XYZ-like Spectra from Laplace sum rules at N2LO in the chiral limit

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- Discoveries of new states on charmonium (bottomium) spectroscopy having unconventionnal hadronic structure (e.g $X_c(3872)$ and $Z_c(3900)$ 1^{++} states ,Y(4260) 1^{--} states).
- Exotic structures, such as molecule and/or fourquark states, could be a possible explanation about their nature.
- QCD spectral sum rules used to determine hadronic parameters.
- Analysis of XYZ-like spectra using this method have been done but only in the lowest order of perturbation.
- Estimation of molecule and four-quark states couplings and masses at higher orders with methods of Laplace Sum Rules.

Outlines

1. Heavy-light molecule and fourquark states currents
2. Expressions of the mass and coupling from QCDSSR.
3. 0^- molecule states.
4. 1^- molecule states.
5. Summary

Masse and Coupling from Laplace Sum Rules

$$\text{Mass}: M_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt\ t\ e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{OPE}(t)}{\int_{4m_Q^2}^{t_c} dt\ e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{OPE}(t)}$$

$$\text{Coupling}: f_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt\ e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{OPE}(t)}{e^{-\tau M_H^2} M_H^8}$$

$$\begin{aligned}\rho^{OPE} &= \frac{1}{\pi}\mathrm{Im}\Pi^{OPE}(t)\\ &= \rho_{pert}\left(1+\frac{NLO+N2LO}{LO}\right)+\rho_{\langle q\bar{q}\rangle}+\rho_{\langle g^2G^2\rangle}+\\ &\quad \rho_{\langle qGq\rangle}+\rho_{\langle qq\rangle^2}+\rho_{\langle g^3G^3\rangle}+\rho_8\end{aligned}$$

Masse and Coupling from Laplace Sum Rules

- Assumption of a factorization of the four-quark currents for the evaluation of perturbative part at NLO and N2LO.
- Plot of curves of M, f as function of τ for differents values of tc
- Optimal results obtained by applying stability criteria(inflexion point, minimum)
- Extraction of values from tc corresponding to the beginning of τ-stability and the one where tc-stability is reached.
- Requirements : the pole contribution is larger than the continuum one, convergence of PT-series.

Scalar	0^{++}	
$\bar{D}D,\ \bar{B}B$		$(\bar{q}\gamma_5 Q)(\bar{Q}\gamma_5 q)$
$\bar{D}^*D^*, \bar{B}^*B^*$		$(\bar{q}\gamma_\mu Q)(\bar{Q}\gamma^\mu q)$
$\bar{D}_0^*D_0^*,\ \bar{B}_0^*B_0^*$		$(\bar{q}Q)(\bar{Q}q)$
$\bar{D}_1D_1, \bar{B}_1B_1$		$(\bar{q}\gamma_\mu\gamma_5 Q)(\bar{Q}\gamma^\mu\gamma_5 q)$
Axial-vector	1^{++}	
$\bar{D}^*D,\ \bar{B}^*B$		$\frac{i}{\sqrt{2}}\Big[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma_5 Q) - (\bar{q}\gamma_\mu Q)(\bar{Q}\gamma_5 q)\Big]$
$\bar{D}_0^*D_1,\ \bar{B}_0^*B_1$		$\frac{1}{\sqrt{2}}\Big[(\bar{q}Q)(\bar{Q}\gamma_\mu\gamma_5 q) + (\bar{Q}q)(\bar{q}\gamma_\mu\gamma_5 Q)\Big]$
Pseudoscalar	$0^{-\pm}$	
$\bar{D}_0^*D,\ \bar{B}_0^*B$		$\frac{1}{\sqrt{2}}\Big[(\bar{q}Q)(\bar{Q}\gamma_5 q) \pm (\bar{Q}q)(\bar{q}\gamma_5 Q)\Big]$
$\bar{D}^*D_1,\ \bar{B}^*B_1$		$\frac{1}{\sqrt{2}}\Big[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma^\mu\gamma_5 Q) \mp (\bar{Q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma^\mu Q)\Big]$
Vector	$1^{-\pm}$	
$\bar{D}_0^*D^*,\ \bar{B}_0^*B^*$		$\frac{1}{\sqrt{2}}\Big[(\bar{q}Q)(\bar{Q}\gamma_\mu q) \mp (\bar{Q}q)(\bar{q}\gamma_\mu Q)\Big]$
$\bar{D}D_1,\ \bar{B}B_1$		$\frac{i}{\sqrt{2}}\Big[(\bar{Q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma_5 Q) \pm (\bar{q}\gamma_\mu\gamma_5 Q)(\bar{Q}\gamma_5 q)\Big]$

