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Measuring CP violation in 3- and 4-body decays

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Multibody charm decays have a rich phenomenology and potentially unique sensitivity to CP violation. In these proceedings we discuss recent results, challenges and prospects in searches for CP violation in three and four body charm decays.

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1 Introduction

In the search for physics beyond the Standard Model, the search for Charge-Parity (CP) violation in charm plays a special role. Charm provides a unique probe of New Physics contributions involving flavour-changing neutral currents in up-type quarks. The Standard Model prediction of zero or small CP violation in charm is rather precise (although slightly less precise than most of us had thought until a couple of years ago). The enormous, clean charm samples available at current and future facilities promise unprecedented precision in charm in a rich variety of decay channels.

CP violation, at least in the Standard Model, is due to CP -violating weak phases. These are observable when at least two amplitudes, A_1 and A_2 , with different strong and weak phases interfere. The resulting differential decay rate to the phase-space point \mathbf{p} is

$$\frac{d\Gamma}{d\mathbf{p}}(\mathbf{p}) = [|A_1(\mathbf{p})|^2 + |A_2(\mathbf{p})|^2 + 2|A_1(\mathbf{p})||A_2(\mathbf{p})|\cos(\Delta\delta_s(\mathbf{p}) + \phi_{CP}(\mathbf{p}))] \frac{d\Phi}{d\mathbf{p}}, \quad (1)$$

where $\frac{d\Phi}{d\mathbf{p}}$ represents the density of states at \mathbf{p} , $\Delta\delta_s$ is the CP -conserving strong interaction phase difference between $A_1(\mathbf{p})$ and $A_2(\mathbf{p})$, while ϕ_{CP} is the CP -violating phase difference that changes sign under CP . For two-body decays, $\frac{d\Phi}{d\mathbf{p}}$ is a δ function as only one (zero-dimensional) phase space point is available. However, for 3, 4, 5, ... body decays, the allowed phase space is at least 2, 5, 8, ... dimensional. The strong phase difference varies across phase space, and so might the CP violating phase. This results in a much richer phenomenology than for two body decays, and motivates the search for local CP violation in different phase space regions, while integrating over all phase space might wash out any effects.

2 Common Analysis Features

2.1 Choice of decay mode - CF, SCS and DCS decays

The sensitivity to CP violation is related to the relative size of the interference term in Eq. 1, which is largest when A_1 and A_2 are of similar magnitude. Significant direct CP violation is therefore most likely in singly Cabibbo suppressed (SCS) decays, as here both tree and penguin amplitudes contribute with comparable magnitude. In Cabibbo favoured (CF) decays, the large SM tree contribution dominates and significant CP violation effects are unlikely. For doubly Cabibbo suppressed (DCS) decay amplitudes such as $D^0 \rightarrow K^+\pi^-$ ^{*}, there is no Standard Model penguin contribution. However, here the amplitude proceeding via $D^0 \rightarrow \bar{D}^0 \rightarrow K^+\pi^-$ is of a comparable magnitude; these “wrong sign” (WS) decays are therefore sensitive to CP violation in D mixing and in the interference between mixing and decay.

*The charge-conjugate mode is always implied unless stated otherwise

2.2 Tagging and the special role of the charm threshold

At the B factories and at hadron colliders, the flavour of neutral D mesons at production is identified by reconstructing D mesons resulting from the process $D^{*+} \rightarrow D^0\pi^+$ and $D^{*-} \rightarrow \bar{D}^0\pi^-$, where the charge of the slow pion identifies the flavour of the D.

