# Triple parton scattering effects in $D$-meson production at the LHC 

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We study triple-parton scattering effects in open charm production in proton-proton collisions at the LHC. Predictions for one, two and three $c \bar{c}$ pairs production are given for $\sqrt{s}=7 \mathrm{TeV}$ and $\sqrt{s}=13 \mathrm{TeV}$. Quite large cross sections, of the order of milibarns, for the triple-parton scattering mechanism are obtained. We suggest a measurement of three $D^{0}$ or three $\overline{D^{0}}$ mesons by the LHCb collaboration. The predicted visible cross sections are of the order of a few nanobarns. The counting rates including $D^{0} \rightarrow K^{-} \pi^{+}$branching fractions are also given. We predict that at $\sqrt{s}=13 \mathrm{TeV}$ a few thousands of events of triple- $D^{0}$ production can be observed by the LHCb collaboration.

## PRESENTED AT

The 17th Conference on Elastic and Diffractive Scattering, EDS Blois 2017, 26th - 30th June 2017, Prague, Czech Republic

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## 1 Introduction

The multi-parton interactions (MPI) got new impulse with the start of the LHC operation [1, 2]. There are several ongoing studies of different processes, so far mostly concentrated on phenomena of double-parton scattering (DPS). Some time ago we have shown that charm production should be one of the best reaction to study doubleparton scattering effects [3]. This was confirmed by the LHCb experimental data [4] and their subsequent interpretation [5, 6, 7].

Very recently also triple parton scattering (TPS) was discussed in the context of multiple production of $c \bar{c}$ pairs [8]. Inspiringly large cross sections were presented there. We followed this first analysis with application to triple $D$ meson production and tried to answer the question whether the triple-parton scattering could be seen in three $D^{0}$ or three $\bar{D}^{0}$ production at the LHCb experiment.

## 2 Formalism

The cross section for TPS in a general form [10] can be written as follows:

$$
\begin{align*}
\sigma_{p p \rightarrow c \bar{c} \bar{c} \bar{c} \bar{c}}^{\mathrm{TPS}}=( & \left.\frac{1}{3!}\right) \int \Gamma_{p}^{g g g}\left(x_{1}, x_{2}, x_{3} ; \vec{b}_{1}, \vec{b}_{2}, \vec{b}_{3} ; \mu_{1}^{2}, \mu_{2}^{2}, \mu_{3}^{2}\right) \\
& \times \hat{\sigma}_{c \bar{c}}^{g g}\left(x_{1}, x_{1}^{\prime}, \mu_{1}^{2}\right) \hat{\sigma}_{c \bar{c}}^{g g}\left(x_{2}, x_{2}^{\prime}, \mu_{2}^{2}\right) \hat{\sigma}_{c \bar{c}}^{g g}\left(x_{3}, x_{3}^{\prime}, \mu_{3}^{2}\right) \\
& \times \Gamma_{p}^{g g g}\left(x_{1}^{\prime}, x_{2}^{\prime}, x_{3}^{\prime} ; \overrightarrow{b_{1}}-\vec{b}, \vec{b}_{2}-\vec{b}, \vec{b}_{3}-\vec{b} ; \mu_{1}^{2}, \mu_{2}^{2}, \mu_{3}^{2}\right) \\
& \times \quad d x_{1} d x_{2} d x_{3} d x_{1}^{\prime} d x_{2}^{\prime} d x_{3}^{\prime} d^{2} b_{1} d^{2} b_{2} d^{2} b_{3} d^{2} b, \tag{1}
\end{align*}
$$

where $\hat{\sigma}_{c \bar{c}}^{g g}\left(x_{i}, x_{i}^{\prime}, \mu_{i}^{2}\right)$ are the partonic cross sections for $g g \rightarrow c \bar{c}$ mechanism, $x_{i}, x_{i}^{\prime}$ are the longitudinal momentum fractions, $\mu_{i}$ are the renormalization/factorization scales and $\frac{1}{3!}$ is the combinatorial factor relevant for the case of the three identical final states. The above TPS hadronic cross section is expressed in terms of the so-called triple-gluon distribution functions $\Gamma_{p}^{g g g}\left(x_{1}, x_{2}, x_{3} ; \vec{b}_{1}, \vec{b}_{2}, \vec{b}_{3} ; \mu_{1}^{2}, \mu_{2}^{2}, \mu_{3}^{2}\right)$.

