Regge-pole models do not know about the $t$ and $Q^2$; $M_V$ dependence of the scattering amplitude. We extend a simple Pomeron pole amplitude by $t$ and $Q^2$; $M_V$ dependencies inspired by geometrical ideas. The experimentally transition from soft to hard dynamics is realized by the introduction of two Pomeron poles with different $Q^2$ $M_V$-dependent residue. A unified description of deeply virtual Compton scattering as well as the elastic electroproduction of all vector meson is suggested.

1 Introduction

The forward slope of the the differential cross sections for elastic scattering is known to be related to the masses/virtualities of the interacting particles. This phenomenon is evident e.g. from Fig. (SLOPES), where the forward slope $B(\tilde{Q}^2) = \frac{d}{d\ln \sigma} \ln \frac{d\sigma}{dt}$ is plotted against the the variable $\tilde{Q}^2 = Q^2 + M_V^2$. The slope, proportional to the interaction radius $R(\tilde{Q}^2)$, decreases with increasing $\tilde{Q}^2$, reaching some saturation value, determined by the finite mass of the nucleons. In this geometric picture, the largest slope (radius) is expected for real Compton scattering $\tilde{Q}^2 = 0$, which may require a separate treatment.

The use of the variable $\tilde{Q}^2 = (M_V^2 + Q^2)$ implies symmetry between the mass $M_V^2$ and virtuality $Q^2$, which should imply equal slopes (radii) for e.g. $J/\psi$ production near $Q^2 = 0$ and $\rho$ electroproduction near $Q^2 \approx 9 \text{ GeV}^2$, which is not supported by the data.
In the present paper we consider exclusive diffractive electroproduction of real photons and vector mesons as well as elastic proton-proton scattering by making use of the above geometric considerations, by writing the scattering amplitude in the form:

$$A(s, t) \sim e^{B(s, M^2) t}$$  \hspace{1cm} (1)

with $B(M^2) \sim 1/f(M^2)$. This approach was used in Ref. 2 for the simpler case of photoproduction, $\tilde{Q}^2 = 0$, excluding real Compton scattering, and without considering nucleon scattering, to be also included below.

While the geometric considerations proved to be efficient 2 for photoproduction, they are not sufficient in the case of electroproduction $Q^2 = 0$, since the relevant cross sections will increase with $\tilde{Q}^2$ contradicting the experimental data. To remedy this deficiency, this rise must be compensated by multiplying the amplitude by a function decreasing with $\tilde{Q}^2$.

Moreover, to cope with the observed trend of hardening the dynamics as $\tilde{Q}^2$ increases, and following Refs. 5, 6, we introduce two components for the diffractive (Pomeron) amplitude of the type Eq. (1), soft $A_s$ and hard $A_h$, each one to be multiplied by a relevant $\tilde{Q}^2$-dependent factor $H_i(\tilde{Q}^2)$, $i = s, h$. These factors are chosen in such a way as provide for the increasing weight of the hard component as the mass (virtuality) increases. To avoid conflict with unitarity, the rise with $\tilde{Q}^2$ of the hard component is finite, and it terminates at some saturation scale $\tilde{Q}^2$, whose value is determined phenomenologically. Explicit examples of these functions will be given below.

Recently 1, 4 a model for exclusive production of vector particles at HERA was suggested and successfully fitted to the HERA data. In that model, the interplay between $t$ and $\tilde{Q}^2$ is achieved by introducing a new variable $z = t - \tilde{Q}^2$.

Good fits were obtained in those papers, however the specific interplay between the variables $t$ and $Q^2$ there awaits for a better understanding and physical interpretation.

