STUDYING GENERALIZED PARTON DISTRIBUTION IN GRAVITY DUAL

Ryoichi Nishio and Taizan Watari
Institute for the Physics and Mathematics of the Universe(IPMU), University of Tokyo, 5-1-5 Kashiwa-no-ha, 277-8683 Chiba, Japan

In this article, we report a recent progress in an attempt to use gauge/gravity duality to study non-perturbative aspects of hadrom scattering. Although it is believed that hadron scattering cannot be studied in a naive picture of scattering of free partons in strongly coupled gauge theories, yet the gravity dual approach allows us to extract parton distributions in a hadron, and their properties turn out not to be qualitatively different from those of the real world QCD. After explaining how the "phase transition" between DGLAP phase and Regge phase in the parton behavior is the gravity dual calculation, we summarize gravity dual predictions on the t-slope parameter in DVCS / vector meson production, and on the parton transverse profile.

1 Introduction: Gravity Dual Approach to Hadron Scattering

Perturbation QCD can be used to obtain more and more precise calculations for some types of observables associated with hadron scattering processes, but not for all of them. Observables need to be inclusive enough, and only high energy scales need to be involved in the definition of the observables. QCD factorization theorems greatly expand the coverage of perturbative QCD by partially using non-perturbative information of hadrons extracted from experimental data.

There are still limitations in this approach, however. It would be ideal if non-perturbative information obtained from a small number of experimental set-ups provided sufficient data in calculating *all other aspects* of observables associated with *any kinds of hadron scattering*, but QCD factorization theorems do not work in that way. Even when center of mass energy of a hadron scattering is much higher than the hadronic scale, yet perturbative QCD has little power in calculating observables that involve small momentum transfer / transverse momentum.

In this proceeding for the 14th Blois conference,[a] which is based on two journal articles [1,2], we report some progress in an attempt of using gauge/gravity duality for hadron scattering. Gravity dual descriptions—superstring theory on warped spacetime geometries—are able to calculate various observables of exclusive processes of hadron scattering and expectation values of gauge-singlet operators. Access to area that is off-limits to perturbative QCD can be provided by this framework.

An important caveat should be mentioned clearly, however, before we get into more detailed description of progress made in this direction. Gauge/gravity duality is a belief (with some evidence supporting the belief to some extent) that superstring theory on a weakly curved warped spacetime geometry is the same as a strongly coupled gauge theory on a flat spacetime. This means that one has to find a corresponding warped spacetime geometry in order to provide a dual

[a] A talk was presented at the conference by TW.

(that is, alternative, yet equivalent) description of a gauge theory. At this moment, concensus has not been reached, to say the least, as to what should be the geometry dual to the QCD of the real world. Thus, gravity dual calculations for hadron scattering based on various warped geometries will share some basic / common features of hadron scattering in strongly coupled gauge theories in general, but it is hard to justify to assume that the result of calculations are quantitatively reliable. The gravity dual approach to hadron *high-energy* scattering also has yet another limitation, in addition to the need for search of an appropriate warped spacetime geometry. It remains a useful framework for calculations only when the geometry is weakly curved, which corresponds to strongly coupled gauge theories. Thus, certainly the gravity dual approach can play a complementary role to perturbative QCD, but it is not that the new approach can cover all sorts of non-perturbative observables for arbitrary choice of kinematical variables. We use gravity dual calculations for hadron scattering only for the purpose of obtaining qualitative results as well as theoretical ideas and better understandings for non-perturbative aspects of hadron scattering.[b]

2 Parton Contributions in Photon–Hadron Scattering in Strongly Coupled Gauge Theories

Presumably the most important concept at the interface between perturbative and non-perturbative aspects of hadron scattering is parton distribution function (PDF), and one of its generalization, generalized parton distribution (GPD). PDF is used as the input data of primary importance in perturbative QCD calculations of sufficiently inclusive observables, and transverse profile of parton densities—Fourier transform of GPD in the two spacial transverse directions—is used in providing MC simulation results for the multiple parton interaction processes in some MC codes. Among these non-perturbative input data, PDF can be determined by DIS experimental data directly, without theoretical modeling or postulates. GPD, on the other hand, cannot be extracted directly from experimental data, because physical processes such as deeply virtual Compton scattering (DVCS) and exclusive vector meson production are described by i) a *convolution of* hadron matrix element $T^{\mu\nu}$

$$(2\pi)^4\delta^4(p_2+q_2-p_1-q_1)T^{\mu\nu} = i\int d^4x d^4y\, e^{iq_2\cdot x}e^{-iq_1\cdot y}\, \langle h(p_2)|T\{J^\mu(x)J^\nu(y)\}|h(p_1)\rangle, \quad (1)$$

not directly by $T^{\mu\nu}$ itself, and furthermore, ii) only $T^{\mu\nu}$ in a limited kinematical regin is used for DVCS and vector meson production. The GPD for the parton transverse profile uses the hadron matrix element $T^{\mu\nu}$ in a different kinematical region. For those reasons, it is inevitable to introduce some form of theoretical modeling of GPD profile in extracting all the nescessary data of GPD from the experimental data. This is certainly one of the places where gravity dual approach can make a unique contribution.

