XYZ-SU3 Breakings from LSR at N2LO

D. Rabetiarivony*,

In collaboration with: *R. Albuquerque***, *S. Narison****, *A. Rabemananjara**,

*G. Randriamanatrika**

**Institute of High Energy Physcs of Madagascar (iHEPMAD)*
***Faculty of Technology, Rio de Janeiro State University (FAT, UERJ), Brazil*
****Laboratoire Univers et Particules de Montpellier (LUPM), CNRS-IN2P3, France*

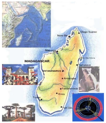

TABLE – Interpolating currents describing the molecule states

Scalar	$\mathbf{0}^{++}$	
$\bar{D}D,\ \bar{B}B$		$(\bar{q}\gamma_5 Q)(\bar{Q}\gamma_5 q)$
$\bar{D}^*D^*,\ \bar{B}^*B^*$		$(\bar{q}\gamma_\mu Q)(\bar{Q}\gamma^\mu q)$
$\bar{D}_0^*D_0^*,\ \bar{B}_0^*B_0^*$		$(\bar{q}Q)(\bar{Q}q)$
$\bar{D}_1D_1,\ \bar{B}_1B_1$		$(\bar{q}\gamma_\mu\gamma_5 Q)(\bar{Q}\gamma^\mu\gamma_5 q)$
Axial-vector	$\mathbf{1}^{++}$	
$\bar{D}^*D,\ \bar{B}^*B$		$\frac{i}{\sqrt{2}}\left[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma_5 Q) - (\bar{q}\gamma_\mu Q)(\bar{Q}\gamma_5 q)\right]$
$\bar{D}_0^*D_1,\ \bar{B}_0^*B_1$		$\frac{1}{\sqrt{2}}\left[(\bar{q}Q)(\bar{Q}\gamma_\mu\gamma_5 q) + (\bar{Q}q)(\bar{q}\gamma_\mu\gamma_5 Q)\right]$
Pseudoscalar	$\mathbf{0}^{-\pm}$	
$\bar{D}_0^*D,\ \bar{B}_0^*B$		$\frac{1}{\sqrt{2}}\left[(\bar{q}Q)(\bar{Q}\gamma_5 q) \pm (\bar{Q}q)(\bar{q}\gamma_5 Q)\right]$
$\bar{D}^*D_1,\ \bar{B}^*B_1$		$\frac{1}{\sqrt{2}}\left[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma^\mu\gamma_5 Q) \mp (\bar{Q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma^\mu Q)\right]$
Vector	$\mathbf{1}^{-\pm}$	
$\bar{D}_0^*D^*,\ \bar{B}_0^*B^*$		$\frac{1}{\sqrt{2}}\left[(\bar{q}Q)(\bar{Q}\gamma_\mu q) \mp (\bar{Q}q)(\bar{q}\gamma_\mu Q)\right]$
$\bar{D}D_1,\ \bar{B}B_1$		$\frac{i}{\sqrt{2}}\left[(\bar{Q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma_5 Q) \pm (\bar{q}\gamma_\mu\gamma_5 Q)(\bar{Q}\gamma_5 q)\right]$

TABLE – Interpolating currents describing the 4-quark states

Scalar	$\mathbf{0}^+$	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\, \gamma_5 C\, \bar{Q}_e^T\right) + k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\, C\, \bar{Q}_e^T\right)\right]$
Axial-vector	$\mathbf{1}^+$	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\, \gamma_\mu C\, \bar{Q}_e^T\right) + k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\, \gamma_\mu\gamma_5 C\, \bar{Q}_e^T\right)\right]$
Pseudoscalar	$\mathbf{0}^-$	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\, C\, \bar{Q}_e^T\right) + k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\, \gamma_5 C\, \bar{Q}_e^T\right)\right]$
Vector	$\mathbf{1}^-$	$\epsilon_{abc}\epsilon_{dec}\left[\left(q_a^T\, C\gamma_5\, Q_b\right)\left(\bar{q}_d\, \gamma_\mu\gamma_5 C\, \bar{Q}_e^T\right) + k\left(q_a^T\, C\, Q_b\right)\left(\bar{q}_d\, \gamma_\mu C\, \bar{Q}_e^T\right)\right]$

- $Q \equiv c$ (resp. b) for $\bar{D}D$ (resp. $\bar{B}B$)-like,
- $q \equiv d$ (resp. s) in the chiral limit (resp. su3 breaking)

Introduction

- Masses and couplings of molecule and four-quark states at N2LO
- evaluation of the effect of SU(3) breakings.

