ELECTRON MOMENTUM ERRORS, SINGLE π^o PHYSICS, AND $2\pi^o$ PHYSICS IN HYDROGEN-NEON MIXTURES IN THE 25-FOOT BUBBLE CHAMBER

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ABSTRACT

The probable error in the bubble-chamber measurement of the momentum of high-energy electrons in hydrogen-neon mixtures is reevaluated in the light of recent advances in measurement techniques and with the assumption of a 500μ setting error. The expected errors are rather larger than those derived by Kalmus.[5] The average error in gamma momenta and in momenta of constrained π^o decays is then derived. With the aid of a Monte Carlo program, ambiguities in sorting out the four gammas from the decay of two π^o's is studied. Explicit comparison is made with the metal plate system suggested by Eisenstein and Kernan.[11] Hydrogen-neon mixtures appear to be superior in all cases. Physics with two π^o's seems very feasible up to $p_{2\pi} = 50$ GeV/c.

I. INTRODUCTION

Extensive discussion has been made of using the 25-foot bubble chamber with hydrogen-neon mixtures, or with a neon mixture surrounding a track-sensitive, pure hydrogen target (hereafter called a "neon sleeve"), in order to detect gamma rays efficiently and measure their energies with sufficient accuracy.[1-3] Some quantitative thought has been given to how well one must measure gamma energies to separate event types.[4] However, very little quantitative thought has been given to the question of how well one might be able to measure gamma-ray energies in the 5-100 GeV region since the work of Kalmus.[5] Virtually no thought has been given to how well one can do experiments detecting two or more π^o's. This note addresses some of these questions, in particular:

1. What electron momentum errors are likely to be obtained, in various mixtures, in light of recent advances in heavy liquid measuring techniques and a more pessimistic assumption about the setting error than Kalmus assumed?

2. How large will the errors in the constrained momentum and angles of a π^o be at various momenta and in various mixtures?

3. In what kinematic regions can $2\pi^0$ physics be done unambiguously?

4. How do neon sleeves compare with metal plate systems?

Physics with more than $2\pi^0$'s was not investigated because of lack of time. The results on $2\pi^0$ physics are not entirely complete, for the same reason.

I adopt the same "neon sleeve" dimensions as that proposed by Roe, for the sake of definiteness, namely, a 14-ft long, 6-ft diameter "target" chamber surrounded by a neon-hydrogen mixture. In the neon, there is roughly 7 ft of useful length in the forward direction and 5 ft in the lateral direction.

II. ELECTRON MOMENTUM ERRORS

Kalmus' Curves

Kalmus assumed a setting error of 100μ in space and used the Behr-Mittner theory[6] to estimate the error due to the radiative loss correction. He used a fixed "cut-off radiation" (defined as the minimum single-gamma radiation which can be detected by change of curvature of the electron track) which depended only on the radiation length of the liquid. This cut-off was 40% of the electron's energy for a radiation length of 50 cm. He considered electrons up to 20 GeV/c.

There is now a broader consensus that the setting error is more likely to be 500μ in space. When one repeats Kalmus' calculations with this assumption, one finds that measurement error is so dominant that his assumptions are no longer valid. For example, if one considers a 20-GeV electron in a mixture with a radiation length (X_0) of 50 cm, the optimum length, at which the sum of the squares of the measuring error and radiative error is a minimum, exceeds the radiation length. Neither the Behr-Mittner, nor any other method of measuring only the trajectory of the electron, is valid at this length because of the rather large probability that the electron has lost most of its energy. Thus one must cut the track length to something more like half a radiation length at most. At that point measuring error is sufficiently large that it would be impossible to detect, by a change in curvature, a single gamma emission of even half the energy of the original electron. Within the context of the simple Behr-Mittner theory, one needs a cut-off radiation which depends on the measurement error contribution to the electron momentum error.

In the low-energy region, in which measuring error is not important, one should take into account the improvement made by Morellet[7] and developed further by Huson of the Brookhaven-Berkeley group.[8] By measuring many points on the track, one can actually determine an average radiation loss for each track much of the time, and detect very small single gamma radiations down to 10% of the electron energy, even in 50-cm radiation length liquids. When a large single gamma radiation is detected in the first part of the track, the method effectively reverts to the Behr-Mittner method.

Error Equation

The error in the momentum of an electron is given by:

$$\left(\frac{\Delta p}{p}\right)^2 = \left(\frac{2p\,\epsilon}{L^2 H}\right)^2 + \frac{3400}{H^2 L X_0} + \frac{1}{6}\,\frac{L y_0^2}{X_0 \ln 2}\,, \qquad (1)$$

where p = momentum (GeV/c)

ϵ = setting error in space (microns)

L = track length measured (cm)

H = magnetic field (kG) = 40 kG

X_0 = radiation length (cm)

y_0 = "cut-off," discussed below (dimensionless).

The formula assumes that the dip of the electron is zero, a reasonable assumption for high-energy gammas. The first term (the setting error) comes from Gluckstern[9] assuming 15 points measured. The second term is the multiple scattering term and is taken from Bethe (Gluckstern's estimate is at least 50% lower, but the term is usually negligible in the energy region of interest here). The third term is the Behr-Mittner error in the radiative correction; $y_0 = \ln\,[E/(E - h\nu_{min})]$, where E is the electron energy and $h\nu_{min}$ is the smallest single gamma radiation detectable by change of curvature. The constant y_0 is usually found to be around 0.35 empirically. In the Morellet method, the same formula holds, but y_0 is determined for each event. The experience of Huson has been that y_0 is at best 0.1, with track lengths no longer than half a radiation length. When a "poor fit" is obtained, indicating large single gamma radiations in the first half-radiation length, the track is usually cut to the quarter of a radiation length and y_0 reverts to the Behr-Mittner value, 0.35. It should be noted that Eq. (1) implies an optimum length. For low energies where the measuring error is negligible, and assuming a constant y_0, this optimum length is given by $L_{opt} = 120/Hy_0$. However, when this optimum length is more than a small fraction of a radiation length, y_0 depends on the length, as mentioned above.

Assumptions About the Cut-Off

In order to extend the use of Eq. (1) to energies where the measuring error is dominant, I have made the following assumptions about y_0 and the maximum length (L_{max}) which one should consider measuring, i.e., one measures to the optimum length or to L_{max}, whichever is smaller. They are consistent with experience at low energies, but somewhat speculative.

If the measuring error is less than 8%, the Morellet method can be used and y_0 is as small as 0.1, with $L_{max} = X_0/2$. The parameter y_0 linearly increases to 0.35 as the measuring error approaches 8%. This gives a lower limit on the error. For an upper limit, I assume $y_0 = 0.35$ and $L_{max} = X_0/4$.

