Nature of the $X(5568)$: a critical Laplace sum rule analysis at N2LO

A. Ch. RABEMANANJARA

Institute of High Energy Physics of Madagascar

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In collaboration with R.M. Albuquerque - S. Narison - D. Rabetiarivony

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- We will also extend our results to the charm channel.

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5. Results an Summary

Exotic mesons

1. Hadrons = Mesons (1 quark + 1 anti-quark $q_1\bar{q}_2$) or Baryons (3 quarks $q_1 q_2 q_3$)
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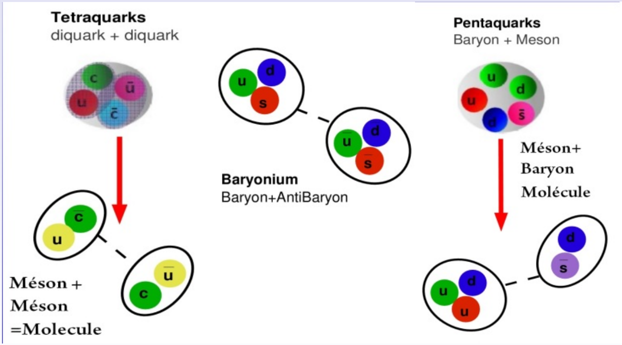
4. QCD allow other configurations



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3. Two point function :

$$\Pi^{\mu\nu}_{mol}(q) = i\int d^4x\ e^{iq.x}\langle 0|TJ^{\mu}(x)J^{\nu\dagger}(0)|0\rangle$$
$$= -(q^2 g^{\mu\nu} - q^{\mu}q^{\nu})\Pi^{(1)}_{mol}(q^2) + q^{\mu}q^{\nu}\Pi^{(0)}_{mol}(q^2)$$

The 2-point function can be evaluate in two ways :

- QCD side : OPE (Operator Product Expansion)
- Phenomenological side $\rightarrow$ f_H : the decay constant and it parametrize the coupling of the hadron to the current.

QCD sum rule

Wilson expansion :

$$\Pi^{\mu\nu}(q) = i\int d^4x\ e^{iq.x}\langle 0|J(x)J^\dagger(0)|0\rangle = \sum_n C_n(q^2)\langle 0| : \mathcal{O}_n(0) : |0\rangle$$

Wick theorem : Normal ordered product + All contractions

$$
\begin{aligned}
:\mathcal{O}_3: &= \quad :\bar{q}(0)q(0) :\Rightarrow \langle\bar{q}q\rangle \\
:\mathcal{O}_4: &= \quad :g_s^2 G^N_{\alpha\beta}(0)G^N_{\alpha\beta}(0) :\Rightarrow \langle g_s^2 G^2\rangle \\
:\mathcal{O}_5: &= \quad :\bar{q}(0) : g_s^2\sigma^{\alpha\beta}G^N_{\alpha\beta}(0)q(0) :\Rightarrow \langle\bar{q}Gq\rangle \\
:\mathcal{O}_6^q: &= \quad :\bar{q}(0)q(0)\bar{q}(0)q(0) :\Rightarrow \langle\bar{q}q\rangle^2 \\
:\mathcal{O}_6^G: &= \quad :f_{NMK}g_s^3)G^N_{\alpha\beta}(0)G^N_{\beta\gamma}(0)G^N_{\gamma\alpha}(0) :\Rightarrow \langle g_s^3 G^3\rangle \\
\rho^{OPE}(t) &= \quad \rho_0(t) + \rho_3(t)\langle\bar{q}q\rangle + \rho_4(t)\langle g^2G^2\rangle + \rho_5(t)\langle\bar{q}Gq\rangle + ...
\end{aligned}
$$

Dispersion relation :

$$\Pi^{OPE}(q) = \int_{t_0}^{+\infty} dt \frac{\rho^{OPE}(t)}{t-q^2}$$

QCD sum rule

- Use the definition of time ordered product
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- $\sum H_q \equiv H_0 + \sum H'$ where H_0 represent the ground state and H' the exited ones.

