Pion form factor in a broad range of momentum transfers from local – duality QCD sum rule

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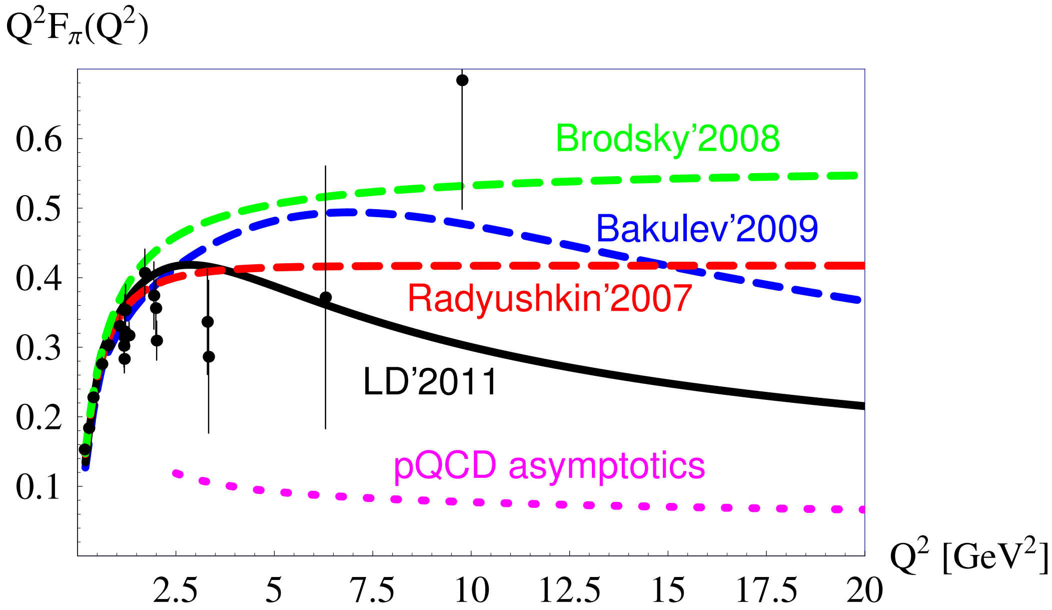
We calculate the pion form factor making use of a local-duality (LD) version of QCD sum rules.

To probe the accuracy of the LD sum rule, we consider in parallel to QCD a potential model with an interaction consisting of Coulomb and confining parts: in this case, the exact form factor is confronted with the result from the quantum-mechanical LD sum rule.

We argue that the LD sum rule gives reliable predictions for the pion form factor in the region $Q^2 \geq 6$ **GeV**^{2}**.**

Based on "Accuracy of local-duality sum rules for the pion elastic form factor" arXiv:1103.3781

Theoretical description of the pion form factor at $Q^2 \sim 5 - 50$ GeV^2 in QCD is a complicated problem:



No conclusive results have been obtained and we still have a strong dispcrepancy between the results form various versions of QCD sum rules.

The goal of the presented work was to study the accuracy of the pion form factor obtained from so-called local-duality version of QCD sum rules.

The value of any hadron observable, in particular of the form factor, extracted from QCD sum rules depends on several ingredients:

(i) the field-theoretic calculation of the OPE series for the relevant correlator.

(ii) the details of the implementation of quark-hadron duality.

(iii) the numerical "extraction procedure".

The errors introduced by the details of the implementation of quark-hadron duality and by the extraction procedure is very hard to control.

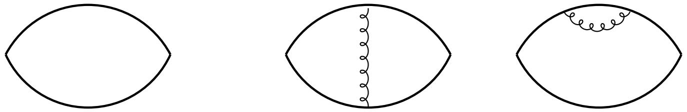
Let us briefly remind the basic ideas leading to the local-duality model:

One starts with the Borel sum rules for two- and three-point correlation functions. Implementing quark-hadron duality in the standard way — as the low-energy cut on the perturbative contributions to the correlators — one obtains the following sum rules in the chiral limit:

• Two-point Borel sum rules for pion decay constant (τ – the Borel parameter):

$$f_\pi^2 = \int_0^{\bar{s}_{\rm eff}(\tau)} ds \exp(-s\tau)\left[\rho_0(s) + \alpha_s\rho_1(s)\right] + \frac{\langle\alpha_s G^2\rangle}{12\pi}\tau + \frac{176\pi\alpha_s\langle\bar{q}q\rangle^2}{81}\tau^2 + \cdots,$$

