Spectra of Light and Heavy Mesons, Glueball and QCD Effective Coupling

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Outline:

Take into account the correct symmetry structure of the quark-gluon interaction in the confinement region.

Find simple forms of the quark and gluon propagators in the hadroniz. region.

Build a relativistic quantum field model of the interacting quarks and gluons with the Analytic Confinement and obtain reasonable description of different processes in hadron physics:

- spectrum of conventional mesons in a wide range of mass scale;
- the lowest-state glueball mass, radius, etc.;
- weak decay constants of light mesons;
- qualitative description of the QCD effective coupling in the low-energy region by exploiting hadron spectrum;

by introducing a minimal set of model parameters. $\{\alpha_s, m_{ud}, m_s, m_c, m_b, \Lambda\}$

QCD at long distances

Many interesting and novel behaviors are expected at **low energies** (IR, infrared region) below 1 GeV.

Confinement and dynamical symmetry breaking are crucial features of QCD. Color confinement is the result of strong interaction and in the hadron scale (~ 1fm ~ 200MeV) QCD becomes non-perturbative.

Green's functions in QCD are tightly connected to confinement and are ingredients for hadron phenomenology. However, any widely accepted and rigorous analytic solutions to these propagators are still missing.

The matrix elements of hadron processes at large distance are integrated characteristics of the vertices and propagators, and the solution should not be too sensitive on the details of propagators. Taking into account the correct global symmetry properties and their breaking (and by introducing additional physical parameters) may be more important than the working out in detail of propagators.

Confinement

There is **no analytic proof** that QCD should be color confining. The reason for confinement may be somewhat complicated. E.g.: **Analytic Confinement**

$$S^{-1}(z) \neq 0 \quad for \quad \forall z \in C$$
$$S^{-1} \cdot \Psi_0(x) = 0 \quad \Rightarrow \quad \Psi_0(x) \equiv 0$$

$\tilde{S}(p^2)$ entire analytic function in complex plane

The QCD vacuum is realized by nontrivial gluon background field with constant strength (the energy of the quark-gluon system is minimal on this background). The GBF leads to the AC of quarks. (H.Leutwyler [1981], G.V.Efimov et al. [1996])

$$\breve{B}_\mu(x) = \Lambda^2 t^a n^a b_{\mu\nu} x_\nu; \qquad n^a n^a = 1; \qquad \tilde{b}_{\mu\nu} \equiv \varepsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} / 2 = \pm b_{\mu\nu};$$
$$\partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) = -2\Lambda^2 b_{\mu\nu} = const; \qquad b_{\mu\alpha} b_{\alpha\nu} = -\delta_{\mu\nu}$$

$$\tilde{S}(p^2) = \langle B | \tilde{S}_{QCD}(p^2) | B \rangle$$
$$\tilde{D}(p^2) = \langle B | \tilde{D}_{QCD}(p^2) | B \rangle$$

entire analytic functions

Model

- Consider a relativistic quantum-field model of quark-gluon interaction.

$$L = -\frac{1}{4}\left(F^A_{\mu\nu} - g f^{ABC} A^B_\mu A^C_\nu\right)^2 + \sum_f \left(\overline{q}^a_f \left[\gamma_\alpha \partial_\alpha - m_f + g\Gamma^\alpha_C A^C_\alpha\right]^{ab} q^b_f\right)$$

$$F^B_{\mu\nu} \equiv \partial_\mu A^B_\nu - \partial_\nu A^B_\mu \qquad \Gamma^\alpha_C \doteq i\gamma_\alpha t^C$$

♣ analytic confinement (AC) takes place.
♣ the coupling remains weak (~1) in the hadronization region.

- Entire Analytic Propagators [G.Ganbold PRD79 (2009)]

$$\tilde{S}^{ab}_\pm(\hat{p}) = \delta^{ab} \frac{1}{2\Lambda} \frac{i\hat{p} + m_f\left[1 \pm \gamma_5 \omega(m_f/\Lambda)\right]}{m_f} \cdot \exp\left\{-\frac{p^2 + m_f^2}{2\Lambda^2}\right\}$$

$$\tilde{D}^{BC}_{\mu\nu}(p) = \delta^{BC}\delta_{\mu\nu} \frac{1}{p^2} \exp\left\{-\frac{p^2}{4\Lambda^2}\right\}$$

$$\omega(z) = 1/(1 + z^2/4)$$

parity-symmetry violation

$$\left\langle \overline{q}_f(0) \cdot q_f(0) \right\rangle = -\frac{6\Lambda^3}{\pi^2} \exp\left\{-\frac{m_f^2}{2\Lambda^2}\right\} \underset{m\to 0}{=} -\frac{6\Lambda^3}{\pi^2} \approx -\left(0.847\bullet\Lambda\right)^3 \neq 0$$

