Heavy quarkonium spectrum and width in a weakly-coupled quark-gluon plasma

Jacopo Ghiglieri
TU München & Excellence Cluster Universe
in collaboration with
N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo

Outline

- Quarkonium in media: the EFT approach

- Non-relativistic EFTs for quarkonium

- Spectrum and width below the dissociation temperature

- Conclusions and outlook

Quarkonium in media: the long story (made short)

J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

T. MATSUI

*Center for Theoretical Physics, Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA*

and

H. SATZ

*Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany
and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark–gluon plasma formation.

Quarkonium in media: the long story (made short)

See talks by M. Laine, A. Vairo, N. Leonardo and I. Selyuzhenkov

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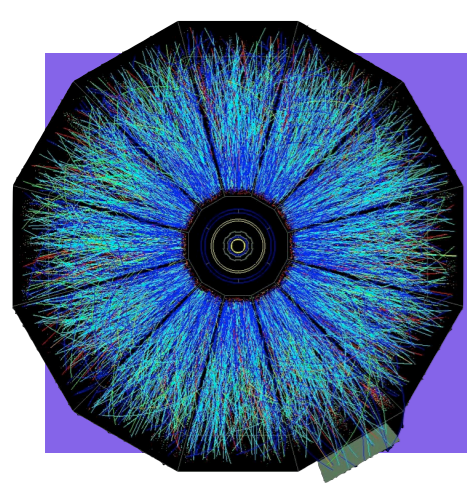*Cambridge, MA 02139, USA*

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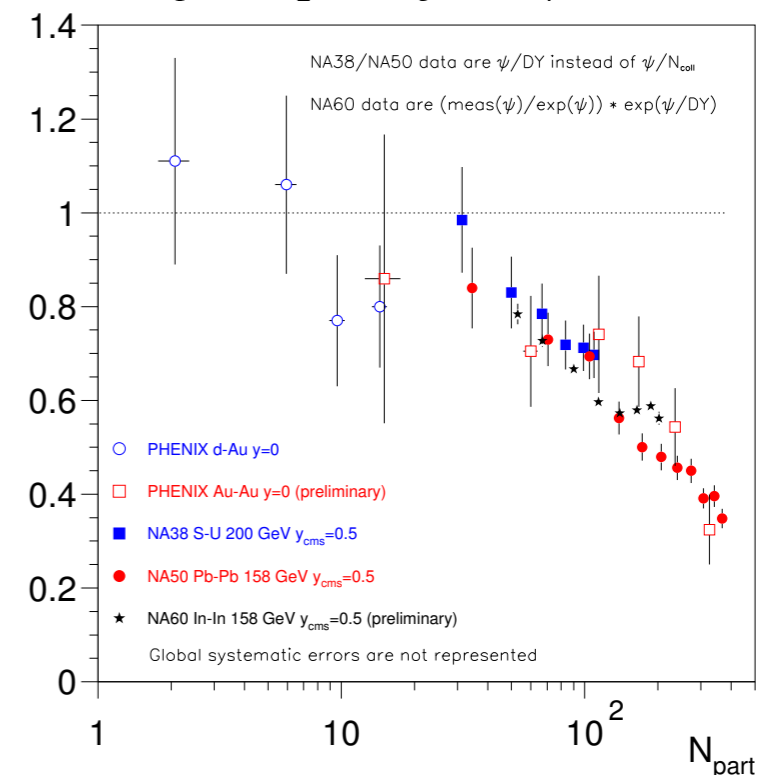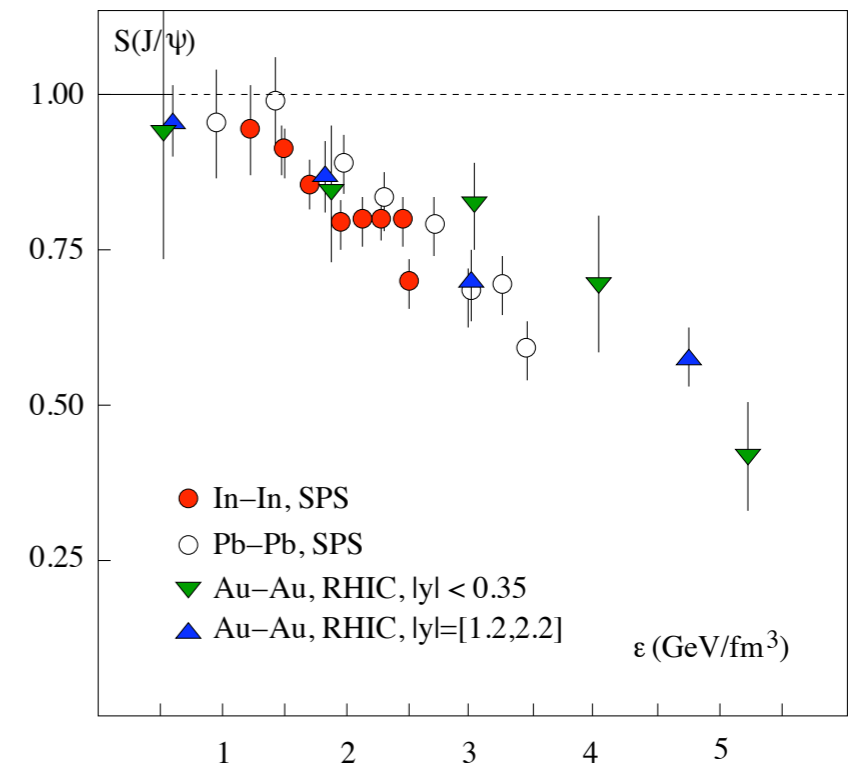*and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

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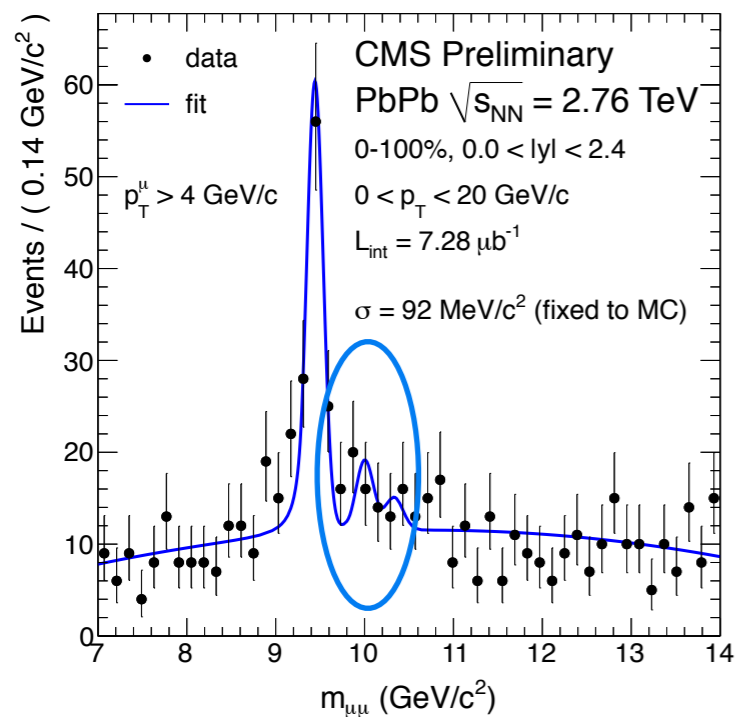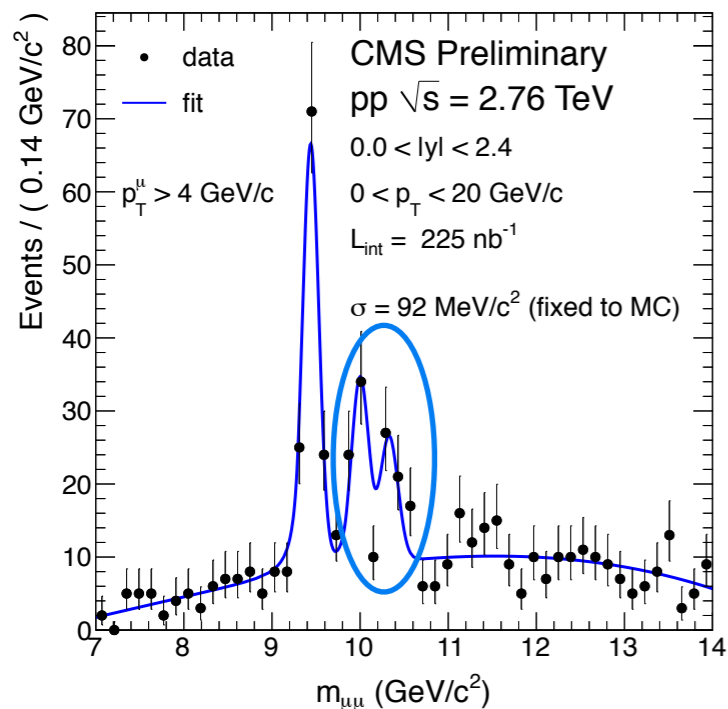
Quarkonium suppression

- Experimental data show a suppression pattern

- A good understanding of suppression requires understanding of

  - In-medium production and cold nuclear matter effects

  - In-medium bound-state dynamics

  - Recombination effects

  - ...

$S(J/\psi)$ vs ε (GeV/fm^3)

- In–In, SPS
- Pb–Pb, SPS
- Au–Au, RHIC, $|y| < 0.35$
- Au–Au, RHIC, $|y|=[1.2,2.2]$

NA38/NA50 data are ψ/DY instead of ψ/N_{coll}

NA60 data are $(meas(\psi)/exp(\psi)) * exp(\psi/DY)$

- PHENIX d-Au y=0
- PHENIX Au-Au y=0 (preliminary)
- NA38 S-U 200 GeV y_{cms}=0.5
- NA50 Pb-Pb 158 GeV y_{cms}=0.5
- NA60 In-In 158 GeV y_{cms}=0.5 (preliminary)

Global systematic errors are not represented

N_{part}

Quarkonium suppression: hot news
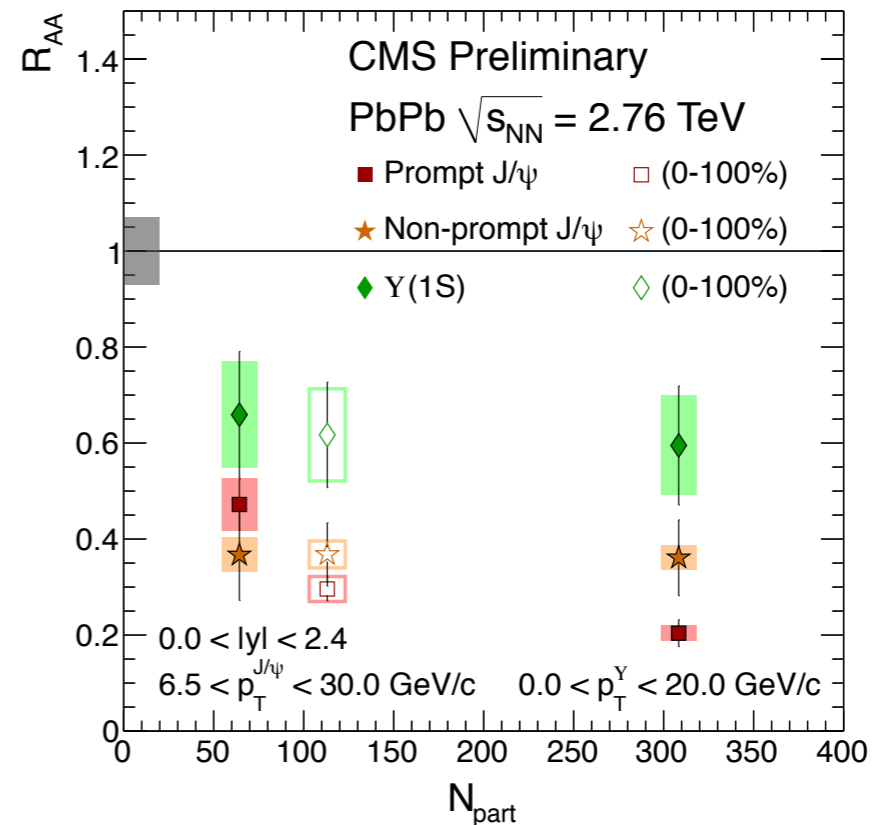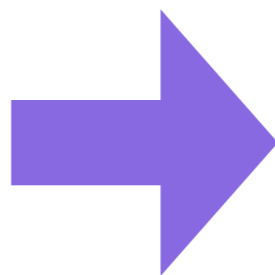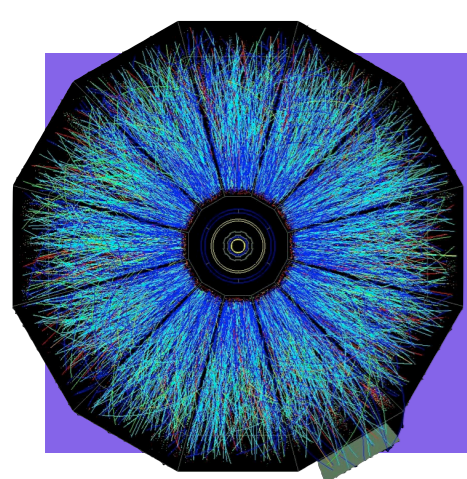
- Significant suppression of the ϒ(2S) and ϒ(3S)

