

Initial energy density of p+p collisions at the LHC

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Accelerating, exact, explicit and simple solutions of relativistic hydrodynamics allow for a simple description of highly relativistic p+p collisions. These solutions yield a finite rapidity distribution, thus they lead to an advanced estimate of the initial energy density of high energy collisions. We show that such an advanced estimate yields an initial energy density in $\sqrt{s} = 7$ TeV p+p collisions at LHC around or above the critical energy density from lattice QCD, and a corresponding initial temperature above the critical temperature from QCD and the Hagedorn temperature. We also show, that several times the critical energy density may have been reached in high multiplicity events, hinting at a non-hadronic medium created in high multiplicity $\sqrt{s} = 7$ TeV p+p collisions.

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Introduction. The interest in relativistic hydrodynamics grew in past years mainly due to the discovery of the almost perfect fluidity of the experimentally created Quark-Gluon-Plasma (QGP) at the Relativistic Heavy Ion Collider (RHIC) [1]. Hydrodynamical models aim to describe the space-time picture of heavy-ion collisions and infer the relation between experimental observables and the initial conditions. Besides numerical simulations there is also interest in models where exact solutions of the hydrodynamical equations are used. It is customary to describe the medium created in heavy ion collisions with hydrodynamic models, however, the proton-proton system is frequently considered as not hot and dense enough to create a supercritical (non-hadronic) medium. Energy densities in $\sqrt{s} = 200$ GeV p+p collisions are definitely below this limit. It is however an interesting question, how high energy densities can be reached in the $\sqrt{s} = 7$ TeV p+p collisions at the LHC. In this paper we will show how pseudorapidity distributions can be calculated from a hydrodynamic solution, compared to data, and how this can be used to estimate the initial energy density of high-energy collisions.

Hydrodynamics. The basic hydrodynamical equations are the local continuity and energy-momentum-conservation equations:

$$\partial_\nu(nu^\nu) = 0, \quad \partial_\nu T^{\mu\nu} = 0, \quad (1)$$

with n being a conserved charge, and T is the energy-momentum tensor. In case of a perfect fluid it is

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}. \quad (2)$$

where ϵ is the energy density and p the pressure. The Equation of State (EoS) closes the set of equations: $\epsilon = \kappa p$ while $p = nT$ defines temperature T . An analytic hydrodynamical solution is a functional form of ϵ , p , T , u^μ and n , which solves the above equations.

We discuss the solution detailed in refs. [2–5]:

$$u^\mu = (\text{ch}\lambda\eta, \text{sh}\lambda\eta), n = n_f \frac{\tau_f^\lambda}{\tau^\lambda}, T = T_f \left(\frac{\tau_f}{\tau}\right)^{\frac{\lambda}{\kappa}}. \quad (3)$$

Here τ is a coordinate proper-time, η the space-time rapidity, subscript f denotes quantities at the freeze-out, while λ controls the acceleration. If $\lambda = 1$, there is no acceleration and we get back the accelerationless Bjorken solution of ref. [6].

Rapidity distributions. The differential rapidity distribution or rapidity density dN/dy (with N being then the total number of particles) was calculated in refs. [2–5]:

$$\frac{dN}{dy} \approx N_0 \cosh^{\frac{\alpha}{2}-1} \left(\frac{y}{\alpha}\right) e^{-\frac{p}{T_f} \cosh^\alpha \left(\frac{y}{\alpha}\right)}, \quad (4)$$

with $\alpha = \frac{2\lambda-1}{\lambda-1}$, and N_0 is a normalization parameter.

The rapidity distribution is approximately Gaussian, if $\lambda > 1$. At $\lambda = 1$, the distribution becomes flat, as this is the Bjorken limit (corresponding to the Hwa-Bjorken solution). Also note that in order to describe experimental data, pseudorapidity distributions have to be calculated as well. See details in refs. [2–5].