At the charm threshold, where CLEO-c and BES III operate, D mesons originate from the decay $\psi(3770) \rightarrow D\bar{D}$. The two D mesons are quantum-correlated and have opposite flavour and opposite CP . This can be used to identify not only flavour eigenstates, but a variety of well-defined $D^0-\bar{D}^0$ superpositions. For example when one D (the tagging D) decays to a CP -even eigenstate such as $D \rightarrow K^+K^-$, the other D is identified as a CP -odd superposition of D^0 and \bar{D}^0 . This is used to study the interference of D^0 and \bar{D}^0 amplitudes in decays such as $D \rightarrow K_S\pi^+\pi^-$ or $D \rightarrow K^+\pi^-\pi^+\pi^-$ [1–5], which is of considerable importance to a variety of analyses, including D mixing and CP violation [6, 7] and the measurement of the CP violating phase γ [8–20]. In Sec. 4.3 we briefly discuss how mixing itself can be used to improve the input from charm to the determination of γ [21].

3 Time-integrated Analyses

3.1 Model-dependent Analyses

Comparing results of amplitude fits to the phase space (Dalitz) distribution of multi-body decays for CP -conjugate decay modes provides a measure CP violation. The advantage of this approach is the physical interpretation of any CP violation observation that such a fit result would allow. Recent searches using this approach include CLEO-c’s amplitude analysis in $D^0 \rightarrow K^+K^-\pi^+\pi^-$ [22] and CDF’s analysis in $D^0 \rightarrow K_S\pi^+\pi^-$ [23]. In the CDF analysis, based on 350 000 $D^0 \rightarrow K_S\pi\pi$ events [23], the statistical uncertainty is already of a similar magnitude as the systematic uncertainty, which is dominated by the model uncertainty. This indicates that with the data samples that will be available at LHCb and its upgrade, and at BELLE II, the model-induced systematic uncertainty would severely limit the precision of the analyses. One option is to use model-independent methods, some of which are described below. An alternative is to improve the theoretical description of amplitude models [24, 25].

3.2 χ^2 based CP violation searches

3.2.1 Technique

This class of model-independent searches for local CP violation in multibody decays follows a statistical method first established by BaBar [26] and developed further

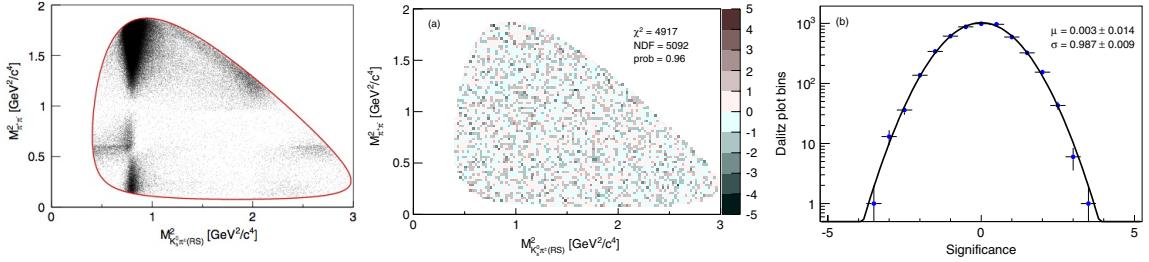


Figure 1: From left to right: CDF’s $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plot; the binning (colour code indicates S_{CP} value); histogram of S_{CP} values, with fit to a Gaussian [23].

in [27, 28]. The $D \rightarrow f$ and $\bar{D} \rightarrow \bar{f}^\dagger$ phase space is divided into bins. For each pair of CP -conjugate bins (with label i) a pull variable $S_{CP}^i = \frac{\Delta N_i}{\sigma(\Delta N_i)}$ is defined, where ΔN_i is the global-rate-adjusted difference between the $D \rightarrow f$ event yield in bin i , N_i , and $\bar{D} \rightarrow \bar{f}$ yield in the CP conjugate bin, \bar{N}_i . The difference is corrected for global rate asymmetries by the factor α , which is the ratio of the total event yields $\alpha = \sum N_i / \sum \bar{N}_i$: $\Delta N_i = N_i - \alpha \bar{N}_i$. This removes any sensitivity to global rate asymmetries, but also makes the analysis robust against global production and detection asymmetries. The denominator in the definition of S_{CP}^i , $\sigma(\Delta N_i)$, is the uncertainty on the numerator. With sufficiently large data samples, the sum over all S_{CP}^i follows a χ^2 statistics. This can be translated into a p -value, which represents the probability to obtain $\chi^2 = \sum_i (S_{CP}^i)^2$ as large as that found in a given experiment, or larger, under the assumption of no CP violation. In the absence of CP violation, the distribution of S_{CP}^i is expected to follow a Gaussian of width 1 and mean zero.

