Eigenfrequency Wire Alignment System for Magnet Fiducialization

C. Zhang¹, C. Mitsuda¹, K. Kajimoto²
1. Japan Synchrotron Radiation Research Institute
2. SPring-8 Service Co. Ltd
About Wire Alignment System

Problems for absolute wire measurement:
- α. determining the curve of the wire
- β. sensor linearity
- γ. sensor offset
- δ. wire straightness error

\[ \text{Sag max} = \frac{\rho g}{8T} L^2 \]

Curve of the wire:
\[ y = \frac{4S}{L^2} x^2 - S \]

<table>
<thead>
<tr>
<th>Sag in 100 m</th>
<th>Diff. of parabola and catenary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 mm</td>
<td>0.13 mm</td>
</tr>
<tr>
<td>100 mm</td>
<td>&lt; 1 μm</td>
</tr>
<tr>
<td>27 mm *</td>
<td>Negligible</td>
</tr>
</tbody>
</table>

* Kevlar carbon wire

<table>
<thead>
<tr>
<th>Density (kg/m)</th>
<th>Tension (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.22E-4</td>
<td>15</td>
</tr>
</tbody>
</table>
About Wire Alignment System

1. Equation of calculating maximum sag of a parabola:

\[ S = \frac{\rho g L^2}{8T} \]

- \( \rho \): density (kg/m);
- \( T \): tension (N);
- \( L \): length (m);
- \( g \): gravity acceleration.

2. Eigenfrequencies of standing wave for the wire with two fixed ends:

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\rho}} \]

- \( n \): n-order of vibration mode

\[ S = \frac{n^2 g}{32f_n^2} \]

- \( \alpha \): Sag depends on eigenfrequency only;
- \( \beta \): Any order of vibration mode gives same value;
- \( \chi \): Usually fundamental frequency is used.

Vibration mode of fixed wire
Eigenfrequency **Wire Alignment System (eWAS)**

* eWAS is developed for absolute measurement. For magnet fiducialization it is composed of four sensor (WPS) assemblies, carbon wire, and wire eigenfrequency measurement devices.
Features of the eWAS

The sag of wire is calculated by measuring the eigenfrequency of wire.

LabVIEW FFT analysis of wire vibration

Test wire:
Mat: Cu-Be
Dia: φ0.2mm
Len: ~2.2m
Ten: 1.5kg
Features of the eWAS

1. Hold up wire then release it, to make wire free oscillation.

2. The oscillation is measured with a laser displacement sensor and an oscilloscope.

3. FFT gives the peaks of vibrations and eigenfrequencies are used to calculate wire sag.
Features of the eWAS

The sensors are embedded into well machined ceramic balls, to translate electrical centres to physical centres.

Sensor Assembly:
Sensor (FOGALE):
Resolution: 0.2 µm
Linearity: ±2 µm @ ±1mm
Mea. Range: ±5 mm

Wire:
Material: Kevlar carbon
Diameter: 0.5 mm
Density: 3.22e-4 kg/m (measured)

Ball (KYOCERA):
Material: ceramic
Diameter: 76.2 mm
Sphericity: ±7 µm
Features of the eWAS

Could wire position sensor be used in this way?

The WPS measures

$$\Delta C = C_1 - C_2$$

The error of $\Delta C$, proportional to the product of wire offset and rotation angle of plates, is small.
Wire system for magnet field measurement device:

α. Distance: 2.2 m

β. Wire fundamental frequency: 99.8 Hz
Verification of the resolution for sag measurement

By the equation

\[ S = \frac{g}{32f_1^2} \]

\( f_1 \) : fundamental frequency

the resolution of sag is:

\[ \Delta S = \frac{g}{16f_1^3} \Delta f_1 \]

It is \( \sim 0.03 \, \mu m \) for our system.

\( @ f_1 = 100 Hz, \Delta f_1 = 50 mHz \)

To verify the resolution, the tension was added with nuts one by one, each weighted about 15 grams, corresponding to 0.12Hz frequency or 0.07\( \mu m \) sag increment.
Experiment results of sag measurement

α. Measured frequency and calculated sag agree with calculation. And, resolution of sag is tested better than 0.1\(\mu\)m.

β. Change of the sag is confirmed by the measurement of WPS.

**Experiment results of frequency measurement and sag resolution.**

**Changes of wire sag measured with the WPS.**
Resolution for 30 meters wire

Sag resolution calculated using fundamental frequency:

\[ \Delta S = \frac{g}{16f_1^3} \Delta f_1 \]

It is low for a 33-m wire. (\(-50\mu m @ f_1 = 8.41Hz, \Delta f_1 = 50mHz\))

Resolution using high order mode frequency:

\[ \Delta S = \frac{g}{16nf_1^3} \Delta f_n \]

It is n times high than 1-order mode.

---

Experiment for 33 meters measurement

Excited eigenfrequencies up to 10-order modes for a 33-m wire *.

* Kevlar carbon wire, tension 10 kg
Resolution for 30 meters wire

A digital force gauge stretches the wire, and increases tension by 10 grams each step, which corresponding to 4µm increment of wire sag.

- **Measured frequency changes for each vibration mode**
- **Calculated sags from frequencies for each mode**

 IWAA2016, Grenoble, France, 3-7 October 2016
Result of 30 meters wire

It is estimated that when utilizing 8-order frequency, resolution of the sag is 6 µm, and measurement error is ~4µm. Measurement results well agree with prospected.

<table>
<thead>
<tr>
<th>Distance</th>
<th>n-order</th>
<th>Frequency</th>
<th>Sag</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(Hz)</td>
<td>(mm)</td>
<td>(µm)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>168.9</td>
<td>0.011</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>202.7</td>
<td>0.067</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>168.9</td>
<td>0.268</td>
<td>0.2</td>
</tr>
<tr>
<td>30</td>
<td>8</td>
<td>90.1</td>
<td>2.415</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>67.6</td>
<td>6.708</td>
<td>10</td>
</tr>
</tbody>
</table>

* Kevlar carbon wire
  Density (kg/m)    3.22E-4
  Tension (kg)      15

**Expectation of resolution**: y = 32.67 + 3.4249x, R= 0.98906

**Expected**: y = 0.008221 - 0.00044455x, R= 0.98894

**Sag measurement error**: about ~4µm (p-p) from prospected.

**Calculation and measurement results for 33 meters wire**
eWAS for Magnetic field measurement device

Capacities of the wire system for magnetic field measurement device:
α. Resolution of the sag : 0.03µm
β. Centre reproducibility : 0.5 µm
γ. Measurement stability : 1 µm/day @ ΔT=1°C

Temperatures of magnet and reference pole during magnetic field measurement.
Change of relative position between magnet and poles during magnetic field measurement.
Conclusion

a. Eigenfrequency wire alignment system is developed for absolute measurement.

b. Features of this system are firstly, wire sag is calculated from the eigenfrequencies of free vibration. Secondly, WPS sensors are embedded into well machined ceramic balls, to translate electrical centres to physical centres.

c. Resolution of sag measurement is tested better than 0.1µm in several meters range. And, utilizing 8-order frequency, it is 6 µm for 30 meters wire.

d. This system can be used in 50 meters with an expected resolution of 10 µm.