CMS, **1105.4894** and CMS-PAS-HIN-10-006 (2011)
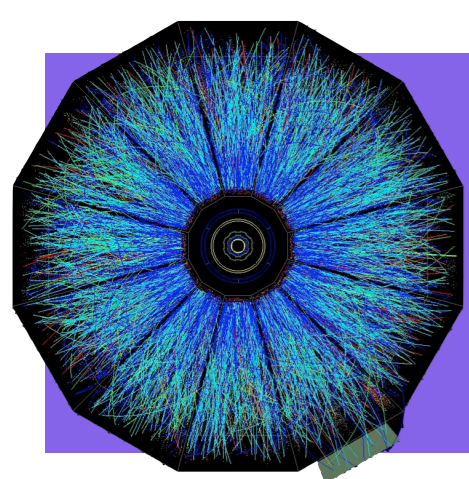
Quarkonium suppression

- Proposed in 1986 as a probe and "thermometer" of the medium produced by the collision
  Matsui Satz **PLB178** (1986)
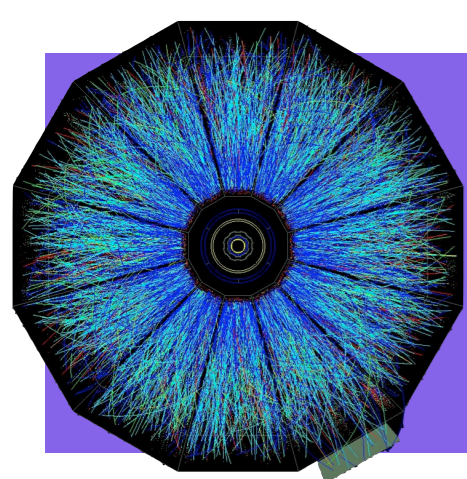
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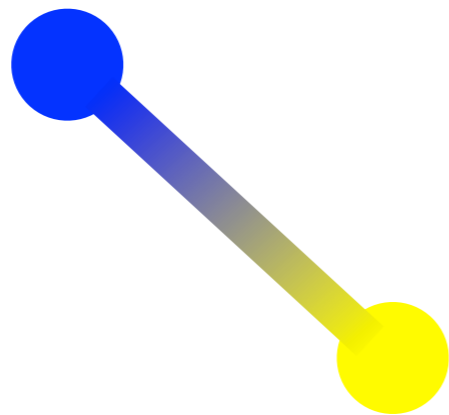
$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

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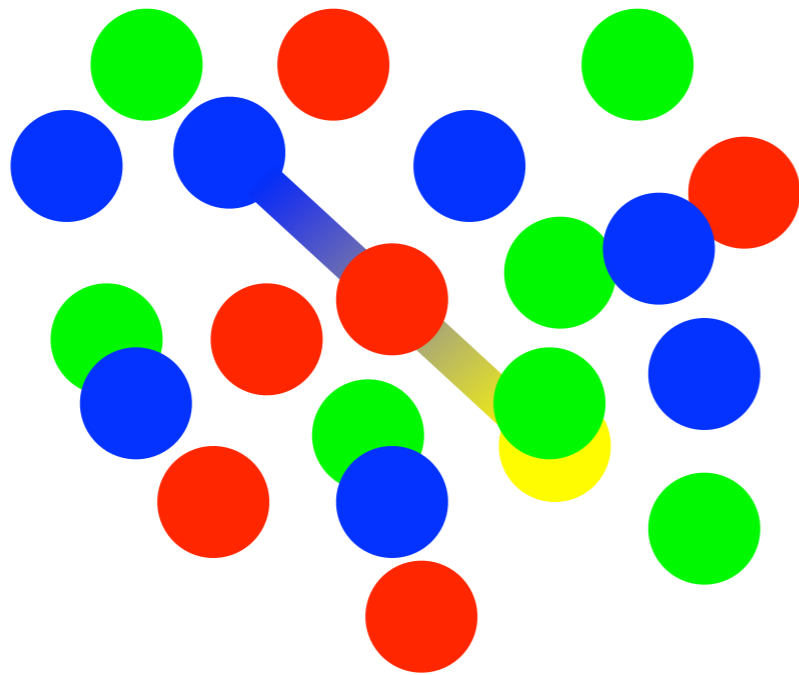
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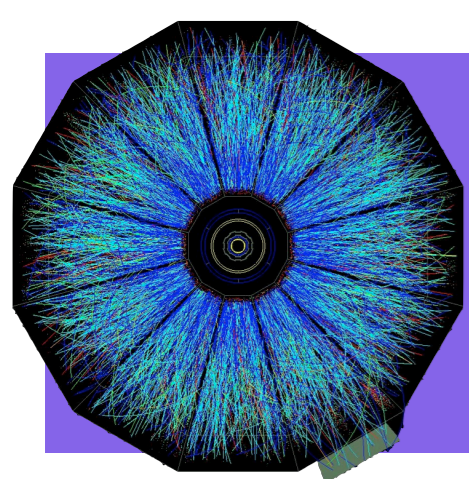$$r \sim \frac{1}{m_D} \longrightarrow \text{Bound state dissolves}$$
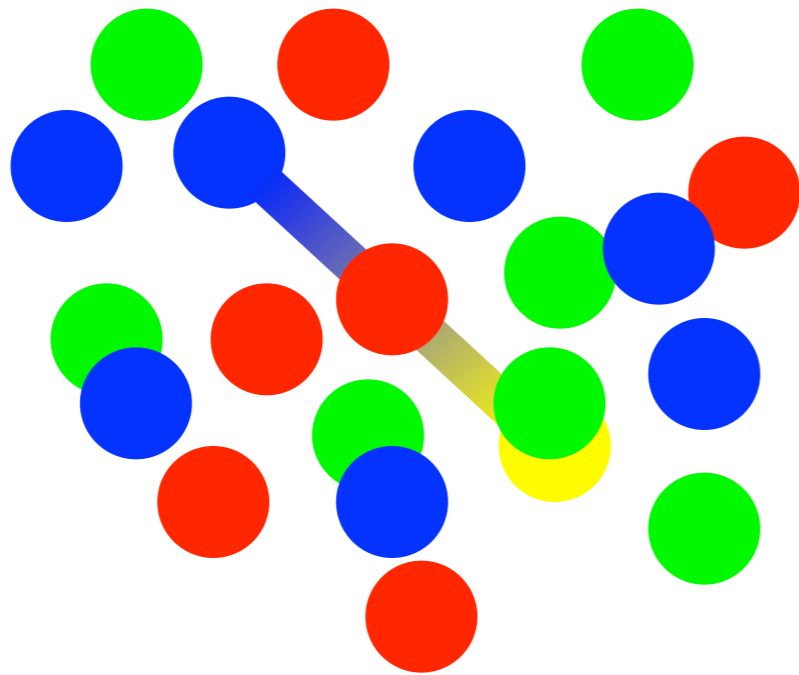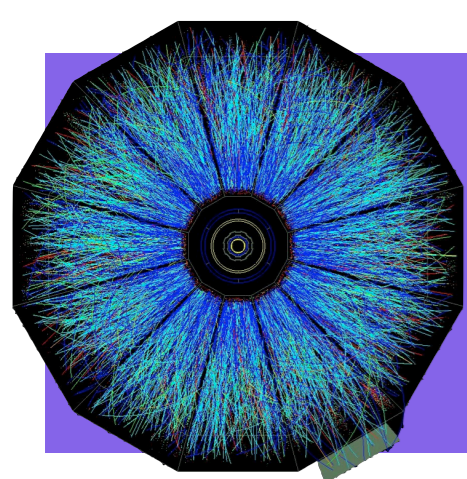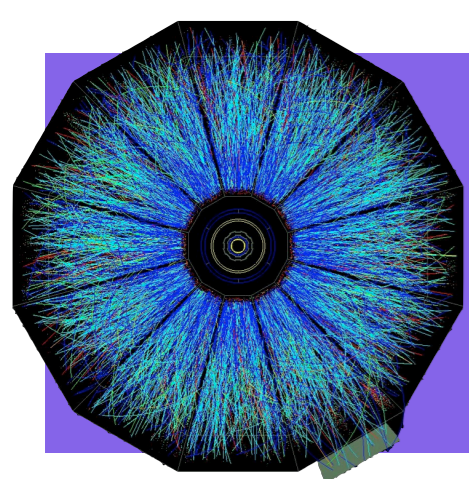
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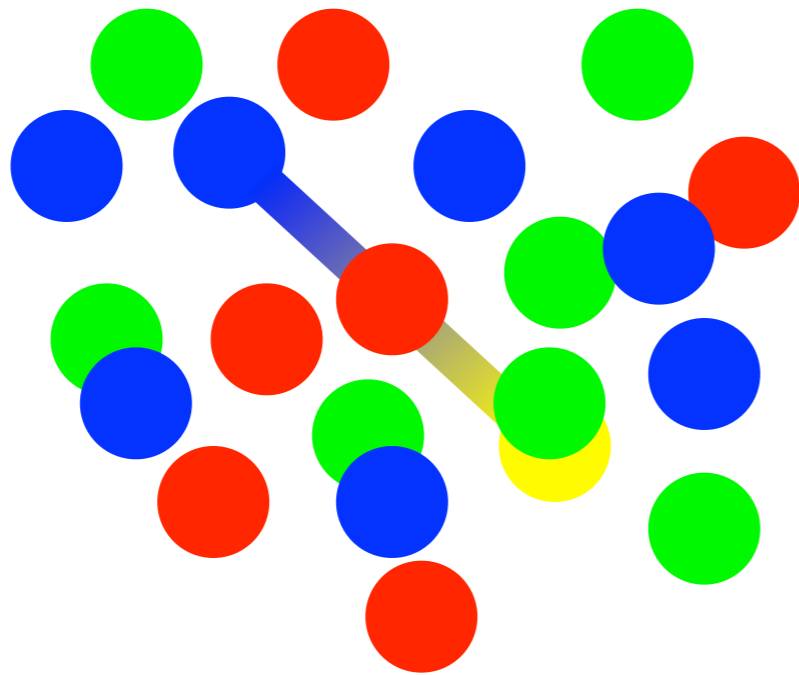
$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \longrightarrow \text{Bound state dissolves}$$

- Studied with potential models, lattice spectral functions, ...

Potential models

Digal, Petreczky, Satz 01
Wong 05-07
Mannarelli, Rapp 05
Mocsy, Petreczky 05-08
Alberico, Beraudo, Molinari, de Pace 05-08
Cabrera, Rapp 2007
Wong, Crater 07
Dumitru, Guo, Mocsy, Strickland 09
Rapp, Riek 10
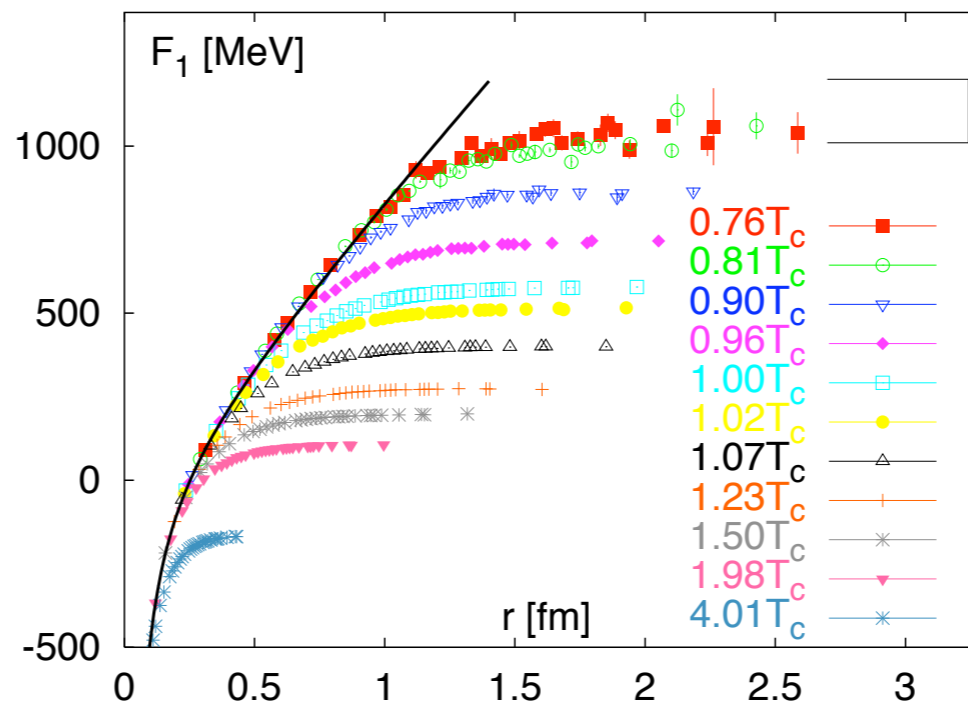
Kaczmarek Zantow
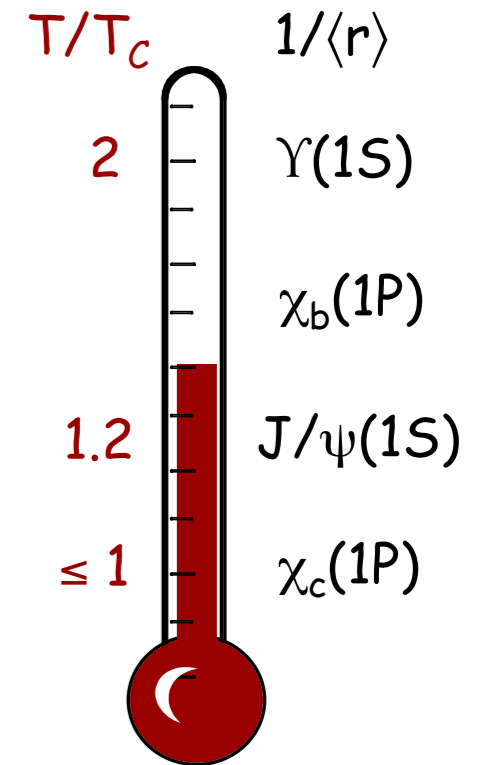
- There is qualitative agreement on this picture of sequential melting