If the measuring error is greater than 8% but less than 23%, y_0 can no longer be determined for each event, and a pure Behr-Mittner approach is assumed with $y_0 = 0.35$. For an upper and lower limit on the errors, I assume $L_{max} = X_0/2$ and $X_0/4$. Again, y_0 gradually increases from the value 0.35 as the measuring error increases to 23%.

If the measuring error is greater than 23%, one cannot even detect single-gamma radiations of, say, 50%, so one is outside of the scope of even the Behr-Mittner method. For an upper and lower limit on the error, I assume that one measures a length between $L_{max} = 0.7X_0$ and $0.4X_0$, and that in either case the radiation error is 33%. This corresponds roughly to the rms spread about the mean energy loss at half the measured length, for either length.[10]

The results of these assumptions are displayed in Figs. 1 and 2, for radiation lengths of 50 and 100 cm. The upper and lower limits of the error band reflect the spread which is expected in the errors owing to the different radiative behavior of individual electrons. The average error is about twice that derived by Kalmus at 5 GeV/c and three times larger at 20 GeV/c. This discrepancy reflects both a more pessimistic assumption about the setting error (500μ instead of 100μ) and the realistic assumption that the parameter y_0 is much larger when the measurement error is large.

In the calculations which follow, I use the average error from Figs. 1 and 2.

III. SINGLE π^0 PHYSICS

Let us consider two hydrogen-neon mixutres, namely, 50 cm and 100 cm radiation length. If there is a 7 foot useful path length in the neon, these mixtures correspond to 4.2 and 2.1 radiation lengths, with single gamma conversion efficiencies of 0.963 and 0.80, respectively. Note that both the total length and the two conversion efficiencies are the same as those adopted by Eisenstein and Kernan[11] in their study of metal plate systems.

Gamma Momentum Errors

In a mixture with a radiation length of 50 cm, the average electron error is represented approximately by the power law (see Fig. 1):

$$\frac{\Delta p}{p} = 0.13\ p_e^{0.63}, \tag{2}$$

at least above 500 MeV/c. Consider now a gamma which produces an $e^{\pm}$ pair. Defining λ by the equations,

$$p_{e^-} = \lambda \, p_\gamma \, , \; p_{e^+} = (1 - \lambda) p_\gamma \, , \tag{3}$$

and propagating errors, one finds for the error in the determination of the gamma momentum,

$$\frac{\Delta p_\gamma}{p_\gamma} = 0.13 \, p_\gamma^{0.63} \left[\lambda^{3.26} + (1 - \lambda)^{3.26} \right]^{1/2} . \tag{4}$$

The square root term varies from 0.45 to 1 and has an average of 0.69. It is appropriate to average since the distribution of λ is approximately constant from 0 to 1. Thus

$$\left. \begin{aligned} & 0.06 \, p_\gamma^{0.63} < \frac{\Delta p_\gamma}{p_\gamma} < 0.13 \, p_\gamma^{0.63} \\ & \text{and} \quad \left(\frac{\Delta p_\gamma}{p_\gamma} \right)_{av} = 0.09 \, p_\gamma^{0.63} \end{aligned} \right\} \text{for } X_0 = 50 \text{ cm.} \tag{5}$$

The result is displayed in Fig. 3. The result of the analogous calculation for X_0 = 100 cm is displayed in Fig. 4.

In Table I, the results for X_0 = 50 cm are compared with the results of Eisenstein and Kernan for an array of metal plates. Comparison is made with several different choices of plate thickness and spacing. In the table, n is the number of radiation lengths of metal used in the array. Column 3, with n = 4, corresponds to the same conversion efficiency as 7 feet of neon-hydrogen with X_0 = 50 cm. My choice of d = 0.36 minimizes the error for a 2.5-BeV gamma, as explained by Eisenstein and Kernan; using thinner plates and more of them decreases the error somewhat for low-momentum gammas while increasing the error for high-momentum gammas. However, for any choice of d, the plate arrangement with n = 4 is always significantly worse than the neon.

Table I. Fractional Error $\Delta p_\gamma / p_\gamma$.

Comparison of gamma momentum errors in a 50-cm radiation length neon-hydrogen mixture (col. 1) with three representative choices of plate thickness (d = plate thickness/radiation length of metal) and total thickness of the array (n = number of radiation lengths of metal), taken from Ref. 11. The total length of plates and hydrogen is 7 feet.

p_γ	neon-H_2 X_0 = 50 cm	plates n = 4, d = 0.36	plates n = 2, d = 0.22	plates n = 2, d = 0.29
0.5	0.06	0.23	0.15	0.18
1.0	0.09	0.23	0.16	0.18
2.5	0.16	0.28	0.18	0.19
5.0	0.25	0.39	0.25	0.22
10.0	0.38	0.67	0.44	0.31
20.0	0.59	1.25	0.84	0.54
30.0	0.76			0.78
40.0	0.92			1.03
50.0	1.06			

With n = 2, d = 0.29, one finds a remarkable similarity in the errors with plates and those of 4 radiation lengths of neon-hydrogen. However, the conversion efficiency of the plate system is then only 0.8 per gamma, compared with 0.96 for the neon-hydrogen mixture.

Error in Constrained π^o Momentum

Let us now turn to the question of the error in the momentum and angle of a π^o, after performing the 1 c fit of the two gammas. Trilling[12] has shown that these errors are given by

$$\left(\frac{\Delta p}{p}\right)_{\pi^o} = \frac{\Delta p_{\pi^o}}{E_{\pi^o}} = \cos\theta^* \left[\left(\frac{p_{\gamma 1}}{\Delta p_{\gamma 2}}\right)^2 + \left(\frac{p_{\gamma 2}}{\Delta p_{\gamma 2}}\right)^2\right]^{-1/2}, \tag{6}$$

$$\Delta\xi = \frac{m_{\pi^o}}{p_{\pi^o}} \sin\theta^* \left[\left(\frac{p_{\gamma 1}}{\Delta p_{\gamma 1}}\right)^2 + \left(\frac{p_{\gamma 2}}{\Delta p_{\gamma 2}}\right)^2\right]^{-1/2}, \tag{7}$$

under the assumption that the errors in the gamma angles play a negligible role compared with the momentum errors.[13] (In the above equations, ξ is the angle of the π^o in the plane of the two gammas; θ^* is the decay angle of the π^o in the π^o rest frame.) Let the error in gamma momenta be written in the form

$$\frac{\Delta p_\gamma}{p_\gamma} = A p_\gamma^{0.63} \qquad \begin{array}{l} A_{max} = 0.13 \\ A_{av} = 0.09 \end{array}, \tag{8}$$

where A is taken from the maximum or average error given by Eq. (5). Using relativistic approximations to the π^- decay kinematics, we obtain

$$\frac{\Delta p_{\pi^o}}{p_{\pi^o}} = A(p_{\pi^o}/2)^{0.63} \quad \cos\theta^* \left[\frac{1}{(1+\cos\theta^*)^{1.26}} + \frac{1}{(1-\cos\theta^*)^{1.26}} \right]^{-1/2} . \tag{9}$$

