FIELD SHAPE TOLERANCES FOR NAL SUPERCONDUCTING BEAM-TRANSPORT ELEMENTS

T. Fields
Argonne National Laboratory

Introduction

In contrast to the situation with conventional beam-transport elements, there is little direct information available on the detailed properties, particularly field shape, of practical superconducting (sc) dipoles and quadrupoles. This note is aimed at estimating field shape tolerances which will be needed by designers of sc beam-transport elements.

Types of Field Errors

Many of the calculational methods used in beam-transport system design assume the following kinds of idealized properties for the transport elements (see Sandweiss' article in Shutt's book):

1. The field is two dimensional in the magnet interior; i.e., there exists a "z-axis".

2. There is a symmetry plane; i.e., the scalar potential is given by

\[ \phi = \sum_{k=1}^{\infty} C_{2k} r^k \sin k\theta \]

with no terms in \( \cos k\theta \).

3. Symmetry about the poles of the magnet, i.e., only multipole moments corresponding to \( 2k + 4nk, n = 0, 1, 2, \ldots \) are
present. Thus the field in a dipole magnet \((2k = 2)\) should have only \(C_2, C_6, C_{10}\), etc., components, and a quadrupole magnet \((2k = 4)\) should have only \(C_4, C_{12}\), etc.

4. The field is not only symmetric about the magnet poles, but the error components given in 3) are also zero.

5. The field has flat, thin edges in the \(z\) direction.

### General Effects of Field Errors

No real magnet, of course, is perfect in any of the above respects. For information on typical field errors, their effects, and other intrinsic aberrations, see pp. 238-240 of Sandweiss' article.

Qualitatively, the situation is that for the best beam optics currently used, a field precision of \(-0.2\%\) is needed in quadrupoles and rather better for dipoles. This refers to the integrated field error in traversing a beam-transport element at maximum usable aperture. As will be discussed below, many beams at NAL will require magnets of similar precision.

### Sources of Field Error in Superconducting-Transport Magnets

Three possible sources of field inaccuracy of sc magnets are:

1. Lacking iron poles, the conductor position and position stability are much more critical than for conventional elements. Since the coil mass and suspension members both have to be kept light, it may be difficult to achieve reproducible high-field precision (and location).
2. The diamagnetic currents will distort the field shape in a history- and field-dependent way.

3. The field source as discrete conductors (within which there will be current sharing) rather than as the smooth surface of iron may cause field problems near the winding.

Remarks

It seems clear that field inaccuracies arising from any of these causes could be made arbitrarily small, by making sufficient investment of time and money. The main problem is to find practical ways to keep field errors low enough. Hence, the object of this note is to develop realistic field shape criteria against which designs for proposed D's and Q's can be compared, and which can also be used to indicate the class of applications for which an actual magnet has tolerable aberrations.

Precision Needed for "Good Optics" Beams at NAL

Consider the rays incident on a point at the edge of the aperture of the first quadrupole in a beam (in either plane). If we take a (full) target height or width of \( t \), and a corresponding Q (full) aperture of \( a \), then the fractional angular spread of the rays at this point is

\[
\pm \epsilon = \pm \frac{2t}{a}.
\]

Taking \( t = 1 \text{ mm} \) and \( a = 75 \text{ mm} \) as typical values for NAL, we obtain \( \epsilon = \pm 2.7\% \). What is then required is that the (incoherent) angular deflection error for rays traversing the maximum aperture of beam
system be less than this intrinsic angular spread, in order that the final image size be determined mainly by the target size. (Because the focusing power, $1/f$, of the several quadrupole pairs or triplets in the beam line are usually comparable, one can often use percentage rather than absolute angular deflection tolerances in estimating the precision needed for each set.)

This is very nearly the same numerical situation as for the "precise" separated beams at present energies, and the implication for magnet precision is known to designers of such present-day beams. Roughly, it is that the quadrupole field errors at the maximum usable part of the Q bore should not exceed ~ 0.2%, that the magnet must thus be located (transversely and in angle) to a corresponding position accuracy, and that the intrinsic edge aberration of a perfect quadrupole is not quite negligible.

The field shape precision in quadrupoles which is required to maintain the phase area originating from the finite target size is dictated mainly by two effects:

1. Because the net deflection imparted to a ray (in the more sensitive of the two deflection planes) by a Q pair is the (small) difference of two large numbers, incoherent field errors in a single member of the pair are magnified (in the net angular deflection) by a factor of 5-10.

2. A practical beam uses several Q pairs or triplets as well as other elements, so that the aberrations per element must
be kept down by a factor of 2-4 compared to the case where a single element or pair is used.

These are not precise criteria, but they can of course be calculated precisely for any given beam-transport system.

For dipoles, the corresponding general requirement is that the field be homogeneous across the aperture to $\sim 10^{-5}$ (see Sandweiss, pp. 239-240). This can only be attained in conventional magnets by using a sextupole pole face winding or shims. For some kinds of beams, this sextupole correction might be performed outside the bending magnet.

**Dependence of Allowable Q Aberrations on Aperture**

We note that the fractional angular accuracy required for the motion of the previously described rays is inversely proportional to the magnet aperture. This implies that: 1) since most aberrations of a given magnet are a rapidly increasing function of radius, there will exist a rather well-defined fraction of the aperture of a real magnet which is useful for a given application, and 2) the allowable limits on average misalignment of the magnet (and its winding) tend to be independent of bore, with a magnitude in the range of $\sim 5-10$ mils.

**Field Stability**

Consider a horizontally parallel beam traversing a bending magnet, and then being brought to a ($\sim 1$ mm) focus by a typical lens of 30 m focal length. We have, in an obvious notation:
\[ \delta X = \delta \theta \quad f = r \theta \quad f \sim r (0.1) (30 \text{ m}), \]

where \( r \) is the fractional regulation of the bending magnet. Setting \( \delta X = 0.3 \text{ mm} \), we obtain \( r = 10^{-4} \). Regulating the field of a superconducting bending magnet to this accuracy should be much easier and cheaper than for a corresponding conventional magnet. However, transients which might be associated with supplying coolant to the sc magnet could conceivably cause troublesome disturbances.

For quadrupoles, field stabilities of \( \approx 10^{-3} \) should be good enough.

**Nonprecise Beam Transport**

Certain applications, such as muon channels, might require a large number of magnets whose aberrations are quite noncritical. Most beams, however, are likely to require precise optics. For example, a momentum bite of 0.1% may become rather standard, and this will demand very accurate control of the "horizontal" as well as the "vertical" motion.

**Iron Shields?**

Some of the ways in which the decision on whether to use either iron shields or yokes interacts with these other questions are:

1. Additional mechanical constraints (coil-iron alignment and possible body forces on the coil) will be occasioned by the iron.
2. A major factor in favor of using iron is the insensitivity of field shape to external fields or iron objects. In particular,
a changing stray field from neighboring beams could cause trouble in maintaining a precise set of orbits.

**Recommended Next Steps**

1. Prototype D and Q magnets should be designed, built, tested, and operated, with a major aim being to achieve field precision and stability comparable to those of high quality conventional magnets.

2. As beam designs for NAL become more definite, quantitative ray-tracing studies of the following should be made for each beam: magnet alignment tolerances, magnet stability tolerances, effects of $C_{12}$ in Q's, possibility of correcting $C_6$ of bending magnets by external sextupoles, the need for octupole correctors for Q edge aberrations, etc. To achieve high performance in both horizontal and vertical motion will be difficult.

**REFERENCE**
