

Overview of Crystalline Color Superconductors

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1 Introduction

Inhomogeneous phases may appear when a stress is applied to a system and the system can minimize the free energy breaking the rotational invariance. Various examples are known in Nature of this sort, as the paramagnetic to ferromagnetic phase transition, or the fluid/solid phase transition. If the rotational symmetry is broken down to a discrete symmetry, the system is typically named a crystal. Crystals generally can form in two different ways, by the presence of an attractive interaction between the elementary constituents or by compression induced by an external agent. Standard crystals, like metallic crystal, form by means of the first mechanism when the Coulomb interaction between ions becomes stronger than the thermal energy. An example of the second mechanism is solid helium. In this case the short range repulsive force between ultracold helium atoms prevents the collapse of the system under an external pressure, inducing the formation of an ordered phase.

2 Inhomogeneous quark matter

Whether one of the two mechanisms described above is at work for quark matter is not obvious. Perturbatively, it is known that the strong interaction between quarks has both an attractive and a repulsive channel, depending on the color degrees of freedom. Thus, both mechanisms can in principle work in quark matter for producing a solid phase.

In the first place, for having deconfined quark matter it is necessary a large energy density, which can be realized in heavy ion collisions (HICs) or in compact stellar objects (CSOs), corresponding to stars having a radius of about 10 km and a mass of about a solar mass. The typical reference energy in strong interactions is $\Lambda_{QCD} \sim 200$ MeV. In HICs the baryonic density is small and the temperature is larger than Λ_{QCD} , meaning that the formation of a solid-like phase is improbable. On the other hand, CSOs are relatively cold stars with a temperature much less than Λ_{QCD} and a quark chemical potential that can reach 400 – 500 MeV. Thus, CSOs are cold and dense, an ideal environment for the formation of a solid-like phase.

It is unclear whether quark matter is present in CSOs, but if it is present it is likely that the attractive color interaction induces the transition to a color superconductor (CSC). In particular, at asymptotic density quark matter is expected to become a CSC in the color-flavor locked (CFL) phase [3]. The reason of color superconductivity is that the estimated critical temperature of CSCs is at least of the order of few MeV, much larger than the tens of KeV temperature of few seconds old CSOs. The reason of CFL phase is that in this phase u, d, s quarks of all colors pair coherently maximizing the free-energy gain. The CFL phase is extremely robust, corresponding to the pairing pattern between all quarks that preserve the largest global symmetry group.

The CFL phase is homogeneous and robust, thus it is unlikely to become a solid, unless a strong stress acts on it. There are two different kinds of stress that can lead to the crystallization. The surface tension of quark matter can lead to the formation of a solid phase close to the surface of strange stars. In this case, clusters of strange matter known as strangelets arrange themselves in a rigid structure. The realization of this crust depends on the value of the surface tension between strange quark matter and the vacuum [8]. A second possibility is that the CSC gap parameter is spatially modulated, forming a crystalline color superconductor (CCSC). In this case, quark matter would form a solid in bulk. The stress responsible for the formation of the CCSC structure relies on the mismatch of the Fermi spheres produced by the combined effect of the electroweak equilibrium and the strange quark mass, M_s . The resulting mismatch between the Fermi spheres is proportional to M_s^2/μ , where μ is the average quark chemical potential. For sufficiently high mismatches the system can minimize the free energy by restricting the pairing to selected quark flavors, as in the two-flavor color superconducting phase, or by quark pairing in restricted regions of the Fermi spheres, as in the CCSC phase [1, 12, 4]. In particular, the restriction of pairing to certain regions of the Fermi sphere results in quark pairs having total momentum, $2\mathbf{q}$, as illustrated in Fig. 1 for the two-flavor case with one couple of pairing regions.