The square-root term is not very independent of $\cos\theta^*$, varying from $\sqrt{2}$ to infinity; however, the factor $\cos\theta^*/\sqrt{}$ varies only from 0 to 0.31 and has an average value of 0.20 (averaging over $\cos\theta^*$). Thus our result for the maximum and average error in the constrained π^o momentum is

$$\begin{aligned} \left(\frac{\Delta p_{\pi^o}}{p_{\pi^o}}\right)_{max} &= 0.31\, A_{max} \left(\frac{p_{\pi^o}}{2}\right)^{0.63} = 0.040 \left(\frac{p_{\pi^o}}{2}\right)^{0.63} \\ \left(\frac{\Delta p_{\pi^o}}{p_{\pi^o}}\right)_{av} &= 0.20\, A_{av} \left(\frac{p_{\pi^o}}{2}\right)^{0.63} = 0.018 \left(\frac{p_{\pi^o}}{2}\right)^{0.63} . \end{aligned} \tag{10}$$

We can gauge the improvement made by the constraint by comparing this error with that resulting from simple scalar addition of the momenta (a good approximation for most fast π^o decays). The result is

$$\frac{\Delta p_{\pi^o}}{p_{\pi^o}} = B\, A \left(\frac{p_{\pi^o}}{2}\right)^{0.63} \qquad \text{(unconstrained)}, \tag{11}$$

where

$$B_{av} = 1.1, \; B_{max} = 2^{0.63} = 1.55.$$

In either the case of the average or the maximum, the error has been decreased by a factor of 5.

The error in ξ can be calculated in a completely analogous fashion. The result is

$$\Delta\xi = \frac{m_\pi}{p_\pi}\, C \left(\frac{p_{\pi^o}}{2}\right)^{0.63} , \tag{12}$$

where

$$C_{max} = 0.093, \ C_{av} = 0.043.$$

It should be reiterated that the above results are for $X_0 = 50$ cm. The "maximum" error is not fully realistic, for only the average electron momentum error (Figs. 1 and 2) has been used. Using maximum electron errors will increase the error by at most 50%. The effect of dip angles, which has also been ignored, will also increase this maximum. The results, for both $X_0 = 50$ and 100 cm, are shown in Figs. 5 and 6.

Comparison With Plates

For a comparison with the constraining process in plate systems, I note that for an array which has been optimized for $p_\gamma = p_{\pi^0}/2$, the average and maximum errors are given [(from Eq. (6) of Eisenstein and Kernan] by:

$$\left(\frac{\Delta p_\pi}{p_\pi}\right) = D\left(\frac{\Delta p_\gamma}{p_\gamma}\right)_0, \quad \begin{matrix} D_{max} = 0.71 \\ D_{av} = 0.35 \end{matrix}, \tag{13}$$

where $(\Delta p_\gamma/p_\gamma)_0$ means the error in the momentum of a gamma of momentum $p_{\pi^0}/2$ and thus has the same meaning as the term, $A(p_0/2)^{0.63}$ in Eq. (10). This gives the surprising impression that one gains nearly a factor of two more from the constraint in neon-hydrogen than in a plate system. Actually, the result merely reflects the fact that when neon-hydrogen and plate-system errors in gamma momentum are equal for $p_\gamma = p_{\pi^0}/2$, then the neon-hydrogen errors are a factor of two or three times smaller than plate-system errors for much lower momentum gammas (see Table I). Inspection of Eq. (6) shows that the lower of the two gamma momentum errors dominates the error in the constrained π^0 momentum. On the other hand, when the two gammas are nearly equal in momentum, and hence have equal errors, $\cos\theta^* \approx 0$, so that the error in the π^0 momentum is much smaller than the average error.

Summary

In summary, a neon-hydrogen mixture with a radiation length of 50 cm, 4 radiation lengths long, yields maximum π^0 momentum errors which are less than 10% for π^0's of momenta less than 10 GeV/c, and average π^0 momentum errors less than 10% up to 30 GeV/c. In the 5-10 GeV/c region, these errors are roughly half of those in a plate system optimized for this momentum region and occupying the same volume, but with only two radiation lengths of metal. For 20 GeV/c π^0's, with an array optimized for that momentum, the errors are more nearly equal, but the necessary plate thickness has risen to 1/3 of a radiation length. It is not clear to me that it is safe to use plates this thick, because of the increasingly non-Gaussian behavior of the radiation loss in the plates themselves.

The question of whether the increased accuracy of the neon is necessary, and other qualitative comparisons of the two systems are taken up in Sec. 5.

IV. $2\pi^0$ PHYSICS

Table II lists the single-gamma and four-gamma conversion efficiencies for three different hydrogen-neon mixtures, assuming a seven-foot length. For $2\pi^0$ physics, one would probably never wish to use a mixture with radiation length greater

Table II. Conversion Efficiencies for 7 Feet of Various Neon-Hydrogen Mixtures.

X_0 = radiation length, n_r = number of radiation lengths, n_c = number of conversion lengths (= 7/9 n_r).

			Conversion Efficiency for n Gammas		
X_0	n_r	n_c	n = 1	n = 2	n = 4
50	4.2	3.3	0.963	0.93	0.85
75	2.8	2.2	0.89	0.79	0.63
100	2.1	1.6	0.80	0.64	0.41

than 100 cm, as the four-gamma efficiency there is only 40%. Exactly what mixture is best is experiment dependent. There is, of course, a trade-off between conversion efficiency and accuracy.

I have examined the question of how well one can pair up the four gammas properly, i.e., determine which gammas come from the same π^0, in the following manner. A simple Monte Carlo program was written to generate $2\pi^0$ events. For a fixed 2π mass and momentum ($M_{2\pi}$, $p_{2\pi}$), both the 2π system and the subsequent π^0's were given isotropic decay distributions. The $e^{\pm}$ pairs resulting from the π^0 decays were given a uniform distribution in the $e^+ - e^-$ energy difference. Errors in the electron momenta were then assigned using the average errors from Figs. 1 and 2. The invariant masses and errors there of all pairs of gammas were then calculated. With four gamma rays, there are three possible pairings. If more than one pairing had both two-gamma masses within two standard deviations of the π^0 mass, the event was declared "ambiguous." The fraction of ambiguous events as a function of $p_{2\pi}$ and $M_{2\pi}$ is shown in Table III.