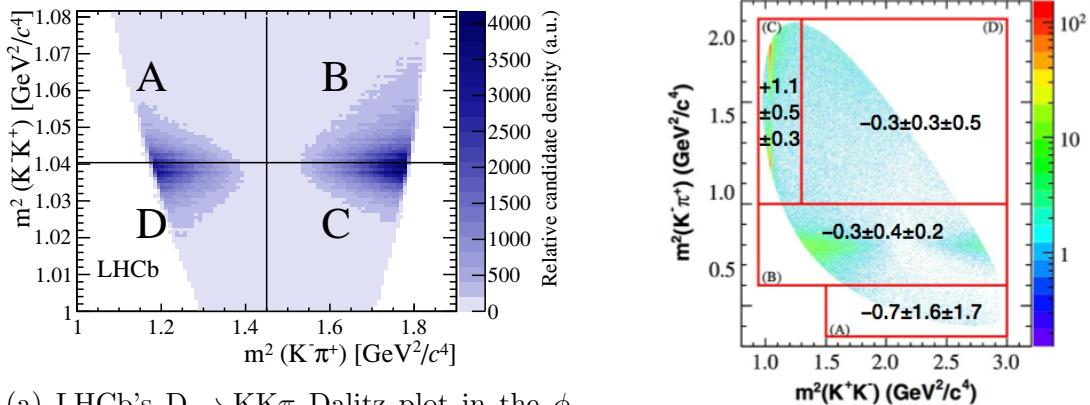
3.2.2 Results for 3-body decays

BaBar published this year a search based on 223 000 $D^\pm \rightarrow K^+ K^- \pi^\pm$ events in its full data sample. LHCb analysed 330 000 signal events in early data (35 pb^{-1}) [29], with recent updates in the $D^\pm \rightarrow \phi \pi^\pm$ region of the Dalitz plot [30] based on a different technique discussed below. CDF applied this method to 350 000 $D^0 \rightarrow K_S \pi^+ \pi^-$ events [23]; the Dalitz plot and S_{CP} distribution for this search are shown in Fig. 1. None of these analyses yielded evidence of CP violation in charm.

3.2.3 Results for 4-body decays

The same principles used to search for CP violation in three-body charm decays have been applied by the LHCb collaboration to four body SCS decays [31], with potentially unique sensitivity to CPV effects [24]. The main difference to the three

[†]where here, D indicates any D meson, charged or neutral, and f indicates a generic final state; the bar indicates the CP conjugate.



(a) LHCb’s $D \rightarrow KK\pi$ Dalitz plot in the ϕ region, split into four regions [30] based on the strong phase across the Dalitz plot. The asymmetries in each bin are used to construct the variable $A_{CP|S}$ described in the text.

(b) BaBar’s $D \rightarrow KK\pi$ Dalitz plot, split into four regions [32]. The measured CP asymmetries with statistical and systematic uncertainties are superimposed on the plot.

Figure 2: Two different binning schemes based on the Dalitz structure of the decay.

body analyses is that the two-dimensional bins in the Dalitz plane are replaced by five-dimensional hypercuboids. Analysing 57 000 $D^0 \rightarrow K^+K^-\pi^+\pi^-$ and 330 000 $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ events, with very small background contamination, no evidence for CP violation is found.

