Modelling in 2D the roughness of the roads in the equation of the vehicle movement

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The roughness situation of the road has serious impact on dynamics stability of vehicles, which can modify their own movements. The modelling of road roughness in 1D, leading to a linear profile, has been performed by many authors such as Kamel Henchi, Mario Fafard and Martin Talbot, Feng Tyan and Yu-Fen Hong, Shun-Hsu Tu and Wes S. Jeng, and so on… In this article, a 2D modelling of road roughness was performed by using a method based on the power spectral density (PSD) principle. This process consists of doing the discretisation of the spectral density function r(x, y) into a set of functions $\{r_x(x_i), r_y(y_i)\}$ in which i indicates the geometrical node index. This way, the description of the continuous repartition of road roughness r(x, y) will be led by the set of the discrete variables $\{r_x(x_i), r_y(y_i)\}$.

Key words: roughness, modelling, discretisation, Power Spectral Density, roads, vehicle

1. INTRODUCTION:

The study of the situation of a road profile represents a great deal of importance for the road construction company, in order the car passengers comfortable. Therefore, the analyses of the origin make of vibrations of a vehicle and also the state of the road have been the topics of several published papers.

Road roughness conditions can be described either using a 1D profile design along an elevated course or, using a 3D profile design which is closer to the real situation or, characterized by a random signal of a large spectrum. However, the usual representation is the use of the distribution of power spectral density (PSD) [1,2,3,5,7,8] in order to obtain a mathematical model. Numerous studies have shown the power spectral density as a function with one variable r(x) and recommended the use of a 1D profile design to describe the state of roughness of the road.

In this article, a 2D profile design of the state of the road roughness is proposed, this way, the interaction of the vehicle wheels with the pavement will be taken into account.

The digital simulation considering both the effect of the origin of he vibrations (action of the roughness of the road on the vehicle wheels) and the dynamics stability of the "shod-vehicle" system uses the functions of the shape r(x,y) before being discreted in two components $\{r_x(x), r_y(y)\}$.

2. MODELLING THE ROAD ROUGHNESS IN A 2D DESIGN

2.1. Road roughness in 1d design

Commonly, the roughness of a pavement can be defined as the distribution of irregularities on its surface. Based on their experimental measurements, Kamel Henchi, Mario Fafard and Martin Talbot [3], were able to propose a mathematical model r(x) to describe the static profile of the road roughness on a $\frac{L}{2}$ distance. As conclusion, the function r(x) can be considered as stationary Gaussian random process whit a null average that can be deducted from the functions of power spectral density, such as:

$$r(x) = \sum_{k=1}^{N} \left[\sqrt{4\, S_r\,(\omega_{sk})\,\Delta\omega_s}\ \cos(\omega_{sk} x - \varphi_k) \right] \tag{1}$$

in which:

$$S_r\ (\omega_{sk}) = A_r \left(\frac{\omega_s}{\omega_{s0}}\right)^{-2}$$

A_r (m^3/cycle), the coefficient of roughness

ω_{sk} (cycle/m), the number of wave

$\omega_{s0} = \frac{1}{2\pi}$ (cycle/m), the frequency of discontinuity

$\Delta\omega_s = \frac{2\pi}{L}$, the number of waves per unit of length on the road

φ_k (rd), the angle of phase

This mathematical model has permitted to represent the profile with such type of graph [3]: (see Figure 1)

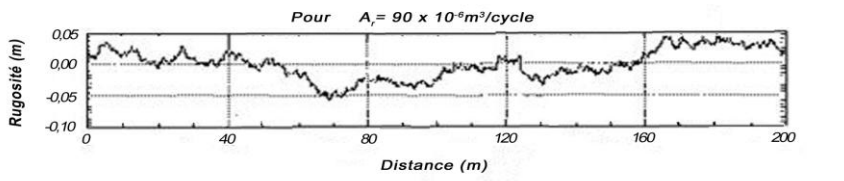

Figure 1: Example of profile of 1D.