- However there are still issues with the definition of an in-medium potential

The EFT approach

- Perturbative computation of the real-time potential between a static quark and antiquark for *T>>1/r*:

$$V_{\mathrm{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)$$

When $r \sim \dfrac{1}{m_D}$ $\mathrm{Im}V \gg \mathrm{Re}V$

Laine Philipsen Romatschke Tassler **JHEP0703** (2007)

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Escobedo Soto **PRA78** (2008), **PRA82** (2010)

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- EFT approach for *static* quark-antiquark pairs in finite T pNRQCD (Non-Abelian, static)
Brambilla JG Petreczky Vairo **PRD78** (2008)

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Laine Philipsen Romatschke Tassler **JHEP0703** (2007)

- EFT approach for *heavy* quark-antiquark pairs in finite *T*
pNRQCD (Non-Abelian, non-static)
Brambilla Escobedo JG Soto Vairo **JHEP1009** (2010)
Brambilla Escobedo JG Vairo **arXiv:1105.4587**

The EFT approach

- Extend the well-established zero temperature EFT framework for heavy quark-antiquark bound states (NRQCD, pNRQCD) to finite temperature

- Systematic approach with modern, rigorous definition of the potential (real-time matching coefficient)

- Transparent connection with quantum mechanics and the Schrödinger equation picture, which appears as the zeroth-order approximation in the EFT framework

NR EFTs for heavy quarkonium: the theoretical landscape

T=0 NR bound states

- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, momentum and energy scales

$$m$$

$$mv \sim m\alpha_{\mathrm{s}} \sim \left\langle \frac{1}{r} \right\rangle$$

$$mv^2 \sim m\alpha_{\mathrm{s}}^2 \sim E$$
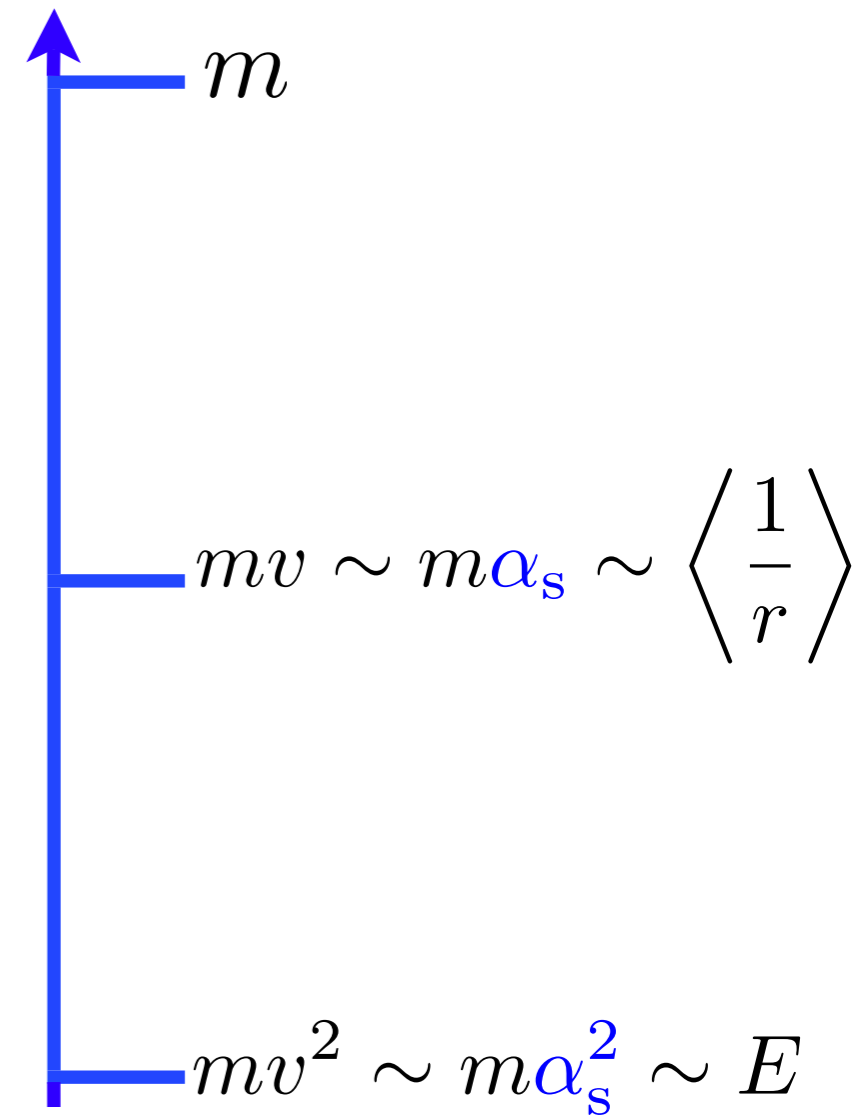
T=0 NR bound states

- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, momentum and energy scales

- One can then expand observables in terms of the ratio of the scales and construct a *hierarchy of EFTs* that are equivalent to QCD order-by-order in the expansion parameter

$$m$$

$$mv \sim m\alpha_{\mathrm{s}} \sim \left\langle \frac{1}{r} \right\rangle$$

$$mv^2 \sim m\alpha_{\mathrm{s}}^2 \sim E$$

T=0 Scales

m

$m\alpha_\mathrm{s} \sim \left\langle \dfrac{1}{r} \right\rangle$

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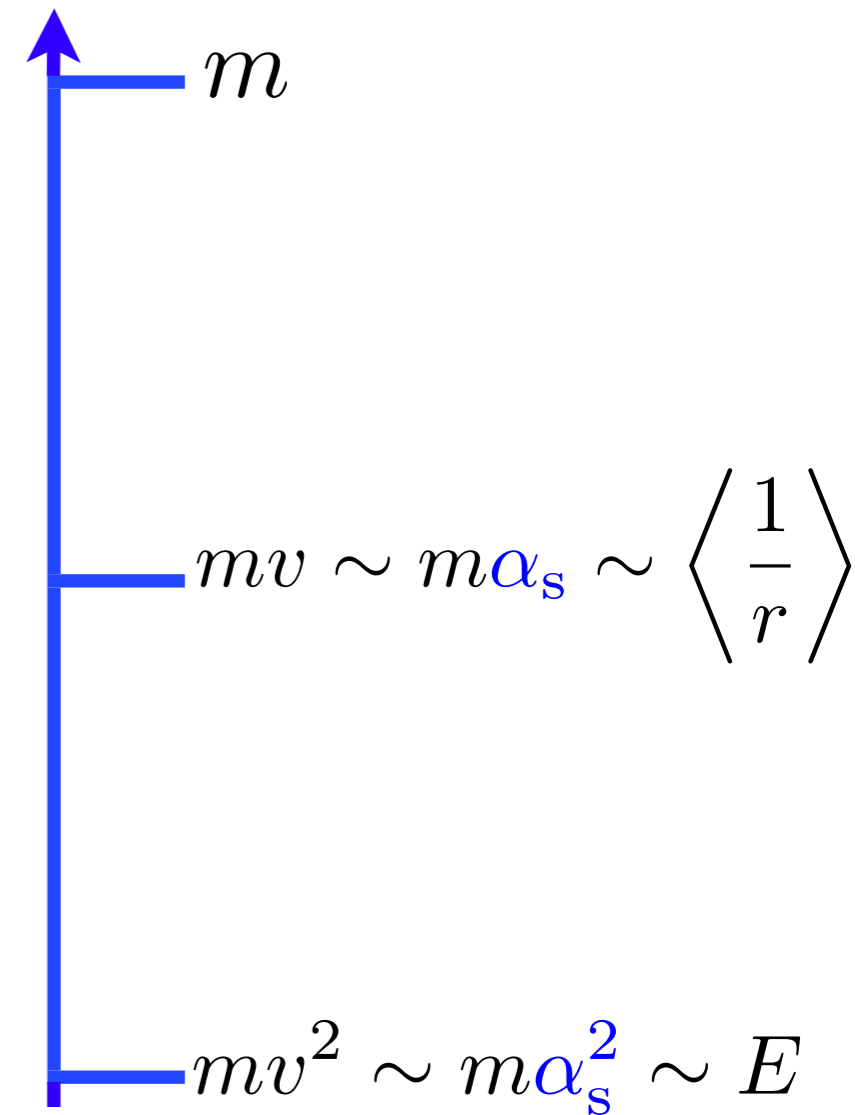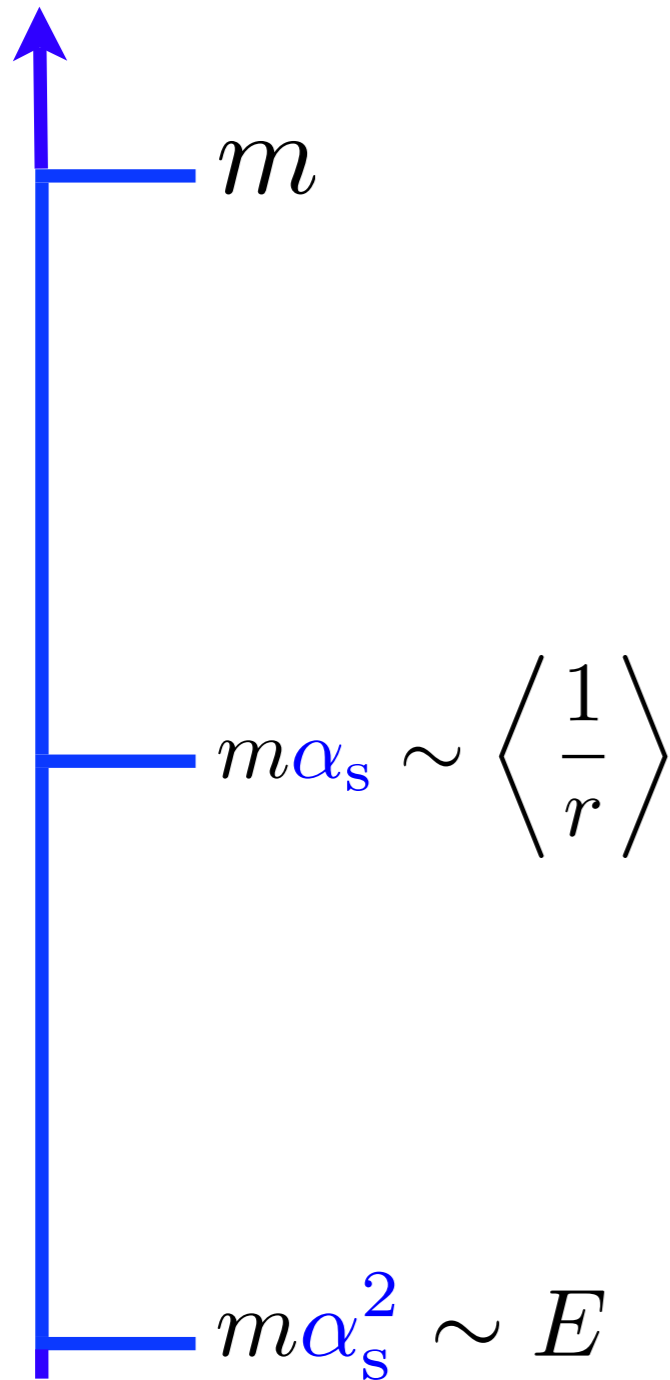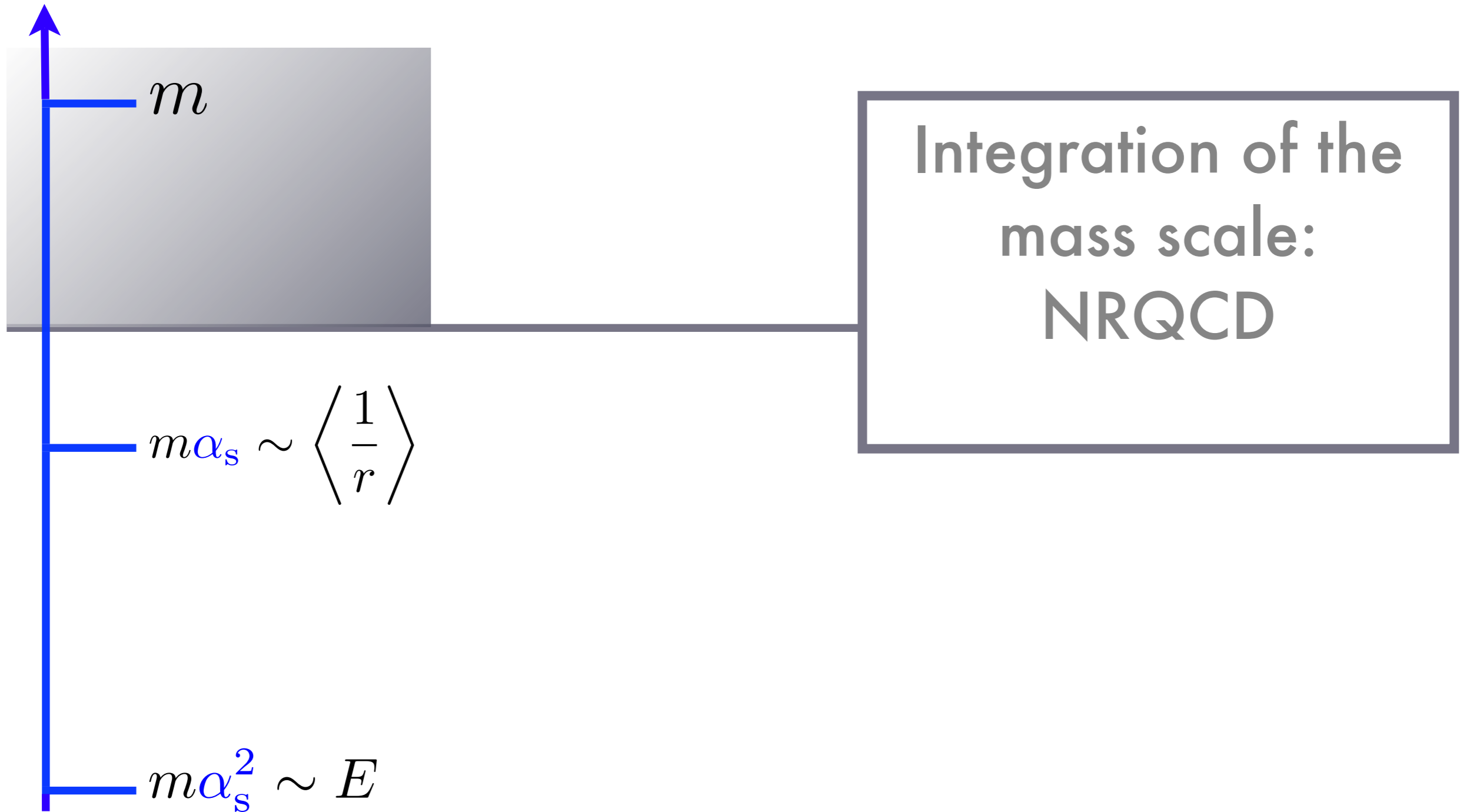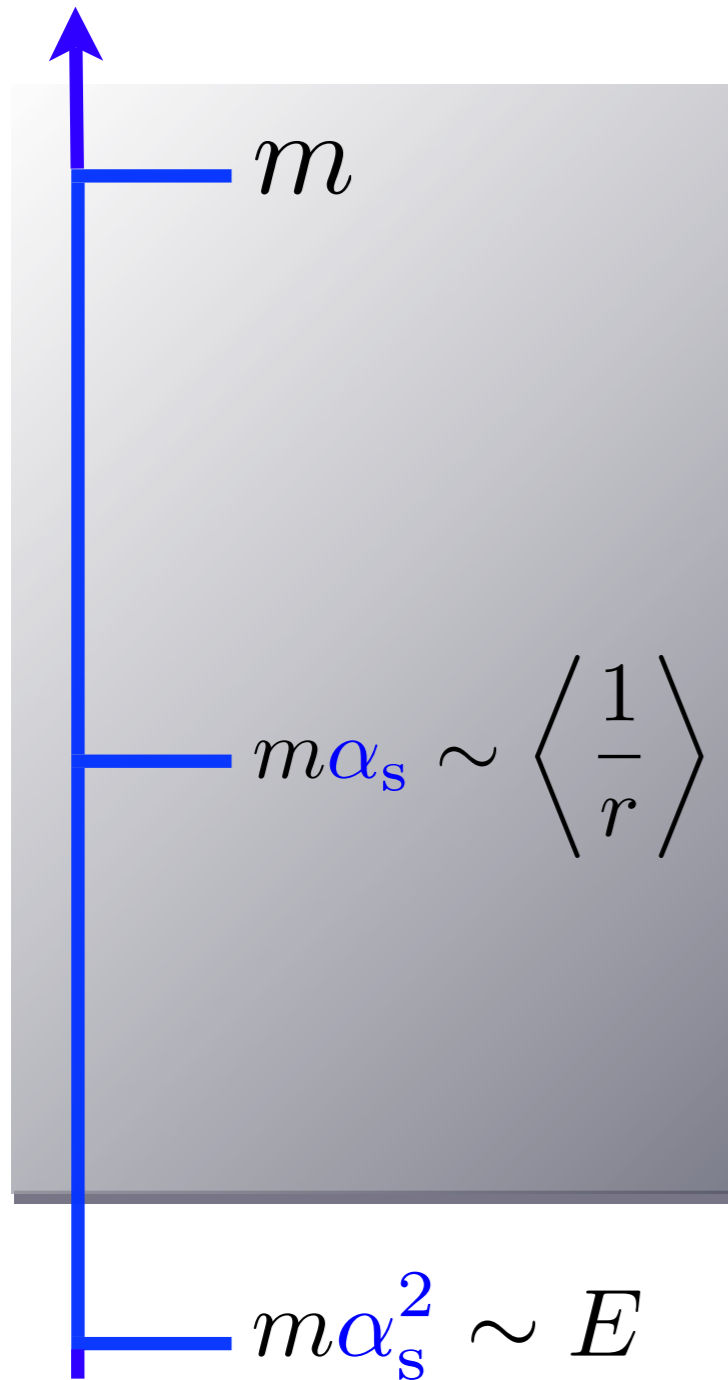
Integration of the soft (momentum transfer) scale: pNRQCD