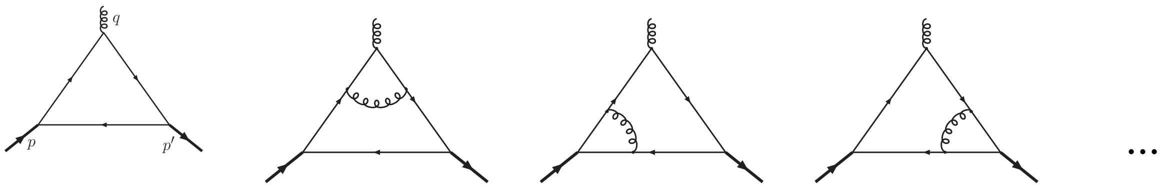
ρ_i are spectral densities of 2-point diagrams of the perturbation theory:



• Three-point sum rule for the pion form factor:

$$\begin{aligned} f_\pi^2 F_\pi(Q^2) &= \int_0^{s_{\rm eff}(\tau,Q^2)} ds_1 \int_0^{s_{\rm eff}(\tau,Q^2)} ds_2 \exp\left(-\frac{s_1+s_2}{2}\tau\right)\left[\Delta_0(Q^2,s_1,s_2) + \alpha_s\Delta_1(Q^2,s_1,s_2)\right] \\ &+ \frac{\langle\alpha_s G^2\rangle}{24\pi}\tau + \frac{4\pi\alpha_s\langle\bar{q}q\rangle^2}{81}(13 + Q^2\tau)\tau^2 + \cdots. \end{aligned}$$

Δ_i are double spectral densities of 3-point diagrams of the perturbation theory:



<u>Problem:</u> Power corrections in three-point function rise with Q (would lead to unphysical behavior of the form factor). To apply sum-rule at large Q: either (model-dependent) summation of power corrections (in terms of so-called non-local condensates) or just set $\tau = 0$.

The Local – duality (LD) limit is $\tau \to 0$. Then ALL power corrections vanish.

To α_s accuracy we have

$$f_\pi^2 F_\pi(Q^2) = \int_0^{s_{\rm eff}(Q^2)} ds_1 \int_0^{s_{\rm eff}(Q^2)} ds_2 \left[\Delta_0(s_1, s_2, Q^2) + \alpha_s \Delta_1(s_1, s_2, Q^2) + O(\alpha_s^2)\right],$$

$$f_\pi^2 = \int_0^{\bar{s}_{\rm eff}} ds \left[\rho_0(s) + \alpha_s \rho_1(s) + O(\alpha_s^2)\right] = \frac{\bar{s}_{\rm eff}}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) + O(\alpha_s^2).$$

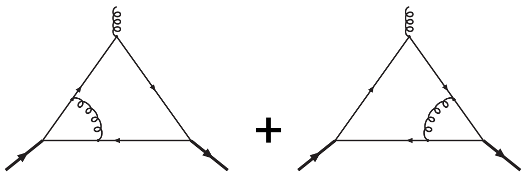
We know f_π, so we can express $\bar{s}_{\rm eff}$ to a given order in α_s through f_π.

The problem is now how to determine $s_{\rm eff}(Q^2)$.

Properties of the spectral functions

- **Vector Ward identity at $Q^2 = 0$:** $\lim_{Q^2\to 0} \Delta_i(s_1, s_2, Q^2) = \rho_i(s_1)\delta(s_1 - s_2)$

- **Factorization at $Q^2 \to \infty$: at large Q, the leading behavior is given by**



$$\Delta_1(s_1, s_2, Q^2) \sim \rho_0(s_1)\rho_0(s_2)\frac{8\pi}{Q^2}, \qquad \Delta_0(s_1, s_2, Q^2) \sim \frac{s_1 + s_2}{Q^4}.$$

Therefore, if we set

$$s_{\rm eff}(Q^2=0)=\frac{4\pi^2 f_\pi^2}{1+\alpha_s/\pi} \qquad s_{\rm eff}(Q^2\to\infty)=4\pi^2 f_\pi^2,$$

then the form factor obtained from the LD sum rule satisfies the correct normalization at $Q^2 = 0$ and reproduces the asymptotic behavior according to the factorization theorem for the form factor at $Q^2 \to \infty$.