According to the description of 1D, the effect of the irregularities affects the properties of dynamic stability of the vehicle on a straight course. In this case, the sources of excitation, applying to the wheels, will act according to a profile of irregularities with longitudinal distribution (see Figure 2)

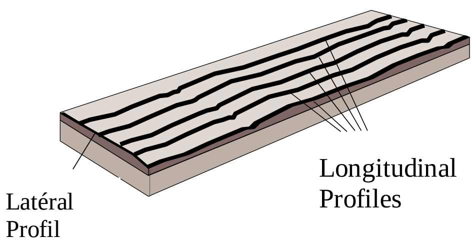

Figure 2: Profile of a road of 1D

2.2. Access to the 2D surface

The description of 2D consists in adopting a profile of surface r(x,y) to represent the roughness. The model mathematical correspondent to this function is obtained by the algebraic sum of two components:

- the component $r_X(x)$, defined on the length of the road
- the component $r_Y(y)$, defined on the width of the road

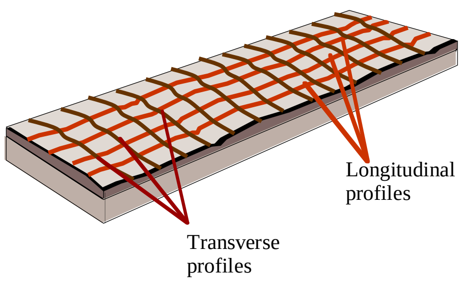

Figure 3 : Example of profile 2D

The two components $r_X(x)$ and $r_Y(y)$ being independent, one can use the expression established for the profile 1D to express their respective mathematical model, as:

$$\left.\begin{aligned} r_x(x) &= \sum_{kx=1}^{N_x}\left[\sqrt{4\,A_{rx}\left(\frac{\omega_{skx}}{\omega_{s0x}}\right)^{-2}\Delta\omega_{sx}}\;\cos\left(\omega_{sx}x-\varphi_{kx}\right)\right] \\ r_y(y) &= \sum_{ky=1}^{N_y}\left[\sqrt{4\,A_{ry}\left(\frac{\omega_{sky}}{\omega_{s0y}}\right)^{-2}\Delta\omega_{sy}}\;\cos\left(\omega_{sy}y-\varphi_{ky}\right)\right]\end{aligned}\right\} \qquad (2)$$

The application of this formulation to the surface of the pavement, discredited geometrically by the Method of the finite differences, conducts to a exact determination of r(x,y) in every, node $P_i(x_i,y_i)$ (see Figure 4), as:

$r(x,y)$ <====> $\{r_x(x_i), r_y(y_i)\}$ wherever is P_i on to the surface

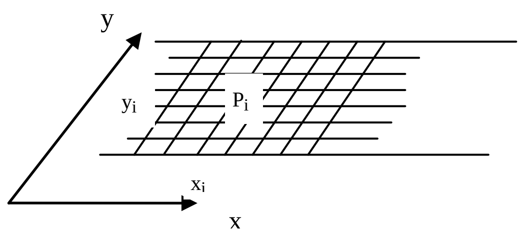

Figure 4: Geometric discretisations

The mathematical model r(x,y) that we propose will have therefore for expression:

$$\begin{aligned} r(x,y) = &\sum_{kx=1}^{N_x}\left[\sqrt{4\,A_{rx}\left(\frac{\omega_{skx}}{\omega_{s0x}}\right)^{-2}\Delta\omega_{sx}}\;\cos\left(\omega_{sx}x_i-\varphi_{kx}\right)\right]+ \\ &\sum_{ky=1}^{N_y}\left[\sqrt{4\,A_{ry}\left(\frac{\omega_{sky}}{\omega_{s0y}}\right)^{-2}\Delta\omega_{sy}}\;\cos\left(\omega_{sy}y_i-\varphi_{ky}\right)\right]\end{aligned} \qquad (3)$$