Interpolating currents with defenite C-parity

Fourquark states

Scalar	0^+	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\,\gamma_5 C\,\bar{Q}_e^T\right)+k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\, C\,\bar{Q}_e^T\right)\right]$
Axial-vector	1^+	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\,\gamma_\mu C\,\bar{Q}_e^T\right)+k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\,\gamma_\mu\gamma_5 C\,\bar{Q}_e^T\right)\right]$
Pseudoscalar	0^-	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\, C\,\bar{Q}_e^T\right)+k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\,\gamma_5 C\,\bar{Q}_e^T\right)\right]$
Vector	1^-	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\,\gamma_\mu\gamma_5 C\,\bar{Q}_e^T\right)+k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\,\gamma_\mu C\,\bar{Q}_e^T\right)\right]$

- Use of the same strategies and approaches for the heavy-light molecule and fourquark states study.
- Analysis of the $\bar{D}D\ 0^{++}$ and $\bar{D}_0^* D\ 0^{-\pm}$ molecule states as illustrations.

$\bar{D}D$ 0^{++} Molécule state

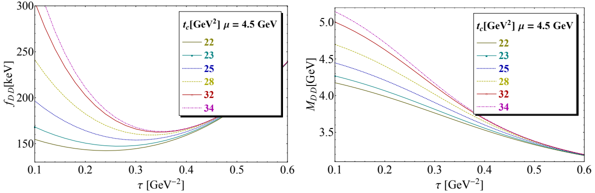

FIGURE – $f_{\bar{D}D}$ and $M_{\bar{D}D}$ at LO as function of τ for different values of t_c,for μ=4.5 GeV

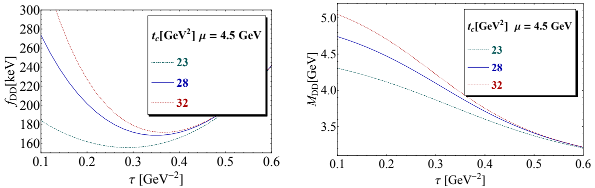

FIGURE – $f_{\bar{D}D}$ and $M_{\bar{D}D}$ at NLO as function of τ for different values of t_c,for μ=4.5 GeV

$\bar{D}D$ 0^{++} Molécule state

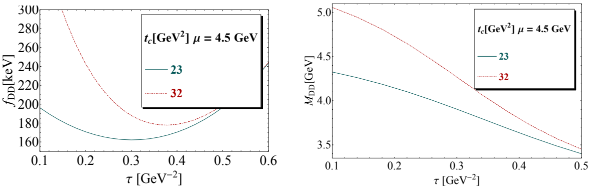

FIGURE – $f_{\bar{D}D}$ and $M_{\bar{D}D}$ at N2LO as function of τ for different values of t_c,for μ=4.5 GeV

- beginning of τ-stability : t_c= 23 GeV^2, $\tau \simeq$ 0.25 GeV^{-2}
- t_c-stability starts to be reached : t_c= 32 GeV^2, $\tau \simeq$ 0.35 GeV^{-2}

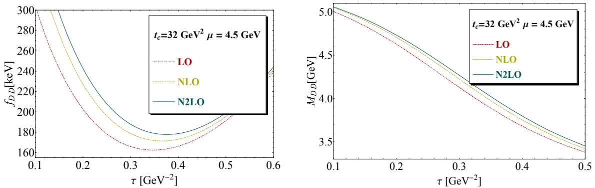

FIGURE – $f_{\bar{D}D}$ and $M_{\bar{D}D}$ as function of τ for $t_c = 32$ GeV^2, for μ=4.5 GeV,for different truncation of the PT series

- NLO $\rightarrow$ N2LO : the coupling increases about 3.6 %
- NLO $\rightarrow$ N2LO : the mass decreases about 1 per mil

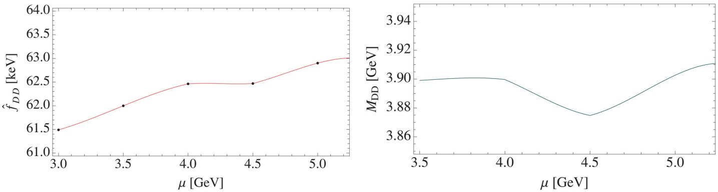

FIGURE – $\hat{f}_{\bar{D}D}$ and $M_{\bar{D}D}$ as function of μ for the corresponding τ-stability region, for $t_c = 32GeV^2$

- The optimal results deduced at $\mu \simeq 4.5\text{GeV}$
- $f_{\bar{D}D} \simeq$ 170(15) keV and $M_{\bar{D}D} \simeq$ 3898(36) GeV