The triple parton distribution functions (triple PDFs) shall account for all possible correlations between the partons. The MPI theory in this general form is well established (see e.g. Ref. [9]) but not yet fully available for phenomenological investigations. The double PDFs in the case of DPS are under intense theoretical studies but their adoption to real process calculations is still limited. On the other hand, the objects like triple PDFs for TPS were discussed so far only in Ref. [10].

As a consequence, in practice one usually follows the factorized Ansatz, where the correlations between partons are neglected and longitudinal and transverse degrees of freedom are separated. According to this approach the formula for inclusive TPS cross section (Eq. (1)) can be simplified to the pocket form [8]:

$$
\begin{equation*}
\sigma_{p p \rightarrow c \bar{c} \bar{c} \bar{c} \bar{c}}^{\mathrm{TPS}}=\left(\frac{1}{3!}\right) \frac{\sigma_{p p \rightarrow c \bar{c}}^{\mathrm{SPS}} \cdot \sigma_{p p \rightarrow c \bar{c}}^{\mathrm{SPS}} \cdot \sigma_{p p \rightarrow c \bar{c}}^{\mathrm{SPS}}}{\sigma_{\mathrm{eff}, \mathrm{TPS}}^{2}} \tag{2}
\end{equation*}
$$

where the triple-parton scattering normalization factor $\sigma_{\text {eff,TPS }}$ contains only information about proton transverse profile.

In principle, the DPS normalization factor $\sigma_{\text {eff,DPS }}$ was extracted experimentally from several Tevatron and LHC measurements (see e.g. Refs. [1, 2] and references therein) and its world average value is $\sigma_{\text {eff,DPS }} \simeq 15 \pm 5 \mathrm{mb}$. Such experimental inputs are not available for $\sigma_{\text {eff,TPS }}$. However, as was shown in Ref. 8] for protonproton collisions, the latter quantity can be expressed in terms of their more known DPS counterpart:

$$
\begin{equation*}
\sigma_{\mathrm{eff}, \mathrm{TPS}}=k \times \sigma_{\mathrm{eff}, \mathrm{DPS}}, \text { with } k=0.82 \pm 0.11 \tag{3}
\end{equation*}
$$

The relation is valid for different (typical) parton transverse profiles of proton. In the numerical calculations below we take $\sigma_{\text {eff,DPS }}=21 \mathrm{mb}$ which corresponds to the average value extracted by the LHCb experiment only from the double charm data [4]. This input gives us the value of $\sigma_{\text {eff,TPS }} \simeq 17 \mathrm{mb}$.

In this paper, each of the single-parton scattering cross sections $\sigma_{p p \rightarrow c \bar{c}}^{\text {SPS }}$ in Eq. (2) is calculated in the $k_{T}$-factorization approach [11]. It was shown in Refs. [12, 5, 6, 7] that within this approach one can get a very good description of the LHCb single and double charm data. In this approach the differential SPS cross section for inclusive single $c \bar{c}$ pair production can be written as:

$$
\begin{align*}
\frac{d \sigma_{p p \rightarrow c \bar{c}}^{\mathrm{SPS}}}{d y_{1} d y_{2} d^{2} p_{1, t} d^{2} p_{2, t}} & =\frac{1}{16 \pi^{2}\left(x_{1} x_{2} S\right)^{2}} \int \frac{d^{2} k_{1 t}}{\pi} \frac{d^{2} k_{2 t}}{\pi} \overline{\mid \mathcal{M}_{\left.g * g * \rightarrow c \bar{c}\right|^{2}}} \\
& \times \delta^{2}\left(\vec{k}_{1 t}+\vec{k}_{2 t}-\vec{p}_{1 t}-\vec{p}_{2 t}\right) \mathcal{F}_{g}\left(x_{1}, k_{1 t}^{2}, \mu^{2}\right) \mathcal{F}_{g}\left(x_{2}, k_{2 t}^{2}, \mu^{2}\right) \tag{4}
\end{align*}
$$

where $\mathcal{M}_{g^{*} g^{*} \rightarrow c \bar{c}}$ is the well-known gauge-invariant off-shell matrix element for $g^{*} g^{*} \rightarrow$ $c \bar{c}$ partonic subprocess and $\mathcal{F}_{g}\left(x_{i}, k_{i t}^{2}, \mu^{2}\right)$ are the so-called unintegrated (transverse momentum dependent) gluon PDFs (uPDFs). Here we use the Kimber-MartinRyskin (KMR) uPDFs [13]. In the perturbative part of the calculations, for the central predictions, we set both the renormalization and factorization scales equal to the averaged transverse mass $\mu^{2}=\frac{m_{i, t}^{2}+m_{j, t}^{2}}{2}$, where $m_{i, t}=\sqrt{p_{i, t}^{2}+m_{c}^{2}}$ and use the charm quark mass $m_{c}=1.5 \mathrm{GeV}$.

The parton-level cross sections for triple charm quark (or charm antiquark) production are further corrected for the $c \rightarrow D$ (or $\bar{c} \rightarrow \bar{D}$ ) hadronization effects via the following procedure:

$$
\begin{equation*}
\frac{d \sigma_{p p \rightarrow D D D}^{\mathrm{TPS}}}{d \xi^{D}} \approx \int \frac{D_{c \rightarrow D}\left(z_{1}\right)}{z_{1}} \cdot \frac{D_{c \rightarrow D}\left(z_{2}\right)}{z_{2}} \cdot \frac{D_{c \rightarrow D}\left(z_{3}\right)}{z_{3}} \cdot \frac{d \sigma_{p p \rightarrow c c c}^{\mathrm{TPS}}}{d \xi^{c}} d z_{1} d z_{2} d z_{3}, \tag{5}
\end{equation*}
$$

where $d \xi^{a}$ stand for $d y_{1}^{a} d y_{2}^{a} d y_{3}^{a} d^{2} p_{1, t}^{a} d^{2} p_{2, t}^{a} d^{2} p_{3, t}^{a}$ taking $a=c$ quark or $D$ meson and $p_{i, t}^{c}=\frac{p_{i, t}^{D}}{z_{i}}$ with meson momentum fractions $z_{i} \in(0,1)$. In the numerical calculations here we use the commonly used in the literature scale-independent Peterson
[14] fragmentation function $D_{c \rightarrow D}(z)$ with the parameter $\varepsilon_{c}=0.05$. In the last step the obtained cross sections for triple-meson production are normalized with the corresponding fragmentation fraction $\mathrm{BR}\left(c \rightarrow D^{0}\right)=0.565$ [15].

## 3 Numerical results

In Fig. 1 we show transverse momentum distribution of one of the two or one of the three $D^{0}$ mesons (all measured by the LHCb detector). In the used multiple parton scattering formalism the distributions for single, double and triple production have the same shape/slope and differ only by normalization. The distributions for triple $D^{0}$ production are about two orders of magnitude smaller than for double $D^{0}$ production, consistent with Table 1. In the left panel, for $\sqrt{s}=7 \mathrm{TeV}$, we also show for reference the LHCb experimental data [4]. We get a good agreement with the LHCb data within the uncertainty bands. The uncertainties for the two and three meson case are propagated from the standard single-parton scattering (SPS) pQCD calculation uncertainties. The chosen uncertainty procedure leads us to the conclusion that our result for TPS is known within a factor $\approx 3$.