T=0 Scales

m

$m\alpha_{\mathrm{s}} \sim \left\langle \dfrac{1}{r} \right\rangle$

Λ_{QCD}

$m\alpha_{\mathrm{s}}^2 \sim E$
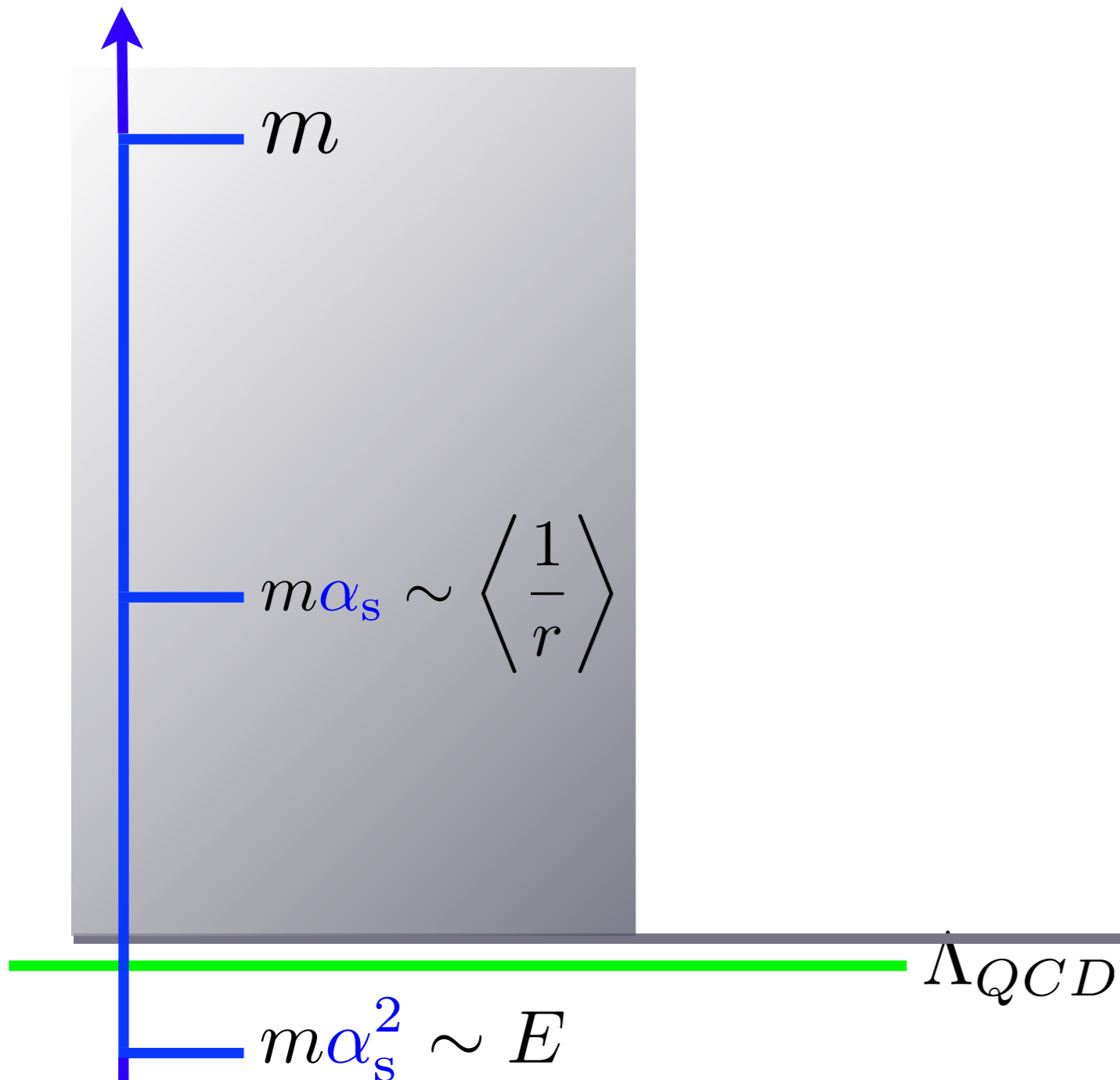
Integration of the soft (momentum transfer) scale: pNRQCD

Thermodynamical scales

- The thermal medium introduces new scales in the physical problem

  - The temperature

  - The electric screening scale (Debye mass)

  - The magnetic screening scale (magnetic mass)

- In the weak coupling assumption these scales develop a hierarchy

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T

$gT \sim m_D$

$g^2 T \sim m_m$
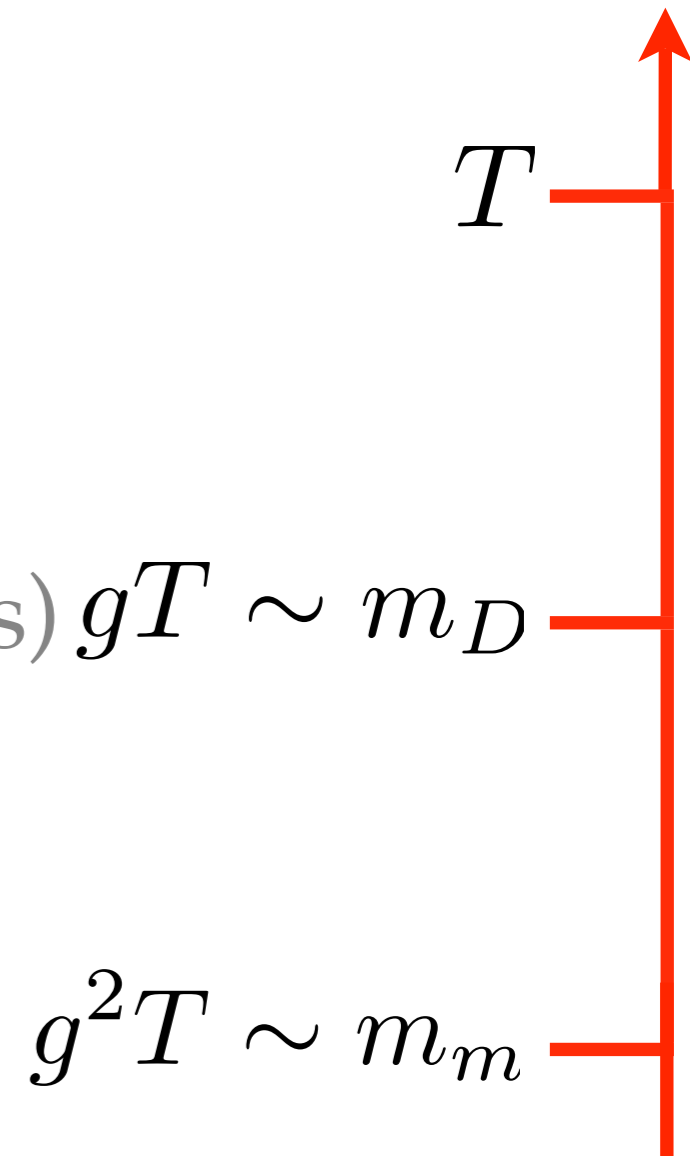
Scales of the problem
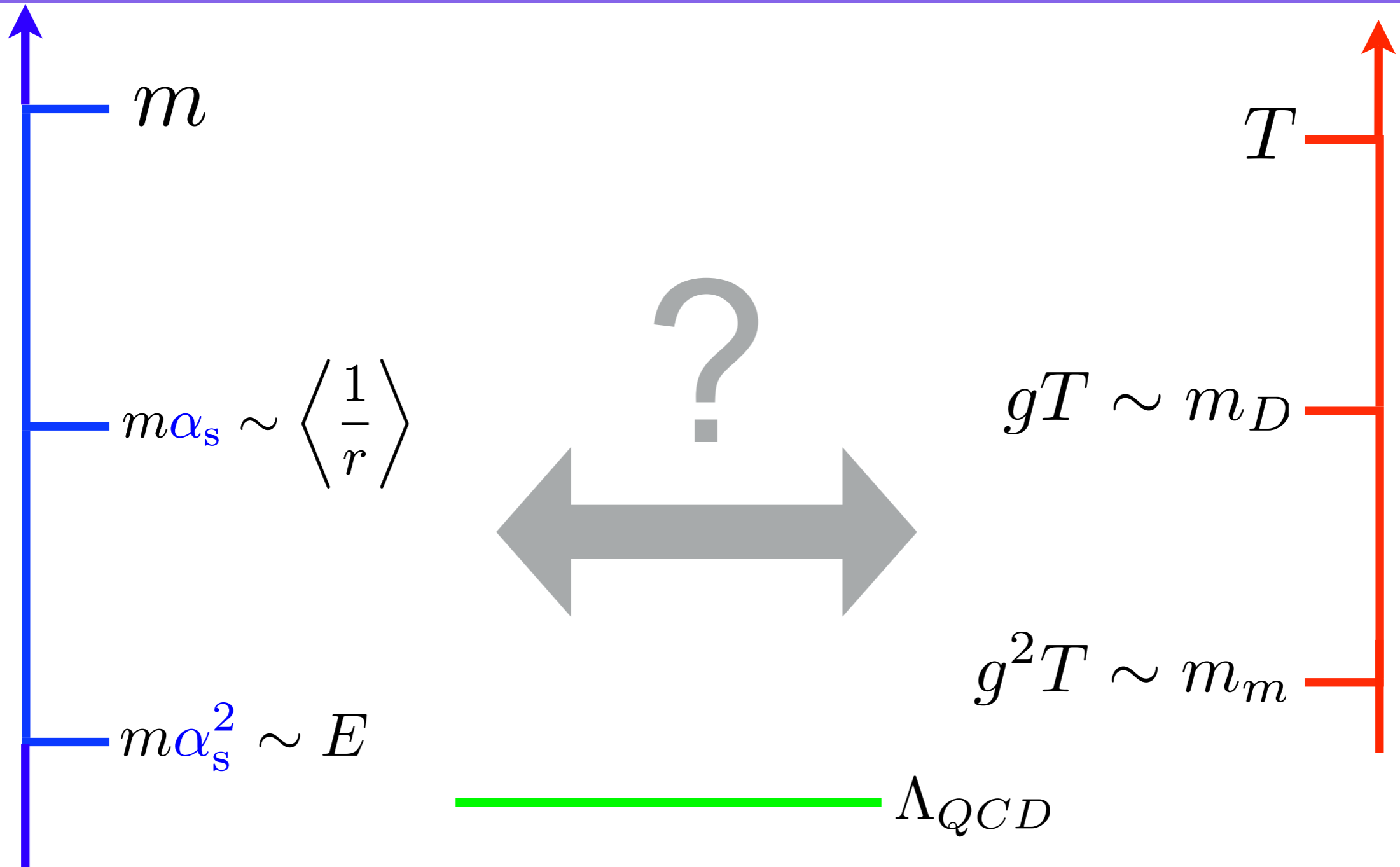
- In our initial work various possibilities have been studied, from $T \ll E$ to $m \gg T \gg 1/r \sim m_D$