Quark-Antiquark Bound States

- Leading-order contributions to quark-antiquark and two-gluon BS

$$Z_{(\bar{q}q)} = \iint \delta\bar{q}\,\delta q\, \exp\left\{-\left(\bar{q}\, S^{-1} q\right) - L_{qq}\right\}$$

$$L_{qq} = \frac{g^2}{2}\sum_{f_1 f_2}\iint dx_1 dx_2\, J^B_{\mu f_1 f_2}(x_1,x_2)\, D^{BC}_{\mu\nu}(x_1,x_2)\, J^C_{\nu f_1 f_2}(x_2,x_1), \qquad J^B_{\mu f_1 f_2}(x_1,x_2) \equiv \bar{q}_{f_1}(x_1)\gamma_\mu t^B q_{f_2}(x_2).$$

- Orthonormalized system U_Q, where Q={n,l, …} are quantum numbers

$$\delta(x-y) = \sum U_Q(x) U_Q(y); \qquad \delta_{QQ'} = \int dy\, U_Q(y) U_{Q'}(y)$$

$$J_{Jf_1 f_2}(x,y) \equiv \sqrt{D(y)}\left(\bar{q}_{f_1}(x+\mu_1 y)\Gamma_J\, q_{f_2}(x-\mu_2 y)\right) = \sum\nolimits_Q J_{QJf_1 f_2}(x) U_Q(y)$$

$$J_{QJ f_1 f_2}(x) \equiv \bar{q}_{f_1}(x) V_{JQ}(\overleftrightarrow{\partial})\, q_{f_2}(x)$$

$$V_{JQ}(\overleftrightarrow{\partial}) \equiv i^l \int dy\, \sqrt{D(y)}\, \Gamma_J\, U_Q(y)\, \exp\left((y/2)\overleftrightarrow{\partial}\right)$$

- A new path integration over auxiliary fields B_N: where N={Q,J,f_1,f_2}

$$e^{L_{qq}} = \iint \delta B^+_N\, \delta B_N\, \exp\left\{-\sum_N (B^+_N B_N) + g\sum_N [(B^+_N J_N) + (J^+_N B_N)]\right\}$$

$$Z_{qq} \to Z_N = \int \prod_N \delta B_N \exp\left\{ -\frac{1}{2}\Big(B_N\,[1+g^2\,Tr(V_N S\; V_N S)]\,B_N\Big) + W_{resid}[B_N] \right\}$$

- Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system {U_N}

$$g^2\,Tr(V_N S\; V_{N'} S) = \big(U_N\;\lambda\;U_{N'}\big) = \alpha_S \cdot \lambda_N(-p^2)\;\delta^{JJ'}\delta^{QQ'}$$

- Symmetric Bethe-Salpeter kernel:

$$\alpha_S \cdot \lambda_N(-p^2) = \frac{2g^2 C_J}{9}\int \frac{d^4k}{(2\pi)^4}\,|V_N(k)|^2 \cdot Tr\left\{\Gamma_J\,\tilde{S}\big(\hat{k}+\mu_1\hat{p}\big)\Gamma_J\,\tilde{S}\big(\hat{k}-\mu_2\hat{p}\big)\right\}$$

- Renormalization:

$$U_{REN}(x) \equiv \sqrt{-\alpha_S \cdot \dot{\lambda}_N(M_N^2)}\cdot\; U_N(x)$$

$$\Big\langle U_N\,|1+\alpha_S\lambda_N(-p^2)\,|\,U_N\Big\rangle = \Big\langle U_N\,|1+\alpha_S\lambda_N(M_N^2) - \alpha_S\dot{\lambda}_N(M_N^2)(p^2+M_N^2)\;U_N\Big\rangle$$
$$= \Big\langle U_{REN}\;\;(p^2+M_N^2)\,|\,U_{REN}\Big\rangle$$

- Meson mass equation: $-p^2 = M_N^2 \;\Leftrightarrow\; 1+\alpha_S\cdot\lambda_N(M_N^2) = 0$

Conventional Meson Spectrum

Fixing model parameters:

[G.Ganbold PRD79 (2009)]

$$\alpha_s = 1.5023, \qquad \Lambda = 416.4\,MeV,$$
$$m_{u,d} = 206.9\,MeV, \qquad m_s = 323.6\,MeV,$$
$$m_c = 1454\,MeV, \qquad m_b = 4699\,MeV.$$

