SOME SIMPLE MUON BEAMS

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ABSTRACT

A few trivial muon beams are discussed. With a single quadrupole lens, one may obtain 10^8 $\mu^+/10^{13}$ interacting protons at a muon momentum of 100 ± 5 GeV/c within a 10 cm diameter area at the experiment, with an angular divergence of 1 mrad.

I. INTRODUCTION

In an attempt to gain a better perspective of the many muon beams proposed for NAL,[1] it has proven useful to just consider a few very naive beam configurations. In fact, such a beam appears adequately matched to the various experimental requirements, questioning the advisability of constructing elaborate (and expensive) multimagnet facilities for first-generation muon experiments.[2]

In the following calculations, μ^+ fluxes are calculated from π^+ production spectra as depicted by Hagedorn and Ranft.[3] Any additional flux from K^+ decays is held in reserve to help compensate for any beam losses not considered here. All fluxes are based on 10^{13} protons interacting in a beryllium target.

The typical beam considered here delivers 100 ± 5 GeV/c muons into a 10-cm aperture. The muon angular divergence is kept as small as possible, since it will be difficult, if not impossible, to determine individual muon trajectories in fluxes greater than 10^7/sec.

II. "PERFECT" BEAM

An upper limit to the fluxes obtainable is first calculated by considering a beam in which all π's decay and every muon of the desired momentum is captured (infinitely long, perfectly focused beam).

Provided the muon momentum bite $\pm \Delta p/p\,|_\mu$ is small compared to $0.43\, p_\pi$ (width of the muon spectrum), all useful muons come from parent pions in a momentum interval $p_0 < p_\pi < p_0/0.57$, where p_0 is the nominal muon momentum. The muon flux is obtained by integrating the pion production over angle and momentum, weighted by the probability that the decay muon lies in the required momentum interval, namely $2\Delta p_\mu / 0.43\, p_\pi$. The flux is then

$$N_\mu = 1.45 \times 10^{10} \sum_{p_\pi = p_0}^{p_0/0.57} \sum_{\theta_\pi = 0}^{\theta_\pi(\max)} \frac{d^2N}{dp_\pi\, d\Omega} \times \frac{\theta_\pi\,[\mathrm{mrad}]}{p_\pi} ,$$

where θ_π is summed in 1 mrad steps and p_π in 10-GeV/c intervals. Figure 1 shows the flux as a function of pion solid angle for $p_0 = 100 \pm 5$ GeV/c. Beyond pion angles of ~5 mrad, one gains flux rather slowly and expensively. For θ_π (max) = 10 mrad, a limit of $3 \times 10^{10}\ \mu^+/10^{13}$ interacting protons is found.

Infinitely long beams are expensive. In a one-kilometer beam, 17% of the π's will decay, reducing the limiting flux to $5 \times 10^9\ \mu^+$. This number should certainly be regarded as the highest flux obtainable at 100 ± 5 GeV/c.

III. NO-QUAD BEAM

Consider now an unfocused but finite length beam. The flux calculation is as before with additional weight to represent the probability that the pion decays within a length L (in meters), namely $0.018\, L/p_\pi$. Now,

$$N_\mu = 2.7 \times 10^{8}\, L \sum_{p_\pi = p_0}^{p_0/0.57} \sum_{\theta_\pi = 0}^{\theta_\pi(\max)} \frac{d^2N}{dp_\pi\, d\Omega} \times \frac{\theta_\pi}{p_\pi^2}$$

The pion spectrum is now weighted by $1/p_\pi^2$, strongly suppressing high momenta.

The beam configuration is shown in Fig. 2. Pions drift a distance L before being absorbed in a 10-meter carbon filter. The dipole pair serves to momentum-analyze the muons to ± 5% and to split them away from the primary proton beam which strikes a heavy hadron shield.