The results can be qualitatively summarized as follows. If one arbitrarily declares 10% ambiguities the maximum tolerable fraction, then one can satisfactorily unscramble $2\pi^0$ events with $M_{2\pi} > 0.4$ GeV at $p_{2\pi}$ = 5 GeV/c, and with $M_{2\pi} > 1.5$ GeV

Table III. Percentage of $2\pi^o$ events in which it is ambiguous which pair of gammas form which π^o. An event is ambiguous if more than one combination has both masses within 2 S. D. of the π^o mass. (The numbers in parentheses are the percentages if the criterion is reduced to 1 S. D.) These results are from a Monte Carlo program with 500 events per bin.

$M_{2\pi}$ (GeV)	0.3	0.4	0.5	0.6	0.8	1.0	1.5	2.0	6.0	10.0
$p_{2\pi}$ (GeV/c)										
				Radiation length = 50 cm						
5	10 (3)	6 (1)	2 (1)	0 (0)	0	0	0	0	0	0
10	16 (5)	15 (4)	4 (1)	1 (0)	0	0	0	0	0	0
15	29 (10)	30 (6)	8 (2)	2 (0)	0	0	0	0	0	0
20	31 (15)	41 (8)	16 (1)	6 (1)	1 (0)	0	0	0	0	0
25	38 (17)	43 (9)	26 (2)	9 (1)	1 (0)	1	0	0	0	0
30	39 (20)	51 (14)	36 (4)	17 (1)	5 (1)	4 (0)	1	0	0	0
35	45 (22)	57 (19)	52 (5)	34 (3)	11 (1)	6 (0)	1	1	0	0
40	61 (30)	59 (24)	43 (4)	18 (1)	8 (1)	3 (1)	1 (0)	0	0	0
45	61 (29)	69 (27)	50 (10)	26 (2)	16 (0)	3	4	0	0	0
50	55 (30)	67 (34)	73 (8)	59 (4)	37 (1)	21 (0)	9	5	0	0
				Radiation length = 100 cm						
5	5 (1)	1 (0)	1	1	0	0	0	0	0	0
10	5 (1)	4 (1)	0 (0)	0	0	0	0	0	0	0
15	9 (2)	5 (1)	2 (1)	0 (0)	0	0	0	0	0	0
20	14 (4)	6 (1)	1 (0)	1	0	0	0	0	0	0
25	13 (3)	7 (7)	2 (2)	1 (1)	0 (0)	0	0	0	0	0
30	17 (4)	11 (3)	2 (0)	1 (0)	1	0	0	0	0	0
35	19 (4)	12 (1)	3 (1)	2 (1)	1 (0)	0	0	0	0	0
40	24 (8)	16 (4)	2 (1)	1 (0)	0	0	0	0	0	0
45	25 (8)	17 (2)	5 (1)	1 (0)	0	0	0	0	0	0
50	26 (8)	23 (5)	6 (1)	2 (1)	1 (0)	0	0	0	0	0

at $p_{2\pi} = 50$ GeV/c, with $X_0 = 50$ cm. For $X_0 = 100$, the corresponding figures are $M_{2\pi} > 0.3$ GeV at $p_{2\pi} = 5$ GeV/c and $M_{2\pi} > 0.45$ GeV at $p_{2\pi} = 50$ GeV/c.

The criterion adopted above for an ambiguous event is quite conservative. If we change the criterion to one standard deviation in both masses (overly optimistic, I believe), the fraction of ambiguities is negligible for $X_0 = 100$ cm and negligible for $M_{2\pi} > 0.5$ for $X_0 = 50$ cm. Furthermore, ambiguous events are not entirely useless, as one still has the four-gamma invariant mass. In any case the results look quite hopeful.

V. THE PROBLEMS OF SHOWERING

The question of whether secondary pairs resulting from the radiation from the primary pairs lead to further ambiguities or biases, in either the case of single-π^0 or $2\pi^0$ physics, has not been carefully studied. Showering creates two different problems. First, it creates a scanning problem: a primary pair can get lost in the shower from the first primary pair, i. e., accidentally identified as part of the shower, even when it does not point tangent to one of the electrons from the first primary pair. Secondly, even if all the pairs are found and properly measured, it may not be clear in all cases whether a particular gamma is a secondary gamma associated with a pair from a first π^0 or a primary pair from a second π^0.

The following semi-quantitative thinking leads me to believe that neither of these problems is important for single-π^0 physics, but that the scanning problem might be troublesome in $2\pi^0$ physics.

Single π^0 Physics

Consider the typical case of a 10 GeV/c π^0. The minimum opening angle of the two gammas is 1.5°. The distribution in opening angle is very peaked towards the minimum; 85% of the decays occur between the minimum and twice the minimum opening angle. Typically, the first gamma will convert half a radiation length into the neon and the second one another radiation length further. With the double chamber which I have adopted (4 meters of pure hydrogen followed by 2 meters of a mixture), the typical distance from the π^0 production vertex and the second gamma conversion point is three meters. (I assume a radiation length of 50 cm.) Thus the conversion point of the second gamma is displaced by 7 cm from the line-of-flight of the first gamma.

An electron in the first pair typically has a momentum of 2.5 GeV/c. In the 50 cm between the first and second gamma's conversion points, such an electron turns through 15° and moves 6 cm from the line-of-flight of the first gamma. Thus the second gamma's conversion point sits roughly on the track of a typical electron from the first gamma; the pair from the second gamma makes a 15° angle with the electron. However, any showering which has occurred is confined approximately to the "horn" inside the first $e^{\pm}$ pair. (Note that the rms emission angle for radiation from a 2.5-GeV/c electron is about 0.06°.)

Thus in the typical case considered, the second pair lies at the edge of the shower from the first pair. If this typical case should happen to represent the average behavior, then in half of the events the two pairs from the π^{0} are cleanly separated; in the other half, the second pair may be intermingled with the shower from the first, but its angle allows further testing of where it came from.

For a pair to be declared the possible result of radiation from another electron, the pair direction must intersect that electron with an angle less than about twice the average radiation emission angle, or at an angle of less than 0.12°, in the case of our 2.5-GeV/c electron. However, this is less than the uncertainty in the electron's direction, so I move the minimum angle up to a more reasonable 0.4°.[14] Let us now consider a "worst case" of π^{0} decay, rather than the typical case above. Let the opening angle between the two gammas be 1.5°; let the second gamma convert two radiation lengths behind the first gamma; let the distance from the π^{0} production vertex and the second gamma be only 1.5 meters. Can this second gamma appear to intersect an electron from the first gamma with an angle of less than 0.4°? With the help of a sketch of the situation, it is obvious that the smallest intersection angle occurs near the vertex of the first pair. However, the line joining the vertices of the two pairs makes an angle 2.3° with the electrons of the first pair. The answer to the question is a fairly definite "no," without even using the direction of the second pair as determined by the direction of its electrons.

