The Orear regime in elastic $pp$-scattering at $\sqrt{s}=7$ TeV

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Abstract

The unitarity condition unambiguously requires the Orear region to appear in between the diffraction cone at low transferred momenta and hard parton scattering regime at high transferred momenta in hadron elastic scattering. It originates from rescattering of the diffraction cone processes. It is shown that such region has been observed in the differential cross section of the elastic $pp$-scattering at $\sqrt{s}=7$ TeV. The Orear region is described by exponential decrease with the scattering angle and imposed on it damped oscillations. They explain the steepening at the end of the diffraction cone as well as the dip and the subsequent maximum observed in TOTEM data. The failure of several models to describe the data in this region can be understood as improper account of the unitarity condition. It is shown that the real part of the amplitude can be as large as the imaginary part in this region. The overlap function is calculated and shown to be small outside the diffraction peak. Its negative sign there indicates the important role of phases in the amplitudes of inelastic processes.

1 Introduction

The TOTEM collaboration has published [1] experimental results on the differential cross section of the elastic $pp$-scattering at the total cms energy $\sqrt{s}=7$ TeV. Among the most interesting features they observe the steepening of the diffraction cone near the squared transferred momentum $0.36$ GeV$^2$, the dip at $0.53$ GeV$^2$ and the maximum at $0.7$ GeV$^2$. We explain them as resulting from the unitarity condition which prescribes the Orear regime characterized by exponential decrease with the scattering angle and imposed on it damped oscillations to start at transferred momenta just above the diffraction cone. There are several other models mostly based on reggeon approach and extensively cited in [1] with published predictions which, however, fail to describe the new data quantitatively. This demonstrates that the unitarity condition was not properly accounted in these models.

At the end of 60s the very first experimental data on elastic $pp$- and $\pi p$- scattering were obtained at energies between 6.8 and 19.2 GeV in the laboratory system [2, 3, 4]. They showed that just after the diffraction cone
which behaved as a Gaussian in the scattering angle there was observed the exponentially decreasing with the angle behavior which was called as the Orear regime in the name of its investigator [3]. Some indications on the shoulder appearing at the beginning of this region (transformed later to the dip at the higher ISR energies) were also obtained. The special session was devoted to these findings at the Rochester conference in Wien in 1968.

The theoretical indications on the possibility of such regime were obtained even earlier [5, 6, 7] but the results did not fit new experimental findings.

At the same time the theoretical explanation based on consequences of the unitarity condition was proposed [8, 9] and the fit to experimental data showed good quantitative agreement with experiment [10]. These ideas are applicable to the TOTEM data which are well fitted also.

2 Theoretical description.

Let us have a look at the unitarity condition which is

\[
\text{Im} A(p, \theta) = I_2(p, \theta) + F(p, \theta) = \frac{1}{32\pi^2} \int \int d\theta_1 d\theta_2 \sin\theta_1 \sin\theta_2 A(p, \theta_1) A^*(p, \theta_2) \frac{1}{\sqrt{|\cos \theta - \cos(\theta_1 + \theta_2)|[\cos(\theta_1 - \theta_2) - \cos \theta]}} + F(p, \theta). \quad (1)
\]

The region of integration in (1) is given by the conditions

\[|\theta_1 - \theta_2| \leq \theta, \quad \theta \leq \theta_1 + \theta_2 \leq 2\pi - \theta. \quad (2)\]

The integral term represents the two-particle intermediate states of the incoming particles. The function \( F(p, \theta) \), called following Van Hove [5] as the overlap function, represents the shadowing contribution of the inelastic processes to the elastic scattering amplitude.

The elastic scattering proceeds mostly at small angles. The real part of the amplitude is negligible there and the amplitude is approximated as

\[ A(p, \theta) \approx 4ip^2\sigma t e^{-Bp^2\theta^2/2} \quad (3) \]
with \( p \) and \( \theta \) denoting the momentum and the scattering angle in the center of mass system, \( B \) known as the diffraction slope and \( \sigma t \) the total cross section.