An example a road pattern, using this formulation is presented in the Figure 5

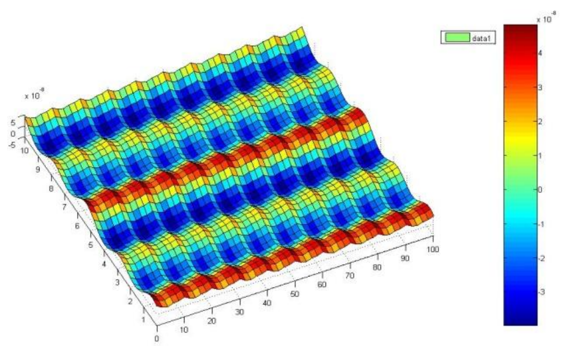

Figure 5: Example of 2D profile stock (Calculus using the Software MATLAB)

2.3. Movement of the vehicle in 3D

The vehicle is modelised by a set of concentrated masses which are interconnected by elastic rubber bands and dissipative (spring). These are 7 masses which are distributed on the following elements:

- body - front suspension - rear suspension - four wheels [2,3,4,5,6].

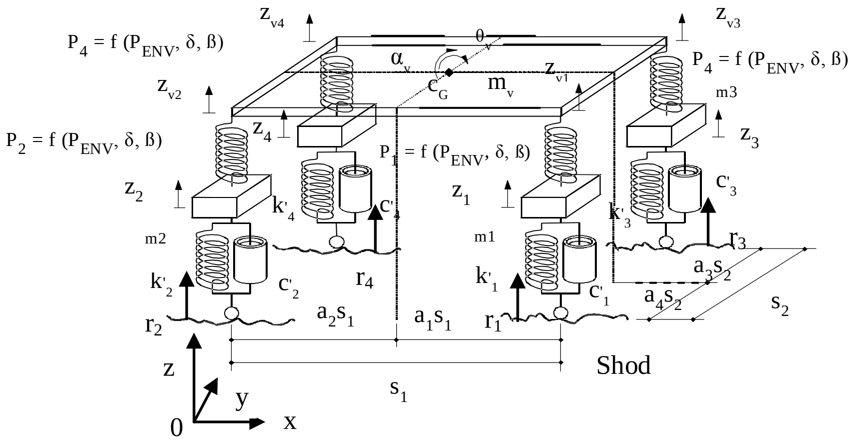

Figure 6: Model of the vehicle in 3D

in which :

k_{pi} rigidity bound to the wheel (tire) i of the vehicle in contact with the surface , C_{pi} amortization of the i wheel, $\dot{r}_i$ speed of the depth and $\dot{Z}_i$ speed of the degrees of liberty of the vehicle

The tires are represented by springs in parallel with the shock absorbers and they stay in permanent contact with the rough surface.

While using the formulation of Lagrange [3,4,9], the equations of the movement of the system are in the shape of:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} - \frac{\partial W_d}{\partial q_i} = Q_i \qquad (4)$$

for every general coordinates q_i of the vehicle.

In this equation,

T and V designate the kinetic energy and the potential energy of the system respectively

W_d, the energy of dissipation of the system,

Q_i, general strength in accordance with W_d,

For the model presented in the Figure 6, the general coordinates $\{q_i\}$ are regrouped in the line vector:

$$<q_i> = <Z_1,\ Z_2,\ Z_3,\ Z_4,\ Z_{1V},\ Z_{2V},\ Z_{3V}> \qquad (5)$$

Thus, the relation (4), applied to the model of the Figure 6 can be written:

$$\sum_{i=1}^{2} P_i\Delta_i^* + \sum_{j=1}^{4}\left[P'_j\,\Delta_j^{'*} + m_j\ddot{Z}_jZ_j^* + m_jgZ_j^* + F_jX_j^* + F_jY_j^*\right] + \left[m_v\ddot{Z}_GZ_G^* + m_vgZ_G^* + m_v\ddot{X}_vX_v^* + m_v\ddot{Y}_vY_v^* + I_{\theta_v}\ddot{\theta}_v\theta_v^* + I_{\alpha_v}\ddot{\alpha}_v\alpha_v^*\right] = 0 \qquad (6)$$

where the different terms are defined like follows:

{ P_i, P'_j } are strengths in the suspensions and in the tires

{ F_j } are the strengths of rubbing of the contact of the tires on the surface

{ m_j } are the masses of the wheels and axles

m_v, I_{θ_v} are the mass and the moment of inertia of the mass of the body of the vehicle

g, the acceleration of the weight

$\Delta_i, \Delta_i^*, \Delta_j, \Delta_j^*$ are respectively the relative displacements and the shifting relative virtual of the suspensions and tires

{ $X_j, X_j^*, Y_j, Y_j^*, Z_j, Z_j^*$ }, { $X_v, X_v^*, Y_v, Y_v^*, Z_v, Z_v^*$ } are respectively the displacements and the shifting virtual of the tires , the displacements and the shifting virtual of the body according to the axes of the reference mark (0x,y,z) θ_v, θ_v^*, α_v, α_v^* are respectively the rotation and the rotation virtual of the body of the vehicle according to the axis 0y and to the axis 0x

While developing the equation (6) and while rejecting the trivial solution for which all displacements, speeds and accelerations are null, one can arrange the equations into the following shape [2]:

$$[M_v]\{\ddot{Z}\}+[C_v]\{\dot{Z}\}+[K_v]\{Z\}=\{Fg\}+\{F^{int}\} \tag{7}$$

where

[Mv] designates the matrix massages, [C_v] designates the matrix of amortization, [K_v] designates the rigidity and {F}= {F_g} + {F^{int}} the vector of load

The model mathematical developed previously is taken in account in the expression of the load:

Vector load

$$\begin{cases} F_1 = m_1 g + k_{p1}\, r(x_1,y_1) + C_{p1}\left(\left(\frac{\partial r}{\partial x}\right)_{x=x_1}\dot{x}_1 + \left(\frac{\partial r}{\partial y}\right)_{y=y_1}\dot{y}_1\right) \\ F_2 = -m_2 g + k_{p2}\, r(x_2,y_2) + C_{p2}\left(\left(\frac{\partial r}{\partial x}\right)_{x=x_2}\dot{x}_1 + \left(\frac{\partial r}{\partial y}\right)_{y=y_2}\dot{y}_1\right) \\ F_3 = -m_3 g + k_{p3}\, r(x_3,y_3) + C_{p3}\left(\left(\frac{\partial r}{\partial x}\right)_{x=x_3}\dot{x}_1 + \left(\frac{\partial r}{\partial y}\right)_{y=y_3}\dot{y}_1\right) \\ F_4 = -m_4 g + k_{p4}\, r(x_4,y_4) + C_{p4}\left(\left(\frac{\partial r}{\partial x}\right)_{x=x_4}\dot{x}_1 + \left(\frac{\partial r}{\partial y}\right)_{y=y_4}\dot{y}_1\right) \\ F_5 = -a_1 m_v g + m_v \left(\frac{h}{S_1}\ddot{x}_1 + \frac{h}{S_2}\ddot{y}_1\right) \\ F_6 = -(a_1 - a_4) m_v g + m_v \left(\frac{h}{S_1}\ddot{x}_1 + \frac{h}{S_2}\ddot{y}_1\right) \\ F_7 = -a_4 m_v g \end{cases} \tag{8}$$

Advancement of the vehicle

The car moves according to the x axis: $x_i = \frac{1}{2}gt^2 + \upsilon_{i0}t + x_{i0}$ (9)

then speed according to the x axis: $\dot{x}_i = gt + \upsilon_{i0}$ (10)

and acceleration according to the x axis: $\ddot{x}_i = g$ (11)