2 The Model

2.1 Regge+geometry=Reggeometry

Quite generally, the Regge-pole scattering amplitude can be written as

$$A(s, t, M, Q^2) = \xi(t) \beta(t, M, Q^2)(s/s_0)^{\alpha(t)},$$  \hspace{1cm} (2)

where $\xi(t) = e^{-i\pi\alpha(t)}$ is the signature factor and $\beta(t, M, Q^2)$ is the residue factor written in the following geometrical form:

$$\beta(t, M, Q^2) = \exp\left[4\left(\frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2}\right)t\right].$$  \hspace{1cm} (3)

Hence

$$A(s, t, M, Q^2) = \tilde{A}_0 \xi(t)(s/s_0)^{\alpha(t)} e^{-4\left(\frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2}\right)|t|}.$$  \hspace{1cm} (4)

The differential cross section is

$$\frac{d\sigma}{d|t|} = \frac{\pi}{s^2}|A(s, t, M, Q^2)|^2 = A_0(s/s_0)^{2(\alpha_0 - 1 - \alpha'|t|)} e^{-8\left(\frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2}\right)|t|},$$  \hspace{1cm} (5)

or

$$\frac{d\sigma}{d|t|} = A_0(s/s_0)^{2(\alpha_0 - 1)} e^{-\left[2\alpha'|\ln(s/s_0)| + 8\left(\frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2}\right)\right]|t|} = C e^{-B|t|}. $$  \hspace{1cm} (6)
The local slope parameter is defined as

\[
B(s, \tilde{Q}^2) = \frac{d}{dt} \ln \frac{d \sigma}{d |t|} = 2\alpha' \ln (s/s_0) + 8\left( \frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2} \right), \tag{7}
\]

and the integrated elastic scattering amplitude is

\[
\sigma_{el} = \left. \frac{1}{B} \frac{d \sigma}{d t} \right|_{t=0} = C/B,
\]

or

\[
\sigma_{el} = \frac{A_0(s/s_0)^{2(a\alpha-1)}}{2\alpha' \ln (s/s_0) + 8\left( \frac{a}{M_V^2 + Q^2} + \frac{b}{2m_N^2} \right)}. \tag{9}
\]

In the case \( a > 0 \) when \( \tilde{Q}^2 = M_V^2 + Q^2 \) grows, \( B(s, \tilde{Q}^2) \) falls, but \( \frac{d\sigma}{d|t|} \) and \( \sigma_{el} \) become larger. While the behavior of \( B(s, \tilde{Q}^2) \) is consistent with the experimental data, the behavior of \( \frac{d\sigma}{d|t|} \) and \( \sigma_{el} \) are not.

### 2.2 Soft and Hard components of the unique Pomeron

We build the scattering amplitude that contain two terms, soft and hard, with two different \( \tilde{Q}^2 \)-dependent factors:

\[
A(s, t, Q^2, M_V^2) = \frac{\tilde{A}_s}{\left(1 + \frac{Q^2}{Q_s^2}\right)} \left(1 + \frac{Q^2}{Q_s^2}\right) e^{-i\frac{\pi}{2} \alpha_s(t)} \left(\frac{s}{s_0s}\right)^{\alpha_s(t)} e^{2\left(\frac{a_s}{Q^2} + \frac{b_s}{2m_N^2}\right)t} + \frac{\tilde{A}_h}{\left(1 + \frac{Q^2}{Q_h^2}\right)} \left(1 + \frac{Q^2}{Q_h^2}\right) e^{-i\frac{\pi}{2} \alpha_h(t)} \left(\frac{s}{s_0h}\right)^{\alpha_h(t)} e^{2\left(\frac{a_h}{Q^2} + \frac{b_h}{2m_N^2}\right)t}. \tag{10}
\]

The \( \tilde{Q}^2 \)-dependent factors were introduced in such a way as to provide for a proper balance between the soft and hard components of the Pomeron, namely that the hard one increases with increasing \( \tilde{Q}^2 \), up to the saturation point, thus securing unitarity. The extra \( \tilde{Q}^2 \)-dependent (but \( t \)-independent!) factor does not violate the geometrical structure of the amplitude.