$2\pi^{0}$ Physics; Ambiguities

In the case of $2\pi^{0}$'s, one has the possibility that gammas from different π^{0}'s have sufficiently equal angles that the one appears to be the result of a radiation from the first. This possibility is obviously highest for low mass, high momentum, $2\pi^{0}$ systems. To evaluate this probability, the Monte Carlo program was once again employed. Let the "critical angle" be defined, as before, as 0.4°, the very approximate angle error in the electron. Following the idea of the simple calculation of the previous paragraph, an event was declared "potentially ambiguous" if any pair of photons in the event (there are six pairings) had an opening angle θ_0 such that $2\theta_0 \leq 0.4°$.

The result of the Monte Carlo program, for $2\pi^{0}$ masses up to 1.0 GeV and $2\pi^{0}$ momenta from 5 to 50 GeV/c, was that 16% of the events were potentially ambiguous. The number of events which would actually be ambiguous is probably less than half this number, for the second gamma must convert a radiation length or more downstream from the first pair for the ambiguity criterion to be truly valid. Futhermore, the above results applies only to low mass (less than 1.0 GeV) $2\pi^o$ masses; for high masses the ambiguity problem is much less.

I conclude that the incidence of ambiguities between direct and secondary electron pairs in $2\pi^o$ physics is of the order of 10%, which is not negligible but is not so large as to be discouraging.

$2\pi^o$ Physics: Scanning

The question of whether one can find all the pairs and digitize them, and how long it will take, whether it be done by manual or automatic scanning, is a more subjective question which probably cannot be answered without real experience at these energies. However, I would like to mention some numbers, based on the Monte Carlo program and shower theory, which might constitute input to any further studies on this question.

With regard to the general appearance of events, the four gammas from a $2\pi^o$ event tend to lie within a 10° cone around the $2\pi^o$ direction. This number, 10°, is the approximate rms value of the largest-angle gamma for the input conditions arbitrarily chosen for the Monte Carlo program. If the conversion points are roughly 2 meters from the production vertex, then they are spread over a 30 cm radius about the $2\pi^o$ direction.

From shower theory, it can be shown that two radiation lengths downstream from the conversion point of a particular gamma, there are, on the average, 0.6 converted gammas from radiations from the first pair with electron energies greater than 1/20 of the primary electron momenta. With four gammas, this means an average of 2.4 extra pairs per event with sizable energy.

On the question of whether human scanners can readily decide whether an electron pair is the result of a direct gamma or a secondary gamma from another pair, it should be noted that for $2\pi^o$ masses less than 1 GeV, and $2\pi^o$ momenta evenly distributed between 5 and 50 GeV/c, 70% of the events have two of the four gammas with an opening angle of less than 2°. When one of these two pairs is well downstream of the first pair, it seems unlikely to me that a human scanner can be relied upon to determine whether it is a direct or secondary pair.

My tentative conclusion is that one will have to rely upon the computer, with some fairly sophisticated and flexible programming, to distinguish primary and secondary pairs. The question of whether a human or automatic scanner can sort out all the pairs and correctly digitize them needs further study. The choice of exactly what mixture of hydrogen and neon to use will probably be influenced by this consideration more than any other.

VI. FURTHER COMPARISONS OF PLATES AND NEON

In Sec. II of this paper, it was pointed out that, using the same volume of the chamber, four radiation lengths of neon-hydrogen yielded smaller errors, often by a factor of two, than two radiation lengths of a plate system. In this section, some further comparisons are made.

Advantages of Neon

In the realm of single-π^0 physics, a neon-hydrogen mixture offers a number of other advantages over a plate system. The higher conversion efficiency provides a better anticoincidence signal again $2\pi^0$ events. Complete visibility of the pairs allows better resolution of pairs which convert near one another. The pairs can be seen in more than two views. One can change mixtures easily between runs, whereas changing the plate system is a major job. With a hydrogen target surrounded by a neon-hydrogen sleeve, one can obtain nearly 4π solid angle detection of gammas, whereas a plate system always has to leave the top and bottom open. A plate system designed to catch wide-angle gammas, such as those resulting from slow π^0's produced at the lower vertex of an exchange reaction, is both complicated to build and view (see Fig. 3 of Eisenstein and Kernan) and less efficient than a neon sleeve.

Advantages of Plates

On the other hand, just because something is better does not mean it is necessary. Kruse[4] has considered what is necessary. For the kind of physics which he chose to discuss, the implication seemed to be that 10% momentum accuracy on π^0's was sufficient. In this case the plates are sufficient in the momentum range $p < 10$ GeV/c. The conversion efficiency is certainly sufficient. The loss of wide-angle gammas is not a serious problem for π^0 momenta above 1 GeV/c for the following reason: 85% of π^0 decays produce pairs of gammas with opening angle between θ_{min} and $2\theta_{min}$, where θ_{min} is given by the equation,

$$\theta_{min} = \frac{15.5}{p} \quad \text{(degrees, GeV/c)}, \tag{14}$$

so that at 1 GeV/c, 85% of the events have both gammas within 30° of the π^0 direction. Thus very few π^0's are lost, provided the π^0's are going largely forward in the lab. Although an array consisting of a large number of plates has a possible problem with turbulence, the double-chamber idea also has a number of possible operational problems, such as turbulence (again) and retaining equal sensitivity in the two chambers.

Lastly, plates are undeniably cheaper at the moment. Neon to half-filled the proposed 25-ft chamber costs \$1.2 million at the moment, whereas a plate system might cost only \$200,000. Cheaper sources of neon are badly needed, and this question is being investigated at Brookhaven.

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[10] This is equivalent to a Behr-Mittner method with no cut off. The distribution function for the energy, E, of an electron at a distance $L = t\,X_0$, where X_0 is the radiation length, is given approximately by the Bethe-Heitler formula

$$u(E)dE = [\ell n\,(E_0/E)]^{t/\ell n\,2^{-1}}\,dE/\Gamma(t/\ell n\,2),$$

where E_0 is the initial energy and Γ is the usual gamma function. This function is plotted in Fig. 7 for $t = 0.42$ and $t = \ell n\,2$. It is nothing like a Gaussian, and thus caution must be exercised in arguments based on the "rms" value of this function. See H. A. Bethe and W. Heitler, Proc. Roy. Soc. A146, 83 (1934).

[11] L. Eisenstein and A. Kernan, Measurement of π^o Momentum with Metal Plate Arrays in the 25-Ft Bubble Chamber, National Accelerator Laboratory 1969 Summer Study Report SS-128, Vol. II.

[12] G. Trilling, SLAC-5-E, 1967, p.65.

[13] Kruse (Ref. 4) has examined the question of where the approximation of negligible errors in the gamma angles breaks down. Assuming that the setting error is 500μ

and the average distance from the π^o production vertex to the gamma conversion point is 1 meter, he finds that p_{π^o} = 27 GeV/c is the momentum at which the angle errors are as important as the momentum errors. He also assumed that $\Delta p_\gamma / p_\gamma$ = 0.2. In this report, gamma momentum errors at 13 GeV/c (which is half of 27 GeV/c) are more typically 0.30, but the average conversion distance is more like 2 meters, so 27 GeV/c is still a reasonable boundary.

