**VISCOSITY AND BLACK HOLES**

D.T. SON
*Institute for Nuclear Theory, University of Washington,*
*Seattle, WA 98195-1550, USA*

We review recent applications of the AdS/CFT correspondence in strongly coupled systems, in particular the quark gluon plasma.

## 1 Introduction

The discovery of Hawking radiation[1] confirmed that black holes are endowed with thermodynamic properties such as entropy and temperature, as first suggested by Bekenstein[2] based on the analogy between black hole physics and equilibrium thermodynamics. For black branes, i.e., black holes with translationally invariant horizons, thermodynamics can be extended to *hydrodynamics*—the theory that describes long-wavelength deviations from thermal equilibrium. Thus, black branes possess hydrodynamic properties of continuous fluids and can be characterized by kinetic coefficients such as viscosity, diffusion constants, etc. From the perspective of the holographic principle[3,4], the hydrodynamic behavior of a black-brane horizon is identified with the hydrodynamic behavior of the dual theory.

In this talk, we argue that in theories with gravity duals, the ratio of the shear viscosity to the volume density of entropy is equal to a universal value of $\hbar/4\pi$. A lot of attention has been given to this fact due to the discovery of a perfect liquid behavior at RHIC. We will also review recent attempts to extend AdS/CFT correspondence to nonrelativistic systems.

## 2 Dimension of $\eta/s$

The standard textbook definition of the shear viscosity is as follows. Take two large parallel plates separated by a distance $d$. The space between the two plates is filled with a fluid. Let one plate moves relative to the other with a velocity $v$. Then the drag force acting on a unit area of the plate is

$$\frac{F}{A} = \eta \frac{v}{d} \tag{1}$$

which defines the viscosity $\eta$. It is measured in kg m$^{-1}$ s$^{-1}$. In $d$ spatial dimensions, shear viscosity $\eta$ is measured in kg m$^{2-d}$ s$^{-1}$. The volume density of entropy $s$ is measured in m$^{-d}$ (in units where the Boltzmann constant $k_B$ is set to one). The ratio $\eta/s$ thus has the dimension of kg m$^2$ s$^{-1}$, i.e., the same as the Planck constant $\hbar$. This observation is more than merely a curiosity, as we shall see shortly.

## 3 Viscosity from dual gravity description

Consider a field theory dual to a black-brane metric. One can have in mind, as an example, the $\mathcal{N}=4$ supersymmetric Yang-Mills theory dual to the the near-extremal D3 brane in type IIB supergravity,

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + dx^2 + dy^2 + dz^2) + \frac{R^2}{r^2 f} dr^2, \qquad f = 1 - \frac{r_0^4}{r^4}, \tag{2}$$

but our discussion is not tied to any specific form of the metric. All black branes have an event horizon ($r = r_0$ for the metric (2)), which is extended along several spatial dimensions ($x$, $y$, $z$ in the case of (2)). The dual field theory is at a finite temperature, equal to the Hawking temperature of the black brane.

The entropy of the dual field theory is equal to the entropy of the black brane, which is proportional to the area of its event horizon,

$$S = \frac{A}{4G}, \tag{3}$$

where $G$ is the Newton constant (we set $\hbar = c = 1$). For black branes $A$ contains a trivial infinite factor $V$ equal to the spatial volume along directions parallel to the horizon. The entropy density $s$ is equal to $a/(4G)$, where $a = A/V$.

The shear viscosity of the dual theory can be computed from gravity in a number of approaches[5,6,7]. Here we use Kubo's formula, which relates viscosity to equilibrium correlation functions. In a rotationally invariant field theory,

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t, \vec{x}),\, T_{xy}(0, \mathbf{0})] \rangle . \tag{4}$$

Here $T_{xy}$ is the $xy$ component of the stress-energy tensor (one can replace $T_{xy}$ by any component of the traceless part of the stress tensor). We shall now relate the right hand side of (4) to the absorption cross section of low-energy gravitons.

According to the AdS/CFT correspondence[8], the stress-energy tensor $T_{\mu\nu}$ couples to metric perturbations at the boundary. Following Klebanov[9], let us consider a graviton of frequency $\omega$, polarized in the $xy$ direction, and propagating perpendicularly to the brane. In the field theory picture, the absorption cross section of the graviton by the brane measures the imaginary part of the retarded Greens function of the operator coupled to $h_{xy}$, i.e., $T_{xy}$,

$$\sigma_{\rm abs}(\omega) = -\frac{2\kappa^2}{\omega} \operatorname{Im} G^{\rm R}(\omega) = \frac{\kappa^2}{\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t, \vec{x}),\, T_{xy}(0, \mathbf{0})] \rangle , \tag{5}$$

where $\kappa = \sqrt{8\pi G}$ appears due the normalization of the graviton's action. Comparing (4) and (5), one finds

$$\eta = \frac{\sigma_{\rm abs}(0)}{2\kappa^2} = \frac{\sigma_{\rm abs}(0)}{16\pi G} . \tag{6}$$

The absorption cross section $\sigma_{\rm abs}$, on the other hand, is calculable classically by solving the linearized wave equation for $h^x_y$. It can be shown (see Appendix) that under rather general assumptions the equation for $h^x_y$ is the same as that of a minimally coupled scalar. The absorption cross section for the scalar is constrained by a theorem[10,11], which states that in the low-frequency limit $\omega \to 0$ this cross section is equal to the area of the horizon, $\sigma_{\rm abs} = a$. Since $s = a/4G$, one immediately finds that

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}, \tag{7}$$

where $\hbar$ is now restored. This shows that the ratio $\eta/s$ does not depend on the concrete form of the metric within the assumptions of[10,11].

