going into neutrals will be somewhat higher. We take 1.45 as a guess for this. Since the muon takes 50 to 60% of the energy of the neutrino, this means that the visible energy must be multiplied by about 1.2 to get E = neutrino energy. If the systematic error on the cross section is to be held to 10%, we must hold the systematic error in the energy going into neutrals to about 6%. Note: this is the systematic error on the estimate not the random error on the measurement of each event. It would seem unreliable to obtain estimates to this accuracy when only a small fraction of the energy going into neutrals materializes in the chamber. Hence, we are led to consider detectors with enhanced gamma-ray detection efficiency.

In a double-chamber arrangement in the 25-foot chamber, with a diaphragm after about 14 feet with hydrogen on the front side and neon on the back side, (or in a chamber with a large diameter cylinder of hydrogen going about 14 feet into the chamber),most of the energy going into gamma rays can be measured and neutrons often detected. (Most of the neutral energy at CERN went into γ's not neutrons.) The measurement accuracy per gamma will be around 25%. It would seem reasonable to be able to obtain a systematic error of about 1/5 this amount and hence obtain the requisite accuracy.

Another limit on accuracy comes from the knowledge of neutrino flux. According to H. Wachsmuth (who supervised the measurement of the ν flux at CERN by means of monitors in the muon shield), it should be possible to know the flux at high ν energies to 10 or 15% if beam-survey data is available.

The discussion of $\bar{\nu}$ total cross sections proceeds similarly except that the rates are smaller; $\bar{\nu}$ and ν cross sections should be similar at high energies and Table II shows rate results for $\bar{\nu}$ cross sections under assumption 1 and assumption 2 given above.

Table II. Total Cross-Section Anti-Neutrino Event Rate With the Same Assumptions as Table I.

	← Assumption 1 →			← Assumption 2 →		
E	25-foot Hydrogen or Deuterium	25-foot Neon off 1 kind of nucleon only	14-foot Hydrogen or Deuterium	25-foot Hydrogen or Deuterium	25-foot Neon off 1 kind of nucleon only	14-foot Hydrogen or Deuterium
5-10	9.9×10^4	9.9×10^5	6.5×10^4	9.9×10^4	9.9×10^5	6.5×10^4
10-15	12.7	12.7	8.3	10.0	10.0	6.6
15-20	4.8	4.8	3.2	2.8	2.8	1.85
20-25	1.9	1.9	1.25	0.85	0.85	0.56
25-30	1.05	1.05	0.68	0.37	0.37	0.25
30-35	0.5	0.5	0.34	0.16	0.16	0.11
35-40	0.26	0.26	0.18	7.4×10^2	7.4×10^3	5.0×10^2
40-50	0.37	0.37	0.23	7.6	7.6	5.2
50-60	0.20	0.20	0.13	3.4	3.4	2.4
60-70	0.16	0.16	0.10	2.3	2.3	1.6
70-80	8.4×10^2	8.4×10^3	5.8×10^2	1.2 .	1.2	0.75
80-90	3.2	3.2	2.2	0.38	0.38	0.26
Total	32×10^4	32×10^5	21×10^4	24×10^4	24×10^5	16×10^4

II. THE INELASTIC NEUTRINO NUCLEON CROSS SECTION

The electron inelastic scattering experiments[1] have been very fruitful. The form factors W_1 and W_2, unlike strong-interaction form factors, do not seem to fall off with increasing momentum transfer but at least one of them remains constant. The results place severe restraints[2, 3] on particle theories. The CERN propane neutrino experiment with very poor statistics finds similar results for the analogous form factors entering into the weak interaction. The variables of interest are E = energy of neutrino, q^2 = momentum transfer squared at the lepton vertex, and $\nu = E - E'$, where E' = energy of muon.