Energy density estimation. In this section we show how this model can be used for improving the famous energy density estimation made by Bjorken [6]. We modify Bjorken's method when acceleration effects become important. Let us focus on the thin transverse slab at mid-rapidity, just after thermalization ($\tau = \tau_0$), illustrated by Fig. 2 of ref. [6]. The radius R of this slab is estimated by the radius of the colliding hadrons or nuclei, and the initial “fireball” volume is $dV = (R^2\pi)\tau_0 d\eta_0$, where $\tau_0 d\eta_0$ is the longitudinal size, as $d\eta_0$ is the pseudorapidity width at τ_0 . See refs. [2–5] for details. The energy content is $dE = \langle E \rangle dN$, where dN is the number of particles and $\langle E \rangle$ is their average energy near $y = 0$. So, as given in Bjorken's paper, the initial energy density is

$$\epsilon_{\text{Bj}} = \frac{\langle E \rangle dN}{(R^2\pi)\tau_0 d\eta_0} = \frac{\langle E \rangle}{(R^2\pi)\tau_0} \frac{dN}{d\eta} \Big|_{\eta=\eta_0}. \quad (5)$$

Here τ_0 is the proper-time of thermalization, estimated by Bjorken as $\tau_0 \approx 1\text{fm}$.

For accelerationless, boost-invariant Hwa-Bjorken flows $\eta_0 = \eta_f = y$, however, for accelerating solutions

one has to apply a correction to take into account the acceleration effects on the energy density estimation, see ref. [4] for details. Thus the initial energy density is given by a corrected estimation ϵ_{corr} as

$$\epsilon_{\text{corr}} = \epsilon_{\text{Bj}} (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda-1} \quad (6)$$

Here ϵ_{Bj} is the Bjorken estimation, which is recovered if dN/dy is flat (i.e. $\lambda = 1$), but for $\lambda > 1$, both correction factors are bigger than 1. Hence the initial energy densities are under-estimated by the Bjorken formula. In refs. [2–5] we performed fits to BRAHMS pseudo-rapidity distributions from ref. [7], and these fits indicate that $\epsilon_{\text{corr}} = 8.5 - 10 \text{ GeV/fm}^3$ in Au+Au collisions at RHIC.

The above corrections are exact results, that were derived in details for a special equation of state (EoS) of $\kappa = 1$ [4]. The correction factors in eq. (6) take into account the work done by the pressure on the surface of a finite and accelerating, hot fireball. However, the relation of the pressure to the energy density is obviously EoS dependent, and as proposed in refs. [2, 3, 5] the effects of a non-ideal equation of state can be estimated with the following formula:

$$\epsilon_{\text{corr}} = \epsilon_{\text{Bj}} (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda-1} \left(\frac{\tau_f}{\tau_0} \right)^{(\lambda-1)(1-c_s^2)} \quad (7)$$

This conjecture satisfies several consistency requirements, for example, it goes back to the exact result of eq. (6) in case of a super-hard EoS of $c_s = 1$ and gives initial energy density values that were checked against numerical solutions [2].

From basic considerations [6], as well as from lattice QCD calculations [8], it follows that the critical energy density, needed to form a non-hadronic medium is around 1 GeV/fm^3 . From the lattice QCD calculations one gets $\epsilon_{\text{crit}} = (6 - 8) \times T_{\text{crit}}^4$ (in $\hbar c = 1$ units), and even with a conservative estimate of $T_{\text{crit}} = 170 \text{ MeV}$, one gets $\epsilon_{\text{crit}} < 1 \text{ GeV/fm}^3$. Thus energy densities above this value indicate the formation of a non-hadronic medium.

Initial energy density at LHC. Let us estimate the quantities in eq. (5). The average transverse momentum in $\sqrt{s} = 7 \text{ TeV}$ p+p collisions is $\langle p_t \rangle = 0.545 \pm 0.005_{\text{stat}} \pm 0.015_{\text{syst}} \text{ GeV}/c$ [9], which corresponds to $\langle E \rangle = 0.562 \text{ GeV}/c^2$ at midrapidity (assuming most of these particles are pions). The radius R can be estimated from the inelastic cross-section, as measured by TOTEM, $\sigma_{\text{inel}} = 73.5 \pm 0.6(\text{stat.})_{-1.3}^{+1.8}(\text{syst.})$ [10], as usually this is double the geometric area, i.e. $\sigma_{\text{inel}} = 2R^2\pi$. From this, $R = 1.081 \text{ fm}$. This is also verified by HBT measurements [11, 12]. The formation time, τ_0 , is conservatively assumed to be $1 \text{ fm}/c$. The only remaining parameter is the rapidity density at midrapidity. This is measured by the LHC experiments, and is found to be $6.01 \pm 0.01(\text{stat})_{-0.12}^{+0.20}(\text{syst.})$ at ALICE [9], while $5.78 \pm 0.01_{\text{stat}} \pm 0.23_{\text{syst}}$ at CMS [13], but in some