[14] An angular error of 0.4° is the measuring error in a 10-cm track. At this short distance one can ignore both the radiative and multiple scattering errors in electrons of 1 GeV and higher.

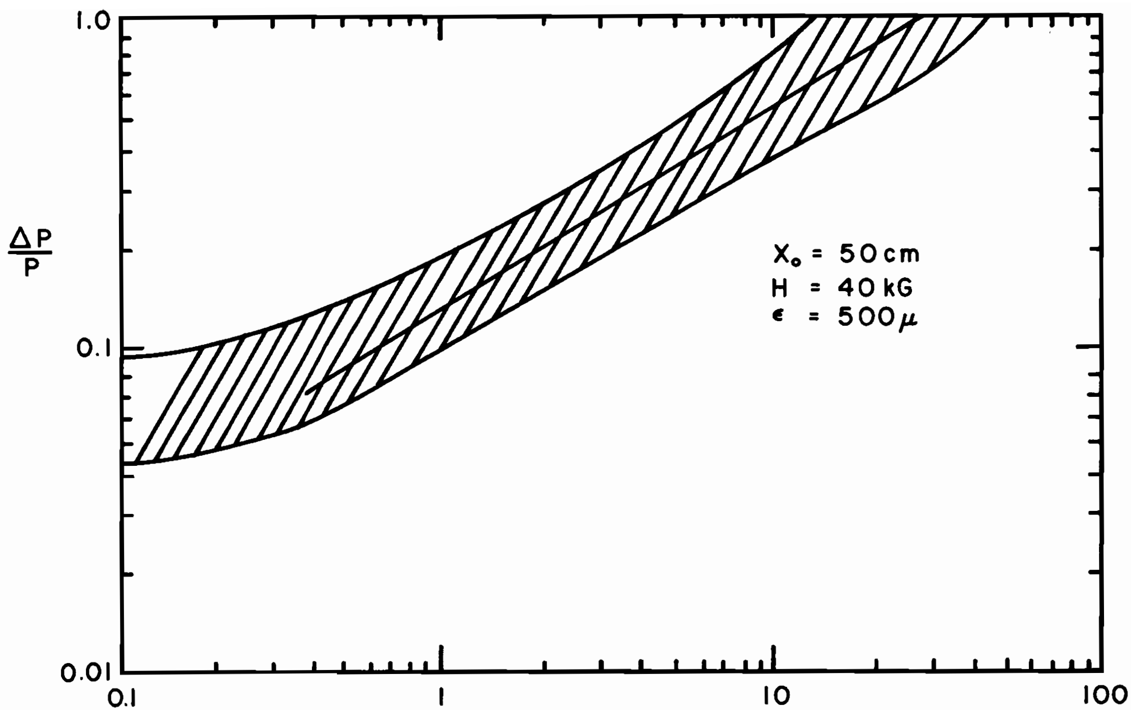

Fig. 1. Fractional error in the momentum of an electron as a function of the electron's momentum, in a hydrogen-neon mixture with a radiation length of 50 cm, a setting error of 500μ, and a field of 40 kG. The two curves represent upper and lower limits, explained in the text. The straight line is a power law approximation to the average, used in the text.

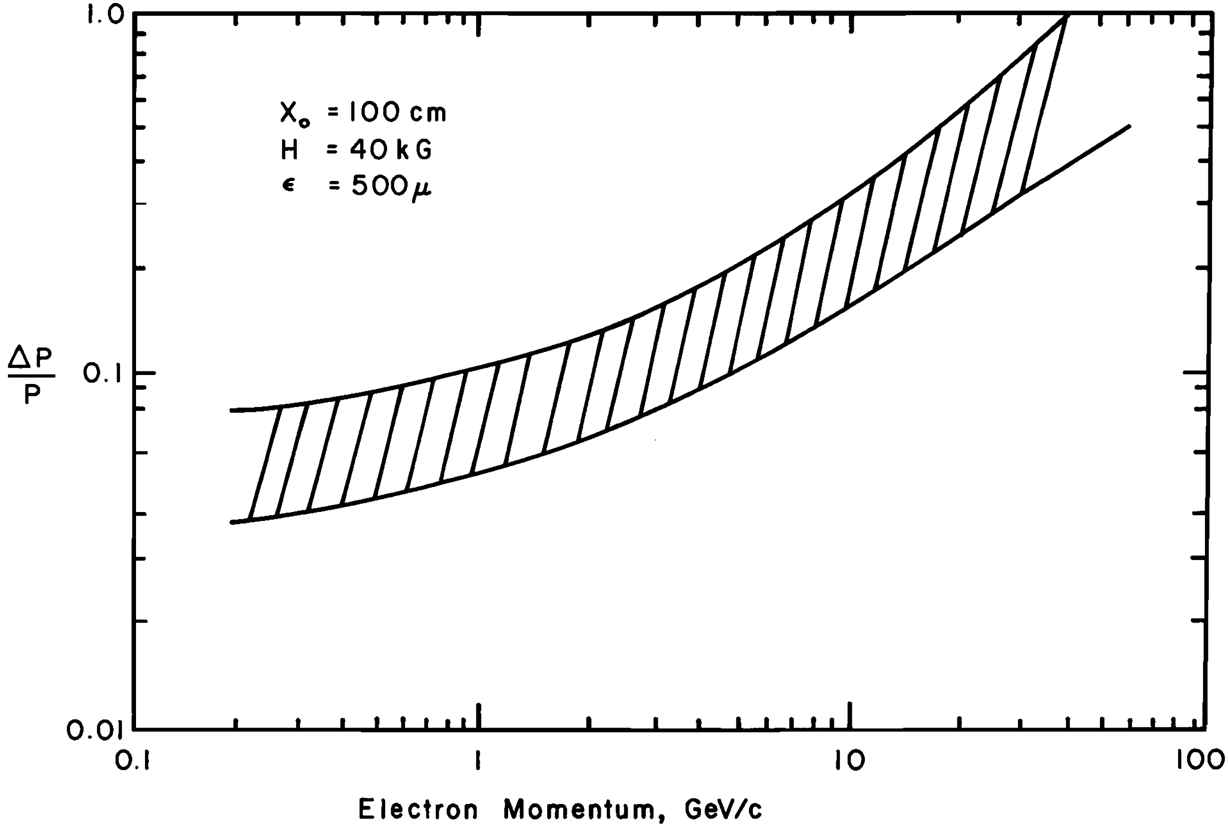

Fig. 2. Same as Fig. 1, with a radiation length of 100 cm.