A general form for the differential cross section is then[3]

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{G^2}{2\pi}\frac{E'}{E}\beta(Q^2,\nu)\left[1 - \frac{Q^2}{4EE'} + \frac{\nu^2 + Q^2}{2EE'}\left(\frac{\sigma_R + \sigma_L}{2\sigma_S + \sigma_R + \sigma_L}\right) + \frac{(E + E')\sqrt{\nu^2 + Q^2}}{2EE'} \times \right.$$

$$\left. \times \left(\frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L + 2\sigma_S}\right)\right] ,$$

where

$$Q^2 = |q^2|$$

G = fermi coupling constant

$$\beta = \frac{\nu}{\pi M}\left(1 - \frac{Q^2}{2M\nu}\right)\left(2\sigma_S + \sigma_R + \sigma_L\right)$$

M = mass of nucleon

$\sigma_R, \sigma_L, \sigma_S$ = cross sections for right, left, and longitudinally polarized terms of weak current (W mesons if they exist). These are functions of q^2 and ν only and are greater than zero.

Furthermore, Bjorken[3] has shown that with rather weak assumptions $\sigma_R, \sigma_L, \sigma_S = 1/Q^2 \times$ functions of $x \equiv Q^2/2M\nu$ only, in the limit $Q^2 \to \infty$, $\nu \to \infty$, x fixed. We will use as variables the dimension less quantities x and $y = \nu/E$, $x \leq 1$ which is the elastic cross-section limit. For large values of x the SLAC results show bumpy W shapes as one is in the resonance region but below x = 1 or 0.5 the data smooths out dramatically.

We can work out some useful kinematics to get a feeling for these reactions. We call the mass of the final hadron system M'. We have for high energy:

$Q^2 \simeq EE'\theta^2$ where θ is the lab angle of the final muon.

Experimentally from the CERN run, the muon takes off around 1/2 the energy of the neutrino on the average. Futhermore from SLAC and CERN results $d\sigma/dx$ shows that generally $x \leq 1/2$. We take $x = 1/4$ as an average. Plugging in we then find:

$$Q^2 \sim 1/4\, ME$$

$$\theta^2 \simeq M/2E$$

$$M'^2 = 3/4\, ME$$

$\phi = \theta$ where ϕ = angle of M' in lab

$$\gamma_{M'} = (E/3M)^{1/2} + (4M/3E)^{1/2}$$

As an example at 30 BeV $\theta \sim \phi \sim 7°$, M' ~ 5 BeV, $\gamma_M' \sim 3.5$. Hence the hadronic component will peak forward in the bubble chamber.

The sidewise gammas will be of low energy and excellent detection efficiency in this direction is not needed for the measurement of $d^2\sigma/dq^2d\nu$. Thus a diaphragm or large diameter cylindrical ($\sim 20\ m^3$) inner chamber is suitable for a double-chamber arrangement. On this point, I am in disagreement with the conclusions stated by

L. Stevenson in his abstract to SS-74 in which he states that a very narrow target will be needed for the $d^2\sigma/dq^2d\nu$ experiment. For study of individual channels, a narrow target may become necessary. However, for the first exploratory experiment I would perfer a large rate to get an overall view. Afterwards, on the basis of examining the first experiment, a second exposure with a narrow target may well be desirable.

In the first part of this note we referred to a bend in the total cross section possibly indicating an intermediate boson. We can now make some crude estimates of the size of this effect. If there is an intermediate boson, one would have a propagator term $F = \left[M_W^2/\left(Q^2 + M_W^2\right)\right]^2$. We took $Q^2 \sim 1/4$ ME as an average value above. Using this we can make the following crude table for the mean value of F as a function of M_W and E.

Table III. Values of F (E, M_W).