multiplicity classes it may reach values of 25-30 (see table I. of ref. [11]). We will take the average of the first two values. Based on eq. (5) one gets:

$$\epsilon_{\text{Bj}} = \frac{0.562 \times 5.895}{1.081^2\pi} \text{ GeV/fm}^3 = 0.902 \text{ GeV/fm}^3. \quad (8)$$

The advanced estimate is based on TOTEM pseudo-rapidity density $dN/d\eta$ data, as these reach out to large enough η values so that the acceleration parameter can be determined. Fits to TOTEM data were performed via eq. (4), as shown in fig. 1. The fit resulted in the acceleration parameter $\lambda = 1.073 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$, where the systematic error is based on the point-to-point systematic error of the data points.

Assuming $c_s^2 = 0.1$ (this is a quite realistic value, at least no harder EoS is expected at LHC, as similar EoS was found at RHIC as well [8, 14, 15]), one only needs a τ_f value. As shown in eq. (3), temperature is proportional to $\tau^{-\lambda/\kappa}$. From this, $\tau_0 = \tau_f (T_f/T_0)^{\kappa/\lambda}$. Thus if the freeze-out temperature (assumed to be around the Hagedorn-temperature or the critical temperature of lattice QCD) is $T_f = 140 \text{ MeV}$, then an initial temperature of $T_0 = 170 \text{ MeV}$ (needed in order to form a strongly interacting quark gluon plasma) corresponds to τ_f being 5-6 times τ_0 , for $\kappa = 10$ (i.e. $c_s^2 = 0.1$) and $\lambda = 1.1$. Even if κ is smaller and λ is higher, τ_f/τ_0 seems to be a rather conservative value. With this, one gets the multiplicative correction factors of 1.146 and 1.101, thus

$$\epsilon_{\text{corr}} = 1.262\epsilon_{\text{Bj}} = 1.139 \text{ GeV/fm}^3 \quad (9)$$

which is just above the critical value. Note that the average p+p multiplicity was used here, so this value represents an average energy density in p+p collisions. Based on table I. of ref. [11], much larger multiplicities have been reached however. The energy density results for these multiplicities is shown on fig. 2. It is clear from this plot, that even for the original Bjorken estimate, supercritical energy densities may have been reached in high multiplicity events. The corrected estimate gives supercritical values even for lower-than-average multiplicities. We also calculated the initial temperature based on the $\epsilon \propto T^4$ relationship, assuming that 175 MeV corresponds to 1 GeV/fm^3 approximately. This is also shown on fig. 2, as well as the reachable pressure values. A temperature of 300-600 MeV may have been reached in 200 GeV central Au+Au collisions of RHIC [16]. Initial temperature values in 7 TeV p+p seem to be lower than that, 300 MeV can be reached in events with a multiplicity of 45. However, 200 MeV may already be reached in events with a multiplicity of 10.

Uncertainty of the estimate. Different sources of uncertainties are detailed in table I. The most important one comes from $dN/d\eta$ at midrapidity. From fig. 2 it is clear that for the Bjorken-estimate, energy density is above the critical value of 1 GeV/fm^3 if the multiplicity is