$\bar{D}_0^* D$ $0^{-\pm}$ Molécule state

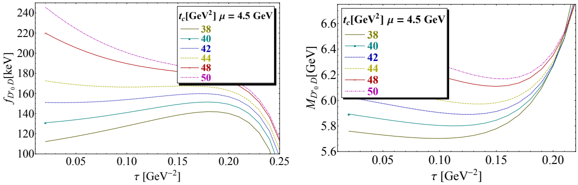

FIGURE – $f_{\bar{D}_0^* D}$ and $M_{\bar{D}_0^* D}$ at LO as function of τ for different values of t_c,for μ=4.5 GeV

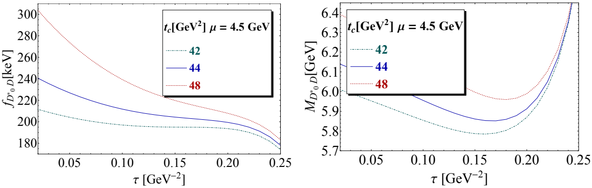

FIGURE – $f_{\bar{D}_0^* D}$ and $M_{\bar{D}_0^* D}$ at NLO as function of τ for different values of t_c,for μ=4.5 GeV

$\bar{D}_0^* D$ $0^{-\pm}$ Molécule state

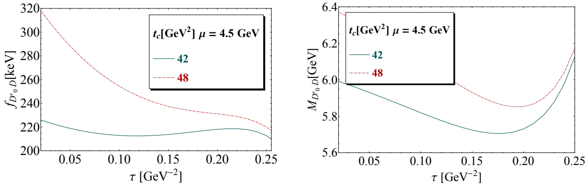

FIGURE – $f_{\bar{D}_0^* D}$ and $M_{\bar{D}_0^* D}$ at N2LO as function of τ for different values of t_c,for μ=4.5 GeV

- beginning of τ-stability : t_c= 42 GeV^2
- t_c-stability starts to be reached : t_c= 48 GeV^2

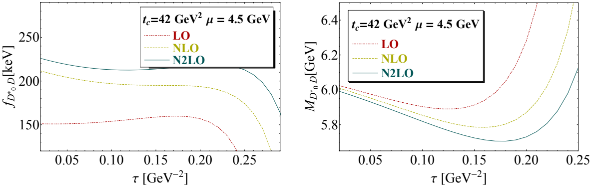

FIGURE – $f_{\bar{D}_0^* D}$ and $M_{\bar{D}_0^* D}$ as function of τ for $t_c = 42$ GeV^2, for μ=4.5 GeV,for different truncation of the PT series

- NLO $\rightarrow$ N2LO : the coupling increases about 7 %
- NLO $\rightarrow$ N2LO : the mass decreases about 2 %

$\bar{D}_0^* D$ $0^{-\pm}$ Molécule state

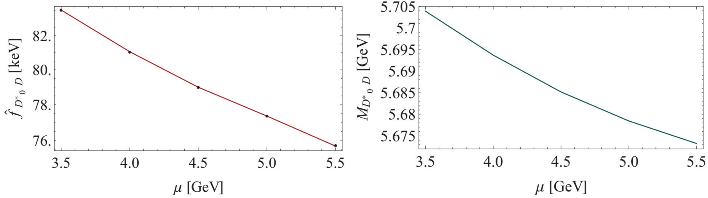

FIGURE – $\hat{f}_{\bar{D}_0^* D}$ and $M_{\bar{D}_0^* D}$ as function of μ for the corresponding τ-stability region, for $t_c = 42GeV^2$

- The optimal results deduced at $\mu \simeq 4.5\mathrm{GeV}$
- $f_{\bar{D}D} \simeq$ 257(19) keV and $M_{\bar{D}D} \simeq$ 5690(140) GeV

Results

TABLE – $\bar{D}D$-like.

Channels	$\hat{f}_M$ [keV]			f_M [keV]			M_M [GeV]		
	LO	NLO	N2LO	LO	NLO	N2LO	LO	NLO	N2LO
Scalar(0^{++})									
$\bar{D}D$	56	60	62(6)	155	164	170(15)	3901	3901	3898(36)
$\bar{D}^*D^*$	-	-	-	269	288	302(47)	3901	3903	3903(179)
$\bar{D}^*_0 D^*_0$					97(15)	114(18)		4003(227)	3954(224)
$\bar{D}_1 D_1$					236(32)	274(37)		3858(57)	3784(56)
Axialvector($1^{+\pm}$)									
$\bar{D}^*D$	87	93	97(10)	146	154	161(17)	3901	3901	3903(62)
$\bar{D}^*_0 D_1$	48	71	83(10)	81	118	137(16)	4394	4395	4401(164)
Pseudo($0^{-\pm}$)									
$\bar{D}^*_0 D$	68	88	94(7)	190	240	257(19)	5956	5800	5690(140)
$\bar{D}^*D_1$	-	-	-	382	490	564(38)	6039	5898	5787(191)
Vector(1^{--})									
$\bar{D}^*_0 D^*$	112	143	157(10)	186	238	261(17)	6020	5861	5748(101)
$\bar{D}D_1$	98	126	139(13)	164	209	231(21)	5769	5639	5544(162)
Vector(1^{-+})									
$\bar{D}^*_0 D^*$	105	135	150(13)	174	224	249(22)	6047	5920	5828(132)
$\bar{D}D_1$	97	128	145(15)	162	213	241(25)	5973	5840	5748(179)

