# Duality relations in proton diffraction dissociation and in DIS 

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We use duality to relate resonances in missing mass $M$ to the largemass diffraction dissociation of protons. In deep inelastic lepton-hadron scattering (DIS), hadronic resonances are related by duality in $Q^{2}$ to the low- $x$, smooth behaviour of the DIS structure functions.

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## 1 Finite mass sum rule (FMSR) in proton diffraction dissociation

Discovery [1] of finite energy sum rules (FESR) and duality between low-energy resonances and asymptotic Regge behaviour stimulated applications of duality in other areas of high-energy physics. Among these are finite-mass sum rules (FMSR) and parton-hadron duality in deep inelastic scattering (DIS). FMSR is an efficient tool in relating the contribution of resonances in the missing mass, $M$ produced in proton diffraction dissociation to the smooth large- $M$ Regge behaviour of the cross sections assumed in the triple-pomeron limit, based on the general optical theorem.

In Refs. [2] a method to include resonances in the missing mass $M$, based on a "reggeized Breit-Wigner" model following from duality was elaborated. Resonances in $M$ are generated by a direct-channel (pole) decomposition of the dual model. The single diffraction (SD) dissociation cross section in our model [2] is :

$$
\begin{align*}
& \frac{d^{2} \sigma_{S D}}{d t d M_{i}^{2}}=F_{P}^{2}(t) F\left(x_{B}, t\right) \frac{\sigma_{T}^{P p}\left(t, M_{i}^{2}\right)}{2 m_{p}}\left(s / M_{i}^{2}\right)^{2(\alpha(t)-1)} \ln \left(s / M_{i}^{2}\right), i=1,2,  \tag{1}\\
& \sigma_{T}^{P p}\left(M_{X}^{2}, t\right)=\operatorname{Im} A\left(M_{X}^{2}, t\right)=\frac{A_{N^{*}}}{\sum_{n} n-\alpha_{N^{*}}\left(M_{X}^{2}\right)}+B g\left(t, M^{2}\right)=  \tag{2}\\
& =A_{n o r m} \sum_{n=0,1, \ldots .} \frac{[f(t)]^{2(n+1)} \operatorname{Im} \alpha\left(M_{x}^{2}\right)}{\left(2 n+0.5-\operatorname{Re} \alpha\left(M_{X}^{2}\right)\right)^{2}+\left(\operatorname{Im} \alpha\left(M_{X}^{2}\right)\right)^{2}}+B g\left(t, M^{2}\right),
\end{align*}
$$

where $\alpha(t)$ is the pomeron trajectory, $\alpha\left(M^{2}\right)$ is the Regge trajectory in the direct $\gamma-p$ channel, $B g$ is the background and $F_{p}$ is the elastic $p P p$ vertex. We use non-linear complex trajectories, typically

$$
\begin{equation*}
\alpha(x)=\alpha_{0}+\alpha_{1} x+\alpha_{2}\left(\sqrt{x_{0}}-\sqrt{x_{0}-x}\right), \quad x=s, t, \mathcal{M}^{2}, \tag{3}
\end{equation*}
$$

where $x_{0}$ is the lightest threshold in the given channel.
The model reproduces the observable sequence of resonances in the missing mass Fig. 1a, see also [5].

The first moment FMSR

$$
|t| \frac{d \sigma}{d t}+\int_{0}^{\nu_{0}} \nu \frac{d^{2} \sigma}{d t d \nu}=\int_{0}^{\nu_{0}} \nu\left(\frac{d^{2} \sigma}{d t d \nu}\right)_{\text {Regge }} d \nu
$$

states that the extrapolation of high $\nu$ behaviour of the function $\nu(d \sigma / d t d \nu)$ into the low $\nu$ region, where $\nu=M_{x}^{2}-M_{p}^{2}-t$ is the crossing-symmetric variable represents the average behaviour of the resonances and vice versa. FMSR were tested against single diffraction dissociation, see Fig 1B.

Duality sum rules work also in deep inelastic scattering (DIS), relating resonances, appearing at large $Q^{2}$ to smooth asymptotic behaviour at low $x$ (parton-hadron duality). Here again, resonances can be described by the above reggeized Breit-Wigner model, see [2].


Figure 1: (a) Double differential cross section of SD at the LHC for different values of $t$ calculated in [6]. (b) A test of the FMSR [4].

## 2 Dual-Regge Structure Function and parton-hadron (Bloom-Gilman) duality

The kinematics of inclusive electron-nucleon scattering, applicable to both high energies, typical of HERA, and low energies as at JLab, is shown in Fig. 2.


Figure 2: Kinematic of deep inelastic scattering.