For a 10-cm aperture and L = 90 m, one finds $2.4 \times 10^7\ \mu^+$ at $p_0 = 100 \pm 5$ GeV/c. A length of 90 meters is near optimum. As the length L increases, the flux drops (by a factor of 2 for L = 200 m). However, the same flux results for L = 40 m but with twice the muon angular divergence. For L = 90 m, the beam divergence is ± 0.5 mrad. As the decay angles are no more than 0.2 mrad, the muons appear to emanate from a small source and could be focused to spot sizes less than 10 cm. The spatial properties of the beam at the experiment are limited by multiple scattering in the filter of ~1 mrad. Any beam losses in the filter have been ignored.

Of course, the flux could be increased by enlarging the channel width with subsequent focusing to restore the spot size. A flux of 2.4×10^8 results from a 35-cm aperture.

Figure 3 shows the muon flux as a function of the momentum p_0. The maximum muon flux available at NAL occurs at 30-40 GeV/c.

IV. SINGLE-LENS BEAM

A substantial increase in flux results from the addition of a single lens placed close to the production target to increase the pion acceptance. This lens focuses pions only, all muons still coming from the drift space of length L. Pions of momentum equal to the desired muon momentum are focused at the end of the drift space. The pion momentum bite $\Delta p/p|_\pi$ is now limited by the channel aperture. If the lens aperture is the same as that of the channel, one finds

$$\Delta p/p|_\pi = \pm \frac{F}{L + 2F} ,$$

where F is the distance from the target to the lens. For a standard geometry (10-cm aperture, F = 10 m, L = 90 m), $\Delta p/p|_\pi = \pm 8\%$. Note that $p_\pi - p_\mu$ is very small; the decay angles are less than 0.2 mrad.

At 100 ± 5 GeV/c, one now obtains 1.6×10^8 muons/10^{13} protons, a factor of 7 improvement over the no-lens case. Again, a length of 90 m is about optimum. For a longer beam, the decay rate is higher but $\Delta p/p|_\pi$ is less. Scaling the beam down to 30 m to fit into a "conventional" target box costs a factor of 2 in flux and has 4 times the angular divergence.

A few final notes:

1. A specific lens configuration was studied only to the extent that a simple quadrupole triplet will suffice with tolerable dimensions and pole fields.

2. The muon polarization is 100%, as only the most energetic muons are accepted for a given pion momentum ($\Delta p/p|_\pi$ is small). However, one may easily reverse the polarization by tuning the momentum analysis dipoles to the low end of the muon spectrum. This is very difficult to do in a conventional multi-quad system that transports both π's and μ's through the same magnets.

3. The dependence of flux on muon momentum is as shown in Fig. 3 for the no-quad beam. The fluxes may be normalized at $p_0 = 100$ GeV/c.

I thank Al Maschke for numerous discussions of some of the problems and limitations in the muon beam business.

REFERENCES

[1] There are a host of unwritten beam suggestions. Some written down are:

T. E. Toohig, Lawrence Radiation Laboratory UCID-10180.

T. Yamanouchi, A Muon Beam at NAL, National Accelerator Laboratory 1968 Summer Study Report B. 2-68-38, Vol. II, p. 1.

L. C. Teng, Possible Layouts for a Muon-Long Spill Neutrino (μ-ν_L) Beam, National Accelerator Laboratory 1969 Summer Study Report SS-64.

[2] For a comparison of various beam possibilities, see J. H. Christenson, Muon Beams Compared, National Accelerator Laboratory 1969 Summer Study Report SS-110.

[3] M. Awschalom and T. White, Secondary Particle Production at 200 GeV, National Accelerator Laboratory FN-191, June 9, 1969.