The dispersion relation become :

$$\Pi^{PHEN}(q) = \frac{\lambda^2}{M_H^2 - q^2} + \int_{t_c}^{+\infty} \mathrm{d}t \frac{\rho^{OPE}(t)}{t - q^2}$$

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QCD Sum Rules : $\Pi_{QCD}(q) = \Pi_{PHEN}(q)$
Excited states $\Longrightarrow$ Inverse Laplace Sum Rules $\Longrightarrow$ masses, couplings

Currents and Spectral Functions

Molecule :

- BK $\quad 0^+ \quad (\bar{b} i\gamma^5 u)(\bar{d} i\gamma^5 s)$
- $B_s\pi$ $\quad 0^+ \quad (\bar{b} i\gamma^5 s)(\bar{d} i\gamma^5 u)$
- B^*K $\quad 1^+ \quad (\bar{b}\gamma^\mu u)(\bar{d} i\gamma^5 s)$
- $B_s^*\pi$ $\quad 1^+ \quad (\bar{b}\gamma^\mu s)(\bar{d} i\gamma^5 u)$

Four-quark :

- $1^- \quad (s^T C\gamma^5 u)(\bar{b}\gamma^\mu\gamma^5 C\bar{d}^T) + k(s^T C u)(\bar{b}\gamma^\mu C\bar{d}^T)$
- $1^+ \quad (s^T C\gamma^5 u)(\bar{b}\gamma^\mu C\bar{d}^T) + k(s^T C u)(\bar{b}\gamma^\mu\gamma^5 C\bar{d}^T)$

Mass and Coupling

$$\text{Mass}: \quad M_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt\ t\ e^{-t\tau} \frac{1}{\pi} \text{Im}\Pi^{OPE}(t)}{\int_{4m_Q^2}^{t_c} dt\ e^{-t\tau} \frac{1}{\pi} \text{Im}\Pi^{OPE}(t)}$$

$$\text{Coupling}: \quad f_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt\ e^{-t\tau} \frac{1}{\pi} \text{Im}\Pi^{OPE}(t)}{e^{-\tau M_H^2} M_H^8}$$

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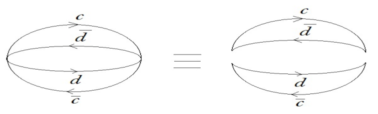
- We also compute the mass and coupling at different scale "μ"
- Noting that the bilinear (pseudo)scalar current acquires an anomalous dimension due to its normalization, thus the decay constants run to order α^2

Convolution



The molecular spectral function can be considered as a convolution of two 2-point functions [1]

$$\frac{1}{\pi}\mathrm{Im}\Pi^{(0,1)}_{mol}(t) = \theta(t-4M_Q^2)\left(\frac{1}{4\pi}\right)^2 t^2 \int_{M_Q^2}^{(\sqrt{t}-M_Q)^2} dt_1 \times \int_{M_Q^2}^{(\sqrt{t}-\sqrt{t_1})^2} dt_2 \times \ldots$$

For spin 0 : $$\ldots = \lambda^{1/2}\left[\left(\frac{t_1}{t}+\frac{t_2}{t}-1\right)^2\right]\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}(t_1)\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}(t_2)$$

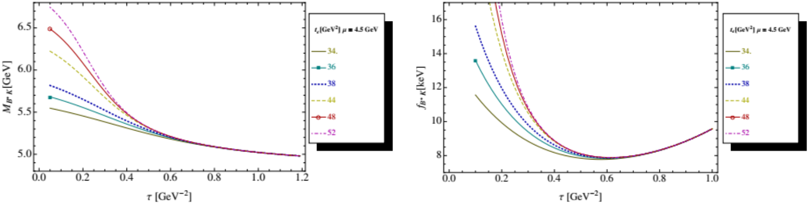
$$\ldots = \lambda^{3/2}\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(t_1)\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(t_2)$$