- In the regime $T \gg \dfrac{1}{r} \sim m_D$ the result of Tassler *et al* 2007 is reobtained (also in the Abelian case)

$$V_{\mathrm{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)$$

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- In the more recent works we adopt this hierarchy

$$m \gg m\alpha_{\mathrm{s}} \gg T \gg m\alpha_{\mathrm{s}}^2 \gg m_D$$

- Thermal effects are thus a perturbation to the Coulombic bound state, but they still modify the potential

- Relevance for bottomonium LHC phenomenology

$m_b \approx 5\,\mathrm{GeV} > m_b\alpha_{\mathrm{s}} \approx 1.5\,\mathrm{GeV} > \pi T \approx 1\,\mathrm{GeV} > m\alpha_{\mathrm{s}}^2 \approx 0.5\,\mathrm{GeV} \gtrsim m_D$

- The temperature is below the *dissociation temperature* $T_d \sim m\alpha_{\mathrm{s}}^{\frac{2}{3}}$

m

$m\alpha_{\mathrm{s}}$

T

$m\alpha_{\mathrm{s}}^2$

m_D

Λ_{QCD}

Our goal

m

$m\alpha_{\rm s}$

T

$m\alpha_{\rm s}^2$

m_D

$\Lambda_{\rm QCD}$
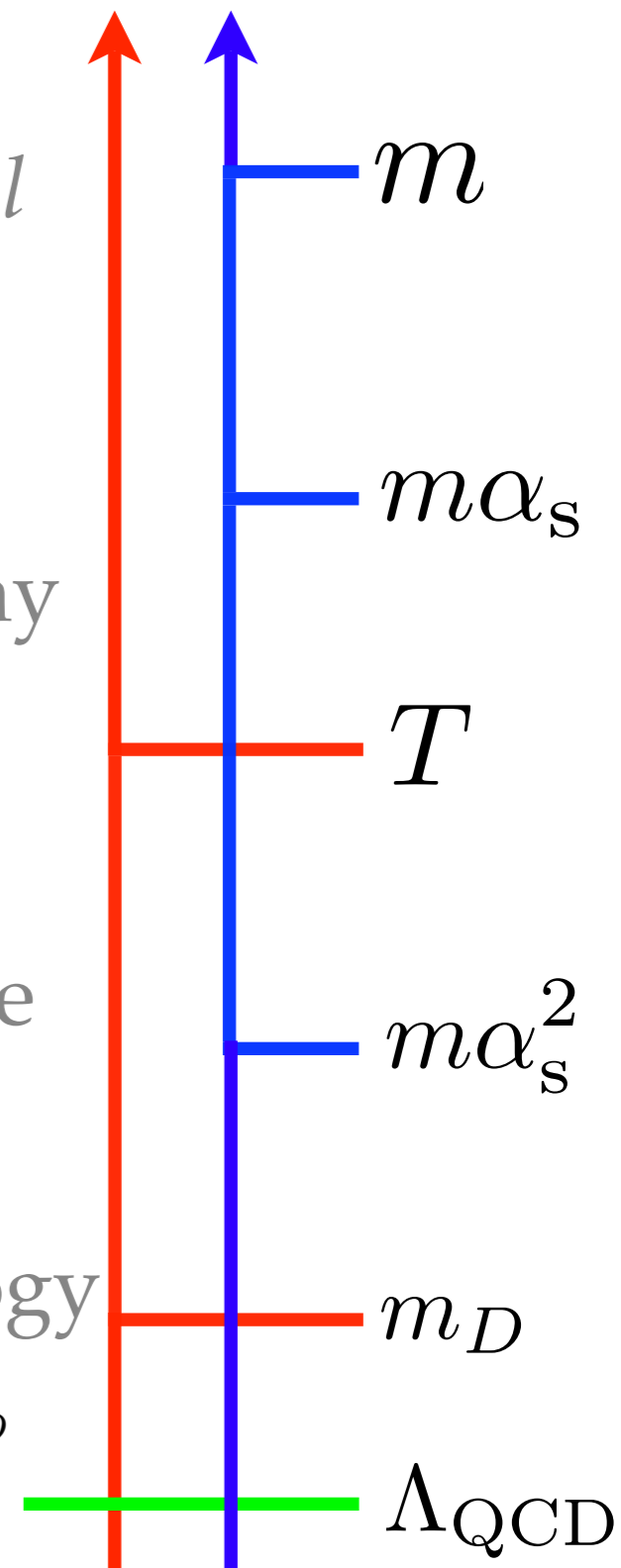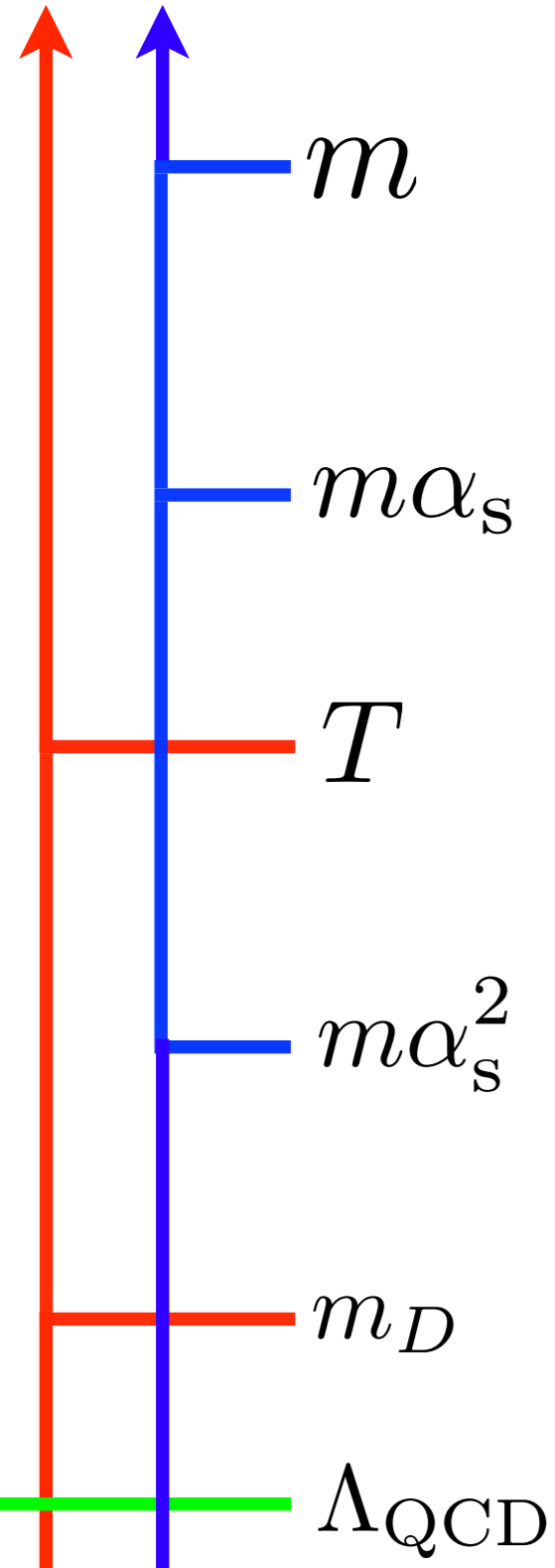
- Our goal is then to compute the spectrum (or thermal corrections to it) and the thermal width in this hierarchy and power counting, up to order $m\alpha_{\rm s}^5$

- This will be achieved by integrating out the heavier scales in succession and creating a series of EFTs

Integrating out m **and** $m\alpha_s$.

Mass scale: NRQCD

m

$m\alpha_{\mathrm{s}}$

T

$m\alpha_{\mathrm{s}}^2$

m_D

Λ_{QCD}
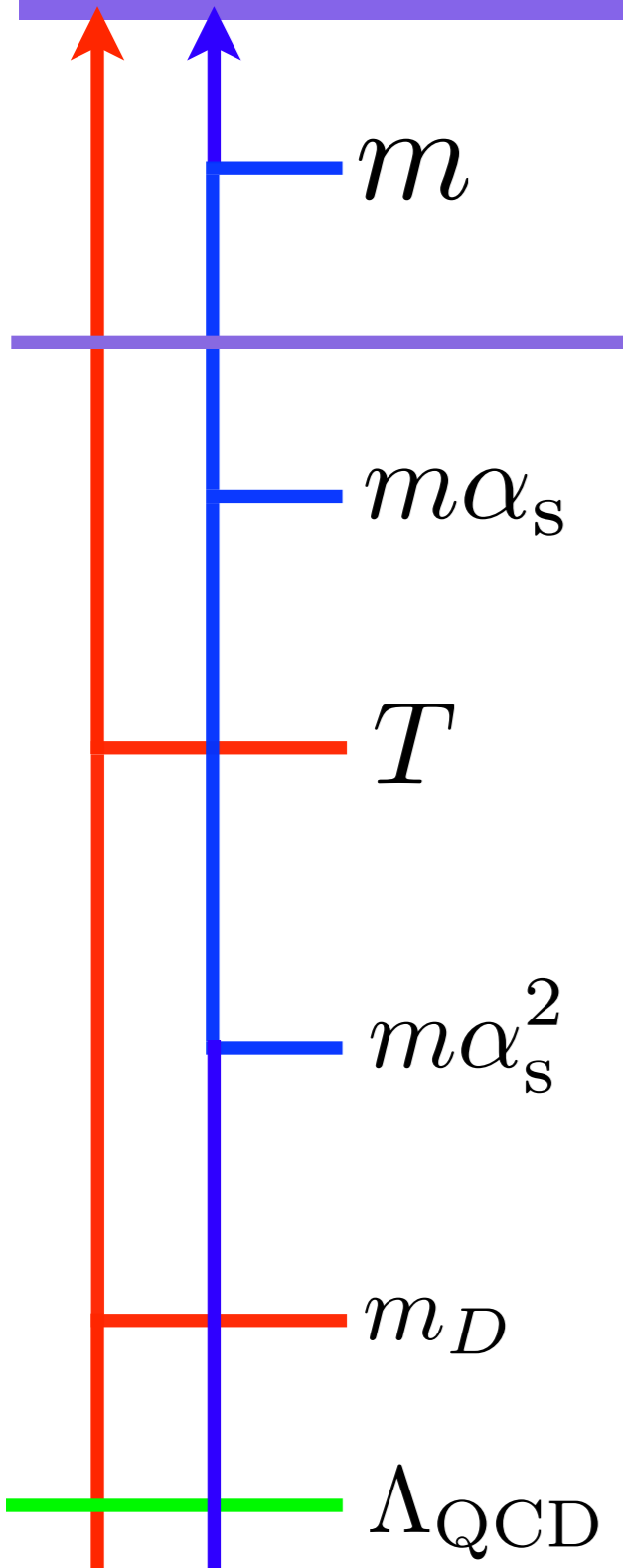
- Smaller scales are put to zero in the matching. This means that the thermal scales do not affect the result of the integration, which yields standard NRQCD

- NRQCD is organized as an expansion in *1/m*

Caswell Lepage **PLB167** (1986)

Bodwin Braaten Lepage **PRD51** (1995)