3.3 Other binned methods

3.3.1 Phase-binning in $D^+ \rightarrow \phi\pi^+$

In a recent search for CP violation in the SCS decay $D^+ \rightarrow \phi\pi^+$ [30] at LHCb, the Dalitz plot around the ϕ resonance is split into four regions, depending on the strong phase δ [33] (Fig. 2a). The bins are chosen such that $\cos(\delta)$ is positive in regions A and C , and negative in regions B and D . If CP violation is induced by an amplitude with zero strong, and approximately constant CP -violating phase, the sign of $\cos(\delta)$ determines the sign of the CP -violating asymmetry. An observable optimised for this case is constructed from the raw asymmetries A_{raw} in each region through $A_{CP|S} \equiv \frac{1}{2} (A_{raw}^A + A_{raw}^C - A_{raw}^B - A_{raw}^D)$. The raw asymmetries include production, detection and background effects. To a good approximation, these effects cancel in $A_{CP|S}$. The result, $A_{CP|S} = (-0.18 \pm 0.17_{\text{stat}} \pm 0.18_{\text{sys}})\%$, shows no sign of CP violation.

3.3.2 Binning around resonances in $D^+ \rightarrow K^+K^-\pi^+$

Several methods have been adopted by BaBar in a search for CP violation in $D^\pm \rightarrow K^+K^-\pi^\pm$ [32] based on 476 fb^{-1} of data with approximately 223 000 selected signal

events. Figure 2b illustrates a local CP violation search where the Dalitz plot is divided into bins around resonances. This optimises the sensitivity e.g. if CP violating new physics affects one resonance and not the others. The measured CP -violating asymmetries in each bin are super-imposed on the Dalitz plot, and show no evidence of CP violation.

3.4 T-odd moments

An alternative method for finding CP violation in four body decays such as $D^0 \rightarrow K^+K^-\pi^+\pi^-$ is based on constructing T -odd variables, i.e. variables that change sign under time-reversal (T) [34]. For D^0 and \bar{D}^0 decays respectively we define

$$C_T = \vec{p}_{K^+} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}), \quad \bar{C}_T = \vec{p}_{K^-} \cdot (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+}). \quad (2)$$

These variables are also parity (P)-odd, which is relevant in so far as the P -reversed process with a sign change of all 3-momenta is directly accessible, while the T -reversed process, resulting in the creation of a D meson from $K^+K^-\pi^+\pi^-$, is generally not. The asymmetry A_T between $C_T > 0$ and $C_T < 0$, and its equivalent \bar{A}_T for \bar{D}^0 events,

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}, \quad \bar{A}_T = \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)}, \quad (3)$$

are parity violating observables, and $\mathcal{A}_T \equiv \frac{1}{2}(A_T - \bar{A}_T)$ is CP violating. Searches for CP violation in this manner have been carried out by BaBar in $D^0 \rightarrow K^+K^-\pi^+\pi^-$, $D^+ \rightarrow K^+K_S\pi^+\pi^-$, and $D_s^+ \rightarrow K^+K_S\pi^+\pi^-$ [35, 36]. No evidence for CP violation has been found. It is interesting to note that the values for A_T , \bar{A}_T vary considerably between the decay channels, with larger values for D^0 and D_s^\pm than for D^\pm :

$$\begin{aligned} D^0 : \quad A_T &= (-68.5 \pm 7.3 \pm 5.8) \cdot 10^{-3} & \bar{A}_T &= (-70.5 \pm 7.3 \pm 3.9) \cdot 10^{-3} \\ D_s^\pm : \quad A_T &= (-99.2 \pm 10.7 \pm 8.3) \cdot 10^{-3} & \bar{A}_T &= (-72.1 \pm 10.9 \pm 10.7) \cdot 10^{-3} \\ D^\pm : \quad A_T &= (-11.2 \pm 14.1 \pm 5.7) \cdot 10^{-3} & \bar{A}_T &= (+35.1 \pm 14.3 \pm 7.2) \cdot 10^{-3} \end{aligned}$$

where the first uncertainty is statistical, and the second systematic.