For spin 1 : $$\ldots = \lambda^{1/2}\left[\left(\frac{t_1}{t}+\frac{t_2}{t}-1\right)^2+\frac{8t_1t_2}{t^2}\right]\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}(t_1)\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(t_2)$$

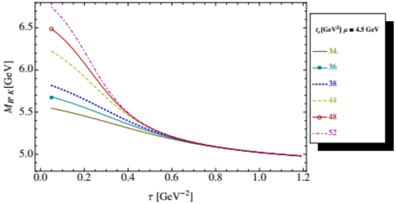
$$\rho_{OPE} = \rho_{pert}\left(1+\frac{NLO+N2LO}{LO}\right)+\rho_{\langle q\bar{q}\rangle}+\rho_{\langle g^2G^2\rangle}+\rho_{\langle qGq\rangle}+\rho_{\langle qq\rangle^2}+\rho_{\langle g^3G^3\rangle}$$

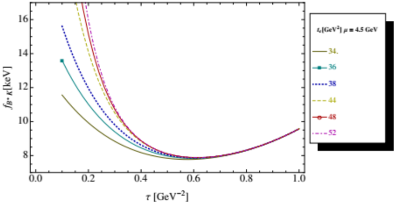
1. A. Pich and E. de Rafael/S. Narison and A. Pivovarov.

Result for B^*K molecule at LO



Stability	t_c (GeV^2)	τ (GeV^{-2})	Mass (GeV)	Coupling (keV)
	$\geq$ 34	0.45 - 0.6	5.19 - 5.20	7.77 - 7.88
	44-48	0.6		





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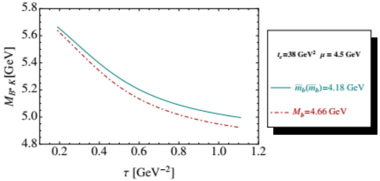
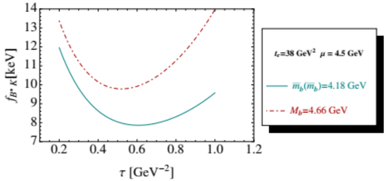
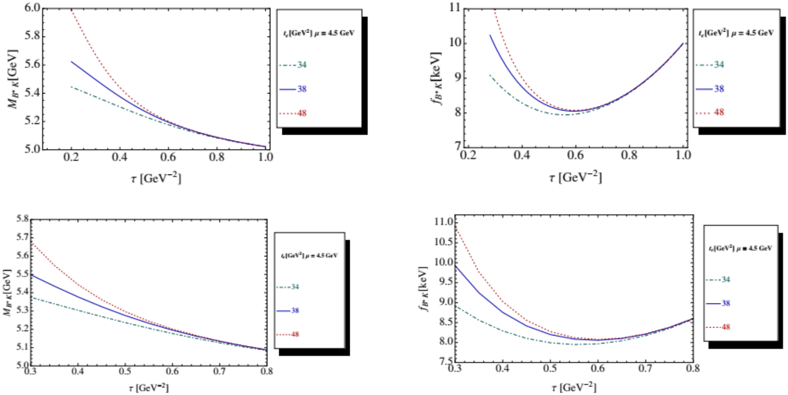




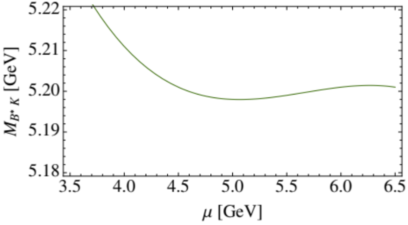
The effect of the definitions (running and pole) of the heavy quark mass used should be added as errors in the LO analysis

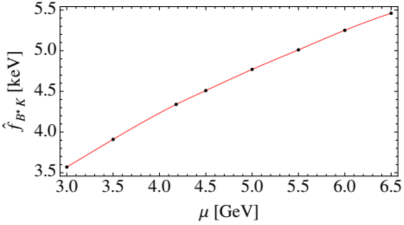
B^*K results at NLO and N2LO



	t_c (GeV^2)	τ (GeV^{-2})	Mass (GeV)	Coupling (keV)
NLO	$\geq$ 34 - 48	0.56 - 0.60	5.200 - 5.201	7.95 - 8.07
N2LO	34 - 48	0.58 - 0.62	5.185 - 5.187	7.95 - 8.09