Mass scale: NRQCD
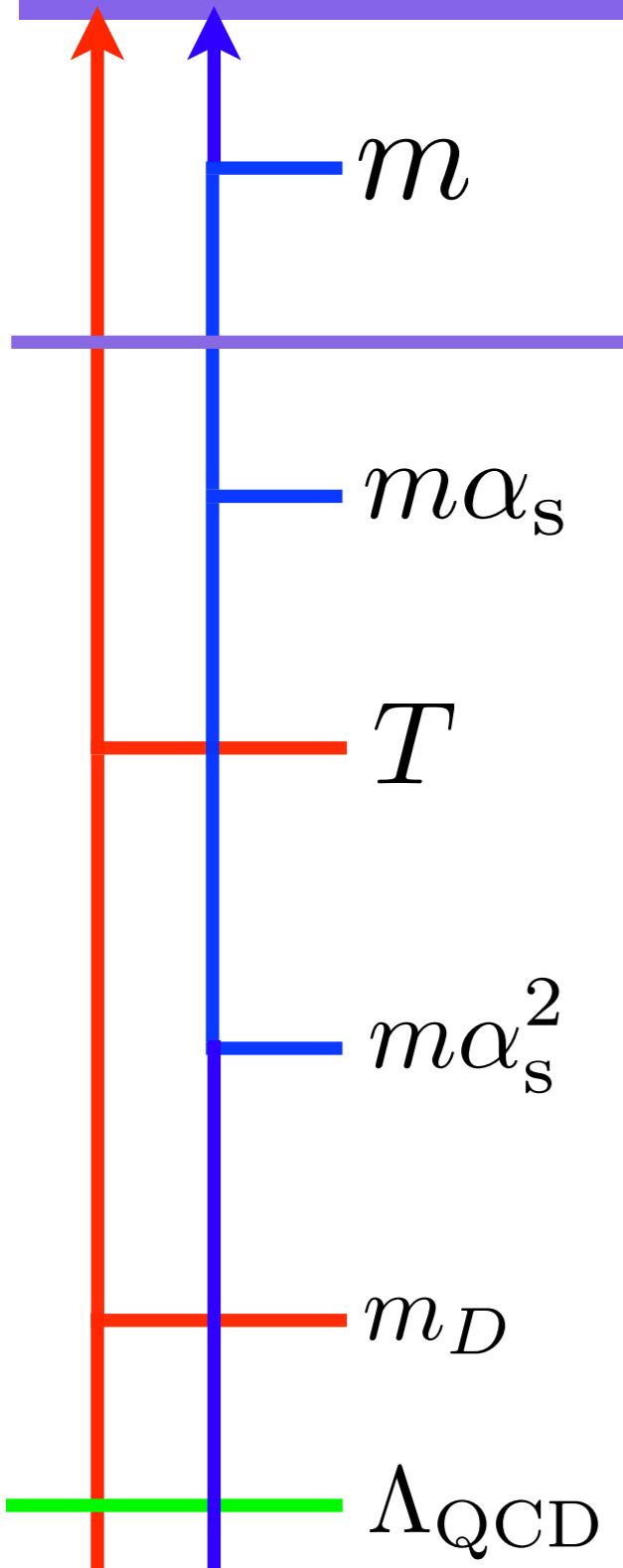
$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{light}} + \psi^\dagger \left(iD_0 + \frac{D^2}{2m} + \dots \right) \psi + \chi^\dagger (iD_0 - \frac{D^2}{2m} + \dots)\chi + \dots$$

m

$m\alpha_{\text{s}}$

T

$m\alpha_{\text{s}}^2$

m_D

Λ_{QCD}
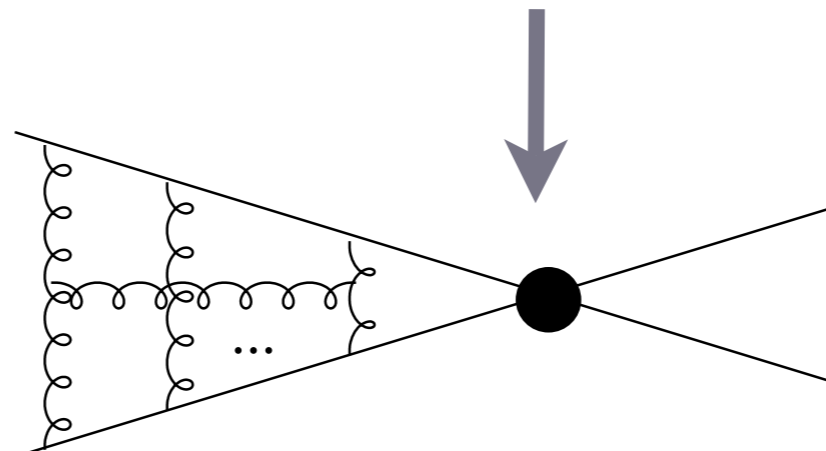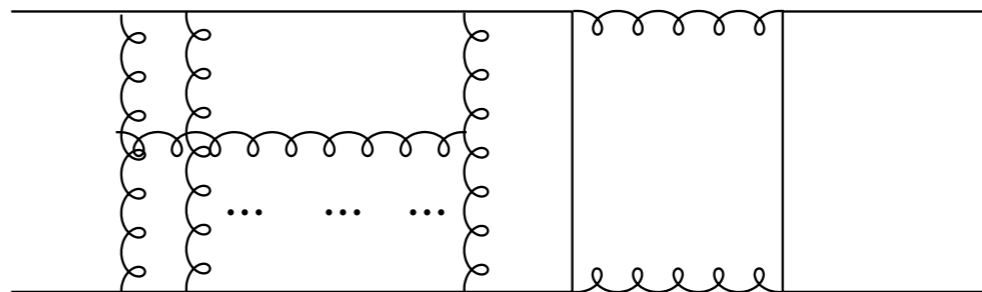
Caswell Lepage **PLB167** (1986)
Bodwin Braaten Lepage **PRD51** (1995)

Relative momentum scale: pNRQCD

$$\frac{1}{E - p^2/m - V(r)}$$
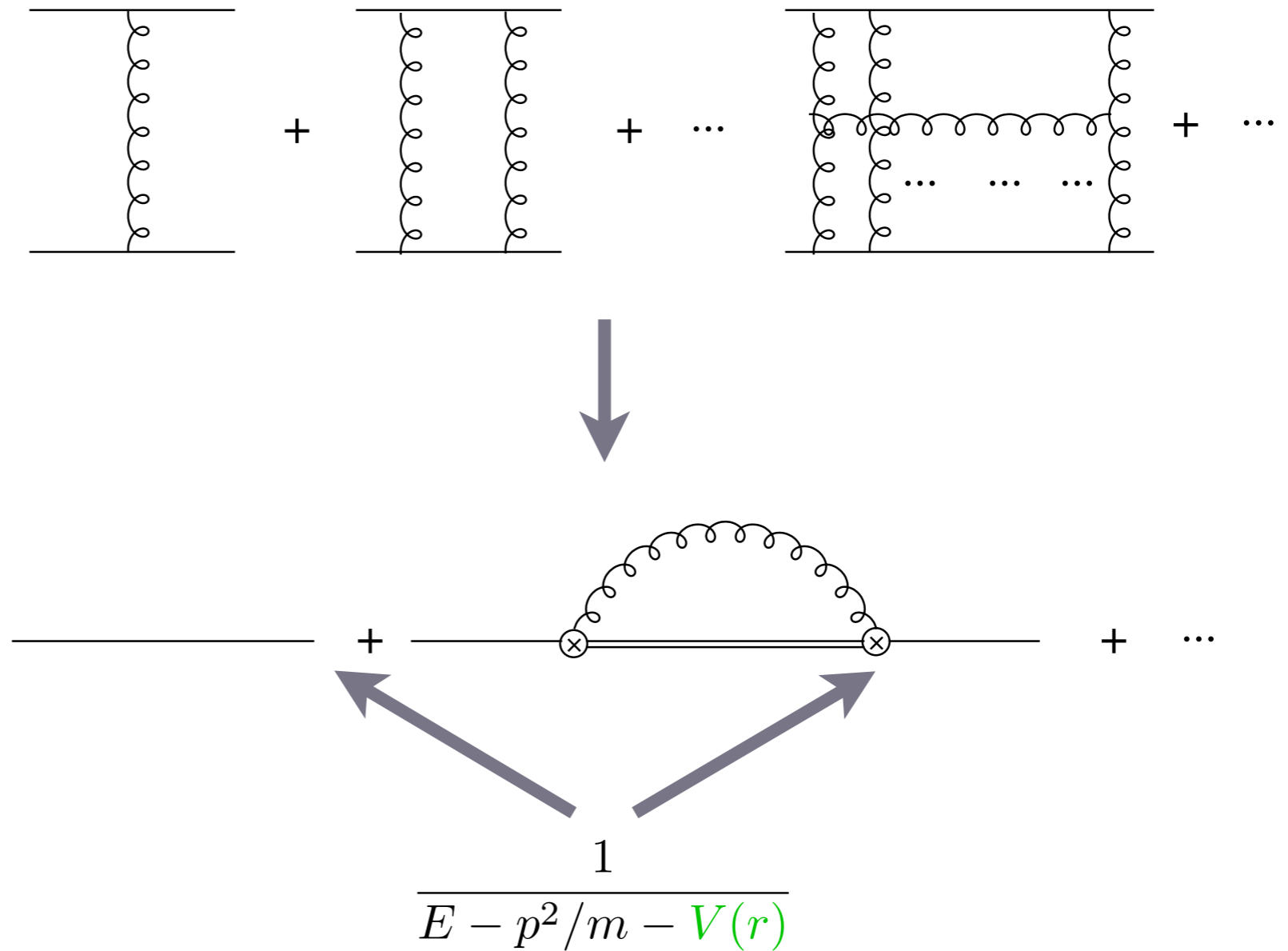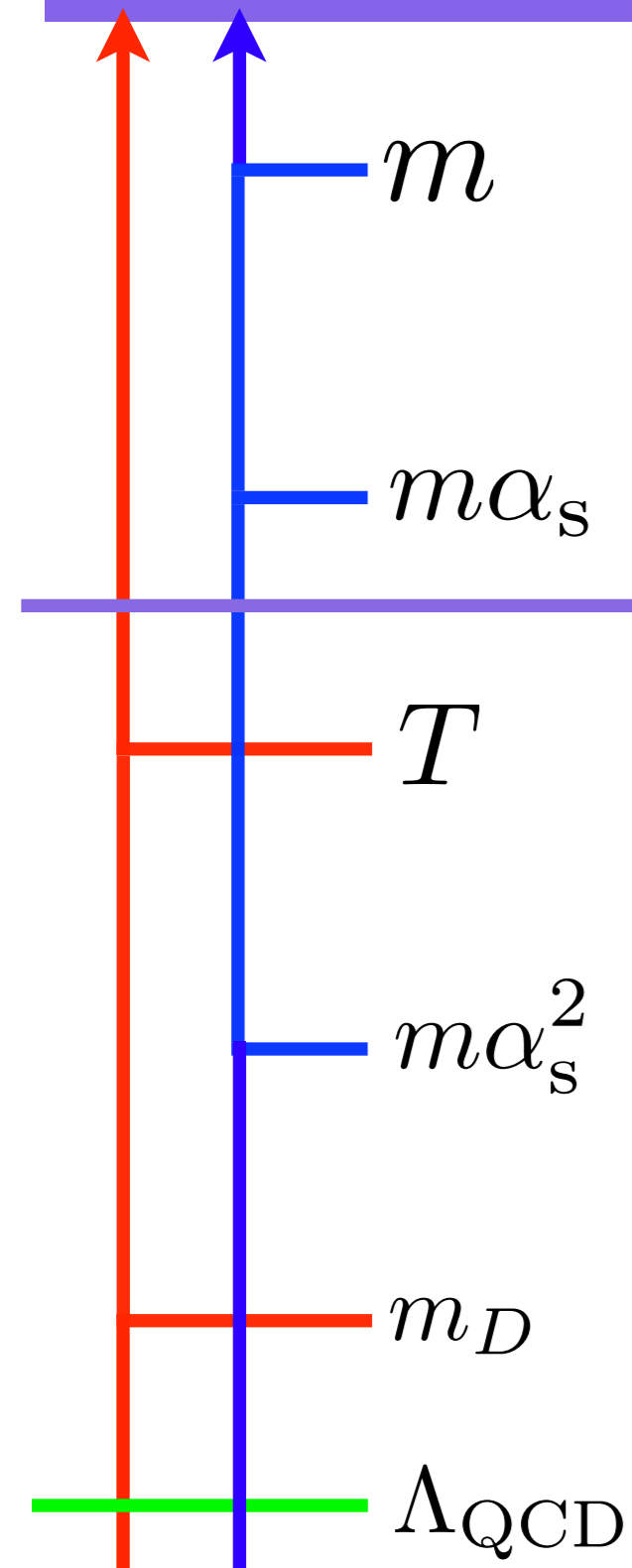
Pineda Soto Nucl. Phys. Proc. Suppl. 64(1998)
Brambilla Pineda Soto Vairo **NPB566** (2000)

Scale markers: m, $m\alpha_{\mathrm{s}}$, T, $m\alpha_{\mathrm{s}}^2$, m_D, Λ_{QCD}

Weakly coupled pNRQCD

$$\mathcal{L}_{\text{pNRQCD}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{light}} + \text{Tr} \left\{ S^\dagger \left[i\partial_0 - H_s \right] S + O^\dagger \left[iD_0 - H_o \right] O \right\}$$

$$+ V_A \, \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} \, S + S^\dagger \mathbf{r} \cdot g\mathbf{E} \, O \right\} + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} \, O + O^\dagger O \mathbf{r} \cdot g\mathbf{E} \right\} + \dots$$