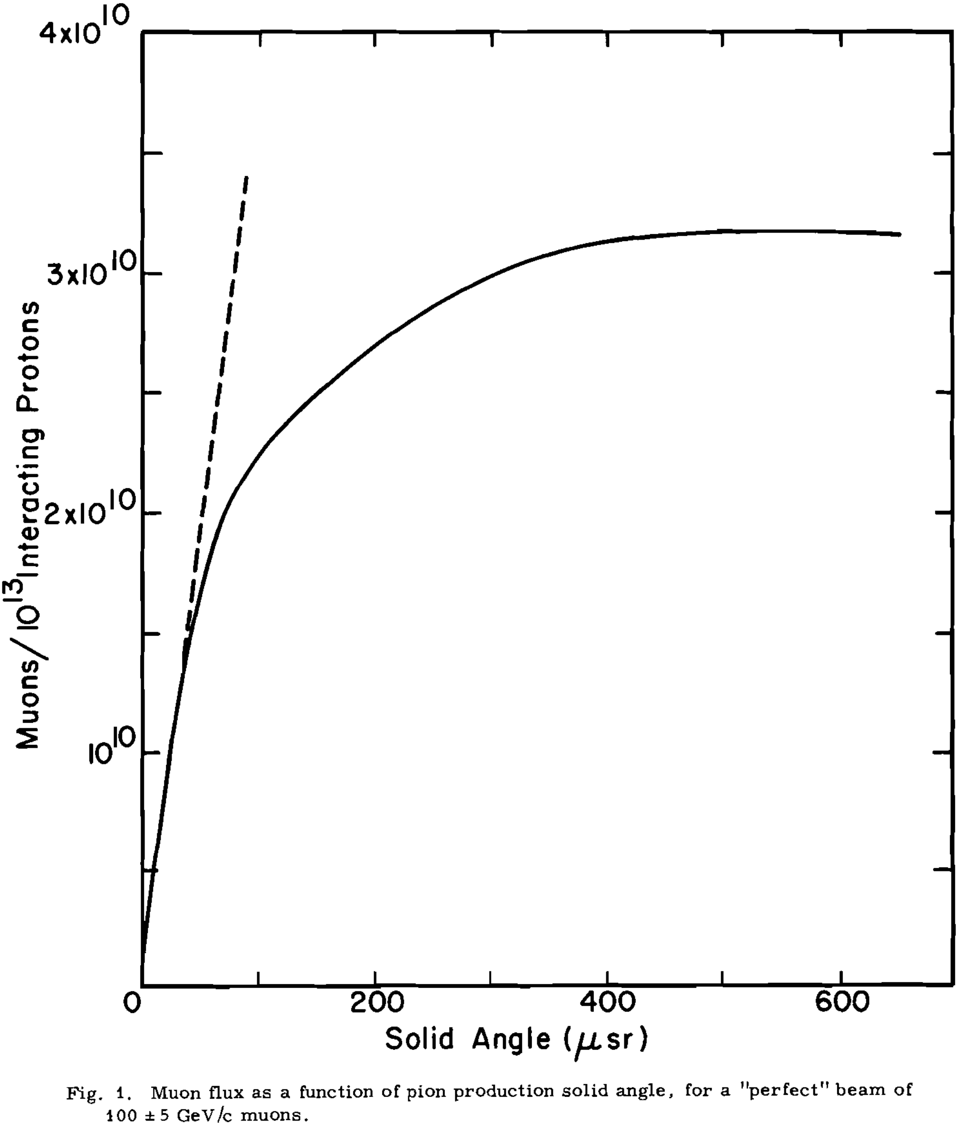

Fig. 1. Muon flux as a function of pion production solid angle, for a "perfect" beam of 100 ± 5 GeV/c muons.