E/M_W (BeV) =	2	5	7	10	15	20
25 (BeV)	0.059	0.44	0.64	0.78	0.90	0.95
50	0.019	0.25	0.44	0.64	0.81	0.88
75	0.0093	0.16	0.33	0.53	0.74	0.84
100	0.0054	0.11	0.25	0.44	0.67	0.79

Bjorken has shown that the total cross section = const × E for $M_W = \infty$ and the factor of F with an appropriate average Q^2 would modify the total cross section. Hence, from the above table the linearity with energy of the total cross section will provide a one way test for an intermediate boson up to about 15 or 20 BeV with reasonable experiment errors on σ_{tot}; i.e. if the cross section is linear, the boson probably does not exist. If the cross section turns over, then it may indicate a boson or a failure in the universal behavior (scaling property) of σ_R, σ_L, σ_S. F will also modify $d^2\sigma/dq^2d\nu$ and allow more detailed tests of a breakdown there. Furthermore, one may obtain a still more sensitive test by looking only at events with high Q^2 as a function of energy.

The preliminary results from CERN are consistent with:

$$\frac{Q^2}{2\pi}\left(\sigma_R + \sigma_L + 2\sigma_S\right) = K_2 \sim \text{constant} \sim 0.9$$

$$\sigma_S \sim 0, \text{ i.e.,} \frac{Q^2}{2\pi}\left(\sigma_R + \sigma_L\right) = K_1 \sim \text{constant} \sim 0.9$$

$$\sigma_R \simeq \sigma_L$$

for $x \lesssim 1/2$ and up to 5 or 6 BeV in energy. We will use these numbers in order to estimate rates. For reference we note that for $1/2 < x < 1$, the values of K_1 and K_2 are given crudely by $0.9/8\,x^3$ for the CERN data.

Then

$$\frac{d^2\sigma}{dxdy} \simeq 0.9\,\frac{G^2 M}{\pi}\,E\left[\left(1 - y - \frac{Myx}{2E}\right) + \frac{y^2}{2} + 1.1\,K_3\,\frac{y}{2}\left(1 - \frac{y}{2}\right)\right].$$

If we integrate from y = 0.5 to 1 (to restrict ourselves to events with large ν) we have:

$$\frac{d\sigma}{dx} = 0.24\,\frac{G^2 M}{\pi}\,E\left(1 - \frac{9\,Mx}{13\,E} + 0.47\,K_3\right).$$

Table IV gives some typical neutrino-deuterium rates expected. We assume a double chamber with an 11-foot fiducial length of deuterium.

Table IV. Deuterium Rates.

E (BeV)	x	Δx	No. of events
50-60	0.33	0.033	100
60-70	0.33	0.033	66
50-60	0.05	0.005	16
20-25	0.05	0.005	72
5-100	0.005	0.0005	115

It is clear that we can go to quite small values of x and look at these processes at quite high neutrino energies.

If we ask what q^2 regions we can investigate, we can integrate $d^2\sigma/dq^2dx$ very crudely for $x < 0.5$ and find:

$$\frac{d\sigma}{dq^2} \simeq 0.9\,\frac{G^2}{2\pi}\left(\log\frac{ME}{q^2} - \frac{3}{4}\right).$$

From this we can see that, for example, for a range of neutrino energies between 40 and 60 BeV, we will have 180 events with $Q^2 = 15\ (\mathrm{BeV}/c)^2 \pm 5\%$. Hence we can search to about $Q^2 = 15\ (\mathrm{BeV}/c)^2$.

If we ask what sorts of limits we can set on K_3 we integrate $d^2\sigma/dxdy$ over x from say 0 to 0.5. We then get:

$$\frac{d\sigma}{dy} = 0.9\,\frac{G^2 ME}{2\pi}\, K_2 \left[1 + \left(\frac{-M}{8E} + \frac{K_3}{2K_2} - 1\right) y + \left(\frac{K_1}{2K_2} - \frac{K_3}{4K_2}\right) y^2\right].$$

Since there are many thousands of events, one should be able to obtain the term linear in y quite well, and find K_3 if it is non-zero, or set very small limits on it if it is 0. Bjorken has shown that a comparison of ν and $\overline{\nu}$ rates is also a sensitive test of $K_3 \sim \sigma_R - \sigma_L$ and does not use the assumptions of the asymptotic limits of the σ's.