- Degrees of freedom: $Q\overline{Q}$ states with energy $E \sim \Lambda_{QCD}, mv^2$ and momentum $p \lesssim mv$
  Singlet and octet color states

- US gluons with energy/momentum $\lesssim mv$

- Expansion in α_s, $\frac{1}{m}$ and r

- Potentials are Wilson coefficients, receive contributions from all scales higher than the energy

Contribution to the spectrum

- Power-counting in the singlet Hamiltonian

$$H_s = \frac{\mathbf{p}^2}{m} - C_F \frac{\alpha_\mathrm{s}}{r} + \mathcal{O}(m\alpha_\mathrm{s}^3)$$

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- Solve the singlet EOM (Schrödinger Eq.) with this Hamiltonian and obtain the Coulomb levels

$$E_n = -\frac{mC_F^2\alpha_{\mathrm{s}}^2}{4n^2} = \frac{1}{ma_0^2 n^2}\,, \qquad a_0 \equiv \frac{2}{mC_F\alpha_{\mathrm{s}}}$$

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- Treat further terms in the expansion in QM perturbation theory, obtaining the $m\alpha_{\mathrm{s}}^3 - m\alpha_{\mathrm{s}}^5$ contributions to the spectrum from this scale

Brambilla Pineda Soto Vairo **PLB470** (1999) Kniehl Penin **NPB563** (1999) Kniehl Penin Smirnov Steinhauser **NPB635** (2002)

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  Brambilla Pineda Soto Vairo **PLB470** (1999) Kniehl Penin **NPB563** (1999) Kniehl Penin Smirnov Steinhauser **NPB635** (2002)

- IR divergences manifest themselves at $m\alpha_s^5$

Integrating out the temperature

pNRQCD_{HTL}

m

$m\alpha_{\rm s}$

T

$m\alpha_{\rm s}^2$

m_D

$\Lambda_{\rm QCD}$

- Integrating out the temperature brings us to a new EFT called pNRQCD_{HTL}

- In this EFT modes with energy and momenta of the order of the temperature are no longer present. This amounts to:

  - Hard Thermal Loop (HTL) resummation in the gauge and light quark sectors
    Braaten Pisarski PRD45 (1992)

  - Thermal modifications to the potentials in the singlet and octet sector

- This EFT and its matching coefficients are independent of the hierarchy of low-lying scales

The pNRQCD_{HTL} Lagrangian

$$\mathcal{L}_{\mathrm{pNRQCD_{HTL}}} = \mathcal{L}_{\mathrm{HTL}} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - H_s - \delta V_s \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - H_o - \delta V_o \right] \mathrm{O} \right\}$$

$$+ \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{1}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\mathbf{r} \cdot g\mathbf{E} \right\} + \dots$$

- The new matching coefficients in the singlet and octet sectors have to be computed by matching pNRQCD to pNRQCD_{HTL}

- We only do this for the singlet sector, which is the only one relevant for the spectrum at our accuracy

- In the matching procedure we have to single out the contribution from the scale T: this will be achieved by appropriate expansions in E/T and m_D/T

Matching pNRQCD to pNRQCD_{HTL}

- The leading contribution to δV_s comes from the dipole vertices
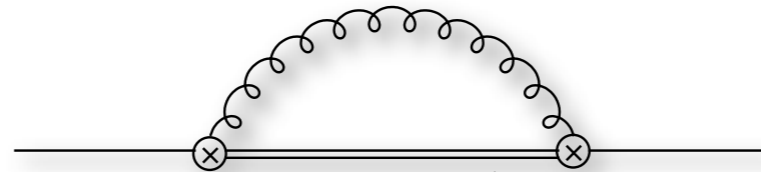
  It gives rise to a correction to the potential and to the spectrum

- Subleading contributions come from the radiative correction to that diagram
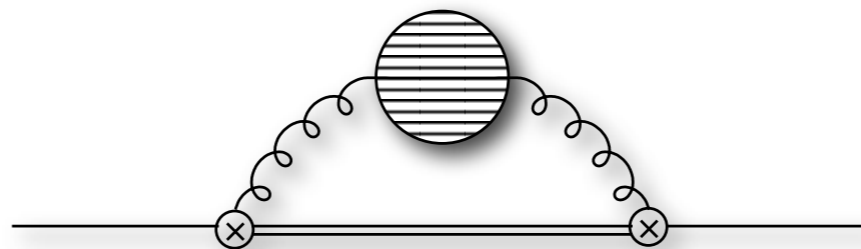
  It gives rise to an imaginary part, corresponding to a thermal width. The phenomenon is called Landau damping

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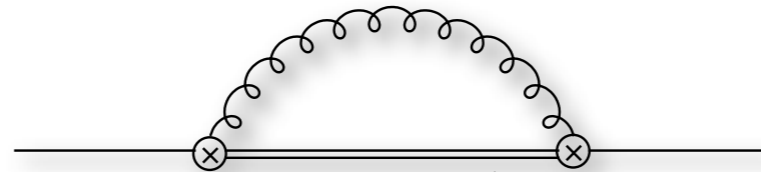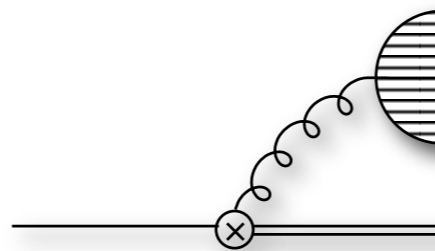 It gives rise to an imaginary part, corresponding to a thermal width. The phenomenon is called Landau damping

Results: the potential

- The correction to the potential from the temperature scale is, when $T \ll m\alpha_{\mathrm{s}}$

$$\delta V_s = \frac{\pi}{9} N_c C_F \alpha_{\mathrm{s}}^2 T^2 r + \frac{2\pi}{3m} C_F \alpha_{\mathrm{s}} T^2 + \frac{\alpha_{\mathrm{s}} C_F I_T}{3\pi} \left[-\frac{N_c^3}{8} \frac{\alpha_{\mathrm{s}}^3}{r} - (N_c^2 + 2N_c C_F) \frac{\alpha_{\mathrm{s}}^2}{mr^2} \right.$$

$$\left. +4(N_c - 2C_F) \frac{\pi \alpha_{\mathrm{s}}}{m^2} \delta^3(\mathbf{r}) + N_c \frac{\alpha_{\mathrm{s}}}{m^2} \left\{ \nabla_{\mathbf{r}}^2, \frac{1}{r} \right\} \right]$$

$$-\frac{3}{2} \zeta(3) C_F \frac{\alpha_{\mathrm{s}}}{\pi} r^2 T m_D^2 + \frac{2}{3} \zeta(3) N_c C_F \alpha_{\mathrm{s}}^2 r^2 T^3$$

$$+i \left[\frac{C_F}{6} \alpha_{\mathrm{s}} r^2 T m_D^2 \left(-\frac{2}{\epsilon} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) \right.$$

$$\left. + \frac{4\pi}{9} \ln 2 \, N_c C_F \alpha_{\mathrm{s}}^2 r^2 T^3 \right]$$

$$\boxed{ m_D^2 = \frac{g^2 T^2}{3} \left(N + \frac{n_f}{2} \right) }$$

- Both the real and the imaginary part are IR divergent; the divergences cancel in the related physical observables, the spectrum and the width, against UV divergences from some lower scale

Results: spectrum and width

- The corresponding correction to the spectrum is

$$
\begin{aligned}
\delta E_{n,l}^{(T)} &= \frac{\pi}{9} N_c C_F \, \alpha_{\mathrm{s}}^2 \, T^2 \frac{a_0}{2}(3n^2 - l(l+1)) + \frac{2\pi}{3m} C_F \, \alpha_{\mathrm{s}} \, T^2 \\
&\quad + \frac{E_n I_T \alpha_{\mathrm{s}}^3}{3\pi} \left\{ \frac{4 C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left(\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right) + \frac{2 N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} \\
&\quad + \left(-\frac{3}{2}\zeta(3)\, C_F \, \frac{\alpha_{\mathrm{s}}}{\pi} \, T \, m_D^2 + \frac{2}{3}\zeta(3) \, N_c C_F \, \alpha_{\mathrm{s}}^2 \, T^3 \right) \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1) \right].
\end{aligned}
$$

- The width is

$$
\begin{aligned}
\Gamma_{n,l} \quad &= \left[-\frac{C_F}{6}\alpha_{\mathrm{s}} T m_D^2 \left(-\frac{2}{\epsilon} + \gamma_E + \ln\pi - \ln\frac{T^2}{\mu^2} + \frac{2}{3} - 4\ln 2 - 2\frac{\zeta'(2)}{\zeta(2)} \right) \right. \\
&\quad \left. -\frac{4\pi}{9}\ln 2 \, N_c C_F \, \alpha_{\mathrm{s}}^2 \, T^3 \right] a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1) \right].
\end{aligned}
$$

Calculating the contribution from the energy scale

Calculating at the energy scale
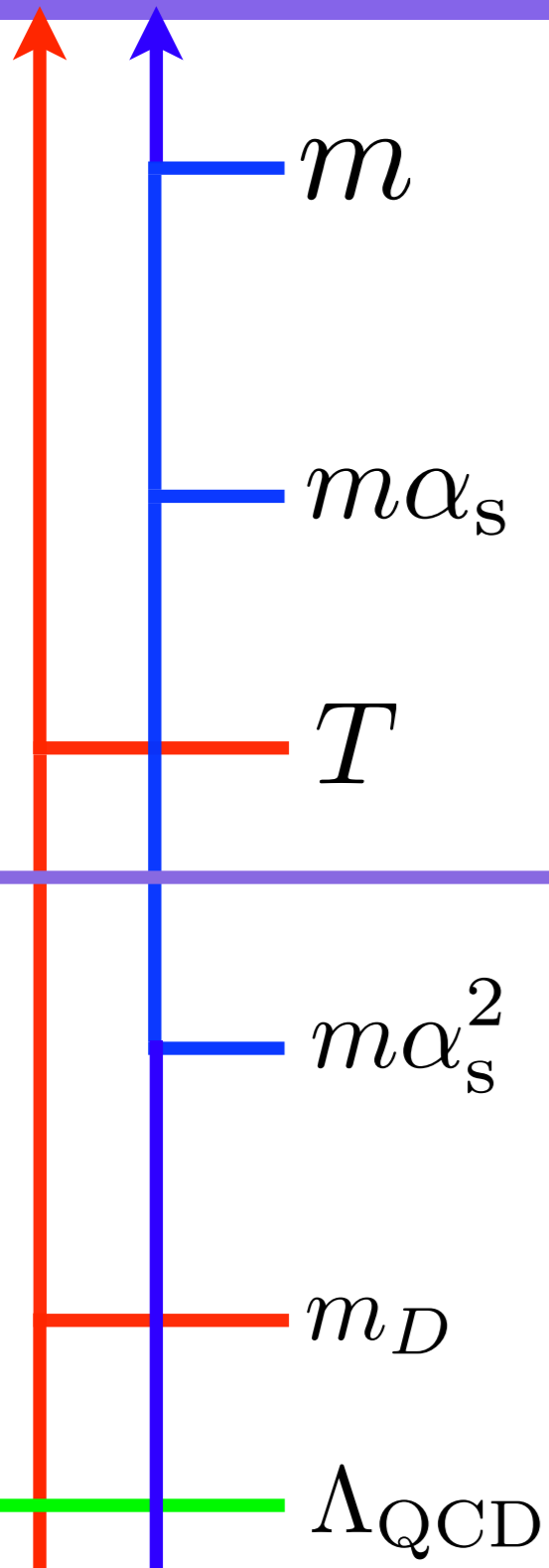
- Since we have reached the binding energy scale we cannot proceed further with EFTs in the heavy quark-antiquark sector

- We thus calculate the contributions to the spectrum and the width *within* pNRQCD$_{\mathrm{HTL}}$

- We thus have to use the HTL-resummed propagators

- We employ the expansions $T \gg m\alpha_{\mathrm{s}}^2 \gg m_D$

The calculation

- Only the leading order diagram contributes within our accuracy
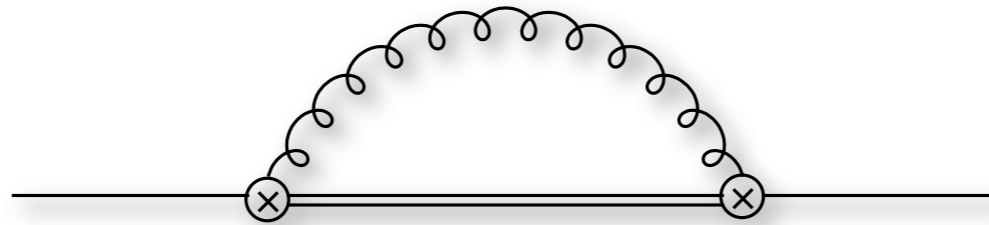
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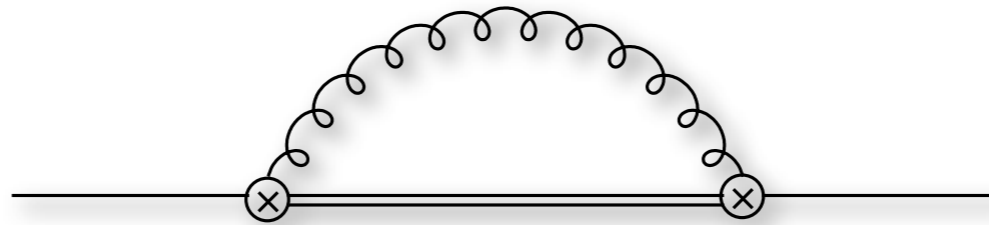
- This diagram introduces a second source of imaginary parts ⇒ thermal width. It is the singlet-to-octet decay width

