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RANGE AND STRAGGLING OF MUONS

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ABSTRACT

The range of muons is calculated including radiation, pair production, and nuclear effects. It is shown that the large individual energy losses characteristic of these processes reduce the average range by a factor of $\ln 2$ from that which one would get on the basis of simply integrating the average energy loss. The fractional straggling first increases as radiation and pair production effects become important and then decreases as the energy is further increased.

I. INTRODUCTION

The study of the passage of fast-moving particles through matter has been important since the early days of nuclear physics.[1] Many of the experimental techniques of detection and measurement of particles depend on such specific properties of penetration as the total range or as the specific energy loss. Protons and pions at energies below a few hundred MeV have a well-defined average range, but the effects of nuclear collisions obscure this definite range at higher energies: The track of the incident proton becomes completely lost in the accompanying nuclear shower of secondary particles at energies higher than a BeV. Individual electrons never show a well-defined range: at low energy, multiple scattering causes them to diffuse through matter, and at high energies a shower conceals the initial electron.

Muons are more satisfactory particles to consider from this point of view because at low energies multiple scattering is not too serious, and at high energies the effect of nuclear collisions is small. On the other hand, at high energies, bremsstrahlung and direct pair production do occur to increase the energy loss above that due to ionization. The nature of the large individual energy losses due to radiation markedly increase the fluctuations of individual track lengths.

The present study is concerned with a quantitative evaluation of the range and straggling of muons at very high energies. Briefly stated, the range is smaller by a factor $\ln 2$ from what one would calculate by neglecting fluctuations; on the other hand, the fluctuations on the ranges of individual tracks are smaller than might be expected intuitively.

II. RANGE CALCULATIONS

First let us calculate the range assuming only bremsstrahlung losses because this can be done more or less rigorously, then we shall include pair production, nuclear absorption, and ionization. The calculation will parallel that made by the author for electrons.[2] Bethe and Heitler[3] give the energy loss of a particle that has traversed a thickness of matter t. They approximate the radiation spectrum by

$$\sigma(k)\,dk = \frac{dk\,dt}{E\,\ln\,[E/(E-k)]}\,, \tag{1}$$

where $\sigma(k)\,dk$ is the probability of the muon energy E radiating a photon of energy k in passing through a distance dt measured in units of shower length, i.e., for this application in muon radiation lengths divided by $\ln 2$. Then they find that the probability of the particle of initial energy E_0 having an energy E after traversing a finite distance t is

$$w(y,t) = (t-1,y)!/(t-1)! \tag{2}$$

in terms of the incomplete gamma-function $(t-1, y)!$ where $y = \ln(E_0/E)$. From this one can find[4] the result we seek, namely

$$w(y,t)\,dt = e^{-y}\,y\,dt/t! \tag{3}$$

For large values of y and t, the above equation can be approximated by the gaussian form

$$w(y,t)\,dt = (2\pi y)^{-1/2}\exp[-(t-y)^2/2y]\,dt\,. \tag{4}$$

From this we see directly that the mean range r is

$$r = y_{max}\,, \tag{5}$$

where y_{max} is the value of y at the peak of the gaussian, and that the root-mean-square straggling of the range s is given by

$$s = y_{max}^{1/2} = r^{1/2}\,. \tag{6}$$

A. Ionization Loss

If only radiation contributed to the energy loss, the range and the straggling would be infinite. However, as the energy degrades, ionization losses become important and allow us to evaluate y_{max}. Expressing Eq. (5) in terms of energy and then differentiating the mean range with respect to the initial energy gives the average radiation loss on traveling a distance dt,

$$-\frac{dE}{dt} = E\,. \tag{7}$$

That this is neither obvious or trivial is clear if we remember that we are using shower units of length which introduce the factor $\ln 2$.

Now let us express the energy in units equal to $\ln 2$ times the critical energy, β, i.e., the energy lost by a muon to ionization in going a distance of one muon radiation length. Here we are making the rough approximation that the ionization loss is independent of energy: we will examine the validity of this later. We add the ionization loss to Eq. (7) to get

$$-dE/dt = E + 1\,, \tag{8}$$

and integrating this over the energy gives the mean range,

$$r = \log(E_0 + 1)\,, \tag{9}$$

which in units of radiation lengths becomes

$$r = \ln 2\, \ln\left[(E_0/\beta \ln 2) + 1\right]. \tag{10}$$

B. Straggling

Now let us turn to straggling which is manifest in the distribution described by Eq. (3). Equation (6) shows us rather surprisingly that on the basis of radiation loss alone the fractional straggling s/r varies as $1/r^{1/2}$, i.e., becomes smaller as the energy increases. Actually the straggling at low energies will be less than given by Eq. (6) because the energy loss due to inoization has been neglected. The energy loss due to ionization for a muon traveling a distance r is just equal to r in the peculiar energy units of the above theory, hence a fraction of the range r/E_0 can be ascribed to ionization loss.

The straggling of this fraction will be less than one per cent; hence for any energy for which this calculation can apply, we can neglect it completely. The remaining fraction of energy that is lost to radiation processes, $(1 - r/E_0)$ will vary roughly as given by Eq. (4), which will only be valid when the fraction $(1 - r/E_0)$ is large. As a rough interpolation formula, suggested by Monte-Carlo calculations,[2] we can write

$$s = \left(1 - \frac{r}{E_0}\right)^{1/2} ,$$

and remember that the gaussian distribution has a cut-off at a range equal to E in shower units or E/β in radiation lengths.