PDF and GPD are characterized as a partial contribution to the hadron matrix element (1), not as the entire $T^{\mu\nu}$. Applying operator product expansion to the time-ordered product $T\{J^\mu(x)J^\nu(y)\}$, we find twist-2 operators (which are bilinear in quark or gluon field) arising as a partial contribution to $T^{\mu\nu}$, and that is what we characterize as the parton contribution to the photon–hadrom scattering (and to DIS, DVCS and vector meson production). The "twist-2" operators parametrized by spin j are all gauge singlet, and hence its hadrom matrix elements can be calculated in gravity dual approach; it has been known that the spin-j field with mass-square $2(j-2)/\alpha'$—fields in the graviton (leading) trajectory of string excitations—corresponds to the "twist-2" spin j operator in the gauge/gravity duality.[3,4] Based on the fact that these fiels are the leading trajectory states, their contributions—parton contributions—dominate at least

[b] An idea for a hybrid use of perturbative QCD calculations and gravity dual calculations has also been proposed for some observables.[1]

at photon–hadrom scattering with higher center of mass energy,[5] (that is, for small Bjorken x), even in gravity dual calculations.

It is therefore possible to calculate PDF / GPD profile from first principle in gravity dual approach; the next question is whether these parton profiles in gravity dual calculations (which is suitable only for strongly coupled theories) are similar to the ones in the real world QCD, which is not strongly coupled at high energy. If the results from gravity dual did not look like those in the real world PDF at all, it would not be possible even to learn a qualitative lesson out of gravity dual calculations. This is a non-trivial question.

In order to answer to this question, it is crucial to note the following. The hadron matrix element tensor $T^{\mu\nu}$ can be decomposed into a small number of structure functions (just like F_1, F_2, F_3 of unnpolarized DIS), and flavor singlet parton contributions to the structure functions are given in the form of integrals over complex j-plane in gravity dual calculations.[5,4,6,7,2]

$$\frac{N_c^2}{R^5}\int\frac{dj}{2\pi i}\int d\nu\frac{1+e^{-\pi i j}}{\sin(\pi j)}\frac{1}{\Gamma(j/2)^2}\int dz\sqrt{g}P_{\gamma\gamma}(z)e^{-jA(z)}\Psi^{(j)}_{i\nu}(t,z) \qquad (2)$$
$$\int dz'\sqrt{g}P_{hh}(z')e^{-jA(z')}\Psi^{(j)}_{i\nu}(t,z')\frac{1}{j-j(\nu)}\left(\frac{\alpha'\tilde{W}^2}{4}\right)^j.$$

See Ref.[2] for definition of these notations. This expression as a j-plane integral is just like in traditional Regge phenomenology or in old dual resonance model, but the extra integral in ν is the new ingredient from the gauge/gravity duality; the Pomeron propagator $1/(j-j(\nu))$ is not as simple as $1/(j-(a_0+\alpha' t))$ linear Pomeron trajectory; these differences directly come from the fact that the dual description involves an extra spacial direction of warping. One can find out that this expression becomes

$$I(x,q^2,t)=\int dj\left(\frac{1}{\sqrt{\lambda}x}\right)^j\left(\frac{\Lambda}{q}\right)^{\gamma(j)}g_j(t) \qquad (3)$$

after using explicit expressions of $\Psi^{(j)}_{i\nu}(t,z)$, $P_{\gamma\gamma}(z)$ etc., and specializing to a case with large photon virtuality $q^2\gg\Lambda^2$. Here, $\gamma(j)$ is the anomalous dimension of the "twist-2" spin-j operators. The j-plane integral expression of this form (3) often appears in perturbative QCD literature as well. The gravity dual approach allows us to calculate $g_j(t)$ in this integral for small momentum transfer; see Eq. (6) in section 4.