The principal contribution to the integral \( I_2 \) arises from a narrow region around the line \( \theta_1 + \theta_2 \approx \theta \). Therefore one of the amplitudes should be inserted at small angles within the diffraction cone while another one is kept at angles outside it. We omit the real part of the amplitude. Thus inserting
the diffraction cone expression for one of the amplitudes in $I_2$ and integrating over one of the angles the linear integral equation is obtained:

$$\text{Im} A(p, \theta) = \frac{p \sigma_t}{4\pi \sqrt{2\pi B}} \int_{-\infty}^{+\infty} d\theta_1 e^{-Bp^2(\theta-\theta_1)^2/2} \text{Im} A(p, \theta_1) + F(p, \theta).$$  \hspace{1cm} (4)

It can be solved analytically (for more details see [8, 9]) under the assumption that the role of the overlap function $F(p, \theta)$ is negligible outside the diffraction cone. The solution contains the term exponentially decreasing with $\sqrt{|t|} = p\theta$ and the exponentially damped oscillating terms. The elastic scattering differential cross section looks like

$$\frac{d\sigma}{p_1 dt} = \left( e^{-\sqrt{2B|t|} \ln \frac{4\pi}{\pi d\rho}} + p_2 e^{-\sqrt{2\pi B|t|}} \cos(\sqrt{2\pi B|t|} - \phi) \right)^2. \hspace{1cm} (5)$$

Here, we have taken into account the real parts of the amplitude simply replacing $\sigma_t$ in Eq. (4) by $\sigma_t f\rho$ where $f\rho = 1 + \rho_d \rho_l$ with averaged values of ratios of real to imaginary parts of the amplitude in the diffraction cone and outside it denoted as $\rho_d$ and $\rho_l$ correspondingly.

Beside the overall normalization constant $p_1$, this formula contains the constants $p_2$ and $\phi$ which determine the strength and the phase of the oscillation\(^2\). They can be found from fits of experimental data. The constant $p_1$ is determined by the transition point from the diffraction cone to the Orear regime. The constants $p_2$ and $\phi$ define the depth of the dip and its position.

We choose $\rho_d \approx 0.14$ as prescribed by the dispersion relations for its value at $t = 0$ [12, 13] and use $p_1$ as a fitted parameter in exponents of Eq. (5). All parameters can depend on energy as well as $B$ and $\sigma_t$. Surely, this is unimportant if the fit is done at a fixed energy as in the present paper.

Apart from comparison of theoretical predictions with experimental data one can get some knowledge about the overlap function $F(p, \theta)$ (see [11]). It is important, in particular, to confirm the assumption about its smallness outside the diffraction peak. Then the equation (1) is used:

$$F(p, \theta) = 16p^2 \left( \frac{\pi d\sigma}{dt} \left( 1 + \rho^2 \right) \right)^{1/2} - \frac{8p^4(1 + \rho_d \rho)}{\pi \sqrt{(1 + \rho_d^2)(1 + \rho^2)}} \int_{-1}^{1} dz_2 \int_{z_1}^{z_1} d\sigma \left[ \frac{d\sigma}{dt_1} \cdot \frac{d\sigma}{dt_2} \right]^{1/2} K^{-1/2}(z, z_1, z_2) \hspace{1cm} (6)$$

\(^{1}\)The results of [11] and our estimates (see Eq. (6) and discussion below) support it.

\(^{2}\)The phase was determined in [8, 9] from the iterative solution of the unitarity equation to be equal $\phi \approx \pm \pi/4$ (actually with somewhat larger absolute value) but we use it here as a free parameter. The first (weaker damped) oscillating term has only been taken into account in Eq. (5). Let us note the same values of the exponential damping and the period of the oscillations.
where $z_i = \cos \theta_i$; $K(z, z_1, z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2$ and $z_1^\pm = zz_2 \pm \sqrt{(1 - z^2)(1 - z_2^2)}^{1/2}$. $\rho = \rho_d$ in the diffraction cone, $\rho = \rho_l$ outside it.