B^*K results (μ), PT-series





μ (GeV)	Mass (GeV)	Coupling (keV)
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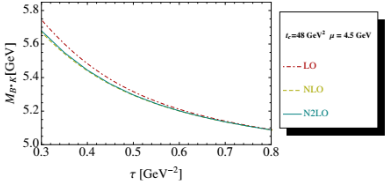

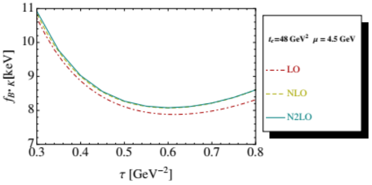

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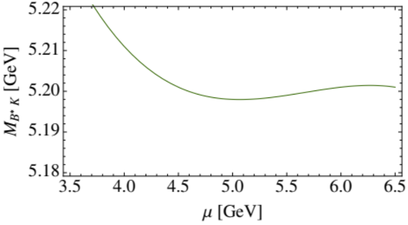

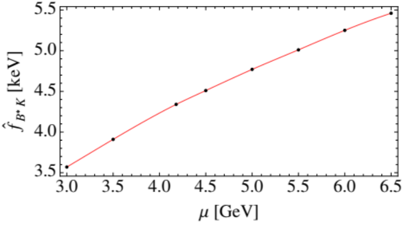

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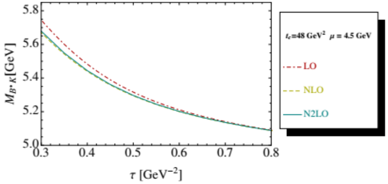

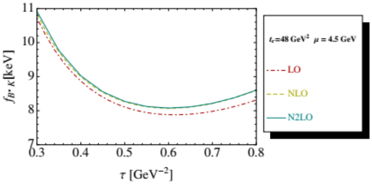

Good convergence of PT-series

Final results

Nature	J^P	Mass [MeV]	$\hat{f}_X$ [keV]	$f_X(4.5)$ [keV]
b-quark channel				
Molecule				
B^*K	1^+	5186 ± 13	4.48 ± 1.45	8.02 ± 2.60
BK	0^+	5195 ± 15	2.57 ± 0.75	8.26 ± 2.40
$B_s^*\pi$	1^+	5200 ± 18	5.61 ± 0.87	10.23 ± 1.59
$B_s\pi$	0^+	5199 ± 24	3.15 ± 0.70	10.5 ± 2.30
Four-quark $(su)(\overline{bd})$				
A_b	1^+	5186 ± 16	5.05 ± 1.32	9.04 ± 2.37
S_b	0^+	5196 ± 17	2.98 ± 0.70	9.99 ± 2.36
c-quark channel				
Molecule				
D^*K	1^+	2395 ± 48	155 ± 36	226 ± 52
DK	0^+	2402 ± 42	139 ± 26	254 ± 48
$D_s^*\pi$	1^+	2395 ± 48	215 ± 35	308 ± 49
$D_s\pi$	0^+	2404 ± 37	160 ± 22	331 ± 46
Four-quark $(su)(\overline{cd})$				
A_c	1^+	2400 ± 47	192 ± 41	260 ± 55
S_c	0^+	2395 ± 68	122 ± 26	221 ± 47

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- We do not include higher contributions (≥ 7) in our estimate but only consider them as a source of the errors.
- Our previous analysis within stability criteria with respect to τ, t_c and μ have done some successful predictions in different hadronic channels. However, we did not predict a mass of a pure exotic $BK, B^*K, B_s\pi$ molecule or four-quark state around the D0's $X(5568)$ wich is not confirmed by LHCb.

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- From our analysis, one may suggest to scan the regions (2327 - 2444) MeV and (5173 - 5226) MeV for detecting these unmixed exotic hadrons.

Thank you

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