4 Time-dependent Amplitude Analyses

4.1 Formalism

D^0 and \bar{D}^0 mesons mix to form mass eigenstates $|D_1^0\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$ and $|D_2^0\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$ with masses M_1, M_2 and widths Γ_1, Γ_2 . We define the usual dimensionless parameters $x = 2\frac{M_1 - M_2}{\Gamma_1 + \Gamma_2}$ and $y = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$, where we follow the convention for

the CP operator used by HFAG, where $CP|D^0\rangle = -|\bar{D}^0\rangle$, so that D_1 is CP even (see [21] for the impact of different conventions on the definition of x and y). The deviation of $|q/p|$ from unity parametrises CP violation in mixing. CP violation in the interference between mixing and decay is parametrised by the phase ϕ_D ; in the usual convention, this is the phase of q/p .

4.2 Amplitude model-dependent analyses

Time-dependent amplitude analyses in decays to final states that are accessible to both D^0 and \bar{D}^0 are sensitive to CP violation in mixing, and CP violation in the interference between mixing and decay. The latest such and most precise measurement in $D^0 \rightarrow K_S\pi^+\pi^-$ was presented by Longke Li on behalf of the BELLE collaboration at this conference [37]. The results for the CP -sensitive parameters are

$$|q/p| = 0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}, \quad \phi_D = -6^\circ \pm 11^\circ \pm 3^\circ {}^{+3^\circ}_{-4^\circ}$$

where the first uncertainty is statistical, the second systematic, and the third represents the uncertainty due to the amplitude model dependence. The results assume no direct CP violation. As in previous measurements [38–40] no evidence for CP violation was found. An example of a mixing and CP violation-sensitive study using a non-self conjugate decay is that of BaBar’s analysis of $D^0 \rightarrow K^+\pi^-\pi^0$. BaBar measured $x'_{K\pi\pi^0}$ and $y'_{K\pi\pi^0}$ (related to x and y through a rotation in the $x-y$ plane) separately for D^0 and \bar{D}^0 ,

$$\begin{aligned} x'_{K\pi\pi^0}(D^0) &= (2.53^{+0.45}_{-0.63} \pm 0.39)\% & y'_{K\pi\pi^0}(D^0) &= (-0.05^{+0.63}_{-0.67} \pm 0.50)\% \\ x'_{K\pi\pi^0}(\bar{D}^0) &= (3.55^{+0.73}_{-0.83} \pm 0.65)\% & y'_{K\pi\pi^0}(\bar{D}^0) &= (-0.54^{+0.40}_{-1.16} \pm 0.41)\%, \end{aligned}$$

also finding no evidence of CP violation.

4.3 Model independence and the charm threshold

The theoretical uncertainty on the amplitude model in multibody analyses potentially limits the precision that can be achieved in measurements of mixing-induced CP violation at LHCb and its upgrade, and at BELLE II. Model-independent methods tend to require input related to the relative phases of the D^0 and \bar{D}^0 decay amplitudes. This information is accessible at the charm threshold [6, 7, 41]. This input is also important for measurements of the CP violation parameter γ in $B^\pm \rightarrow D K^\pm$ and related decays [1–5, 8, 11, 16–20]. The CLEO-c collaboration provides such input for $D^0 \rightarrow K_S\pi\pi$, $D^0 \rightarrow K_S K K$ [1, 2], $D^0 \rightarrow K^-\pi^+\pi^0$, $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ [4] (for a recent update see Guy Wilkinson in these proceedings) and $D^0 \rightarrow K_S K^- K^+$ [3]. Rather than seeing the dependence of charm mixing on charm coherence parameters as an obstacle to precision mixing measurements, it can in fact be used to constrain these

coherence parameters and thus improve the charm input to the measurement of γ in $B^- \rightarrow DK^-$ and related decays. A recent study [21], also discussed in Samuel Harnew's contribution to these proceedings [42], indicates that with existing LHCb data, input from charm mixing can substantially reduce the uncertainties on the coherence parameters in $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$, with the potential to significantly improve the precision of future γ measurements.