The measurement problems are much the same as with the total cross section. The relative errors in q^2 are about the same as those in E and the relative error in ν is about twice that in q^2. Again a double-chamber arrangement would seem very useful. (A mixture of neon and hydrogen might be considered as an alternative, but the measurement accruacy is reduced and there is considerable problem determining which interactions occur in neon, which in hydrogen, and generally in comparing n and p cross sections which is of great interest.)

There are many other things which can be tested here. For instance

$$\sigma^{\nu p}_{S,R,L} = \sigma^{\overline{\nu} n}_{S,R,L} \quad ,$$

from saying the weak current is an isotopic vector. This relation can be tested in the inelastic experiments and one can see whether this isotopic vector structure is maintained at high momentum transfer.

If one looks at the momentum transfer to the recoil nucleon as well, one can hope to investigate further weak interaction properties. For instance, Bjorken has noted at the NAL seminar that this would distinguish between present theories some of which want W bosons to interact inside and others at the surface of the nucleon or outside it. Pais and Treiman have noted that these inelastic events are also very useful for locality tests of the weak vertex.[4]

One word might be said about specific channels. For πp and Kp interactions the region of 2.4 BeV/c incoming particle momentum has been especially fruitful for bubble-chamber analysis; many resonances can be made, but one has few enough open channels that enough events can be accumulated to analyze each individually. Since the muon takes around half the neutrino energy, this range in hadrons corresponds to about 4 to 8 BeV neutrinos. These energies cannot be explored with reasonable statistics at BNL but will be the domain of NAL (and Serpukov).

III. ADLER TEST

S. L. Adler[5] has shown that based on the PCAC hypothesis and CVC, there exists a relation between $\nu + N \rightarrow \mu + \alpha$ where α is any set of strongly interacting particles and $\pi + N \to \alpha$ provided that the muon is almost forward ($\cos\theta \sim 1$) and the momentum transfer, q^2, at the lepton vertex is quite small.

In terms of the previous formulas, the theorem says that as $Q^2 \to 0$, σ_R/σ_S and $\sigma_L/\sigma_S \to 0$ and $\sigma_S \to F_\pi^2/Q^2 \left(m_\pi^2/Q^2 + m_\pi^2\right)^2 \sigma_\pi$ where $F_\pi \sim 0.9\, m_\pi$ is the pion decay constant. This provides a critical test of the PCAC hypothesis. The CERN propane chamber results with low statistics find agreement within a factor of 2 but are limited by statistics. In the inelastic cross-section experiment described in Sec. II, enough events should be avilable to form a definitive test of PCAC and possibly to find out how fast the vector current contributions enter as q^2 is increased.

We note $Q^2 = -m_\mu^2 + 2E(E' - p'\cos\theta) \sim -m_\mu^2 + 2EE'(1 - \cos\theta)$. If we ask that $Q^2 < 0.10$ then $(1 - \cos\theta) < 0.11/2EE'$, and if $E' > 1$ BeV and $E > 10$ BeV, then $1 - \cos\theta < 0.005$. If the low Q^2 events also possess the indicated property of high Q^2 events that $d\sigma/dQ^2 \sim 1/(Q^2)^3$ then we should have adequate numbers of events. If we just scale up from the CERN results even without allowing for the higher energies at NAL, we would have more than 4,000 events with $\cos\theta > 0.975$ and $Q^2 < 0.1(\mathrm{BeV}/c)^2$ which is the most restrictive interval (10 events) of the CERN experiment. We expect that the effect of the higher energies at NAL is to put even more of the events in this interval. Hence the event rate should be quite adequate for this test even if we cut to $Q^2 < 0.01$ so $Q^2 < m_\pi^2$. However, if we integrate the Adler formula given above and assume $\sigma_\pi = 30$ mb, we find about 160 events in the hydrogen of a double chamber using $Q^2 < 0.01$ which is smaller than the above but can test PCAC to about 10% level.

APPENDIX I. DIFFERENT SETS OF NOTATION

There are several sets of notation for σ_R, σ_L, σ_S extant. I attempt to correlate some of them below.