P-mesons	PDG	Our estim.	V-mesons	PDG	Our estim.
π	138	138	ρ	770	770
K	495	495	ω	782	785
D	1870	1840	K*	892	909
D_s	1970	1970	Φ	1019	1022
η_c	2979	3012	D*	2010	1942
B	5279	5337	Ds*	2112	2078
B_s	5370	5451	J/ψ	3097	3097
B_c	6286	6422	B*	5325	5464
η_b	9302	9434	Y	9460	9460

$\omega \leftrightarrow \Phi$

$(u\bar{u} + d\bar{d})/\sqrt{2} \leftrightarrow s\bar{s}$

$\theta_V = 74^0$

|relat. errors| < 2%

Decay Constants of Light Mesons

- these are important value in particle physics:

$$\pi^+ \to \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma$$

$$D_s^+ \to l^+ \nu$$

$$i\, f_P p_\mu = \langle 0 | J_\mu(0) | U_{renorm}(p) \rangle$$

$$i f_p p_\mu = \frac{g}{6} \int \frac{d^4k}{(2\pi)^4} \int dx\, e^{ikx}\, U_R(x) \sqrt{D(x)}\, Tr\left\{ i\gamma_5\, \tilde{S}\left(\hat{k} + \xi_1 \hat{p}\right) i\gamma_5 \gamma_\mu\, \tilde{S}\left(\hat{k} - \xi_2 \hat{p}\right) \right\}$$

$$\alpha_s = 1.5023, \qquad \Lambda = 416.4\, MeV,$$
$$m_{u,d} = 206.9\, MeV, \qquad m_s = 323.6\, MeV.$$

→

$$f_\pi = 128.8\ MeV$$
$$f_K = 157.7\ MeV$$

[G.Ganbold PRD79 (2009)]

PDG: →

$$f_\pi^{\exp} = 130.4 \pm 0.04 \pm 0.2\ MeV$$
$$f_K^{\exp} = 155.5 \pm 0.2 \pm 0.8\ MeV$$

Glueball Lowest State

Theoretical status:

The existence of glueballs is predicted by QCD because of the self-interaction of gluons.

The lightest glueball is a scalar.

$$J^{PC} = 0^{++}$$

1710±50±58 MeV	Particle Data Group
1750±50±80 MeV	C.J.Morningstar, M.Peardon (2000).
1500-1800 MeV	C.Amsler, N.A.Tornqvist (2004); S.Narison(2000);D.V.Bugg (2004);
1475 MeV	H.B.Meyer, M.J.Teper (2005)

Experimental status:

Signatures for glueballs:

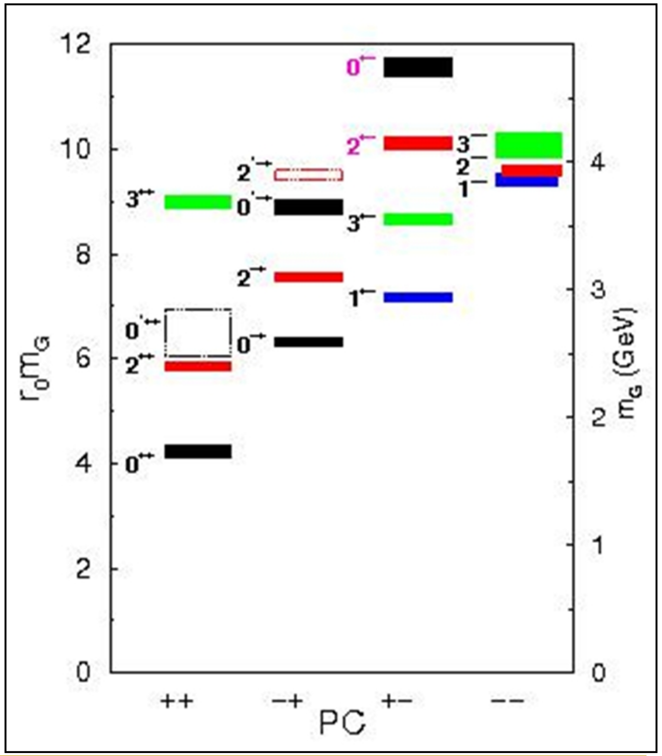
-- no place in (q-qbar) nonets,

-- enhanced production in gluon-rich (short distance) channels of rad.decays,

-- decay branching fractions incompatible with (q-qbar) states

LEP and LHC: see talk by W.Ochs

PDG:



Two-Gluon Bound States

$$Z_{(AA)} = \exp\{-L_{AA}\}$$

$$L_{AA} = \frac{g^2}{12}\iint dx_1\, dx_2 \left(J^{BB}_{\mu\mu'}(x_1,x_2)\, J^{CC}_{\nu\nu'}(x_1,x_2) - J^{BB}_{\mu\nu'}(x_1,x_2)\, J^{CC}_{\nu\mu'}(x_1,x_2)\right)$$

$$\cdot\left(\delta^{\nu\nu'} W_{\mu\mu'}(x_1,x_2) - \delta^{\mu\nu'} W_{\nu\mu'}(x_1,x_2) - \delta^{\nu\mu'} W_{\mu\nu'}(x_1,x_2) + \delta^{\mu\mu'} W_{\nu\nu'}(x_1,x_2)\right),$$

$$J^{BC}_{\mu\nu}(x_1,x_2) \equiv A^B_\mu(x_1)\, A^C_\nu(x_2), \qquad W_{\mu\nu}(x_1,x_2) \equiv \frac{\partial^2}{\partial x_1^\mu\, \partial x_2^\nu} D(x_1 - x_2)$$

- The glueball mass is derived from:

$$1 = \frac{8g^2}{3}\int dz\, e^{izp}\, \Pi(z), \quad p^2 = -M_G^2$$

$$\Pi(z) \equiv \iint dt\, ds\, U(t)\sqrt{W(t)}\, D\left(\frac{t+s}{2} + z\right) D\left(\frac{t+s}{2} - z\right)\sqrt{W(s)}U(s)$$

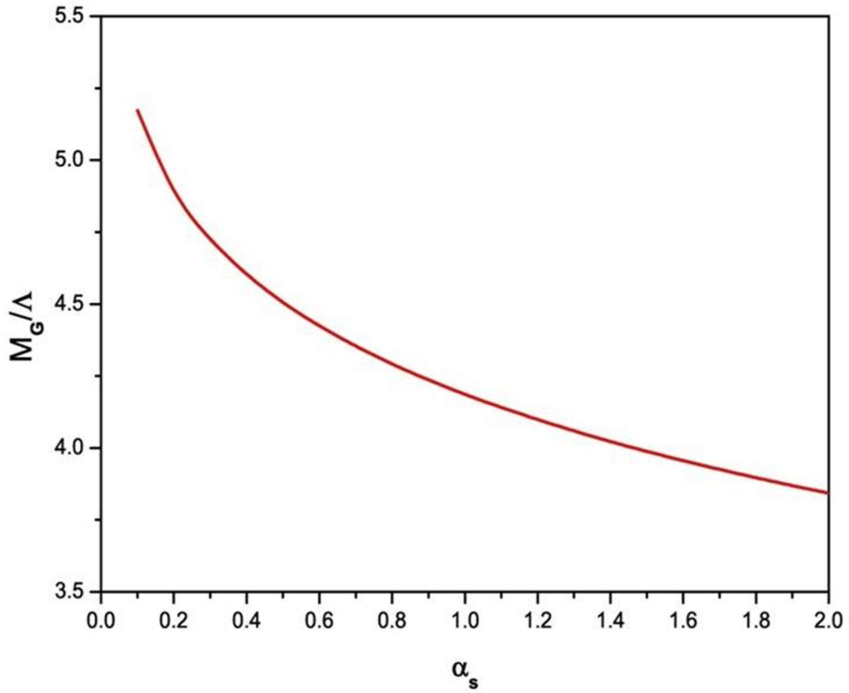
$$W(t) \equiv \exp(-t^2)/(2\pi)^2$$

Lowest-State (Scalar) Glueball

$$M_G = 2\Lambda\sqrt{\ln\left(\frac{\alpha_{crit}}{\alpha_s}\right)}$$

$$\alpha_{crit} \equiv \frac{3\pi}{4}\left(3+2\sqrt{2}\right)^2$$

[G.Ganbold PRD79 (2009)]



$$\alpha_s = 1.5023$$
$$\Lambda = 416.4\,MeV$$

⇒

$$M_G = 1661\,MeV$$

$$r_G = \sqrt{\frac{\int d^4x \cdot x^2 \cdot D(x)}{\int d^4x \cdot D(x)}} = \frac{\sqrt{2}}{\Lambda} \approx \frac{1}{295MeV} \sim 0.67\,fm$$

$$r_G M_G = 2\sqrt{2}\sqrt{\ln\left(\frac{\alpha_{crit}}{\alpha_s}\right)} \approx 5.64$$

~ 4.6 in quenched lattice, Y.Chen (2006)

QCD Effective Coupling

Due to the polarization of QCD vacuum, the color charge ***g*** is
a) **screened** by the virtual quark-antiquark pairs
b) **antiscreened** by the polarization of virtual gluons.
a variation of the physical coupling under changes of distance **1/Q**, or energy scale **Q**.