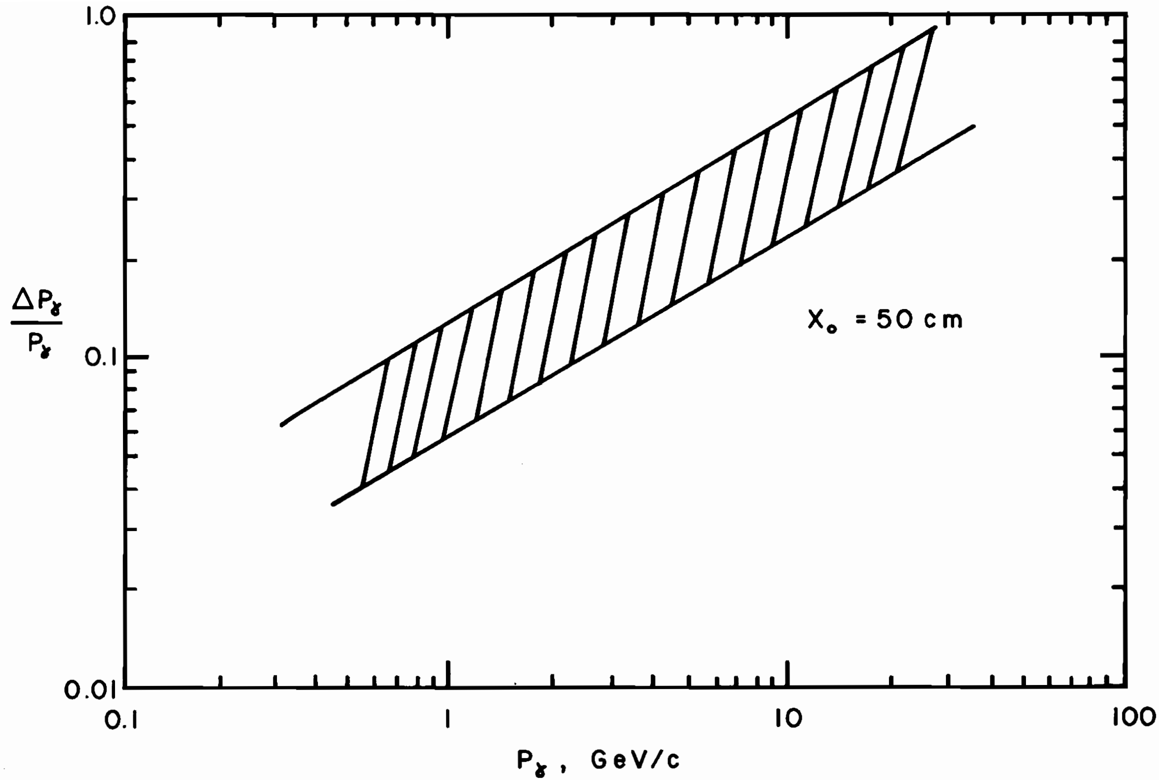

Fig. 3. Fractional error in the momentum of gammas for a radiation length of 50 cm, based on the power-law approximation of Fig. 1. The upper and lower curves represent the upper and lower limits resulting from the variation in the energy division between the two electrons of the pair.

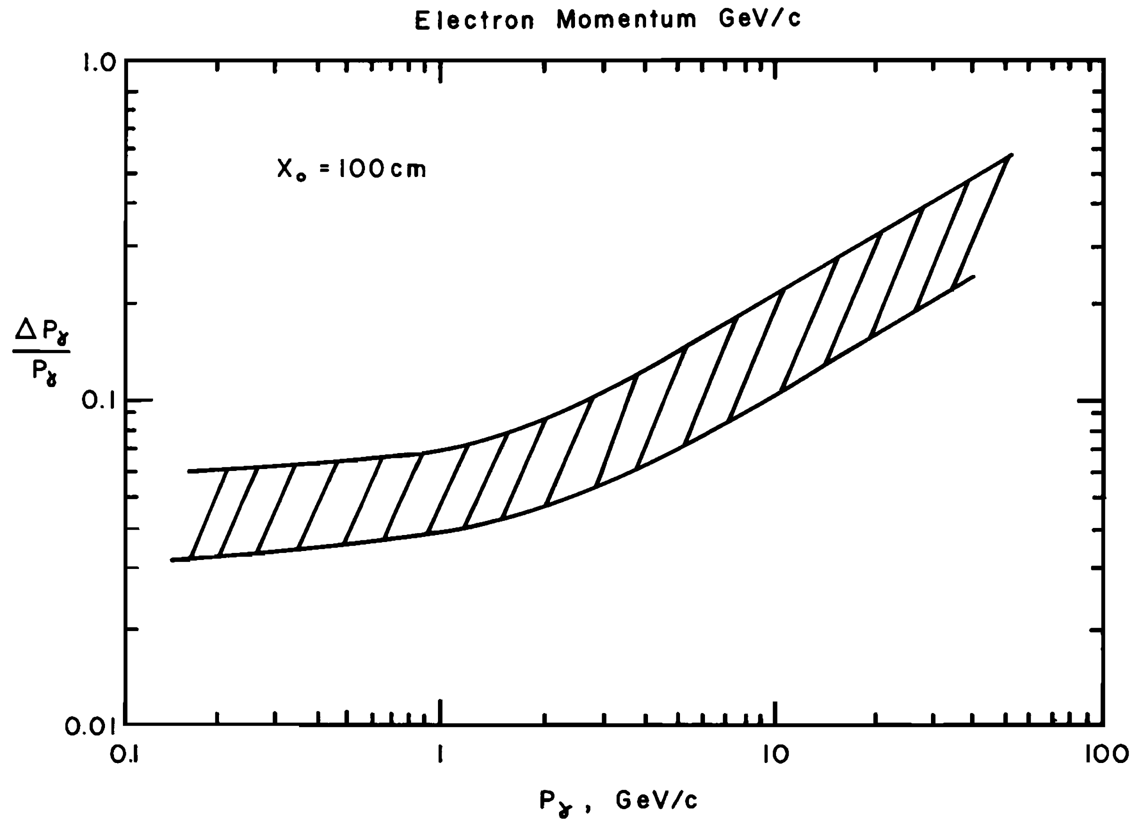

Fig. 4. Same as Fig. 3, with a radiation length of 100 cm.