Formally, the general three-flavor CCSC condensate is given by

$$\langle 0|\psi_{iL}^\alpha\psi_{jL}^\beta|0\rangle = -\langle 0|\psi_{iR}^\alpha\psi_{jR}^\beta|0\rangle \propto \sum_{I=1}^3 \Delta_I \varepsilon^{\alpha\beta\gamma} \varepsilon_{ijk} \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I\}} e^{2i\mathbf{q}_I^m \cdot \mathbf{x}}, \quad (1)$$

where $\psi_{jL(R)}^\alpha$ are left (right) handed fermionic fields with flavor i and color α and $\varepsilon^{\alpha\beta\gamma}$ and ε_{ijk} are the completely antisymmetric Levi-Civita symbols in color and flavor space, respectively. The color structure is determined by the requirement of interaction in the antitriplet channel and the flavor structure is determined by the requirement of s -wave interaction. The condensate with index I corresponds to pairing between quarks whose flavor and color is not I . The modulation of the I 'th condensate is defined by the vectors \mathbf{q}_I^m , where m is the index which identifies the elements of the set $\{\mathbf{q}_I\}$. In position space, this corresponds to condensates that vary like

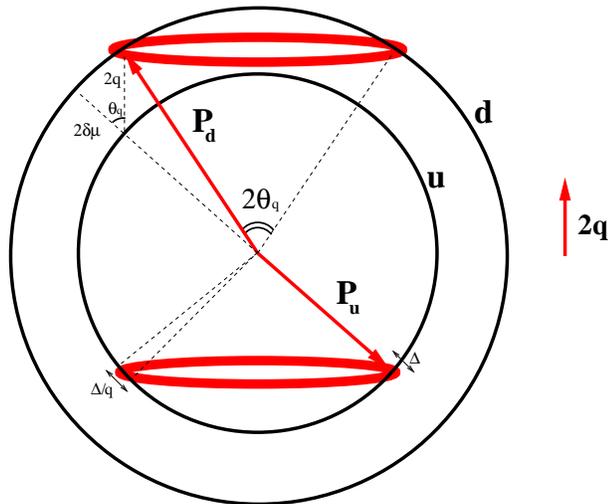


Figure 1: (color online). Pictorial description of the CCSC pairing regions for a single plane wave structure in two-flavor quark matter. The quark pairs have momentum $\mathbf{P}_u + \mathbf{P}_d = 2\mathbf{q}$. The pairing regions correspond to the two red ribbons on the top the Fermi spheres and are characterized by an opening angle $2\vartheta_q \simeq 67^\circ$, thickness $\sim \Delta$ and angular width $\sim \Delta/q$.

$\sum_m \exp(2i\mathbf{q}_I^m \cdot \mathbf{x})$, meaning that the \mathbf{q}_I^m 's are the reciprocal vectors which define the crystal structure of the condensate. The CCSC phase is the QCD analogue of a form of non-BCS pairing first proposed by Larkin, Ovchinnikov, Fulde and Ferrell (LOFF) [9, 7] for systems with Fermi mismatches exceeding the Chandrasekhar-Clogston limit [5, 6]).

Distinguishing the phenomenological signature of standard nuclear matter and deconfined quark matter is hard, because we have poor knowledge of both the properties of nuclear matter and of quark matter at densities above the nuclear saturation density. In particular, the poorly constrained equation of state of quark matter can easily reproduce the properties of the poorly known equation of state of nuclear matter above saturation density [2]. In [10] and [11] we have investigated the possible signature of torsional oscillations of the crust of strange stars. These oscillations are particularly interesting because the restoring force is the elastic shear stress and the shear modulus of the CCSC phase is much larger than the shear modulus of the neutron star crust. The typical frequency obtained are of the order of the kHz. In particular in [11] a nonbare strange star model having a CCSC crust surmounted by a standard ionic crust was considered. We found that even if a small fraction of the energy of a Vela-like glitch is conveyed to a torsional oscillation, the ionic crust will likely break. The reason is that the very rigid and heavy CCSC crust layer will absorb only a small fraction of the oscillation energy, leading to a large torsional oscillation

of the ionic crust eventually exceeding the breaking strain of nuclear matter.

Acknowledgement

We express our thanks to the organizers of the CSQCD IV conference for providing an excellent atmosphere which was the basis for inspiring discussions with all participants. We have greatly benefitted from this.

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