THE ELECTROMAGNETIC FORM FACTOR
OF THE CHARGED PION (MUON, KAON, ELECTRON)

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Introduction

Extensive work has been invested in understanding the electromagnetic form factor, EFF, of the proton in the space-like region and also in the time-like region. The ρ, ω, and φ should contribute to this form factor but do not predict the $q^{-4}$ fall off of the EFF at high $q$. The charged pion EFF should be understood in terms solely of the ρ meson; thus it should be a much simpler beast to treat theoretically.

Different techniques have been employed in the past to obtain the EFF of the pion. Table I lists the results. Theory predicts the following behavior:

$$
\text{EFF}(\pi) = \left(1 + q^2/m_{\rho}^2 \right)^{-1}
$$

which would give $r_\pi = 0.63 \times 10^{-13}$ cm. We propose a direct measurement of EFF (π) via elastic electron scattering using atomic electrons as the target material in a beam of high momentum pions.

Experiment

Liquid hydrogen is an ideal target since it will maximize the number of electrons per nucleon and also number of electrons per radiation length. The number of nucleons should be kept low to minimize the
background of strongly interacting processes; the number of radiation lengths should be kept low to minimize the bremsstrahlung energy loss of the recoil electron.

The available 100-GeV $\pi^+$ beams at NAL might have $\pi^+$ intensities in the beam $\sim 5 \times 10^7$/burst with 3 to 4 times as many protons. If all counters and spark chambers could be kept out of the main beam, full utilization of this intensity might be realized. Two things speak against this approach, however:

1. The number of strong interactions in the hydrogen target are about 6%; one begins to be limited by the rate in the anticounters.
2. The desired cross section is high enough that the running time does not become excessive even if the incident beam is lowered to $\sim 10^6$/pulse.

A midway philosophy is shown in Fig. 2 with two wire chambers in the beam as if the intensity were lowered but with an extra bending magnet in the electron arm of the spectrometer so that all variables of the $\pi^+$ and $e^-$ can be obtained without any information from the wire chambers in the beam. Thus if the background problems are not severe, one might use this technique without the first two wire chambers in the beam to study the end point of the spectrum where the cross section becomes zero and to study the $\mu^+$, $e^+$, and $K^+$ scattering from $e^-$'s where the fraction of these particles in the beam might be appreciably lower than the $\pi^+$'s.

The region of available momentum transfer $q$ via this technique is limited. One should easily obtain the rms radius of the pion defined as
The range of $q$ may be high enough that one can distinguish among models which have the same $R_\pi$. For example

$$\text{EFF} = e^{-\frac{1}{6} q^2 R_\pi^2}$$

and

$$\text{EFF} = \left(1 + \frac{1}{6} q^2 R_\pi^2 \right)^{-1}$$

will have the same slope at $q^2 = 0$ but (1) gives an effect of 0.756 at the maximum available $q^2$ while (2) gives 0.770 each with $R_\pi^2 = \frac{6}{m_p^2} = (0.62 f)^2$. Thus an experiment of statistical precision of $< 1\%$ could tell the difference between these two shapes which are characterized by the same rms radius. Figure 1 shows the maximum available kinematic quantities for various incident particles as a function of incident momentum.

The incident beam is focused to a 1 cm spot with 0.1 mrad angular divergence as it enters the LH target. This is sufficiently precise that no additional measurement need be made. Anticounters around the target insure that the charged products of a reaction are well collimated in the forward direction. Wire spark chambers measure the angle and position of the $\pi^+$ and $e^-$ particles after they have been bent away from
the incident beam. The electron's momentum is measured, its identity finally determined by a lead-scintillator counter sandwich. The trigger will be

1. no anticounters being triggered
2. a charged $^+$ strongly interacting particle
3. a negative electron.

The final fit to the event will be a 4c fit (overconstrained by 4 equations) with all quantities in the initial and final state particles measured or known.

(A 4c fit at high energies, however, becomes effectively a 3c fit.)

$$T_{e\text{max}} \text{ for } 100\text{-GeV/c incident } \pi^+ \text{ is } 84 \text{ GeV. The maximum momentum transfer, } q_{\text{max}}, \text{ is}$$

$$q_{\text{max}} = \sqrt{-t_{\text{max}}} = \sqrt{2M e T_e(\text{max})} = 0.29 \text{ GeV/c}.$$

The experiment covers a range of momentum transfers from $q(\text{max})$ to $1/2 q(\text{max})$; the predicted effect for rho dominance is:

$$\frac{d\sigma}{dt} \text{ (meas)} = \left| \text{EFF (}\pi\text{)} \right|^2 = \left( \frac{1}{1 + \frac{q^2}{M^2 \rho}} \right)^2,$$

which is 0.77 at $q(\text{max})$ and 0.94 at one half $q(\text{max})$. One day's running will be 24 hours $\times$ 60 minutes $\times$ 15 pulses/minute $= 21,600$ pulses $\times$ 1 real event/pulse $= 20,000$ events. One real event can be obtained
with $10^6 \pi^+$, so this gives a comfortable safety factor for background (could stand maybe 50 background trigger rate and still not sacrifice the real event rate) and a comfortable incident beam intensity of $\sim 4 \times 10^6$ particles of $3 \times 10^6 p's$ and $1 \times 10^6 \pi's$. The high proton background cannot fake a real event in a direct manner, since $T_{e\text{max}}$ for an incident proton is less than 6 GeV. If a good experiment consisted of 20 bins of $10^4$ events each (1% statistics) or $\sim 2 \times 10^5$ events, then 10 days of data taking time would suffice for the $\pi^+ e^-$ experiment. Five days would allow a $p e^-$ experiment as a check; additional time would allow a $K^+ e^-$ and $\mu^+ e^-$ experiment, although special enriched beams may be required for these latter experiments.

A good monitor for $\pi^+'s$ is needed, one that has an absolute precision of better than 1%. An idea that might allow this precision might be collecting and integrating the Cerenkov light from the beam transport pipe from the production target to the experiment where a small amount of gas (e.g. 0.043 atm of $H_2$) is admitted to be above threshold for $\pi'$s, but below $K'$s and $p'$s.

Equipment: The following list of items would be needed for the experiment and requested of NAL:

1. H-magnet 1 ft wide $\times$ 4 in. high $\times$ 3 m long 18 kG
2. C-magnet 1 ft wide $\times$ 8 in. high $\times$ 3 m long 18 kG
3. Beam monitor of $<$1% precision
4. 1/4 m $\times$ 2 cm diameter liquid hydrogen target
5. Small computer with mag tape storage
6. DISC Cerenkov counter to measure the fraction of $\pi^+$ in the beam
REFERENCES


Table I. Techniques for Measuring the Pion Form Factor.

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<th>Present Results</th>
<th>Method</th>
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<td>1. $\pi e$ elastic scattering 1</td>
<td>$R_\pi &lt; 3 \times 10^{-13}$ cm</td>
<td>direct experiment</td>
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<tr>
<td>2. $\pi \pm \alpha$ elastic</td>
<td>$R_\pi &lt; 1 \times 10^{-13}$ cm</td>
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<td>scattering 2</td>
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<td>3. $e^- p \rightarrow e^- \pi^+$</td>
<td>$R_\pi = 0.86 \pm 0.14 \times 10^{-13}$ cm</td>
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Fig. 1. Elastic scattering from electrons. The mass assumed in the expression for the form factor FF is that of the rho meson.
Fig. 2. Experimental layout for measuring scattering from electrons.