The older notation used by Bjorken[3] and by the CERN experimental neutrino group in its Physics Letters article (submitted) was W_1, W_2, W_3. Below I list W_1, W_2, W_3 CERN Bjorken agrees with CERN for the neutrino case but for the analogous e-m scattering $W_{\text{Bjorken}} = W_{\text{CERN}}/M$*. I use the σ_R, σ_L, σ_S as defined by Bjorken in his 1969 Aspen lectures. These differ from the more standard ones used by Bjorken in his Physical Review article by $\sigma_{PR} = \sigma_{\text{Aspen}}/4\pi\alpha$,

$$W_1 = \frac{\nu M}{2\pi}\left(1 - \frac{Q^2}{2M\nu}\right)\left(\sigma_R + \sigma_L\right)$$

$$W_2 = \frac{\nu M}{2\pi}\left(1 - \frac{Q^2}{2M\nu}\right)\frac{1}{1 + \frac{\nu^2}{Q^2}}\left(2\sigma_S + \sigma_R + \sigma_L\right)$$

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*Bjorken's Eq. 5 in his Phys.Rev. article is inconsistent by a factor of M in the transformation from $d\Omega\, dE'$ to $dq^2 d\nu$. I use the $dq^2 d\nu$ expression as standard.

$$W_3 = \frac{M^2}{\pi}\left(1 - \frac{Q^2}{2M\nu}\right)\frac{1}{\sqrt{1 + \frac{Q^2}{\nu^2}}}\left(\sigma_R - \sigma_L\right)$$

(Q^2 is defined to be greater than 0)

$$\sigma_R = \frac{\pi}{2M\left(1 - \frac{Q^2}{2M\nu}\right)}\left[\frac{2}{\nu}\ W_1 + \frac{\sqrt{1 + \frac{Q^2}{\nu^2}}}{M}\ W_3\right]$$

$$\sigma_L = \frac{\pi}{2M\left(1 - \frac{Q^2}{2M\nu}\right)}\left[\frac{2}{\nu}\ W_1 - \frac{\sqrt{1 + \frac{Q^2}{\nu^2}}}{M}\ W_3\right]$$

$$\sigma_S = \frac{\pi}{M\nu\left(1 - \frac{Q^2}{2M\nu}\right)}\left[\left(1 + \frac{\nu^2}{Q^2}\right)\ W_2 - W_1\right] .$$

Furthermore the variables used in the CERN experiment are

$$\rho_{CERN} = \frac{1}{2\,x_{NEW}}$$

$$x_{CERN} = y_{NEW} \quad ,$$

$W_1, W_2, W_{3\ CERN}$ were related to the function $F_1, F_2, F_{3\ CERN}$ in the deep inelastic region by

$$W_1 = F_1$$

$$W_2 = \frac{M}{\nu} F_2$$

$$W_3 = \frac{M}{\nu} F_3 .$$

The same relation holds between the Bjorken Phys. Rev. W's and F's, since the Bjorken F's differ from the CERN by the same factor of M. F_1, F_2, and F_3 are functions only of x_{NEW} in the deep inelastic region.

Finally Adler uses α, β, γ where (again using CERN W_1, W_2, W_3)

$$\alpha = \frac{W_1 \nu^2}{Q^2 + \nu^2} \rightarrow \frac{W_1}{M} \quad \text{if } \nu^2 >> Q^2$$

$$\beta = \frac{W_2}{M} \; \frac{1}{1 - \frac{Q^2}{4EE'}}$$

$$\gamma = \frac{W_3 \nu^2}{2M^2 \left(Q^2 + \nu^2\right)} \rightarrow \frac{W_3}{2M^2} \quad \text{if } \nu^2 >> Q^2 \; .$$

REFERENCES

[1]B. Richter Proceedings 1968 Vienna Conference.
[2]J. D. Bjorken SLAC-PUB-571, May 1969.
[3]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
[4]Pais and Treiman, Bogoliubov Anniversary Volume.
[5]S. L. Adler, Phys. Rev. 135, B963 (1964).