Double Ratio Sum Rules

$$f^{sd} \equiv \frac{f_H^s(\tau, t_c, \mu)}{f_H^d(\tau, t_c, \mu)}\ , \quad r^{sd} \equiv \frac{M_H^s(\tau, t_c, \mu)}{M_H^d(\tau, t_c, \mu)},$$

the upper indices s, d indicate the s and d quark channels.

XYZ-SU3 Breakings

The analysis will be illustrated,

- in the case of $\bar{D}_s D_s$ for the 0^{++} & 1^{++} molecule and four-quark states,
- in the case of $\bar{D}^*_{s0} D_s$ for the $0^{-\pm}$ & $1^{-\pm}$ molecule and four-quark states.

The results for the others will only be quoted.

$\bar{D}_s D_s$ Molecule states

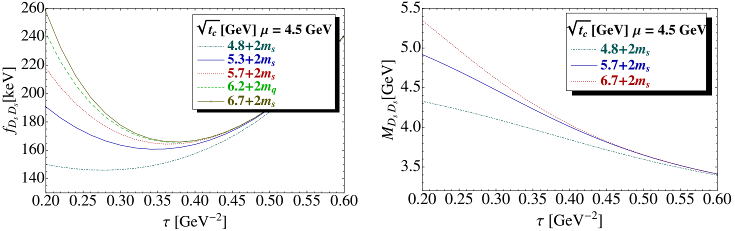

FIGURE – τ-behaviour of coupling and mass at NLO for different values of t_c and for $\mu = 4.5$GeV

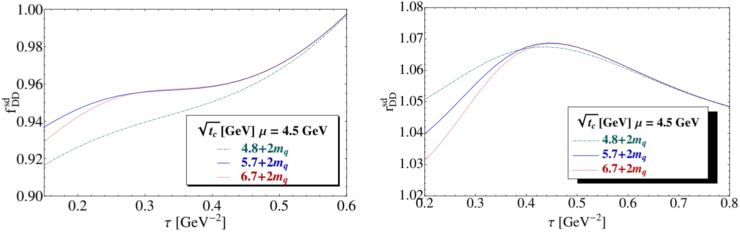

FIGURE – τ-behaviour of SU3 ratios of couplings (masses) f^{sd}_{DD} (resp r^{sd}_{DD}) at NLO for different values of t_c and for $\mu = 4.5$GeV

Final results : mean value of $f_{\bar{D}_s D_s}$ (resp $M_{\bar{D}_s D_s}$) obtained from a direct determination and from their SU3 ratios $f_{\bar{D}D}^{sd}$ (resp $r_{\bar{D}D}^{sd}$), at the minimum or inflection point for the common range of t_c.

$$f_{DD}^{sd} = 0.950(4)_f(6)_{t_c}(0)_\tau ... \quad \Rightarrow \quad f_{D_s D_s} = 156(8)_f(1)_{t_c}(0)_\tau ... \text{ keV}$$

$$r_{DD}^{sd} = 1.069(1)_{t_c}(0)_\tau ... \quad \Rightarrow \quad \mathsf{M}_{D_s D_s} = 4169(6)_M(4)_{t_c}(0)_\tau \text{ MeV}$$

$\bar{D}_s^* D_{s1}$ Molecule states

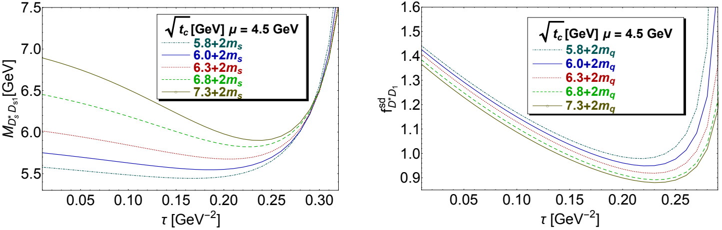

FIGURE – τ-behaviour of su3 ratio of couplings and mass at NLO for different values of t_c and for $\mu = 4.5$GeV

By a direct determination :

$$\mathrm{M}_{\bar{D}_s^* D_{s1}} = 5724(176)_{t_c}(14)_{\tau} \ldots \text{MeV}$$