3 Fit of the experimental data

Having at our disposal Eq. (5) we try to fit experimental distribution of elastic $pp$-scattering at $\sqrt{s} = 7$ TeV. The result is shown in Fig. 1.

It is seen that the fit is quite successful in the expected applicability region of $|t|$ from 0.36 GeV$^2$ to 1.5 GeV$^2$. First of all, we notice the steeper decrease compared to the slope of the diffraction cone at $|t| < 0.3$ GeV$^2$ as observed in experiment. It is explained here as the negative contribution of the oscillating term in Eq. (5). That determines the phase $\phi$. The dip develops at $|t| = 0.53$ GeV$^2$ where cos-term vanishes. Then this term becomes positive and increases leading to the maximum at $|t| \approx 0.7$ GeV$^2$. The positions of the dip and the maximum are uniquely determined by the period of oscillations $\Delta t = 2\pi/B$ which is predicted by the unitarity condition and depends only on the well measured slope of the diffraction peak $B$. The damping exponent in front of the cos-term becomes so strong at larger $|t|$ that the simple Orear regime with the first term in Eq. (5) prevails. Let us note that the exponent in this term is extremely sensitive to the parameter $\rho_l$ because the ratio $4\pi B/\sigma_t$ is very close to $1^4$. That helps determine this parameter. It would require very high precision to observe the next weak minimum at $|t| \approx 0.9 - 1.0$ GeV$^2$ because the exponent in the second term is very strong and damps the oscillations at larger $|t|$. It is interesting to note that the damping increases with energy due to increase of the slope $B$. At the same time the shrinkage of the cone leads to the shift of the Orear regime (and the dip) to smaller angles and the oscillations are still noticeable there. Let us list the parameters in Eq. (5) which we found by the fitting procedure: $p_1 = 18.71$; $p_2 = 115.6$; $\phi = -0.845$; $\rho_l \approx -2$. Up to now the only possible estimate of the ratio of real to imaginary parts of the elastic scattering amplitude was by dispersion relations at $t = 0$. It is for the first time that it is done at large $|t|$ and shows that this ratio is of the order of 1 there. The parameter $\phi$ is so close to its theoretical estimate that it was not even necessary to use it as a free one.

The overlap function has been calculated and shown to be rather similar

\footnote{This happens at all energies!}
to its shape at lower energies [11] so that we do not demonstrate it here.

4 Conclusions

Thus we conclude that at intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations. The experimental data on elastic $pp$ differential cross section at $\sqrt{s}=7$ TeV in this region are fitted by it with well described position of the dip at $|t| \approx 0.53$ GeV$^2$, its depth and subsequent damped oscillations with the predicted period about 0.3 GeV$^2$. The large amplitude of the oscillations and their negative sign explain the steepening of the slope above $|t|=0.36$ GeV$^2$. The positive sign of the oscillating term at $|t| \approx 0.7$ GeV$^2$ leads to the maximum. Strong damping of the oscillations at higher values of $|t|$ results in clear signature of the simple exponential (in $\sqrt{|t|}$) behavior observed first by Orear which extends up to $|t| \approx 1.5$ GeV$^2$.

The fit allows without using any definite model for the first time to estimate the ratio of real to imaginary parts of the elastic scattering amplitude in this region far from forward direction $t=0$. It is of the order of 1.

The overlap function at 7 TeV has been calculated using only the experimental differential cross section and the above estimate of the ratio of real to imaginary parts. As at low energies, it is small and negative in the Orear region. That confirms the assumption used when solving the unitarity equation and shows that the phases of inelastic amplitudes become crucial in any model of inelastic processes.

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References