5 Conclusion

CP violation in charm has unique sensitivity to physics beyond the Standard Model. Multibody analyses offer a rich phenomenology and unique sensitivity to CP violation, but also pose unique experimental and theoretical challenges. In these proceedings, we give a summary of recent experimental results, with impressive examples in terms the precision achieved, and the complexity of the analyses required to achieve it. In the face of rapidly increasing, high quality data samples, a potential limitation for future analyses is the theoretical uncertainty in the description of soft hadronic effects in multibody decays. Model-independent methods, many relying on input from the charm threshold, offer a way around this limitation in many cases. At the same time, work is ongoing to reduce the theoretical uncertainty in the description of multibody decays. The very rich phenomenology accessible in multibody charm decays, combined the prospect of large new data samples at LHCb (upgrade), BELLE II and at the charm threshold with BES III, justify considerable optimism for new and exciting results in multibody charm analyses in the near future.

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References

- [1] CLEO Collaboration, J. Libby *et al.*, *Model-independent determination of the strong-phase difference between D^0 and $\bar{D}^0 \rightarrow K_{S,L}^0 h^+ h^-$ ($h = \pi, K$) and its impact on the measurement of the CKM angle γ/ϕ_3* , Phys. Rev. **D82** (2010) 112006, [arXiv:1010.2817](https://arxiv.org/abs/1010.2817).
- [2] CLEO Collaboration, R. A. Briere *et al.*, *First model-independent determination of the relative strong phase between D^0 and $\bar{D}^0 \rightarrow K_s^0 \pi^+ \pi^-$ and its impact on the CKM Angle γ/ϕ_3 measurement*, Phys. Rev. **D80** (2009) 032002, [arXiv:0903.1681](https://arxiv.org/abs/0903.1681).

- [3] CLEO Collaboration, J. Insler *et al.*, *Studies of the decays $D^0 \rightarrow K_S^0 K^- \pi^+$ and $D^0 \rightarrow K_S^0 K^+ \pi^-$* , Phys. Rev. **D85** (2012) 092016, [arXiv:1203.3804](#).
- [4] CLEO Collaboration, N. Lowrey *et al.*, *Determination of the $D^0 \rightarrow K^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ Coherence Factors and Average Strong-Phase Differences Using Quantum-Correlated Measurements*, Phys. Rev. **D80** (2009) 031105, [arXiv:0903.4853](#).
- [5] CLEO Collaboration, D. M. Asner *et al.*, *Determination of the $D^0 \rightarrow K^+ \pi^-$ relative strong phase using quantum-correlated measurements in $e^+ e^- \rightarrow D^0 \bar{D}^0$ at CLEO*, Phys. Rev. **D78** (2008) 012001, [arXiv:0802.2268](#).
- [6] S. Malde and G. Wilkinson, *$D^0 - \bar{D}^0$ mixing studies with the decays $D^0 \rightarrow K_S^0 K^\mp \pi^\pm$* , Phys. Lett. **B701** (2011) 353, [arXiv:1104.2731](#).
- [7] A. Bondar, A. Poluektov, and V. Vorobiev, *Charm mixing in the model-independent analysis of correlated $D^0 \bar{D}^0$ decays*, Phys. Rev. **D82** (2010) 034033, [arXiv:1004.2350](#).
- [8] D. Atwood and A. Soni, *Role of charm factory in extracting CKM phase information via $B \rightarrow DK$* , Phys. Rev. **D68** (2003) 033003, [arXiv:hep-ph/0304085](#).
- [9] M. Gronau and D. Wyler, *On determining a weak phase from CP asymmetries in charged B decays*, Phys. Lett. **B265** (1991) 172.
- [10] M. Gronau and D. London, *How to determine all the angles of the unitarity triangle from $B_d \rightarrow D K_S$ and $B_s^0 \rightarrow D \phi$* , Phys. Lett. **B253** (1991) 483.
- [11] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining γ using $B^\pm \rightarrow D K^\pm$ with multibody D decays*, Phys. Rev. **D68** (2003) 054018, [arXiv:hep-ph/0303187](#).
- [12] D. Atwood, I. Dunietz, and A. Soni, *Enhanced CP violation with $B \rightarrow K D^0 (\bar{D}^0)$ modes and extraction of the Cabibbo-Kobayashi-Maskawa angle γ* , Phys. Rev. Lett. **78** (1997) 3257.
- [13] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining γ using $B^\pm \rightarrow D K^\pm$ with multibody D decays*, Phys. Rev. D **68** (2003) 054018.
- [14] Belle Collaboration, A. Poluektov *et al.*, *Measurement of ϕ_3 with Dalitz plot analysis of $B^\pm \rightarrow D^{(*)} K^\pm$ decays*, Phys. Rev. D **70** (2004) 072003.
- [15] J. Rademacker and G. Wilkinson, *Determining the unitarity triangle γ with a four-body amplitude analysis of $B^+ \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^\pm$ decays*, Phys. Lett. **B647** (2007) 400, [arXiv:hep-ph/0611272](#).