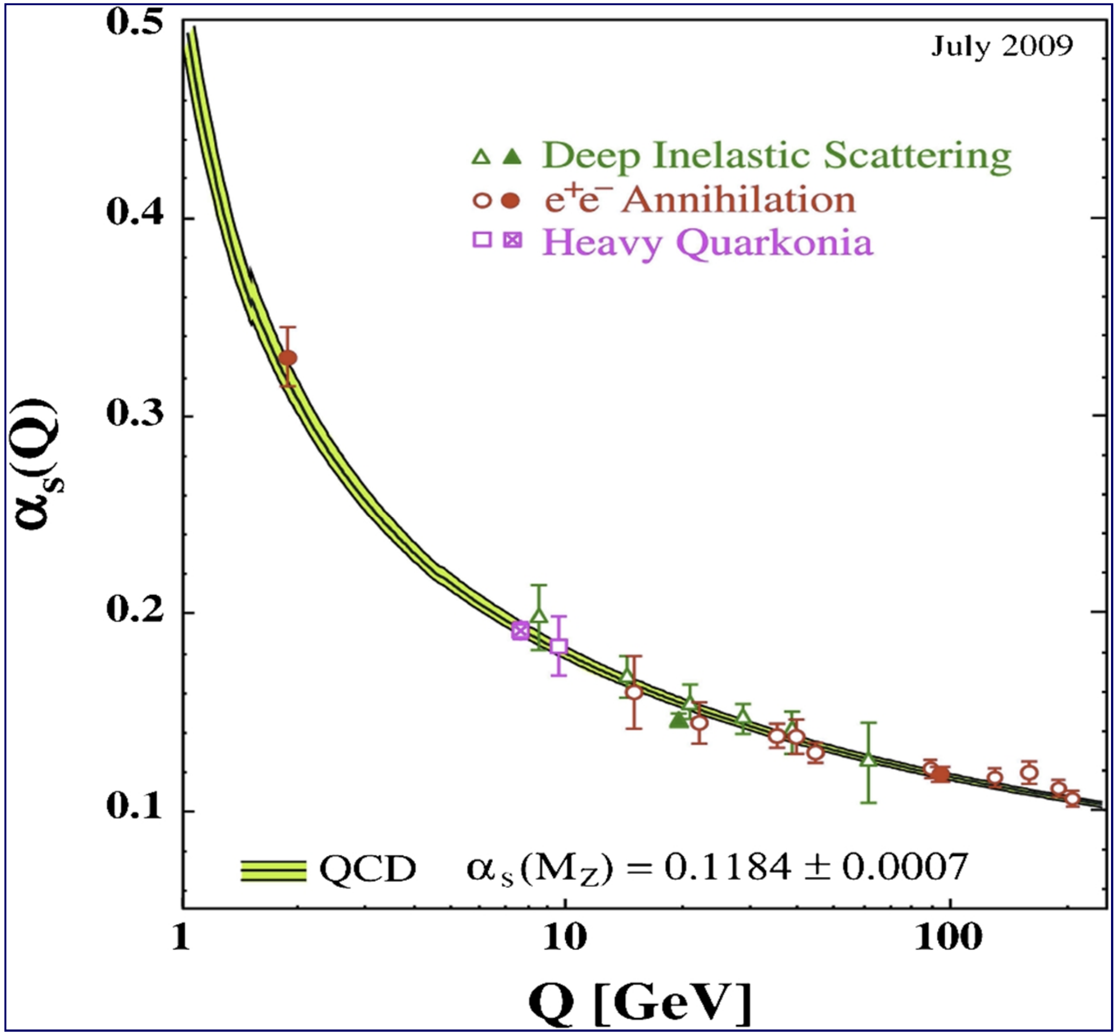
$$\alpha_s = \frac{g^2}{4\pi} \quad \Rightarrow \quad \alpha_s(Q)$$

Many quantities in hadron physics are affected by the IR behavior of alpha. However, the latter is not well defined yet.

Determination of QCD coupling remains at forefront of experimental studies.