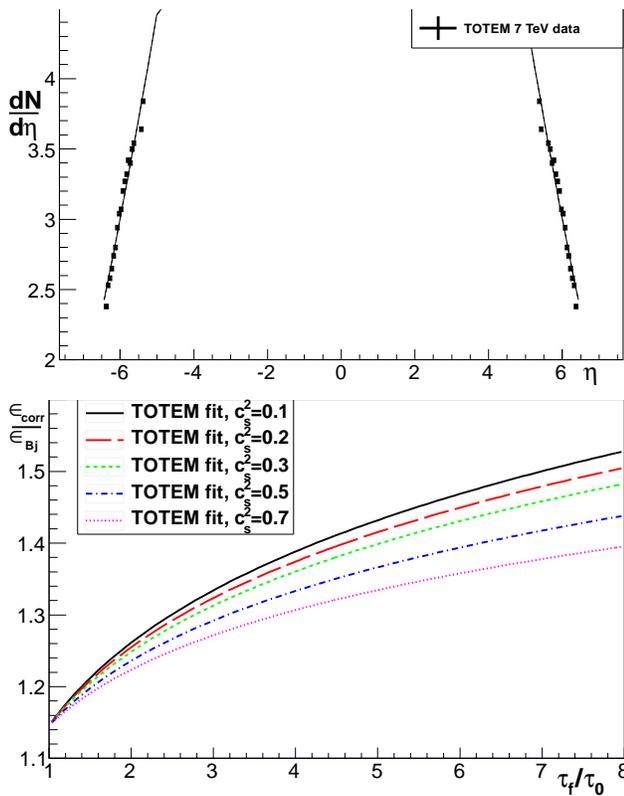


Figure 1: Top: charged particle $\frac{dN}{d\eta}$ distributions from TOTEM fitted with the result of the relativistic hydro solution described in this paper. Bottom: the correction factor as a function of freeze-out time versus thermalization time (τ_f/τ_0). At a reasonable value of 2, the correction factor is around 25%.

larger than 6-7, while the corrected initial energy density is always above the critical value. Taking all sources of uncertainty into account, the final result for the energy density corresponding to mean multiplicity density is

$$\epsilon = 1.14 \pm 0.01(\text{stat})_{-0.16}^{+0.21}(\text{syst}) \text{ GeV}/\text{fm}^3 \quad (10)$$

and the main systematic error comes from the estimation of the ratio τ_f/τ_0 . A source of systematic uncertainty is the use of the given hydrodynamic solution. This uncertainty may be estimated by using other hydrodynamic models that contain acceleration: the Landau model [17], the Bialas-Peschanski model [18], or numeric models of hydrodynamics, however, in the current paper we focus on the analytic results that can improve on Bjorken's famous initial energy density estimate. A more detailed numerical hydrodynamical investigation is outside the scope of the present manuscript.

Summary. We have shown, that based on an accelerating hydro solution and TOTEM LHC data, the advanced estimate of the initial energy density yields a value that is not inconsistent with a supercritical state in 7 TeV proton-proton collisions. The energy density is proportional to the measured multiplicity, and so in high-multiplicity events, energy densities several times the

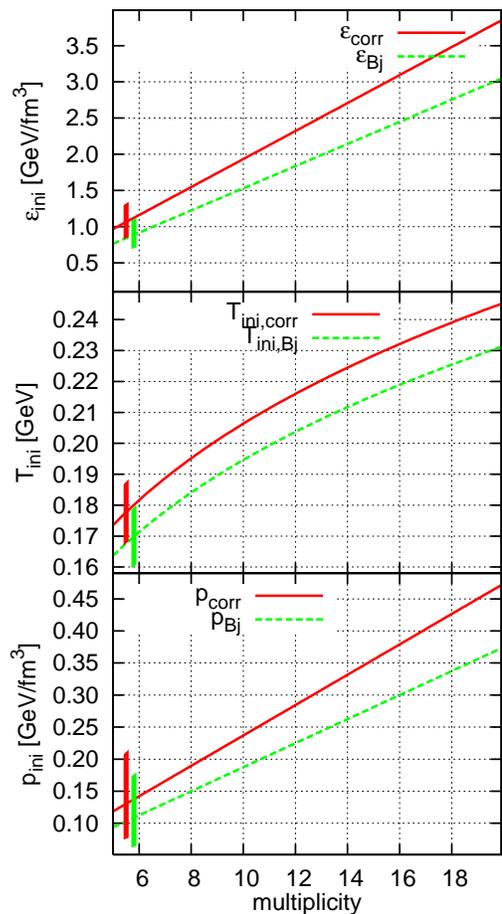


Figure 2: Initial energy density (based on table I), temperature and pressure (based on the $\epsilon \propto T^4$ relationship), is indicated as a function of central multiplicity density. The Bjorken-estimate (dashed curve) is above the critical energy density of 1 GeV/fm³ if the multiplicity is larger than 6-7. Corrected initial energy density (solid curve) is always above the critical value. Boxes show systematic uncertainty.