These two values are not far from each other, so, it is easy to construct an interpolation function $s_{\rm eff}(Q^2)$ for all Q^2.

The local – duality model for hadron elastic form factors :

a. Based on a dispersive three-point sum rule at $\tau = 0$ (i.e. infinitely large Borel mass parameter). In this case all power corrections vanish and the details of the non-perturbative dynamics are hidden in one quantity — the effective threshold $s_{\rm eff}(Q^2)$.

b. Makes use of a model for $s_{\rm eff}(Q^2)$ based on a smooth interpolation between its values at $Q^2 = 0$ determined by the Ward identity and at $Q^2 \to \infty$ determined by factorization. Since these values are not far from each other, the details of the interpolation are not essential.

Obviously, the LD model for the effective continuum is a crude model which does not take into account the details of the confinement dynamics. The only property of theory relevant for this model is factorization of hard form factors.

The model may be tested in quantum mechanics for the case of the potential containing the Coulomb and Confining interactions.

• **In this case the form factor satisfies the factorization theorem**

$$F(Q) \to F_\infty(Q) \equiv \frac{16\pi\,\alpha\, m\, R_g}{Q^4}, \qquad R_g \equiv |\Psi(r=0)|^2.$$

The LD sum rules is very similar to QCD; the spectral densities are calculated from 2- and 3-point diagrams of NR field theory.

• **The exact form factor may be calclated and confronted with the LD model thus probing its accuracy.**

We consider a set of confining potentials

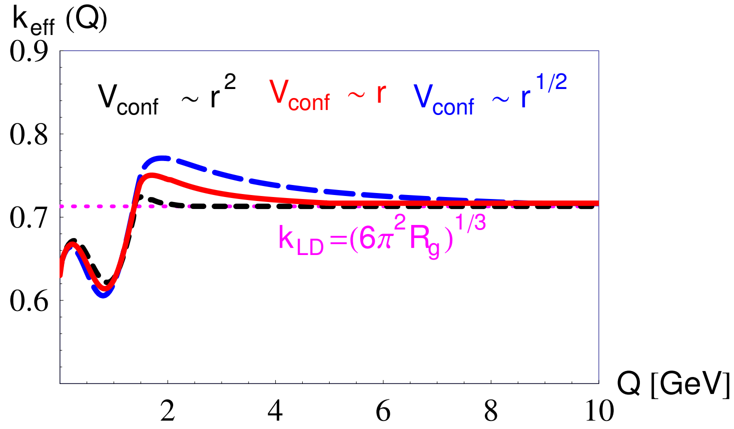
$$V_{\rm conf}(r) = \sigma_n\,(m\,r)^n, \qquad n = 2, 1, 1/2$$

and choose model parameters relevant for hadron physics: $m = 0.175$ **GeV – reduced constituent light-quark mass;** $\alpha = 0.3$**; the strengths** σ_n **of the confining potentials such that the Schrödinger equation for each of the potentials yields the same** $\Psi(r = 0) = 0.078$ **GeV**$^{-3/2}$**. This requires** $\sigma_2 = 0.71$**,** $\sigma_1 = 0.96$**,** $\sigma_{1/2} = 1.4$**, respectively.**

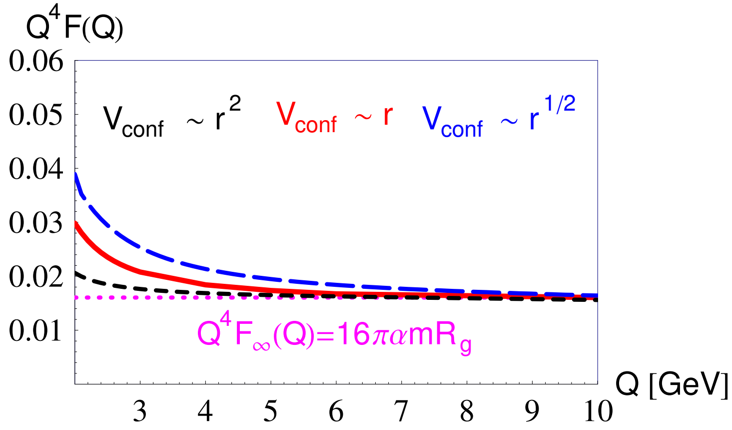
Then, the asymptotic behaviour of the form factor is the same for all three cases; respectively, the LD model for the form factor is also the same. The exact ffs at finite Q **are of course different.**