Process	Q (GeV)	α_s	Reference
τ decays	1.78	0.330 ± 0.014	S. Bethke (2009)
$Q\bar{Q}$ states	4.1	0.239 ± 0.012	S. Davies (2003)
Υ decays	4.75	0.217 ± 0.021	A. Penin (1998)
$Q\bar{Q}$ states	7.5	0.1923 ± 0.0024	S. Bethke (2009)
Υ decays	9.46	0.184 ± 0.015	S. Bethke (2009)
e^+e^- jets	14.0	0.170 ± 0.021	P. A. M. Fernandez (2002)

Measurements of α_s as a function of energy scale Q versa QCD predictions.



A self-consistent and physically meaningful prediction of the QCD effective charge in the IR regime remains one of the actual problems in particle physics.

Long-distance behavior

Recent theoretical results predict an IR behavior of the gluon propagator.

- We consider a gluon propagator:

$$\tilde{D}^{BC}_{\mu\nu}(p) = \delta^{BC}\delta_{\mu\nu}\frac{1-\exp\left(-p^2/\Lambda^2\right)}{p^2} = \delta^{BC}\delta_{\mu\nu}\int_0^{1/\Lambda^2} ds\, e^{-s\,p^2}$$

- The quark propagator remains the same.
- Meson mass is defined from equation:

$$1 = \alpha_s \cdot \lambda\left(J, \frac{m_1}{\Lambda}, \frac{m_2}{\Lambda}, \frac{M_J}{\Lambda}\right), \qquad J = \{P, V, S, A, T\}$$

- Fixing model parameters:

$$\begin{aligned}\alpha_s(9460) &= 0.1817,\\ \alpha_s(3097) &= 0.2619,\\ \alpha_s(2112) &= 0.3074,\\ \alpha_s(2010) &= 0.3138\end{aligned}$$

⇒

$$\Lambda = 345\,MeV$$

$$m_{ud} = 192.56, \qquad m_s = 293.56,$$
$$m_c = 1447.59, \qquad m_b = 4692.51$$

• Solving Inverse Problem:
Derive effective coupling in region below 1 GeV:

[G.Ganbold PRD81 (2010)]

$$\alpha_s(138) = -\lambda_P^{-1}(\Lambda, 138, m_{ud}, m_{ud}) = 0.7131,$$
$$\alpha_s(495) = -\lambda_P^{-1}(\Lambda, 495, m_{ud}, m_s) = 0.6086,$$
$$\alpha_s(770) = -\lambda_V^{-1}(\Lambda, 770, m_{ud}, m_{ud}) = 0.4390,$$
$$\alpha_s(892) = -\lambda_V^{-1}(\Lambda, 892, m_{ud}, m_s) = 0.4214.$$