For large photon virtuality $q^2\gg\Lambda^2$ and small Bjorken $x\ll 1/\sqrt{\lambda}$, the j-plane integral (3) can be evaluated by the saddle point approximation.[5,6,7] As long as the momentum transfer $|t|$ is not much larger than the hadron mass scale Λ^2,

$$I(x,q^2,t)\simeq\left(\frac{1}{x}\right)^{j^*}\left(\frac{\Lambda}{q}\right)^{\gamma(j^*)}g_{j^*}(t),\qquad\frac{\partial\ln(xI)}{\partial\ln(q/\Lambda)}=\gamma(j^*),\quad\frac{\partial\ln(xI)}{\partial\ln(1/x)}=j^*-1=:\lambda_{\text{eff.}}=\delta/4. \quad (4)$$

From this, it follows that the q-evolution and x-evolution of the parton contributions to the structure functions in gravity dual calculation remain qualitatively the same as in perturbative QCD.[1,2] (note that we should not (and do not have to) take q/Λ, λ and N_c literally infinite) The essence of this success is in the fact that (2) becomes (3).

3 Phase Diagram

Although we employed saddle point approximation in evaluating the complex j-plane integral (3), it is not always appropriate. Certainly the Pomeron pole $1/(j-j(\nu))$ is absent in (3), because ν integration has been carried out in passing from (2) to (3), but the factor $g_j(t)$ may still contain some poles in j for some range of kinematical variables. When the integrand of (3)

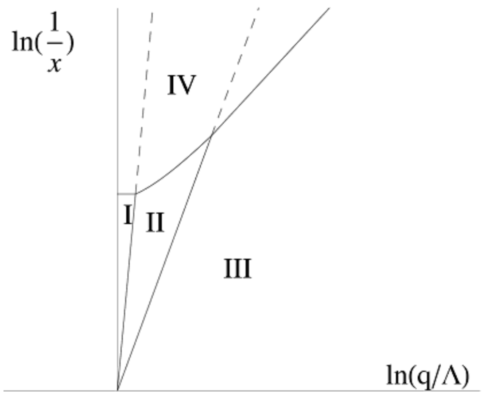


Figure 1: Phase diagram of UV conformal theories with large 't Hooft coupling for a given value of momentum transfer t. The boundary between the phase IV (where non-linear effects (higher order terms in $1/N_c$ expansion) are not negligible) and the other phases is determined by following the logic of Ref. [6]. [This is Fig. 9 of Ref. [2].]

has a pole, and if the real part value of the pole $j = \alpha_{IP,1}(t)$ is larger than that of the saddle point j^*, then the integration contour should be deformed so that it splits into the one encircling the pole and the other one passing through the saddle point. In this case, the contour integral from the one through the saddle point is not the dominant component; the one from the pole,

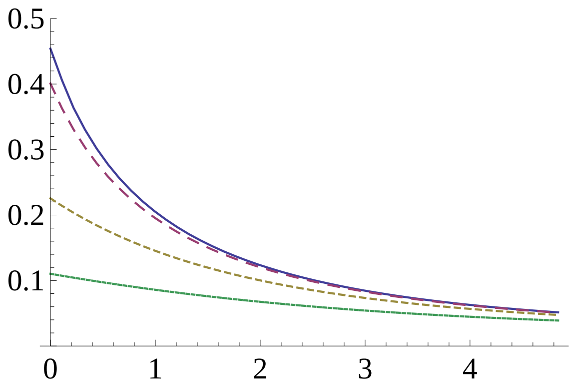
$$I(x, q, t) \simeq \left(\frac{1}{\sqrt{\lambda}x}\right)^{\alpha_{IP,1}(t)} \left(\frac{\Lambda}{q}\right)^{\gamma(\alpha_{IP,1}(t))} \mathrm{Res}_{j=\alpha_{IP,1}(t)} \left(g_j(t)\right), \tag{5}$$

provides the approximation of structure functions. Depending upon whether there is a pole (or singularity) whose real part is larger than the saddle point in the j-plane or not, the *parton contributions* to the structure functions qualitatively change its behavior.

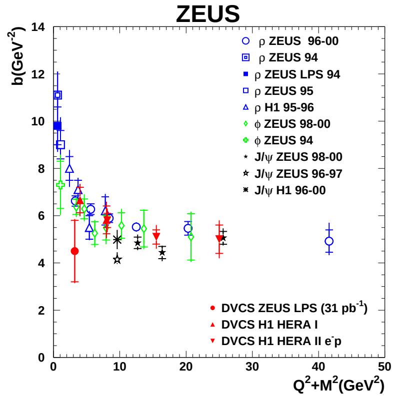
The transition between DGLAP phase with (4)—saddle point domination—and Regge phase with (5)—pole domination—in the parton contribution is now perfectly understood theoretically with an explicit expression of $g_j(t)$ (see section 4) obtained by first principle calculation in gravity dual. The transition between these two phases is not a sharp phase transition, however, but is a crossover. This is because the two phases are characterized by using the saddle point approximation, which cannot be exact without literally taking the small x limit.