Pair Production and Other Energy Losses

Up until now we have neglected direct pair production as well as nuclear interactions. In fact, the loss of energy due to the direct production of electron pairs is comparable (within about ten percent) to the loss due to bremsstrahlung.[5] The energy loss due to nuclear interactions amounts at most to a few percent of the total loss, being relatively larger for light elements and at high energies. On the basis of a Weissacker-Williams consideration of direct pair production, we can expect the atomic shielding factor to be almost identical to that for bremsstrahlung. Thus, except for the lowest energy losses where the mass of the created pair becomes significant, we can expect the two processes to be roughly the same.

In order to include these effects, I suggest that we define a muon interaction length in place of the muon radiation length that we have been using thus far so that the above theory will be valid. Until a more exact calculation of the muon interaction length is made, I suggest that we use simply one half of the muon radiation length. On this basis, the range and straggling in various materials has been computed and is plotted in Fig. 1. The range in μ is significantly smaller than that given in Ref. 5 where the effects of fluctuations were neglected.

REFERENCES

[1] H. Bethe, Ann. der Physik. 5, 325 (1930); N. Bohr, Phil. Mag. 25, 10 (1913).
[2] R. R. Wilson, Phys. Rev. 84, 100 (1951).
[3] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) 146, 94 (1934).
[4] Using

$$\int_0^y x^t e^{-x} dx = t! - e^{-y} \sum^{t} t!/\mu!\, y^{\mu} ,$$

which can be derived by successive partial integrations (for integral t), one obtains Eq. (3) by taking the difference between t and (t - 1). This procedure was shown to me by S. Pasternack: orginally, I simply guessed the result (3).

[5]R. H. Thomas, Lawrence Radiation Laboratory UCID-10010, July 2, 1964.

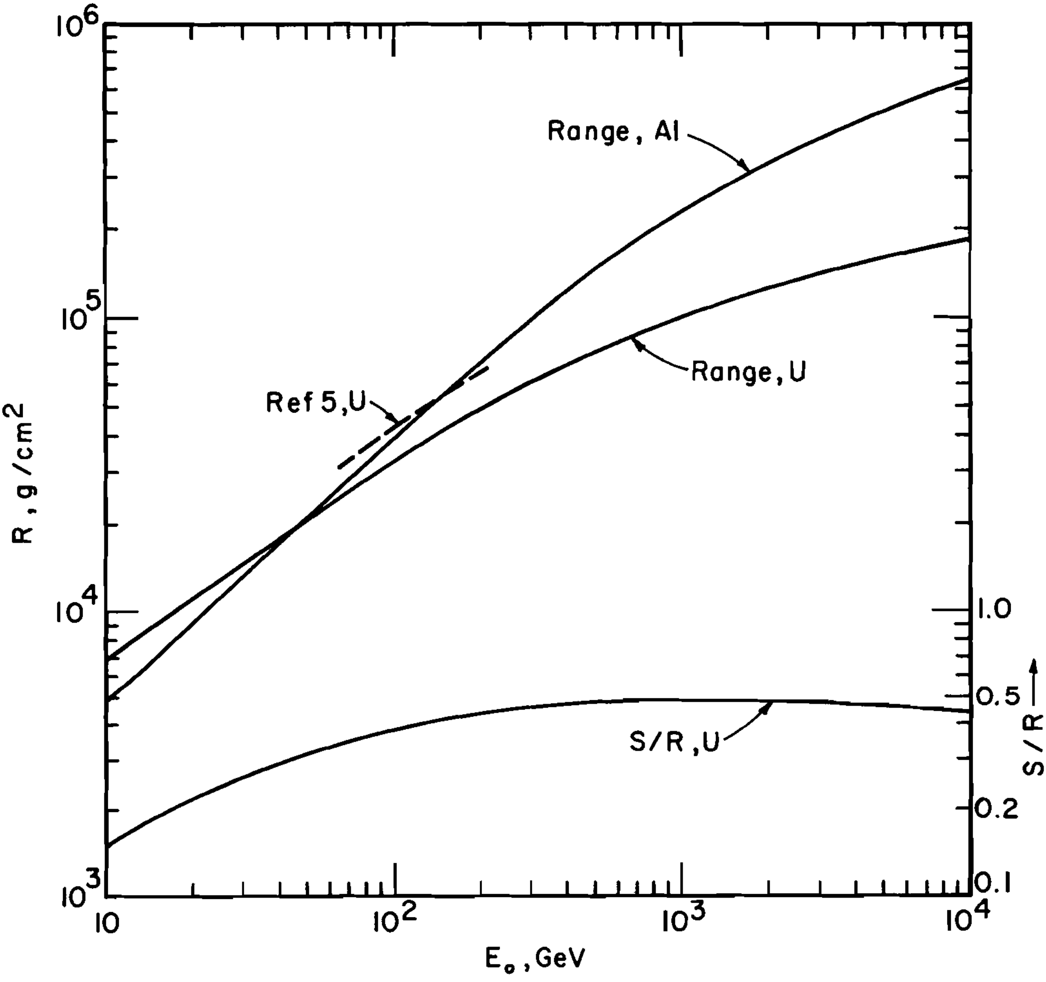

Fig. 1. Range and straggling of muons.