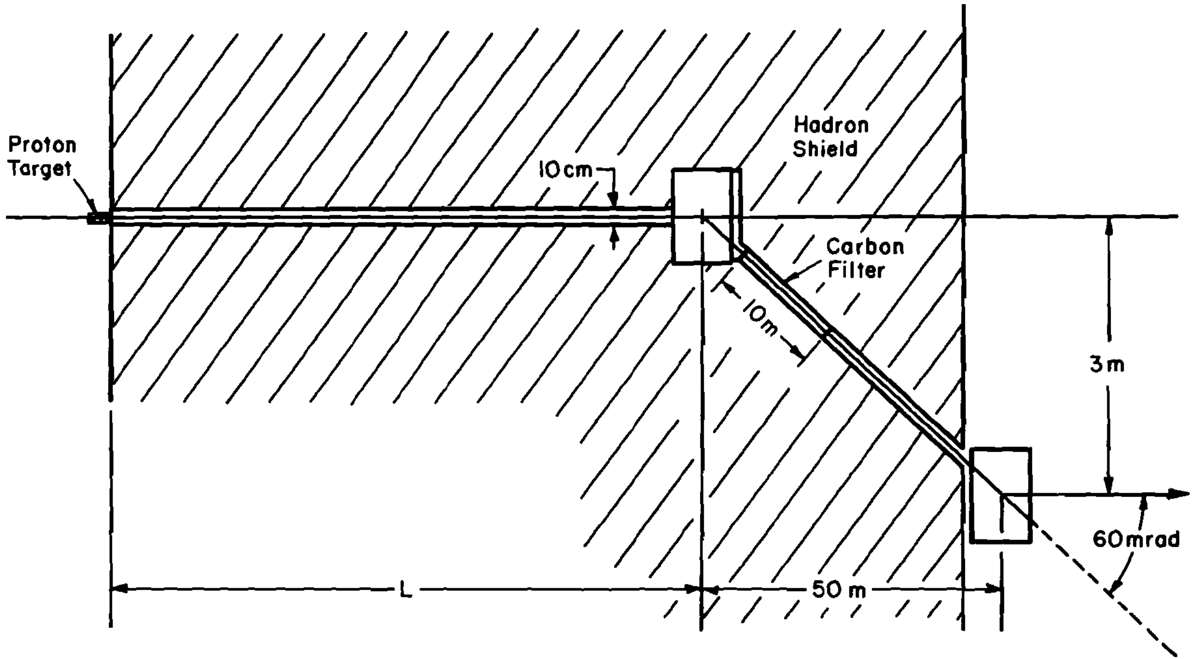

Fig. 2. Configuration of no-quad beam. In the case of the single-lens beam, a lens is installed F meters downstream of the target and the decay length L is maintained.

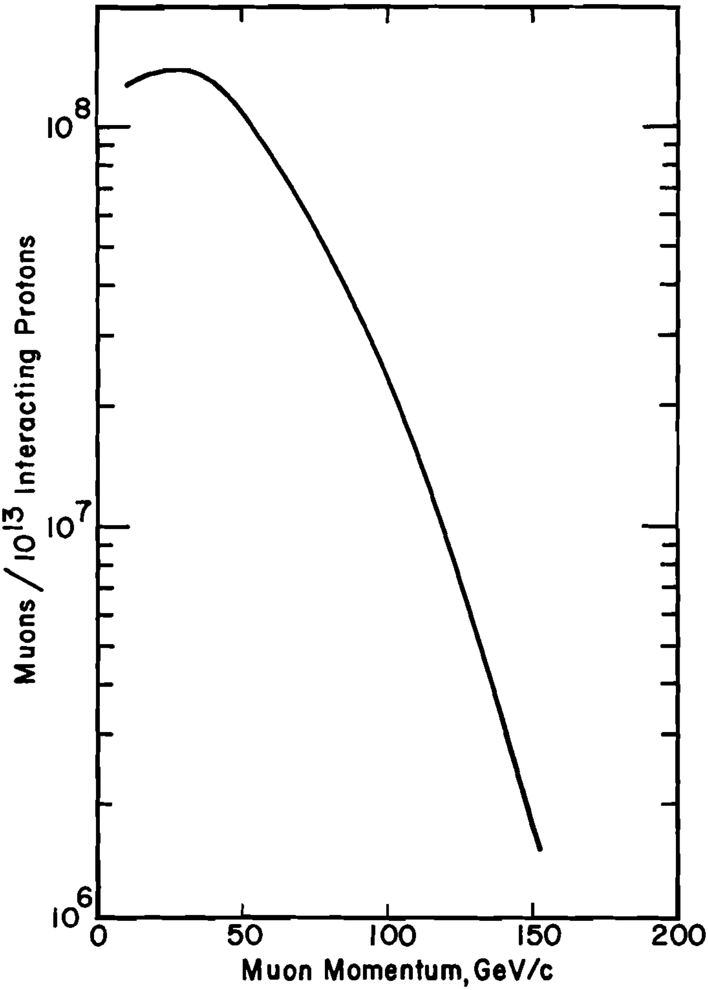

Fig. 3. Muon flux as a function of muon momentum for the no-quad beam.