The calculation

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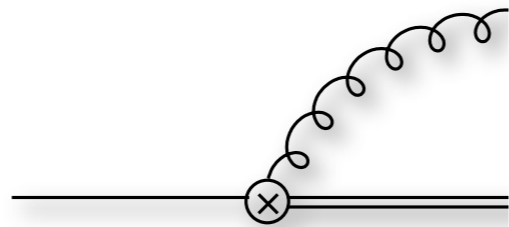
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The calculation

- Only the leading order diagram contributes within our accuracy

- This diagram introduces a second source of imaginary parts ⇒ thermal width. It is the singlet-to-octet decay width

- The calculation is quite involved, requiring expansions in the propagators and in the Bose distribution

- UV divergences appear and cancel the IR divergencies from higher scales

Summary and conclusions

The spectrum

- The contribution *of the thermal medium only* to the spectrum is

$$\delta E_{n,l} = \frac{\pi}{9} N_c C_F \, \alpha_{\rm s}^2 \, T^2 \frac{a_0}{2} \left[3n^2 - l(l+1) \right] + \frac{\pi}{3} C_F^2 \, \alpha_{\rm s}^2 \, T^2 \, a_0 \, {\color{blue}\sim m\alpha_{\rm s}^5 \frac{T^2}{E^2}}$$

$${\color{blue}m\alpha_{\rm s}^5 \sim} \left\{ + \frac{E_n \alpha_{\rm s}^3}{3\pi} \left[\log \left(\frac{\color{red}2\pi T}{\color{red}E_1} \right)^2 - 2\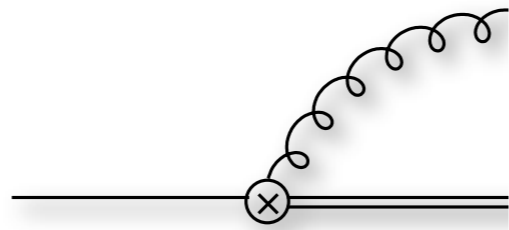gamma_E \right] \left\{ \frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right] \right. \right.$$

$$\left. \left. + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} + \frac{2E_n C_F^3 \alpha_{\rm s}^3}{3\pi} {\color{blue}L_{n,l}} \right.$$

$${\color{blue}m\alpha_{\rm s}^6 \frac{T^3}{E^3} \sim} \quad + \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1) \right] \left\{ - \left[\frac{3}{2\pi} \zeta(3) + \frac{\pi}{3} \right] C_F \, \alpha_{\rm s} \, T \, m_D^2 \right.$$

$$\left. + \frac{2}{3} \zeta(3) \, N_c C_F \, \alpha_{\rm s}^2 \, T^3 \right\}$$

The spectrum

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$$\delta E_{n,l} = \frac{\pi}{9} N_c C_F \, \alpha_{\mathrm{s}}^2 \, T^2 \frac{a_0}{2} \left[3n^2 - l(l+1) \right] + \frac{\pi}{3} C_F^2 \, \alpha_{\mathrm{s}}^2 \, T^2 \, a_0 \sim m\alpha_{\mathrm{s}}^5 \frac{T^2}{E^2}$$

$$m\alpha_{\mathrm{s}}^5 \sim \left\{ + \frac{E_n \alpha_{\mathrm{s}}^3}{3\pi} \left[\log \left(\frac{2\pi T}{E_1} \right)^2 - 2\gamma_E \right] \left\{ \frac{4 C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right] \right. \right.$$

$$\left. + \frac{2 N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} + \frac{2 E_n C_F^3 \alpha_{\mathrm{s}}^3}{3\pi} L_{n,l}$$

$$m\alpha_{\mathrm{s}}^6 \frac{T^3}{E^3} \sim \quad + \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1) \right] \left\{ - \left[\frac{3}{2\pi} \zeta(3) + \frac{\pi}{3} \right] C_F \, \alpha_{\mathrm{s}} \, T \, m_D^2 \right.$$

$$\left. + \frac{2}{3} \zeta(3) \, N_c C_F \, \alpha_{\mathrm{s}}^2 \, T^3 \right\}$$

- The leading terms thus indicate that the mass of the bound state increases quadratically with the temperature.

- Up to this order the thermal shift to the spectrum is independent of the spin

The thermal width

- The contribution of the thermal medium to the width is

$$\Gamma_{n,l} = \frac{1}{3} N_c^2 C_F \alpha_{\mathrm{s}}^3 T + \frac{4}{3} \frac{C_F^2 \alpha_{\mathrm{s}}^3 T}{n^2} (C_F + N_c)$$

$$+ \frac{2 E_n \alpha_{\mathrm{s}}^3}{3} \left\{ \frac{4 C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right] + \frac{2 N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\}$$

$$- \left[\frac{C_F}{6} \alpha_{\mathrm{s}} T m_D^2 \left(\ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9} \ln 2 \, N_c C_F \alpha_{\mathrm{s}}^2 T^3 \right]$$

$$\times a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1) \right]$$

$$+ \frac{8}{3} C_F \alpha_{\mathrm{s}} T m_D^2 \, a_0^2 n^4 \, I_{n,l}$$

- Two sources of decay: singlet-to-octet thermal breakup and Landau damping

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$$\Gamma_{n,l} = \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2 \alpha_s^3 T}{n^2}(C_F + N_c) \ \sim m\alpha_s^5 \frac{T}{E}$$

$$m\alpha_s^5 \sim \ + \frac{2E_n \alpha_s^3}{3}\left\{ \frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\}$$

$$m\alpha_s^6 \frac{T^3}{E^3} \sim \begin{cases} - \left[\frac{C_F}{6}\alpha_s T m_D^2 \left(\ln \frac{\textcolor{red}{E_1^2}}{\textcolor{red}{T^2}} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9}\ln 2 \, N_c C_F \, \alpha_s^2 \, T^3 \right] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1) \right] \\ + \frac{8}{3}C_F \alpha_s \, T m_D^2 \, a_0^2 n^4 \, I_{n,l} \end{cases}$$

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$$m\alpha_s^6 \frac{T^3}{E^3} \sim \left\{ \begin{array}{l} - \left[\frac{C_F}{6}\alpha_s T m_D^2 \left(\ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9}\ln 2 \, N_c C_F \, \alpha_s^2 \, T^3 \right] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times \, a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1) \right] \\ + \frac{8}{3}C_F \alpha_s \, T m_D^2 \, a_0^2 n^4 \, I_{n,l} \end{array} \right.$$

- Two sources of decay: singlet-to-octet thermal breakup and Landau damping

- In our hiearchy the former is larger than the latter

- Up to this order the width as well is independent of the spin

Conclusions

- We have shown how to construct a series of EFT to integrate out in succession the many scales that characterize a non-relativistic bound state at finite temperature

- We have obtained the contribution to the singlet potential from the scale T, when $T \ll m\alpha_s$

- We have obtained the contribution of the thermal bath to the spectrum and the width of heavy quarkonia, in a scale setup that can be relevant for the ground state of bottomonium at the LHC

Other EFT results

- Spin-orbit interaction
  Brambilla Escobedo JG Vairo **arXiv:1105.4587**

- Velocity-dependence of the potential and the width
  Escobedo Mannarelli Soto **arXiv:1105.1249**

Backup

The real-time formalism
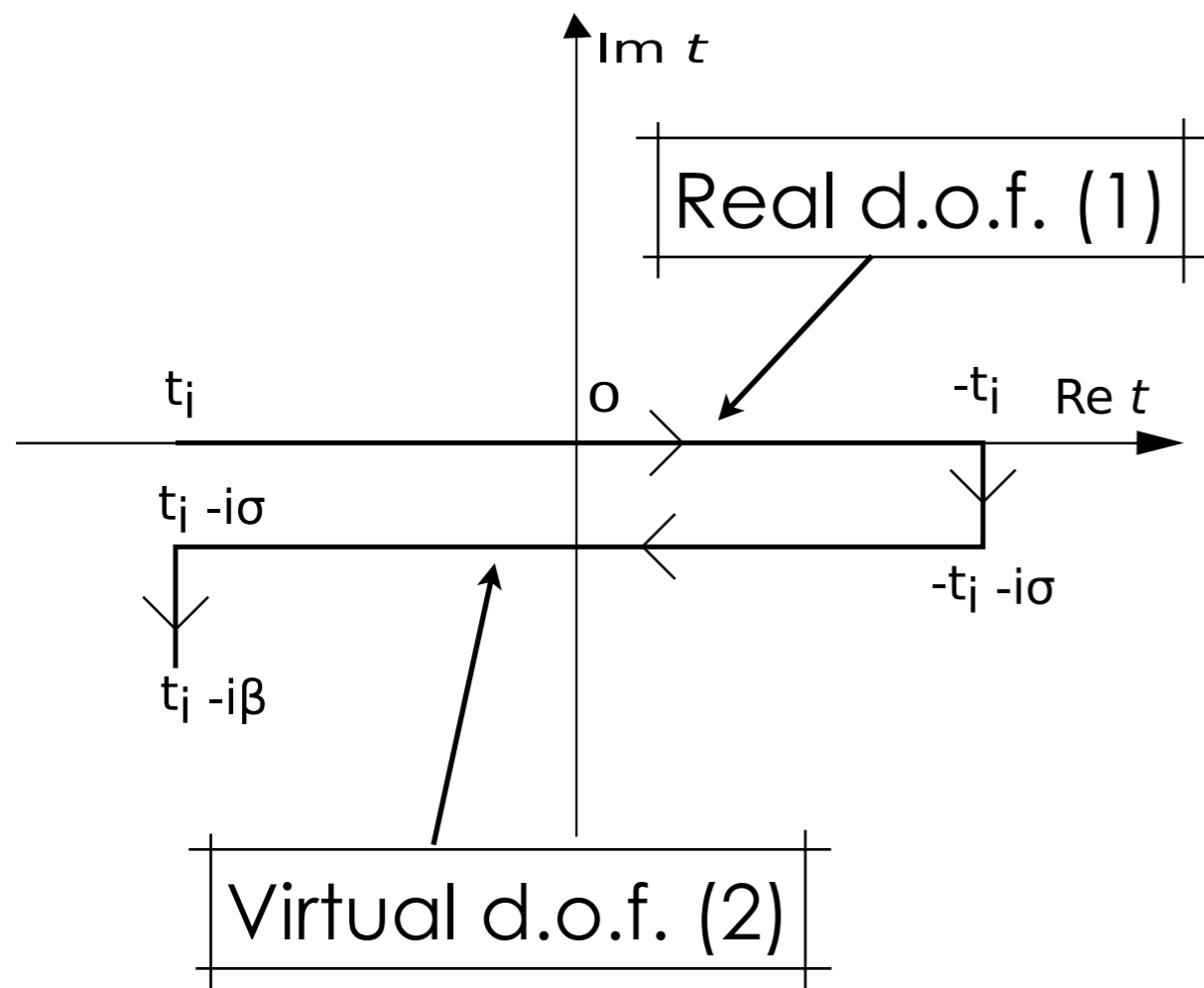
- Time evolution along this Schwinger-Keldysh path

- $t_i \to -\infty$ limit

- Doubling of the degrees of freedom

The real-time formalism

- Closer to the T=0 EFT approach

- In the static quark sector the virtual DOF decouple

- Simpler than the imaginary-time + analytical continuation (potential as pole of the propagator in the large real-time limit)

- Propagators as 2X2 matrices, vertices of type "2" have opposite sign

Free propagators

- Propagators written as $D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$

- Static quark

$$S_{\alpha\beta}^{(0)}(k_0) = \delta_{\alpha\beta} \begin{pmatrix} \dfrac{i}{k_0 + i\eta} & 0 \\ 2\pi\delta(k_0) & \dfrac{-i}{k_0 - i\eta} \end{pmatrix}$$

(static quark of type 2 decouple)

- Gluons (Coulomb gauge)

$$D_{00}^{(0)}(k) = \begin{pmatrix} \dfrac{i}{k^2} & 0 \\ 0 & -\dfrac{i}{k^2} \end{pmatrix}$$

$$D_{ij}^{(0)}(k_0, k) = \left(\delta_{ij} - \frac{k^i k^j}{k^2} \right) \left\{ \begin{pmatrix} \dfrac{i}{k_0^2 - k^2 + i\eta} & \theta(-k_0)\,2\pi\delta(k_0^2 - k^2) \\ \theta(k_0)\,2\pi\delta(k_0^2 - k^2) & -\dfrac{i}{k_0^2 - k^2 - i\eta} \end{pmatrix} + 2\pi\delta(k_0^2 - k^2)\,n_{\mathrm{B}}(|k_0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

The static potential

$$V_s(r, \mu) = -C_F \frac{\alpha_{V_s}(1/r)}{r}$$

$$= -C_F \frac{\alpha_s(1/r)}{r} \left\{ 1 + \frac{\alpha_s(1/r)}{4\pi} a_1 + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 a_2 \right.$$

$$+ \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \left[\frac{16\,\pi^2}{3} C_A^3 \ln r\mu + a_3 \right]$$

$$+ \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \left[a_4^{L2} \ln^2 r\mu \right.$$

$$\left. + \left(a_4^L + \frac{16}{9} \pi^2 C_A^3 \beta_0 (-5 + 6\ln 2) \right) \ln r\mu + a_4 \right]$$

$$\left. + \qquad \cdots \right\}$$

Fischler 77 Peter 97 Schröder 99 Brambilla et al.
03/08 Sumino et al. 2009 Steinhauser et al. 2009