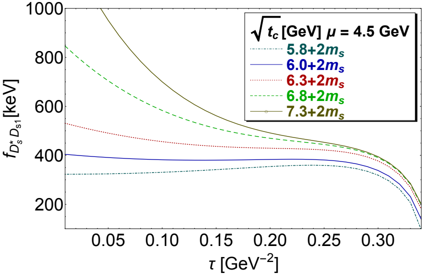

FIGURE – τ-behaviour of coupling at NLO for different values of t_c and for $\mu = 4.5$GeV

Taking the mean value of $f_{\bar{D}_s^* D_{s1}}$ obtained from a direct determination and from the su3 ratio $f^{sd}_{\bar{D}^* D_1}$:

$$
\begin{aligned}
f_{\bar{D}_s^* D_{s1}} &= 455(22)... \text{ keV} \\
f^{sd}_{\bar{D}^* D_1} &= 0.93(1)...
\end{aligned}
$$

Results

TABLE – $\bar{D}D$-like molecules couplings, masses and their corresponding SU3 ratios from LSR within stability criteria at NLO to N2LO of PT.

Channels	$f_M^{sd} \equiv f_{M_s}/f_M$		f_{M_s} [keV]		$r_M^{sd} \equiv M_{M_s}/M_M$		M_{M_s} [MeV]	
	NLO	N2LO	NLO	N2LO	NLO	N2LO	NLO	N2LO
Scalar(0^{++})								
$\bar{D}_s D_s$	0.95(3)	0.98(4)	156(17)	167(18)	1.069(4)	1.070(4)	4169(48)	4169(48)
$\bar{D}_s^* D_s^*$	0.93(3)	0.95(3)	265(31)	284(34)	1.069(3)	1.075(3)	4192(200)	4196(200)
$\bar{D}_{s0}^* D_{s0}^*$	0.88(6)	0.89(6)	85(12)	102(14)	1.069(69)	1.058(68)	4277(134)	4225(132)
$\bar{D}_{s1} D_{s1}$	0.906(33)	0.93(34)	209(28)	229(31)	1.097(7)	1.090(7)	4187(62)	4124(61)
Axial(1^{++})								
$\bar{D}_s^* D_s$	0.93(3)	0.97(3)	143(16)	156(17)	1.070(4)	1.073(4)	4174(67)	4188(67)
$\bar{D}_{s0}^* D_{s1}$	0.90(1)	0.82(1)	87(14)	110(18)	1.119(24)	1.100(24)	4269(205)	4275(206)
Pseudoscalar($0^{-\pm}$)								
$\bar{D}_{s0}^* D_s$	0.94(5)	0.90(4)	225(24)	232(25)	0.970(50)	0.946(40)	5604(223)	5385(214)
$\bar{D}_s^* D_{s1}$	0.93(4)	0.90(4)	455(34)	508(38)	0.970(50)	0.972(34)	5724(195)	5632(192)
Vector(1^{--})								
$\bar{D}_{s0}^* D_s^*$	0.87(4)	0.86(4)	208(11)	216(11)	0.980(33)	0.956(32)	5708(184)	5571(180)
$\bar{D}_s D_{s1}$	0.97(3)	0.93(3)	202(12)	213(13)	0.970(33)	0.951(31)	5459(122)	5272(120)
Vector(1^{-+})								
$\bar{D}_{s0}^* D_s^*$	0.98(5)	0.92(5)	219(17)	231(18)	0.963(32)	0.948(32)	5699(184)	5528(179)
$\bar{D}_s D_{s1}$	0.92(3)	0.88(3)	195(13)	212(14)	0.959(34)	0.955(34)	5599(155)	5487(152)

TABLE – $\bar{B}B$-like molecules couplings, masses and their corresponding SU3 ratios from LSR within stability criteria at NLO to N2LO of PT.