- [16] BaBar Collaboration, J. Lees *et al.*, *Search for $b \rightarrow u$ transitions in $B^\pm \rightarrow [K^\mp\pi^\pm\pi^0]_D K^\pm$ decays*, Phys. Rev. **D84** (2011) 012002, [arXiv:1104.4472](#).
- [17] LHCb collaboration, R. Aaij *et al.*, *A model-independent Dalitz plot analysis of $B^\pm \rightarrow DK^\pm$ with $D \rightarrow K_S^0 h^+ h^-$ ($h = \pi, K$) decays and constraints on the CKM angle γ* , Phys. Lett. **B718** (2012) 43, [arXiv:1209.5869](#).
- [18] LHCb collaboration, R. Aaij *et al.*, *A measurement of γ from a combination of $B^\pm \rightarrow DK^\pm$ analyses including first results using $2fb^{-1}$ of 2012 data*, .
- [19] LHCb collaboration, R. Aaij *et al.*, *A measurement of γ from a combination of $B^\pm \rightarrow Dh^\pm$ analyses*, [arXiv:1305.2050](#), submitted to Phys. Lett. B.
- [20] LHCb collaboration, R. Aaij *et al.*, *Observation of the suppressed ADS modes $B^\pm \rightarrow [\pi^\pm K^\mp\pi^+\pi^-]_D K^\pm$ and $B^\pm \rightarrow [\pi^\pm K^\mp\pi^+\pi^-]_D \pi^\pm$* , Phys. Lett. **B723** (2013) 44, [arXiv:1303.4646](#).
- [21] S. Harnew and J. Rademacker, *Charm mixing as input for model-independent determinations of the CKM phase γ* , Physics Letters B **728** (2014) 296 , [arXiv:1309.0134](#).
- [22] CLEO Collaboration, M. Artuso *et al.*, *Amplitude analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$* , Phys. Rev. **D85** (2012) 122002, [arXiv:1201.5716](#).
- [23] CDF Collaboration, T. Aaltonen *et al.*, *Measurement of CP-violation asymmetries in $D^0 \rightarrow K_S \pi^+ \pi^-$* , Phys. Rev. **D86** (2012) 032007, [arXiv:1207.0825](#).
- [24] I. Bigi, *3- and 4-Body Final States in B , D and tau Decays about Features of New Dynamics with CPT Invariance or "Achaeans outside Troy"*, [arXiv:1306.6014](#).
- [25] C. Hanhart, *Modelling low-mass resonances in multi-body decays*, [arXiv:1311.6627](#).
- [26] BaBar Collaboration, B. Aubert *et al.*, *Search for CP Violation in Neutral D Meson Cabibbo-suppressed Three-body Decays*, Phys. Rev. **D78** (2008) 051102, [arXiv:0802.4035](#).
- [27] I. Bediaga *et al.*, *On a CP anisotropy measurement in the Dalitz plot*, Phys. Rev. **D80** (2009) 096006, [arXiv:0905.4233](#).
- [28] I. Bediaga *et al.*, *Second Generation of 'Miranda Procedure' for CP Violation in Dalitz Studies of B ($\&$ D $\&$ τ) Decays*, Phys. Rev. **D86** (2012) 036005, [arXiv:1205.3036](#).
- [29] LHCb collaboration, R. Aaij *et al.*, *Search for CP violation in $D^+ \rightarrow K^- K^+ \pi^+$ decays*, Phys. Rev. **D84** (2011) 112008, [arXiv:1110.3970](#).