The Regge phase is absent in physical kinematical region $t < 0$, when $g_j(t)$ is calculated in hard wall model (in the leading order in $1/N_c$). But in other gravity dual models (that is faithful to string theory even in the treatment of IR end of the geometry), this may not nescessarily be the case. In gravity dual models with asymptotic free running, indeed, $g_j(t)$ has isolated poles even for physical kinematical region $t < 0$, and the saddle point has smaller real part than the leading pole when the Bjorken x is sufficiently small. The boundary between the two phases (I vs II + III) is like the one in Figure 1 in nearly conformal gauge theories with large 't Hooft coupling.

The DGLAP phase can be further separated into two phases (II and III in Figure 1). The phase III is where $j^* > 2$, and the phase II $j^* < 2$. From this definition, it immediately follows that GPD (and PDF) increaces in DGLAP evolution, $\gamma(j^*) < 0$ in phase II, and GPD begins to decreace in the DGLAP evolution in the phase III. This distinction between the two phases coincides with the phase distinction in Ref.[6] based on the behavior of the real part of the amplitudes. In phase III, the real part of the amplitude is understood as spin-2 hadron exchange in the t-channel, while in phase II, such a spin-2 interpretation is inappropriate, and the real part of the amplitude should be understood as a systematic summation of t-channel and u-channel exchange of hadrons with spin $j = 2, 4, 6, \cdots$.



(a) theory calculation [Fig. 10 of Ref. [2]]



(b) data [Fig. 5 of Ref. [9]]

Figure 2: The left pannel (a) is a theoretical calculation of slope parameter B made dimensionless by multiplying Λ^2. It is presented as a function of $i\nu^*$, which is further a function of the saddle point value j^*. Multiple curves coresopnd to different choices of the value of t/Λ^2. The right pannel (b) is the slope parameter data from HERA experiments presented as a function of q^2; the data reflect the averaged value of the slope in the range $0 \leq |t| \leq \Lambda^2$.

4 Slope Parameter and GPD Modeling

While one can derive q-evolution and $\ln(1/x)$ evolution of PDF / GPD in perturbative QCD (that is, DGLAP/ERBL and BFKL equations), the momentum transfer dependence cannot be determined by perturbative QCD within the $|t| \leq \Lambda^2$ region. Thus, the t-slope parameter $B := \partial \left[\ln(d\sigma/dt)\right] / \partial t$ of DVCS and vector meson production is truely a non-perturbative observable.

In gravity dual approach, the factor $g_j(t)$ can be calculated, and so is the slope parameter, once a warped geometry is given. In the hard wall model, an analtyic expression of $g_j(t)$ is

$$\begin{aligned} g_j(t) &= \frac{N_c^2}{R^3} \int dz \sqrt{-g} P_{hh}(z) (z\Lambda)^j \left(\frac{\sqrt{-t}}{\Lambda}\right)^{i\nu_j} \frac{2}{\Gamma(i\nu_j)} \\ & \quad e^{-2A(z)} \left[K_{i\nu_j}(\sqrt{-t}z) - \frac{K_{i\nu_j}(\sqrt{-t}/\Lambda)}{I_{i\nu_j}(\sqrt{-t}/\Lambda)} I_{i\nu_j}(\sqrt{-t}z) \right], \end{aligned} \tag{6}$$

which corresponds, through the gauge/gravity duality, to a reduced matrix element of spin-j "twist-2" operators renormalized at $\mu = \Lambda$.

The slope parameter can be calculated by evaluating the matrix element $g_j(t)$ at the saddle point value $j = j^*$. The result is shown in Figure 2 (a). It is a decreacing function of j^*, which means that the slope parameter decreaces for larger $\ln(q/\Lambda)$, and it should be less sensitive to $\ln(1/x)$ for small value of x. This theoretical result from gravity dual approach is indeed confirmed in the experimental data of DVCS and exclusive vector meson production obtained at the HERA experiment[8] (see Figure 2 (b)).

The matrix element (6) obtained in this gravity dual calculation is used in (3) to yield a theoretical model of GPD. The integrand may contain singularity; the zero in the factor $I_{i\nu_j}(\sqrt{-t}/\Lambda)$ in the denominator of (6) indeed gives rise to t-dependent poles in the complex j-plane. These poles are interpreted as a Kaluza–Klein tower of Pomeron trajectories obtained after dimensional reduction of the graviton trajectory of string theory on the holographic radius of the gravity dual models.[2] As we already stated in the previous section, the GPD profile shows qualitatively different behavior, depending upon whther j^* has larger real part than the leading singularity in the j-poane. This GPD model is close to the dual parametrization / colinear factorization models[10,11] in phase II + III (DGLAP phase).