Indeed, explicit calculations of the viscosity using the AdS/CFT correspondence or the "membrane paradigm" technique show that the ratio $\eta/s$ is the $1/(4\pi)$ for D$p$[5,7], M2 and M5[12] branes and $\mathcal{N}=2^*$ deformations of the D3 metric[7,13].

Dual gravity description of gauge theories is valid in the regime of infinitely strong coupling. As Eq. (7) shows, in this regime the ratio $\eta/s$ appears to be universal (independent of the coupling constant and other microscopic details of the theory). Let us now argue that the ratio $\eta/s$ approaches infinity in the limit of vanishing coupling.

The entropy density $s$ of a weakly coupled system is proportional to the number density of quasiparticles $n$,

$$s \sim n\,. \tag{8}$$

The shear viscosity is proportional to the product of the energy density and the mean free time (time between collisions) $\tau$

$$\eta \sim n\epsilon\tau\,, \tag{9}$$

where $\epsilon$ is the average energy per particle (which is of the order of the temperature $T$). Therefore

$$\frac{\eta}{s} \sim \epsilon\tau\,. \tag{10}$$

Now, in order for the quasiparticle picture to be valid, the width of the quasiparticles must be small compared to their energies, i.e., one should have

$$\frac{\hbar}{\tau} \ll \epsilon \tag{11}$$

which means that

$$\frac{\eta}{s} \gg \hbar\,. \tag{12}$$

The observation that $\eta/s$ is a constant in strongly coupled theories with gravity dual and is large in weakly coupled theories prompts us to formulate the "viscosity bound" conjecture: in any finite-temperature field theory, the ratio of shear viscosity to entropy density cannot be smaller than the value of this ratio in theories with gravity duals:

$$\frac{\eta}{s} \geqslant \frac{\hbar}{4\pi}\,. \tag{13}$$

As we have seen, the bound (13) can be understood as a consequence of the uncertainty principle: the product of the energy and the mean free time of a quasiparticle cannot be smaller than $\hbar$. The precise numerical coefficient $1/(4\pi)$ cannot, however, be obtained from the uncertainty principle alone.

It turns out that the viscosity bound is can be violated by corrections proportional to $1/N_c$, where $N_c$ is the number of colors [14,15]. It still seems that there is a lower bound on $\eta/s$. In one particular model, causality is violated when one tries to make $\eta/s$ smaller than 64% of $1/4\pi$ [16].

## 4 Nonrelativistic holography

The nonrelativistic equivalence of $\mathcal{N}=4$ SYM theory is the Fermi gas at unitarity. The symmetry of the system is the Schrödinger symmetry. Recently, a metric has been found that realizes this symmetry [17,18]:

$$ds^2 = -2\frac{dx^+}{z^4} + \frac{-2dx^+dx^- + d\vec{x}^2 + dz^2}{z^2} \tag{14}$$

Furthermore, it has been found that this metric can be realized in string theory [19,20,21]. The shear viscosity satisifies $\eta/s = 1/4\pi$. The thermal conductivity has also been calculated; its value corresponds to the Prandtl number equal to 1 [22].

**Acknowledgments**

This work is supported, in part, by DOE Grant No. DE-FG02-00ER41132.

**References**

1. S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
2. J. D. Bekenstein, *Phys. Rev.* D **7**, 2333 (1973).
3. G. 't Hooft, gr-qc/9310026.
4. L. Susskind, *J. Math. Phys.* **36**, 6377 (1995).
5. G. Policastro, D. T. Son and A. O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001).
6. G. Policastro, D. T. Son and A. O. Starinets, *J. High Energy Phys.* **0209**, 043 (2002).
7. P. Kovtun, D. T. Son and A. O. Starinets, *J. High Energy Phys.* **0310**, 064 (2003).
8. For review, see: O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, *Phys. Rep.* **323**, 183 (2000).
9. I. R. Klebanov, *Nucl. Phys.* **B496**, 231 (1997).
10. S. R. Das, G. W. Gibbons and S. D. Mathur, *Phys. Rev. Lett.* **78**, 417 (1997).
11. R. Emparan, *Nucl. Phys.* **B516**, 297 (1998).
12. C. P. Herzog, *J. High Energy Phys.* **0212**, 026 (2002).
13. A. Buchel and J. T. Liu, *Phys. Rev. Lett.* **93**, 090602 (2004).
14. Y. Kats and P. Petrov, *J. High Energy Phys.* **0901**, 044 (2009).
15. A. Buchel, R. C. Myers and A. Sinha, *J. High Energy Phys.* **0903**, 084 (2009)
16. M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, *Phys. Rev. Lett.* **100**, 191601 (2008).
17. D. T. Son, *Phys. Rev.* D **78**, 046003 (2008).
18. K. Balasubramanian and J. McGreevy, *Phys. Rev. Lett.* **101**, 061601 (2008).
19. C. P. Herzog, M. Rangamani and S. F. Ross, *J. High Energy Phys.* **0811**, 080 (2008).
20. J. Maldacena, D. Martelli and Y. Tachikawa, *J. High Energy Phys.* **0810**, 072 (2008).
21. A. Adams, K. Balasubramanian and J. McGreevy, *J. High Energy Phys.* **0811**, 059 (2008).
22. M. Rangamani, S. F. Ross, D. T. Son and E. G. Thompson, *J. High Energy Phys.* **0901**, 075 (2009).