Results for quantum-mechanical potential model:

(a) Exact effective thresholds for various confining potentials; (b) The exact vs LD form factors



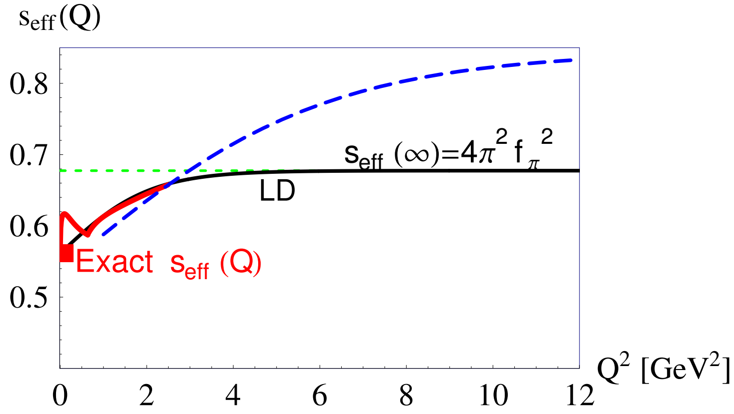
(a)

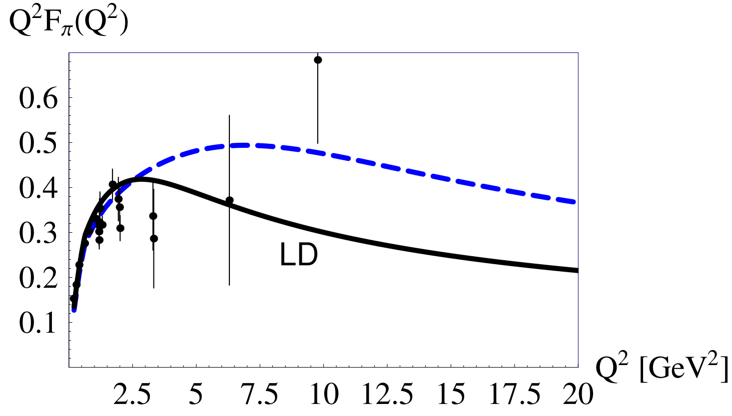


(b)

Results for the pion form factor in QCD:

(a) Various models for the effective threshold; (b) The corresponding F_π





Summary and conclusions

We investigated the elastic pion form factor by means of a LD model which may be formulated in any theory where hard exclusive amplitudes satisfy the factorization theorem (in essence, any theory where the interaction behaves as a Coloumb-like interaction at small distances and as a confining interaction at large distances).

Our main conclusions are as follows:

1. For $Q^2 \le 4$ GeV^2, the exact effective threshold exhibits a rapid variation with Q. Respectively, the accuracy of the LD model in this region depends on subtle details of the confining interaction and cannot be predicted in advance.

2. For $Q^2 \ge 4$ GeV^2, independently of the details of the confining interaction, the predictions of the LD model exhibit maximal deviations from the exact form factor in the region $Q^2 \approx 4 - 8$ GeV^2. As Q increases further, the accuracy of the LD model increases rather fast. For arbitrary confining interaction, the LD model gives very accurate results for $Q^2 \ge 20 - 30$ GeV^2.

3. The accurate data on the pion form factor indicate that the LD limit for the effective threshold $s_{\rm eff}(\infty) = 4\pi^2 f_\pi^2$ is reached already at relatively low values $Q^2 = 5 - 6$ GeV^2; thus, large deviations from the LD limit at $Q^2 = 20 - 50$ GeV^2 appear to us unlikely.

Obviously, our analysis does not provide a proof but rather an argument for the accuracy of the LD model in QCD and the expected behavior of the pion elastic form factor at large values of Q^2. Therefore, an accurate measurement of F_π in the range $Q^2 = 4 - 10$ GeV^2 will have important implications for the behavior of the form factor at larger Q^2, up to asymptotically large values.