5 Transverse Profile

Finally, let us briefly state the result on the parton distribution profile in the transverse spacial directions. This distribution is obtained[12] by Fourier transformation of non-skewed GPD with respect to the transverse momentum transfer $\vec{\Delta}_T = (\vec{p}_2 - \vec{p}_1)_T$.

Carrying out the Fourier transformation and ν integration in (2) first, we obtain a j-plane integral expression for the transverse profile

$$\tilde{I}(x, q^2; b) \sim \int dj \left(\frac{1}{\sqrt{\lambda}x}\right)^j \left(\frac{\Lambda}{q}\right)^{\gamma(j)} e^{-m_1^{(j)} b}; \tag{7}$$

here, $m_1^{(j)}$ is the mass of spin-j hadrons forming the first level of Kaluza–Klein expansion of the leading Pomeron trajectory of superstring theory in 10-dimensions. In the hard wall model with Dirichlet boundary condition at IR, an expression for $m_n^{(j)}$ (that is analytic in j) is given by $\Lambda j_{i\nu_j,n}$, where $j_{\mu,n}$ is the n-the zero of Bessel function of order μ, and ν_j is the inverse function of $j = j(\nu)$.

For small x and large (q/Λ) (but still finite), the integral above is evaluated by the saddle point approximation. The saddle point $j^*(x, q^2; b)$ is determined by the following condition:

$$\frac{\partial\gamma(j^*)}{\partial j} \ln(q/\Lambda) + \ln(1/\sqrt{\lambda}x) = b\frac{\partial m_1^{(j)}}{\partial j}. \tag{8}$$

After a little analysis, it turns out that the transverse profile becomes

$$\tilde{I} \propto e^{-\frac{(b\Lambda)^2}{\frac{2}{\sqrt{\lambda}}\ln(1/x)}} \quad \left(\frac{1}{\sqrt{\lambda}}\ln(1/x) \ll b\Lambda\right), \qquad \tilde{I} \propto e^{-m_1^{(j^*)} b} \quad \left(1 \ll b\Lambda \ll \frac{1}{\sqrt{\lambda}}\ln(1/x)\right); \tag{9}$$

here, we assume sufficiently small x, $\ln(q/\Lambda) \ll (\lambda)^{-1/2}\ln(1/x)$. The transverse profile becomes approximately Gaussian at large impact parameter region, but it is not Gaussian at the core (region with smaller b) as above. The parton density at the core is higher than that naively expected from interpolation of the Gaussian profile, and the profile is given by linear exponential e^{-mb}, with the characteristic mass scale $m = m_1^{(j^*)}$ gradually getting larger for larger impact parameter, so that the profile smoothly continues to the Gaussian profile for larger b.

References

1. R. Nishio and T. Watari, *Phys. Lett.* B **707**, 362 (2012).
2. R. Nishio and T. Watari, *Phys. Rev.* D **84**, 075025 (2011).
3. S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Nucl.Phys.* B **636**, 99 (2002).
4. R.C. Brower et.al. *JHEP* **0712**, 005 (2007).
5. J. Polchinski and M.J. Strassler, *JHEP* **0305**, 012 (2003).
6. Y. Hatta, E. Iancu and A.H. Mueller, *JHEP* **0801**, 026 (2008).
7. R.C. Brower, M. Djuric, I. Sarcevic, and C.-I. Tan, arXiv:1007.2259.
8. A. Aktas et.al. [H1 Collaboration], *EPJ* C **44**, 1 (2005), F.D. Aaron et.al. [H1 Collaboration], *Phys. Lett.* B **659**, 796 (2008), *Phys. Lett.* B **681**, 391 (2009).
9. S. Chekanov et.al. [ZEUS Collaboration], JHEP **0905**, 108 (2009).
10. A.V. Belitsky et.al., *Phys. Lett.* B **421**, 312 (1998), M.V. Polyakov, *Nucl. Phys.* B **555**, 231 (1999), M.V. Polyakov and A.G. Shuvaev hep-ph/0207163, D. Mueller and A. Schafer, *Nucl. Phys.* B **739**, 1 (2006), K. Kumericki, D. Mueller, K. Passek-Kumericki," *Nucl. Phys.* B **794**, 244 (2008),
11. D. Mueller, hep-ph/0605013.
12. M. Burkardt, *Int. J. Mod. Phys.* A **18**, 173 (2003).