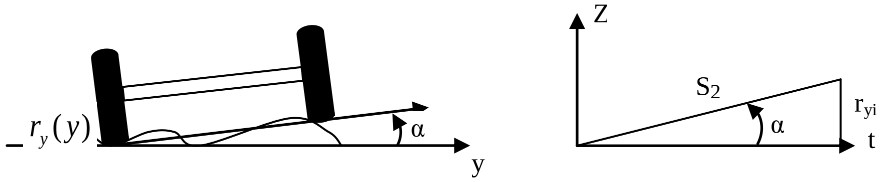

Figure 7: Slant of vehicle according to the y axis

The angular speed $\omega_i = \frac{d\alpha_i}{dt} = \dot{\alpha}_i$ (12)

and $v_{yi} = \omega_i S_2$, $\sin(\alpha_i) = \frac{r_{yi}}{S_2}$ (13)

then the component of speed according to y is $\dot{y}_i = \omega_i S_2$ (14)

and the component of acceleration according to y is $\ddot{y}_i = \dot{\omega}_i S_2$ (15)

3. CONCLUSION

The modelling in 2D the state of roughness of the roads using a model mathematical r(x,y) permits to calculate the loads applied to the vehicle, using “wheels-shod” interactions

The following calculations of simulations are using:

- the decomposed expression of the function r(x,y) presented as:

  $r(x,y)=r_X(x) + r_Y(y)$

- the discretisation of the surface of the pavement by the method of the finite differences.

This type of approach drives to a description of the state of dynamic stability of the “vehicle- shod” system by discrete variables that are defined to the nodes of the surface discredited.

In the term of load {F}:

$\dot{r}_{Xi}$ represent the contribution of the shod to the variation of the speed of the vehicle as linear component (effect on the shock absorber), $\dot{r}_{Yi}$ represent the contribution as angular component (effect on the roll)

An extension of the applications of the proposed method is foreseeable for other mathematical 1D model (polynomial, ect…)

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References

[1] Feng Tyan and Yu-Fen Hong , Shun-Hsu Tu and Wes S. Jeng, “Generation of Random Road Profiles”, CSME-1373, 2006

[2] Akoussah K. Esenam, Fafard Mario et Henchi Kamel, “Détermination du facteur d'amplification, dynamique des ponts par éléments finis”, Université du Bénin ; Ecole Nationale Supérieure d'Ingénieurs, B.P. 1515 Lomé- TOGO, Université Laval ; Faculté des Sciences et Génies, Département de Génie Civil, Québec, Canada, GlK 7P4, 2000

[3] Kamel Henchi, Mario Fafard et Martin Talbot, “Analyse dynamique de l’interaction pont véhicules pour les ponts routiers”, Can. J. Civ. Eng. Vol. 25, 1998.

[4] Olegas Prentkoviskis, Marijonas Bogdevicius, “Dynamics of motor vehicle taking into consideration the interaction of wheels and road pavement surface”, Transport, Vol XVII, No 6, 244-253, 2002

[5] González, Arturo; O'brien, Eugene J.; Li, Yingyan; Cashell, K., “The use of vehicle acceleration measurements to estimate road Roughness ”, Vehicle System Dynamics, 46 (6): 483-499, http://dx.doi.org/10.1080/00423110701485050, http://hdl.handle.net/10197/2382. 2008-06

[6] G. Lombaert, G. De Grande, “Numerical modelling and in situ measurements of free field traffic induced vibrations”, 4th PIARC International Symposium on Surface Characteristics of Roads and Airfields, Nantes, France, May 2000

[7] A.M. TRON, T. GORLOV, “Photocathode roughness impact on photogun beam characteristics”, Moscow, Russia, Proceedings of EPAC, Edinburgh, Scotland, 2006

[8] Arturo González, “Vehicle-Bridge Dynamic Interaction Using Finite Element Modelling”, Sciyo, http://sciyo.com/articles/show/title/vehicle-bridge-dynamicinteraction-using-finite-element-modellingUniversity, http://hdl.handle.net/10197/2527, 2010

[9] Broquet Claude, “ Comportement dynamique des dalles de roulement des ponts en béton sollicités par le trafic routier ”, Thèse, Département de Génie Civil, Ecole Polytechnique de Lausanne, 1999