The basic idea in our approach is the use the off-mass-shell continuation of the dual amplitude with non-linear complex Regge trajectories. We adopt the two-component picture of strong interactions, according to which direct-channel resonances are dual to cross-channel Regge exchanges and the smooth background in the $s$-channel is dual to the pomeron exchange in the $t$-channel.

The cross section is related to the structure function by

$$
\begin{equation*}
F_{2}\left(x, Q^{2}\right)=\frac{Q^{2}(1-x)}{4 \pi \alpha\left(1+4 m^{2} x^{2} / Q^{2}\right)} \sigma_{t}^{\gamma^{*} p} \tag{4}
\end{equation*}
$$

, and use the norm where

$$
\begin{equation*}
\sigma_{t}^{\gamma^{*} p}(s)=\mathcal{I} m A\left(s, Q^{2}\right) \tag{5}
\end{equation*}
$$

The center of mass energy of the $\gamma p$ system, the negative squared photon virtuality $Q^{2}$ and the Bjorken variable $x$ are related by

$$
\begin{equation*}
s=W^{2}=Q^{2}(1-x) / x+m^{2} . \tag{6}
\end{equation*}
$$

In the Regge-dual approach with vector meson dominance implied, Compton scattering can be viewed as an off-mass shell continuation of a hadronic reaction, dominated in the resonance region by non-strange ( $N$ and $\Delta$ ) baryonic resonances. The scattering amplitude can be written as a pole decomposition of the dual amplitude and factorizes as a product of two vertices (form factors) times the propagator:

$$
\begin{equation*}
\left[A\left(s, Q^{2}\right)\right]_{t=0}=N\left\{\sum_{r, n} \frac{f_{r}^{2\left(n-n_{r}^{m i n}+1\right)}\left(Q^{2}\right)}{n-\alpha_{r}(s)}+\left[A\left(s, Q^{2}\right)\right]_{t=0}^{B G}\right\}, \tag{7}
\end{equation*}
$$

where $N$ is an overall normalization coefficient, $r$ runs over all trajectories allowed by quantum number conservation (in our case $r=N_{1}^{*}, N_{2}^{*}, \Delta$ ) while $n$ runs from $n_{r}^{\min }$ (spin of the first resonance) to $n_{r}^{\max }$ (spin of the last resonance - for more details see next section), and $\left[A\left(s, Q^{2}\right)\right]_{t=0}^{B G}$ is the contribution from the background. The functions $f_{r}\left(Q^{2}\right)$ and $\alpha_{r}(s)$ are respectively form factors and Regge trajectory corresponding to the $r^{\text {th }}$-term.

As seen from Fig. 3a, the model fits almost perfectly the complicated resonance pattern, however its rise towards small is $x$ too steep (see also [5]), which may be corrected by the use of asymptotically flatter Regge trajectories, namely: $\alpha(s)=$ $\alpha_{0}-\sum_{i} \ln \left(1+\beta_{i} \sqrt{t_{i}-t}\right.$ instead of (3) .

Below we check the validity of parton-hadron duality for our Regge-dual model by calculating the duality relation

$$
\begin{equation*}
I\left(Q^{2}\right)=\frac{I^{\text {res }}}{I^{\text {scale }}} \tag{8}
\end{equation*}
$$

where

$$
I_{\text {res. }}\left(Q^{2}\right)=\int_{s_{\text {min. }}}^{s_{\max }} d s F_{2}^{\text {res. }}
$$

$$
I_{\text {scale }}\left(Q^{2}\right)=\int_{s_{\text {min. }}}^{s_{\text {max. }}} d s F_{2}^{\text {scale }}
$$

using the model (7).
We fix the lower integration limit $s_{\min }=s_{0}$, varying the upper limit $s_{\max }$ equal $5 \mathrm{GeV}^{2}$ and $10 \mathrm{GeV}^{2}$. These limits imply "global duality", i.e. a relation averaged over some interval in $s$ (contrary to the so-called "local duality", assumed to hold at each resonance position). For fixed $Q^{2}$ the integration variable can be either $s$ (as in our case), $x$ or any of its modifications $\left(x^{\prime}, \xi, \ldots\right)$ with properly scaled integration limits. The difference may be noticeable at small values of $Q^{2}$ due to the target mass corrections (for details see e.g. [3]). These effects are typically non-perturbative and, apart from the choice of the variables, depend on detail of the model.

The function $F_{2}^{\text {Res }}$ is our SF The results of the calculations for different values of $s_{\text {max }}$ are shown in Fig. 3b.

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Figure 3: (a) Model predictions [2] for the structure function $F_{2}(x)$ at $Q^{2}=0.225 \div$ $0.925 \mathrm{GeV}^{2}$. The data are from [3]. (b) Global parton-hadron duality test for different values of $s_{\max }$.


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