- [30] LHCb collaboration, R. Aaij *et al.*, *Search for CP violation in $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow K_S^0\pi^+$ decays*, JHEP **1306** (2013) 112, [arXiv:1303.4906](#).
- [31] LHCb Collaboration, R. Aaij *et al.*, *Model-independent search for CP violation in $D^0 \rightarrow K^-K^+\pi^-\pi^+$ and $D^0 \rightarrow \pi^-\pi^+\pi^+\pi^-$ decays*, Phys. Lett. **B726** (2013) 623, [arXiv:1308.3189](#).
- [32] BaBar Collaboration, J. Lees *et al.*, *Search for direct CP violation in singly Cabibbo-suppressed $D^\pm \rightarrow K^+K^-\pi^\pm$ decays*, Phys. Rev. **D87** (2013), no. 5 052010, [arXiv:1212.1856](#).
- [33] CLEO Collaboration, P. Rubin *et al.*, *Search for CP Violation in the Dalitz-Plot Analysis of $D^\pm \rightarrow K^+K^-\pi^\pm$* , Phys. Rev. **D78** (2008) 072003, [arXiv:0807.4545](#).
- [34] I. I. Bigi and A. I. Sanda, *CP Violation*, Cambridge University Press, May, 2009.
- [35] BaBar Collaboration, P. del Amo Sanchez *et al.*, *Search for CP violation using T-odd correlations in $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decays*, Phys. Rev. **D81** (2010) 111103, [arXiv:1003.3397](#).
- [36] BaBar Collaboration, J. Lees *et al.*, *Search for CP violation using T-odd correlations in $D^+ \rightarrow K^+K_S^0\pi^+\pi^-$ and $D_s^+ \rightarrow K^+K_S^0\pi^+\pi^-$ decays*, Phys. Rev. **D84** (2011) 031103, [arXiv:1105.4410](#).
- [37] L. Li, *$D^0 - \overline{D}^0$ Mixing and CP Violation in $D^0 \rightarrow K_{Shh}$ measurements*, [arXiv:1310.6142](#).
- [38] CLEO Collaboration, D. Asner *et al.*, *Search for D^0 - anti- D^0 mixing in the Dalitz plot analysis of $D^0 \rightarrow K_S^0\pi^+\pi^-$* , Phys. Rev. **D72** (2005) 012001, [arXiv:hep-ex/0503045](#).
- [39] BaBar Collaboration, P. del Amo Sanchez *et al.*, *Measurement of D^0 -anti- D^0 mixing parameters using $D^0 \rightarrow K_S^0\pi^+\pi^-$ and $D^0 \rightarrow K_S^0K^+K^-$ decays*, Phys. Rev. Lett. **105** (2010) 081803, [arXiv:1004.5053](#).
- [40] BELLE Collaboration, K. Abe *et al.*, *Measurement of $D0$ - anti- $D0$ Mixing Parameters in $D0 \rightarrow K(s)0$ pi+ pi- decays*, Phys. Rev. Lett. **99** (2007) 131803, [arXiv:0704.1000](#).
- [41] C. Thomas and G. Wilkinson, *Model-independent $D^0 - \overline{D}^0$ mixing and CP violation studies with $D^0 \rightarrow K_S^0\pi^+\pi^-$ and $D^0 \rightarrow K_S^0K^+K^-$* , JHEP **1210** (2012) 185, [arXiv:1209.0172](#).
- [42] S. Harnew, *CP Violation and Mixing in Multi-body D decays*, [arXiv:1311.4729](#).