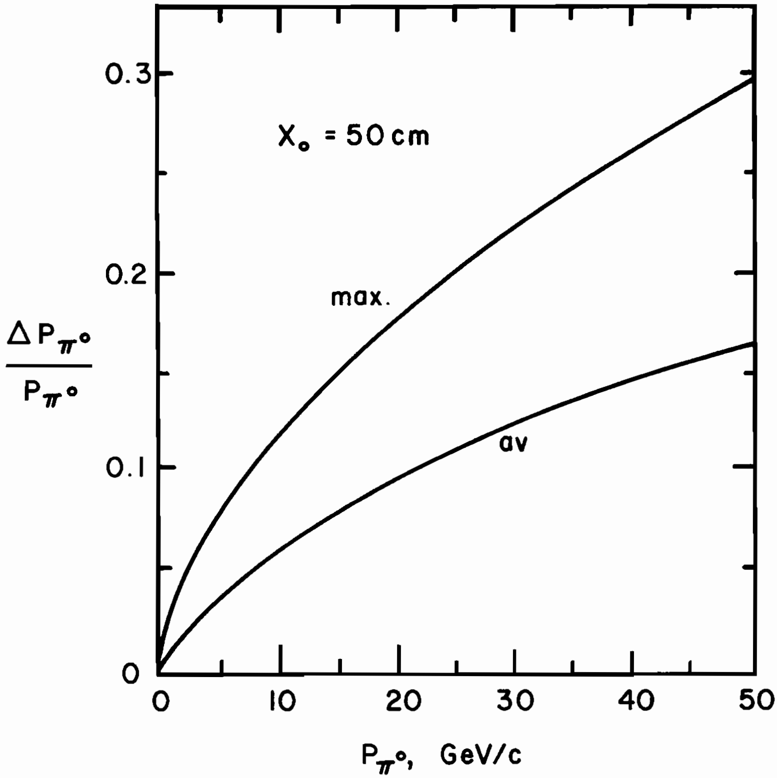

Fig. 5. Fractional error in the momentum of a π^o determined from a 1c fit using the two decay gammas. The upper curve is the maximum error which results when the decay angle is zero degrees and the gamma momentum errors are the maximum, from Fig. 3. Radiation length = 50 cm.

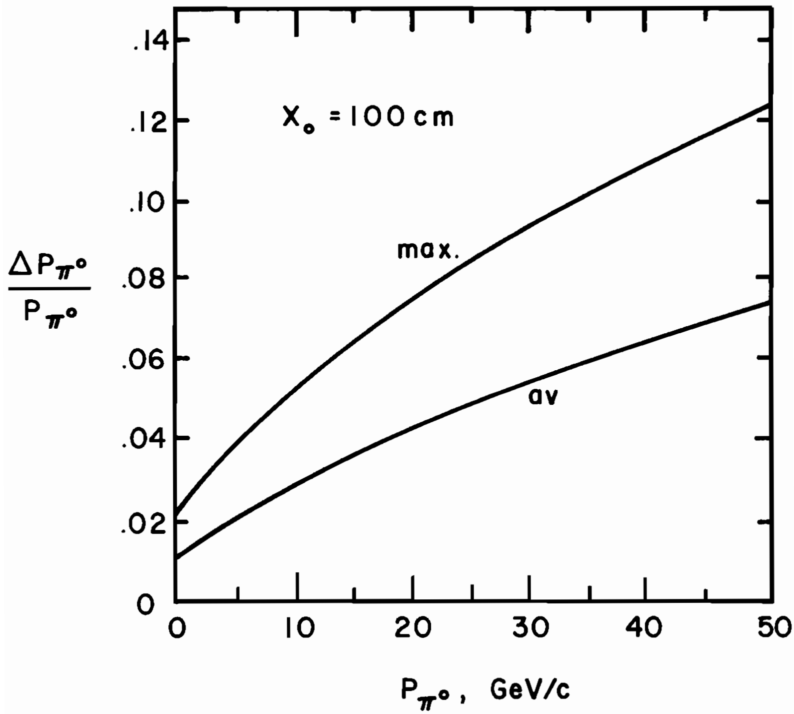

Fig. 6. Same as Fig. 5, with a radiation length of 100 cm.

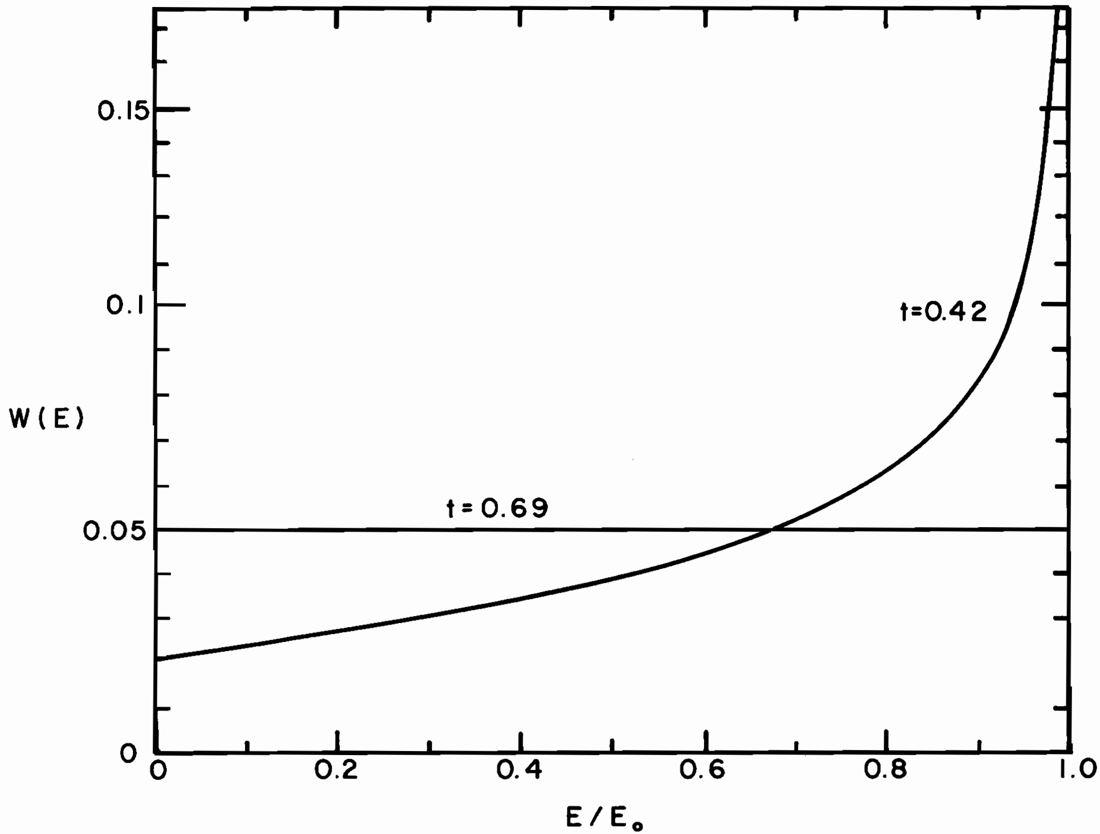

Fig. 7. The probability, W, that an electron of initial energy E_0 will have an energy E after traversing a distance $L = tX_0$, for t = 0.42 and 0.69.