Channels	$f_M^{sd} \equiv f_{M_s}/f_M$		f_{M_s} **[keV]**		$r_M^{sd} \equiv M_{M_s}/M_M$		M_{M_s} **[MeV]**	
	NLO	N2LO	NLO	N2LO	NLO	N2LO	NLO	N2LO
Scalar(0^{++})								
$\bar{B}_s B_s$	1.04(4)	1.15(4)	17(2)	20(2)	1.027(4)	1.029(4)	10884(74)	10906(74)
$\bar{B}_s^* B_s^*$	1.00(3)	1.12(3)	31(5)	36(6)	1.028(5)	1.029(5)	10944(134)	10956(134)
$\bar{B}_{s0}^* B_{s0}^*$	1.11(5)	1.07(5)	13(3)	17(4)	1.050(11)	1.034(11)	11182(227)	11014(224)
$\bar{B}_{s1} B_{s1}$	1.197(73)	1.214(74)	24(5)	29(6)	1.040(2)	1.035(2)	10935(170)	10882(169)
Axial($1^{+\pm}$)								
$\bar{B}_s^* B_s$	1.01(3)	1.18(4)	16.7(2)	20(2)	1.028(4)	1.030(4)	10972(195)	10972(195)
$\bar{B}_{s0}^* B_{s1}$	0.80(4)	0.79(4)	9.1(2.2)	10.7(2.6)	1.052(14)	1.031(14)	11234(208)	11021(204)
Pseudo($0^{-\pm}$)								
$\bar{B}_{s0}^* B_s$	1.06(3)	1.02(3)	58(3)	68(4)	1.00(3)	1.00(3)	12725(217)	12509(213)
$\bar{B}_s^* B_{s1}$	0.96(4)	0.95(4)	100(11)	118(13)	1.00(3)	1.00(3)	12726(295)	12573(292)
Vector(1^{--})								
$\bar{B}_{s0}^* B_s^*$	0.95(3)	0.90(3)	51(4)	59(5)	1.00(3)	0.99(3)	12715(267)	12512(263)
$\bar{B}_s B_{s1}$	0.83(4)	0.77(3)	45(3)	50(3)	0.99(3)	0.99(3)	12615(236)	12426(233)
Vector(1^{-+})								
$\bar{B}_{s0}^* B_s^*$	0.94(3)	0.92(3)	51(5)	59(6)	1.00(3)	0.99(3)	12734(262)	12479(257)
$\bar{B}_s B_{s1}$	0.89(4)	0.85(3)	48(5)	55(6)	0.99(3)	0.98(3)	12602(247)	12350(242)

TABLE – 4-quark couplings, masses and their corresponding SU3 ratios from LSR within stability criteria at NLO and N2LO of PT.

Channels	$f_M^{sd} \equiv f_{M_s}/f_M$		f_{M_s} **[keV]**		$r_M^{sd} \equiv M_{M_s}/M_M$		M_{M_s} **[MeV]**	
	NLO	N2LO	NLO	N2LO	NLO	N2LO	NLO	N2LO
c-quark								
$S_{sc}(0^+)$	0.91(4)	0.98(4)	161(17)	187(19)	1.085(11)	1.086(11)	4233(61)	4233(61)
$A_{sc}(1^+)$	0.80(4)	0.87(4)	141(15)	160(17)	1.081(4)	1.082(4)	4205(112)	4209(112)
$\pi_{sc}(0^-)$	0.88(7)	0.86(7)	256(29)	267(30)	0.97(3)	0.96(3)	5671(181)	5524(176)
$V_{sc}(1^-)$	0.91(10)	0.87(10)	245(31)	258(33)	0.96(4)	0.96(4)	5654(239)	5539(234)
b-quark								
$S_{sb}(0^+)$	0.78(3)	0.83(3)	22(5)	26(6)	1.044(4)	1.048(4)	11122(149)	11133((149)
$A_{sb}(1^+)$	0.92(3)	0.98(3)	22(4)	26(5)	1.042(6)	1.046(6)	11150(172)	11172(172)
$\pi_{sb}(0^-)$	0.80(7)	0.76(4)	66(12)	71(13)	0.985(2)	0.975(2)	12730(215)	12374(209)
$V_{sb}(1^-)$	0.97(6)	0.90(6)	64(8)	68(9)	0.996(3)	0.984(30)	12716(272)	12411(266)

1

1. R. Albuquerque et al., Int. J. Mod. Phys. A33 (2018) 1850082

- SU3 breakings are relatively small for the masses ≤ 10(resp.3)% for the charm (resp. bottom) channels while it can be large for the couplings ($\leq 20\%$).
- The 0^{++} X(4700) experimental candidate can be identified with a $D^*_{s0}D^*_{s0}$ molecule state.
- The masses of 1^{++} X(4147) and X(4273) are compatible within the error with the one of $D^*_s D_s$ and the axial-vector A_{sc} four-quark state.
- For the bottom sector, experimental checks of our predictions are required.