The differential and integrated cross sections now are:

\[
\frac{d\sigma_{el}}{d|t|} = H_s^2 e^{2L_s(\alpha_s(t) - \phi_s t)} + H_h^2 e^{2L_h(\alpha_h(t) - \phi_h t)} + 2H_s H_h e^{L_s(\alpha_s(t) - \phi_s t) + L_h(\alpha_h(t) - \phi_h t)} \cos\left(\frac{\pi}{2}(\alpha_s(t) - \alpha_h(t))\right), \tag{11}
\]

\[
\sigma_{el} = \frac{H_s^2 e^{2L_s(\alpha_s(t) - \phi_s t)}}{2(\alpha_s'(L_s + \phi_s))} + \frac{H_h^2 e^{2L_h(\alpha_h(t) - \phi_h t)}}{2(\alpha_h'(L_h + \phi_h))} + 2H_s H_h e^{L_s(\alpha_s(t) - \phi_s t) + L_h(\alpha_h(t) - \phi_h t)} \frac{\mathcal{B} \cos \phi_0 + \mathcal{L} \sin \phi_0}{\mathcal{B}^2 + \mathcal{L}^2}, \tag{12}
\]

where the notation \( H_s = \frac{A_s}{(1 + \frac{Q^2}{Q_s^2})^{n_s + 1}}, \quad H_h = \frac{A_h}{(1 + \frac{Q^2}{Q_h^2})^{n_h + 1}} \).
\( L_s = \ln \left( \frac{s}{s_0s} \right), \quad g_s = 2 \left( \frac{a_s}{Q^2} + \frac{b_s}{2m_s^2} \right), \quad \alpha_s(t) = \alpha_{0s} + \alpha'_s t, \)

\( L_h = \ln \left( \frac{s}{s_0h} \right), \quad g_h = 2 \left( \frac{a_h}{Q^2} + \frac{b_h}{2m_h^2} \right), \quad \alpha_h(t) = \alpha_{0h} + \alpha'_h t, \)

\( B = L_s \alpha'_s + L_h \alpha'_h + (g_s + g_h), \)

\( \Sigma = \frac{\pi}{2} (\alpha'_s - \alpha'_h), \)

\( \phi_0 = \frac{\pi}{2} (\alpha_{0s} - \alpha_{0h}) \)

was introduced.

For the soft and hard components of the Pomeron trajectory we use the Donnachie-Landshoff parameterization \(^{5,6}\):

\[ DL \text{ pomerons} : \quad \alpha_s(t) = 1.08 + 0.25t, \]

\[ \alpha_h(t) = 1.44 + 0.01t. \]

### 3 Fits to the data

Although there are 16 fitting parameters, almost all of them can be fixed.

\[
\begin{pmatrix}
A_s, & \tilde{Q_s^2}, & n_s, & s_{0s}, & \alpha_{0s}, & \alpha'_s, & a_s, & b_s \\
A_h, & \tilde{Q_h^2}, & n_h, & s_{0h}, & \alpha_{0h}, & \alpha'_h, & a_h, & b_h
\end{pmatrix}
\]

The values of the parameters obtained form fitting the DVCS differential elastic cross section are:

**soft**:

- \( A_s \): 3.437 ± 3.587; 23.724 ± 36.520; 2; 7.0; 1.08; 0.25; 1.0; 1.0;
- \( \tilde{Q_s^2} \): 2.466 ± 27.216; 1.535 ± 7.243; 2; 7.0; 1.44; 0.01; 1.0; 1.0;

and those form fitting the integrated elastic DVCS cross section are:

**soft**:

- \( A_s \): 2.448 ± 2.118; 29.683 ± 38.633; 2; 7.0; 1.08; 0.25; 1.0; 1.0;
- \( \tilde{Q_s^2} \): 2.458 ± 2.744; 1.705 ± 2.089; 2; 7.0; 1.44; 0.01; 1.0; 1.0;

These two fits were performed separately, giving similar results. The experimental data for the fits are from Refs. \(^{7,8,9,10}\).

The results of the fit are shown in the figures appended.
Figure 1: Results of our fit to $\gamma^* p \to \gamma p$ elastic cross-section as a function of $Q^2$ for ZEUS 99-00 (upper, left icon) and H1 96-00 (upper, right icon); for ZEUS 96-00 (lower left and right icons).

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Figure 2: Results of our fit to $\gamma^* p \rightarrow \gamma p$ elastic cross-section as a function of $W$ for ZEUS 99-00 (upper left and middle left icons) and H1 05-06 (upper right, icon); $\gamma^* p \rightarrow \gamma p$ differential cross-section as a function of $t$ for ZEUS 99-00 (down-left) and H1 05-06 (middle and lower, right icons).
References