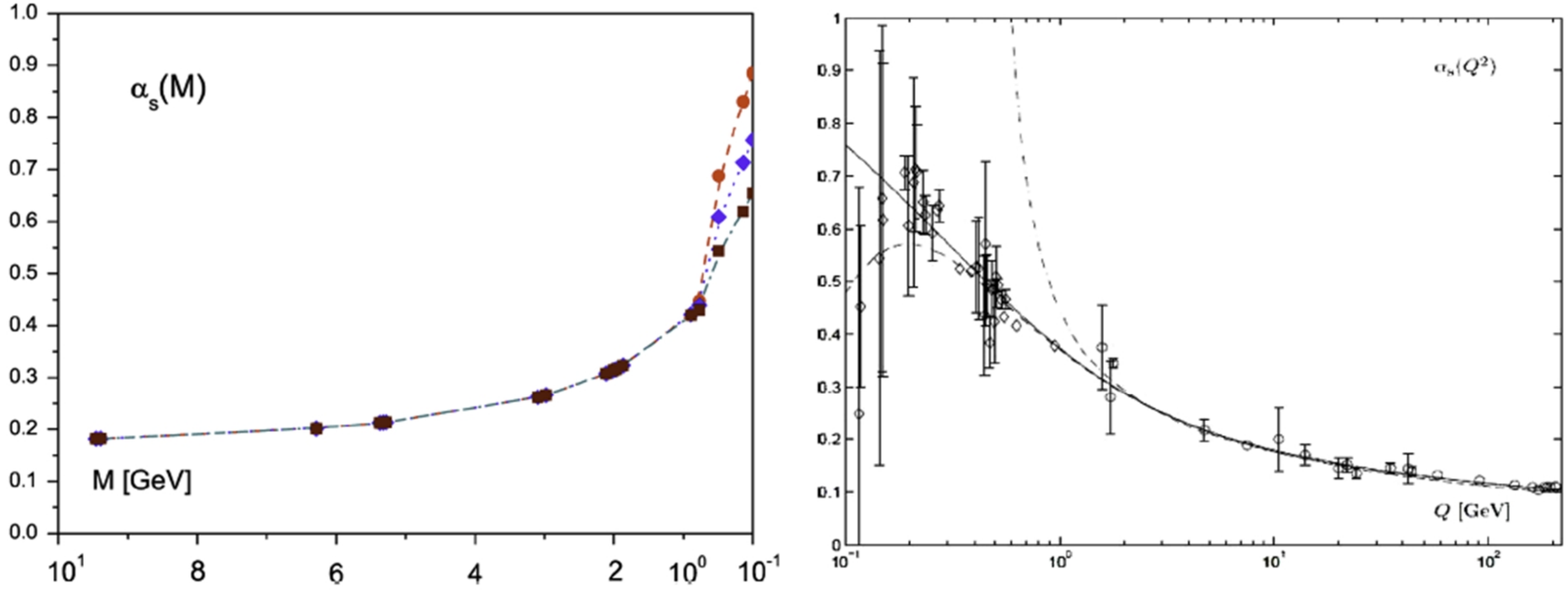
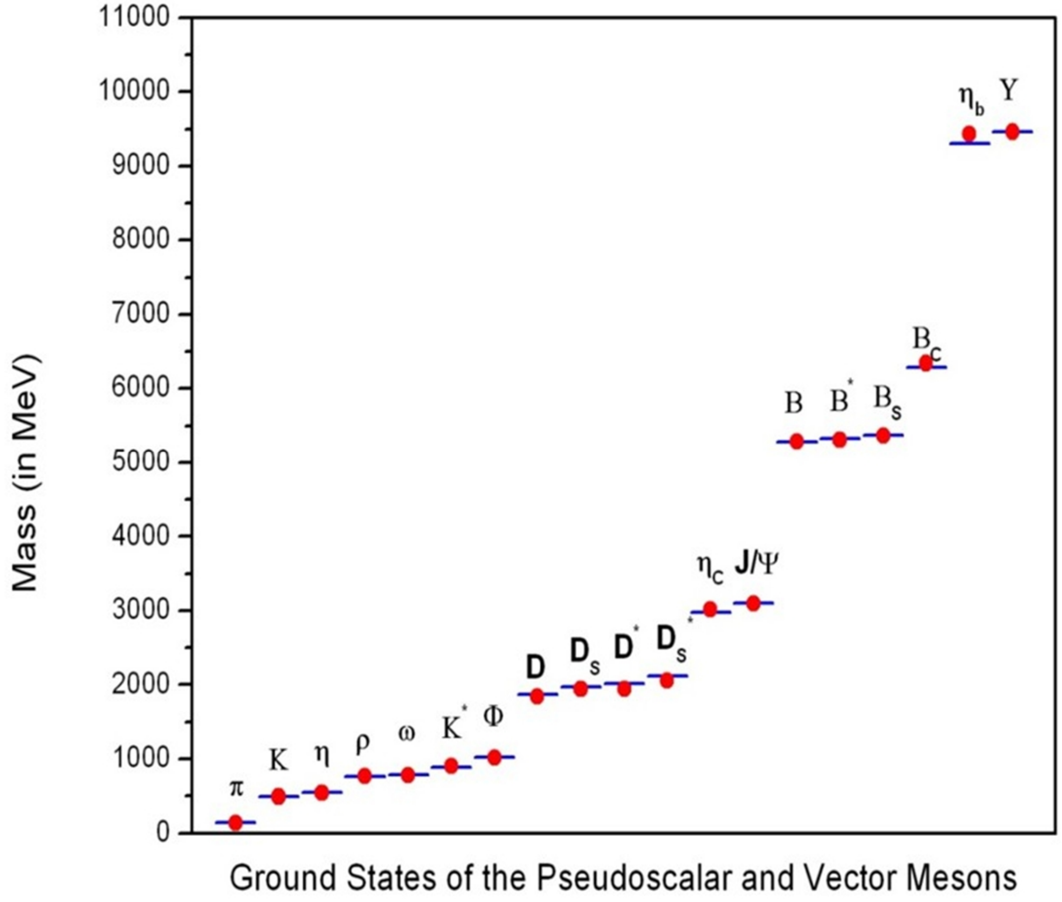


FIG. 5 (color online). Summary of estimates of $\hat{\alpha}_s(M)$ in interval from 0 to 10 GeV at different values of confinement scale. In the left panel, $\Lambda = 330$ MeV (red dots), $\Lambda = 345$ MeV (blue diamonds), and $\Lambda = 360$ MeV (black squares) compared with $\alpha_s(Q)$ (in the right panel) defined in low-energy (open diamonds) and high-energy (open circles) experiments. Also shown are the three-loop analytic coupling (solid curve), its perturbative counterpart (dot-dashed curve), both normalized at the Z-boson mass, and the massive one-loop analytic coupling (dashed curve) (for details see Ref. [31]).