Figure 1: Transverse momentum distributions of one of the measured $D^{0}$ mesons for double- $D^{0}$ (upper long-dashed lines) and triple- $D^{0}$ (lower solid lines) production for $\sqrt{s}=7$ (left panel) and 13 TeV (right panel). Details are specified in the figure. The data points for double- $D^{0}$ production in the left panel are taken from Ref. [4]. The lower and upper limits of uncertainty bands were obtained from the lower and upper limits fors single $D^{0}$ $\left(\overline{D^{0}}\right)$ production.

In Table 1 we show our predicted cross sections for two and three mesons within the fiducial volume of the LHCb detector. The predicted value at $\sqrt{s}=7 \mathrm{TeV}$ for $D^{0} D^{0}+\overline{D^{0} D^{0}}$ final state is consistent with the measured one: $\sigma_{L H C b}=690 \pm 40 \pm 70$ (see Table 12 in Ref. [4]).

Table 1: The integrated cross sections for double and triple $D^{0}$ meson production (in nb ) within the LHCb acceptance: $2<y_{D^{0}}<4$ and $3<p_{T}^{D^{0}}<12 \mathrm{GeV}$, calculated in the $k_{T}$-factorization approach. The numbers include also the charge conjugate states.

| Final state | $\sqrt{s}=7 \mathrm{TeV}$ |  | $\sqrt{s}=13 \mathrm{TeV}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| DPS: $\sigma\left(D^{0} D^{0}+X\right)$ | 784.74 | max: 1538.09 | 2992.91 | max: 5866.10 |
| TPS: $\sigma\left(D^{0} D^{0} D^{0}+X\right)$ | 2.38 | $\begin{aligned} & \min : 282.51 \\ & \max : 6.53 \\ & \min : 0.51 \\ & \hline \end{aligned}$ | 17.71 | mina: $18.5 .5{ }^{\text {ma }}$ min: 3.83 |

Table 2: Number of events for different values of the feasible integrated luminosity in the LHCb experiment for the central predictions of cross sections from Table 1. The branching fractions for $D^{0} \rightarrow K^{-} \pi^{+}\left(\overline{D^{0}} \rightarrow K^{+} \pi^{-}\right)$are included here.

|  | Integrated <br> Luminosity | DPS $\left(D^{0} D^{0}\right)$ | TPS $\left(D^{0} D^{0} D^{0}\right)$ |
| :--- | :---: | :---: | :---: |
|  | $355 \mathrm{pb}^{-1}$ | $0.43 \times 10^{6}$ | 51 |
| 7 y 7 TeV | $1106 \mathrm{pb}^{-1}$ | $1.34 \times 10^{6}$ | 159 |
| 13 | $1665 \mathrm{pb}^{-1}$ | $7.70 \times 10^{6}$ | 1789 |
| TeV | $5000 \mathrm{pb}^{-1}$ | $23.11 \times 10^{6}$ | 5374 |

Finally in Table 2 we show the number of counts for different realistic values of the integrated luminosity for the LHCb experiment. The predicted numbers of events for DPS double- and TPS triple- $D^{0}$ production correspond to the central predictions for cross sections from Table 1. Here we have included in addition the relevant decay branching fraction $\operatorname{BR}\left(D^{0} \rightarrow K^{-} \pi^{+}\right)=0.0393$ [16]. In the case of triple $D^{0}$ production we predicted about 100 counts at $\sqrt{s}=7 \mathrm{TeV}$ and a few thousands of counts at $\sqrt{s}=13 \mathrm{TeV}$ for realistic integrated luminosities. We hope the LHCb collaboration will be able to verify our predictions soon. More details of the analysis can be found in our original paper [17].

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[^0]:    ${ }^{1}$ This study was partially supported by the Polish National Science Center grant DEC2014/15/B/ST2/02528 and by the Center for Innovation and Transfer of Natural Sciences and Engineering Knowledge in Rzeszów.