parameter	value	stat.	syst. eff. on ϵ
λ	1.073	0.1%	0.4% (from data)
c_s^2	0.1	-	-2%+0.2% (if $0.05 < c_s^2 < 0.5$)
τ_f/τ_0	2	-	-4%+10% (for τ_f/τ_0 in 1.5-4)
τ_0 [fm/c]	1	-	underestimates ϵ
R [fm]	1.081	0.5%	1.5% (from σ_{inel})
$\langle E \rangle$ [GeV/c ²]	0.562	0.5%	3%
$dN/d\eta$	5.895	0.2%	3%

Table I: Sources of statistical and systematic errors.

critical energy density of 1 GeV/fm³ has been reached. This however does not mean that these collisions cannot be used as a baseline to ion collision results. It means, that an important and necessary condition is satisfied for the formation of a non-hadronic medium in 7 TeV p+p collisions at CERN LHC, however, the exploration of additional signatures (radial and elliptic flow, volume

or mean multiplicity dependence of the signatures of the nearly perfect fluid in p+p collisions, scaling of the HBT radii with transverse mass, and possible direct photon signal and low-mass dilepton enhancement) can be a subject of detailed experimental investigation even in p+p collisions at the LHC.

The application of hydrodynamical expansion to data analysis in high energy p+p collisions is not an unprecedented or new idea, as Landau worked out hydrodynamics for p+p collisions [19], and Bjorken also notes this possibility in his paper [6] describing his energy density estimate.

It is also noteworthy that Hama and Padula assumed [20] the formation of an ideal fluid of massless quarks and gluons in p+p collisions at CERN ISR energies of $\sqrt{s} = 53\text{--}126$ GeV. Alexopoulos *et al.* used Bjorken's estimate to determine the initial energy density of $\sim 1.1 \pm 0.2$ GeV/fm³ at the Tevatron in $\sqrt{s} = 1.8$ TeV p+p collisions in the E735 experiment [21], while Lévai and Müller argued [22], that the transverse momentum spectra of pions and baryons indicate the creation of a fluid-like quark-gluon plasma in the same experiment at the same Tevatron energies. However, these earlier works considered the quark-gluon plasma as an ideal gas of massless quarks and gluons, while the RHIC experiments pointed to a nearly perfect fluid of quarks where the speed of sound is measured to be $c_s \approx 0.35 \pm 0.05$ that is significantly different from that of a massless ideal gas of quarks and gluons, characterized by a $c_s = 1/\sqrt{3} \approx 0.57$. Recently, Shuryak and Zahed also proposed [23] the application of hydrodynamics for high multiplicity p+p and p+A collisions at CERN LHC.

The main result of our study indicates, that the initial energy density is apparently large enough in average or even low multiplicity p+p collisions at the $\sqrt{s} = 7$ TeV LHC energies to create a strongly interacting quark-gluon plasma, so a smooth evolution with increasing multiplicity is expected, as far as hydrodynamical phenomena are considered.

Probably the most important implication of our study is the need for an e+p and e+A collider: as far as we know only in lepton induced proton and heavy ion reactions can one be certain that a hydrodynamically evolving medium is not created even at the TeV energy range. The results of lepton-hadron and lepton-nucleus interactions thus will define very clearly the particle physics background to possible collective effects. For example, recently azimuthal correlations were observed in high multiplicity p+p and p+A as well as in heavy ion reactions (the ridge effect [24, 25]), whose origin is currently not entirely clear. If such a ridge effect appears also in e+p and e+A collisions, then most likely this effect is not of a hydrodynamical origin, while if it does not appear in e+p and e+A collisions in the same multiplicity range as in p+p and p+A reactions, then the ridge is more likely a hydrodynamical effect.

If indeed a strongly interacting non-hadronic medium is formed in high multiplicity p+p collisions, then purely the jet suppression in heavy ion collisions does not reveal the true nature of these systems: the proper measure would be energy loss per unit length (as proposed in ref. [26]), which may be quite similar in these systems, even if the total suppression is different.

We are looking forward to measurements unveiling the nature of the matter created in proton-proton collisions. In experimental p+p data, one should look for the enhancement of the photon to pion ratio in high multiplicity events (as compared to low multiplicity ones) [27], for a hydrodynamic scaling of Bose-Einstein correlation radii or that of azimuthal asymmetry [28], or even the enhancement of low mass dileptons [29].

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