Meson Masses Estimated with Running Coupling



|relative errors| < **3%**

• our estimate

-- PDG

[G.Ganbold PRD81 (2010)]

IR-finite Behavior of Effective Coupling

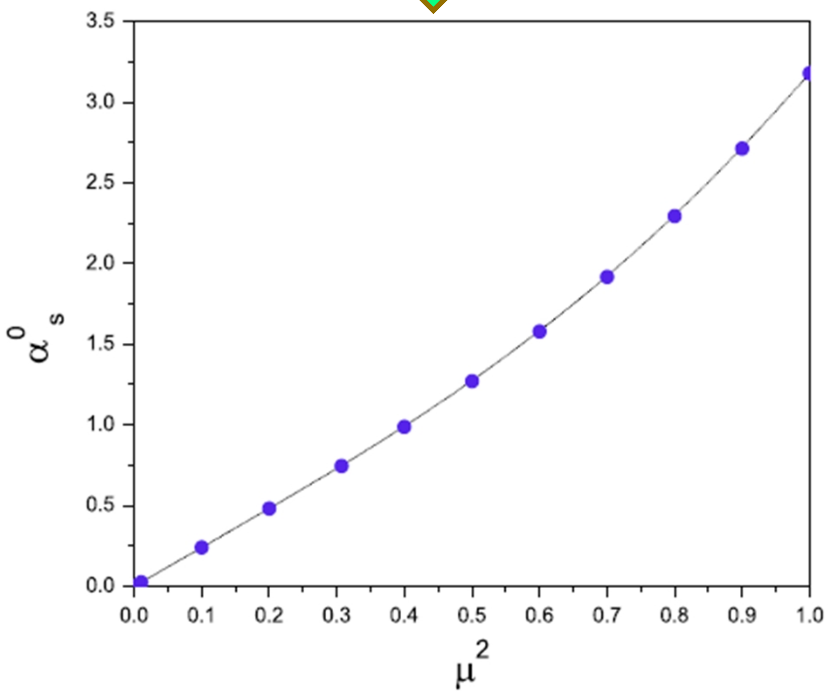
The possibility that the QCD coupling constant features an IR-finite behavior has been extensively studied in recent years.

Consider the origin point:
M=0 , m1= m2=m_{ud}

$$\alpha_s^0 = \alpha_s(0)$$

Particularly, for
m=192.56 MeV
and Λ=345 MeV

$$\mu \equiv m/\Lambda$$



$$\mu \equiv m/\Lambda \approx 0.558$$

$$\alpha_s^0 = 0.757, \quad or \quad \alpha_s^0/\pi = 0.241$$

$$\alpha_s^0/\pi = 0.19 - 0.25 \quad [S.Godfrey\ 1985],$$
$$\alpha_s^0/\pi = 0.265 \quad [T.Zhang\ 1991],$$
$$\alpha_s^0/\pi = 0.26 \quad [F.Halzen\ 1993],$$
$$\left\langle \alpha_s^0/\pi \right\rangle_{1GeV} = 0.2 \quad [M.Baldicchi\ 2008]$$

Conclusion and Outlook

♣ Our guess about the symmetry structure of the quark-gluon interaction in the confinement region has been tested and the use of simple forms of propagators has resulted in quantitatively reasonable estimates in di□erent sectors of the low-energy particle physics.

♣ We provide a new, independent and analytic estimate of the lowest (scalar) glueball mass.

♣ Despite its pure model origin, the approximations used, our model gives a new glance at the long-distance behavior of QCD coupling. Particularly, we found a specific IR behaviour of QCD coupling below 1 GeV.

♣ The consideration can be extended to actual problems in hadron physics:

- Other mesons (scalar, axial …: c-cbar, b-bbar, X, Yb, Z …)
- Exotic, mixed, many-body states (qqG, GG+qq, qqqq, …)
- Other glueball states (pseudoscalar, tensor), pomeron exchange …
- Hadronic decay processes …
- Further analyses for the behaviour of QCD coupling below 1 GeV.