Results

TABLE – $\bar{B}B$-like.

Channels	$\hat{f}_M$ [keV]			f_M [keV]			M_M [GeV]		
	LO	NLO	N2LO	LO	NLO	N2LO	LO	NLO	N2LO
Scalar(0^{++})									
$\bar{B}B$	4.0	4.4	5(1)	14.4	15.6	17(4)	10605	10598	10595(58)
$\bar{B}^*B^*$	-	-	-	27	30	32(5)	10626	10646	10647(184)
$\bar{B}_0^*B_0^*$	2.1	3.2	4(1)	7.7	11.3	14(4)	10653	10649	10648(113)
$\bar{B}_1B_1$					20(3)	28.6(4)		10514(149)	10514(149)
Axialvector($1^{+\pm}$)									
$\bar{B}^*B$	7	8	9(3)	14	16	17(6)	10680	10673	10646(150)
$\bar{B}_0^*B_1$	4	6	7(1)	8	11	14(2)	10670	10679	10692(132)
Pseudo($0^{-\pm}$)									
$\bar{B}_0^*B$	11	16	20(3)	39	55	67(10)	12930	12737	12562(260)
$\bar{B}^*B_1$	-	-	-	71	105	136(19)	12967	12794	12627(225)
Vector(1^{--})									
$\bar{B}_0^*B^*$	21	29	35(6)	39	54	66(11)	12936	12756	12592(266)
$\bar{B}B_1$	21	29	35(7)	39	54	65(12)	12913	12734	12573(257)
Vector(1^{-+})									
$\bar{B}_0^*B^*$	20	29	34(4)	38	54	64(8)	12942	12774	12617(220)
$\bar{B}B_1$	20	29	35(5)	37	53	65(9)	12974	12790	12630(236)

Results

TABLE – Fourquark Q=c.

Channels	$\hat{f}_M$ [keV]			f_M [keV]			M_M [GeV]		
	LO	NLO	N2LO	LO	NLO	N2LO	LO	NLO	N2LO
c-quark									
$S_c(0^+)$	62	67	70(7)	173	184	191(20)	3902	3901	3898(54)
$A_c(1^+)$	100	106	112(18)	166	176	184(30)	3903	3890	3888(130)
$\pi_c(0^-)$	84	106	113(5)	233	292	310(13)	6048	5872	5750(127)
$V_c(1^-)$	123	162	178(11)	205	268	296(19)	6062	5904	5793(122)

TABLE – Fourquark Q=b.

Channels	$\hat{f}_M$ [keV]			f_M [keV]			M_M [GeV]		
	LO	NLO	N2LO	LO	NLO	N2LO	LO	NLO	N2LO
b-quark									
$S_b(0^+)$	4.6	5.0	5.3(1.1)	16	17	19(4)	10652	10653	10654(109)
$A_b(1^+)$	8.7	9.5	10(2)	16	18	19(3)	10730	10701	10680(172)
$\pi_b(0^-)$	18	23	27(3)	62	83	94(11)	13186	12920	12695(254)
$V_b(1^-)$	24	33	40(5)	45	62	75(9)	12951	12770	12610(242)

Summary

- Improvement of all previous works about the masses and couplings of exotic mesons at LO using QCD spectral sum rules.
- New compact and integrated expressions of the heavy light molecule and fourquark states spectral functions at the lowest order of perturbation and up to d=8 condensates of the OPE.
- The masses of $X_c(3872)$, $Z_c(3900,4200,4430)$ observed states are compatible with (almost) pure $1^{+\pm}$, 0^{++}molecule or/and four-quark states.
- The masses of $1^{-\pm}$ molecule and/or fourquark states in c-chanel in the range of (5646-5961)MeV are about 1.5 GeV above the $Y_c(4260,4360,4660)$ and the ones in b-chanel (12326-12829) MeV are also too high compared with the $Y_b(9898,10260,10870)$ experimental candidates.
- The molecule and/or fourquark states couplings are much weaker than ordinary couplings such as $f_{D,B}$

Thanks

THANK YOU ! ! !