Electric dipole transitions of heavy quarkonium

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Outline

1. Motivation
2. Basic formalism
   - Effective Field Theory approach to heavy quarkonium
   - Quarkonium states and transitions
3. E1 transitions
   - Definition & non-relativistic limit
   - Matching of the Lagrangian
   - Wave-function corrections
   - Results

Why should one study EM transitions?

- information about the quarkonium spectrum and the wave-functions
- significant contributions to the decay rate (at least for E1)
- new experimental data provided in the last and next few years (CLEO, BES, B factories)

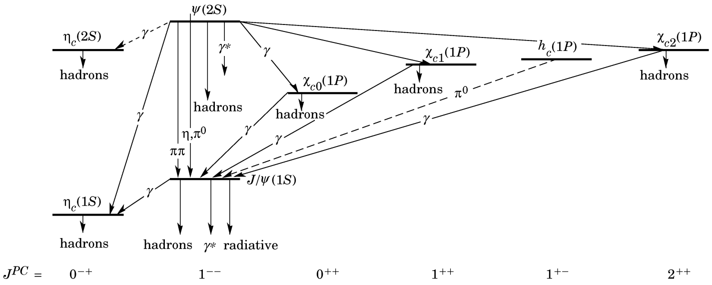

Figure: K. Nakamura et al. (PDG), J. Phys. G 37 (2010)

What has been done?

- phenomenological approach: QCD motivated potential models
  `Grotch et al., Phys. Rev. D 30 (1984)`
  `Eichten et al., Rev.Mod.Phys. 80 (2008)`
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  BUT: strict model-independent derivation missing, systematic procedure for relativistic corrections desirable
- lattice QCD (quenched):
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- EFT treatment of radiative decays: pNRQCD
  $\longrightarrow$ M1 transitions
  `Brambilla et al., Phys. Rev. D 73 (2006)`
  $\longrightarrow$ still missing: treatment of E1 transitions

Basic formalism

EFT for heavy quarkonium
Description of decay processes

Scales in quarkonium

- separation of scales in heavy quarkonium

$$m \gg p \sim mv \gg E \sim mv^2$$

where $v^2 \ll 1$ ($v^2 \approx 0.1$ for $b\bar{b}$, $v^2 \approx 0.3$ for $c\bar{c}$)
$\longrightarrow$ systematic treatment of relativistic corrections in powers of v
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- weakly coupled quarkonia ($E \gtrsim \Lambda_{QCD}$)
$\longrightarrow$ perturbative treatment with Coulomb potential at leading order (valid for the ground states J/ψ, $\Upsilon(1S)$, η_c, η_b)

$$\begin{aligned} \alpha_s(m) &\sim v^2 \\ \alpha_s(mv) &\sim v \\ \alpha_s(mv^2) &\sim 1 \end{aligned}$$

Effective field theories for quarkonium

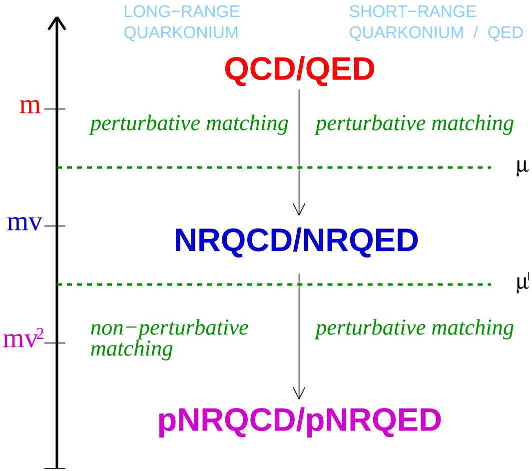

Figure: A. Vairo, arXiv 0902.3346 (2009)

NRQCD

- integrate out energy & momentum modes of order m from QCD
- Lagrangian

$$\begin{aligned}\mathcal{L} \;=\;& \varphi^\dagger\left(iD_0+\frac{\mathbf{D}^2}{2m}+\frac{\mathbf{D}^4}{8m^3}+\ldots\right)\varphi\\ &+g\varphi^\dagger\left(\frac{c_F}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}+i\frac{c_s}{8m^2}\boldsymbol{\sigma}\cdot[\mathbf{D}\times,\mathbf{E}]+\ldots\right)\varphi\\ &+ee_Q\varphi^\dagger\left(\frac{c_F^{em}}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}^{em}+i\frac{c_s^{em}}{8m^2}\boldsymbol{\sigma}\cdot[\mathbf{D}\times,\mathbf{E}^{em}]+\ldots\right)\varphi\\ &+c.c.+\mathcal{L}_{\rm light}+\mathcal{L}_{\rm YM}\end{aligned}$$

coefficients by matching with QCD

pNRQCD (for weak coupling)

- integrate out
  $\longrightarrow$ quarks with energy & momentum $\sim mv$
  $\longrightarrow$ gluons & photons of energy or momentum $\sim mv$
- new degrees of freedom: $Q\bar{Q}$ color singlet and octet fields

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- new degrees of freedom: $Q\bar{Q}$ color singlet and octet fields
- Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Bigg\{ & S^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_S \right) S \\
& + O^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_O \right) O \\
& + gV_A (O^\dagger \mathbf{r}\cdot\mathbf{E} S + S^\dagger \mathbf{r}\cdot\mathbf{E} O) \\
& + gV_B \frac{\{O^\dagger, \mathbf{r}\cdot\mathbf{E}\}}{2} O + \ldots \Bigg\} \\
& + \mathcal{L}_{\gamma\mathrm{pNRQCD}} + \mathcal{L}_{\mathrm{light}} + \mathcal{L}_{\mathrm{YM}}
\end{aligned}
$$

pNRQCD (for weak coupling)

- now: Only relevant degrees of freedom present
- high energy dynamics encoded in Wilson coefficients (obtained by matching with NRQCD at energy mv)
- definite power counting of operators

$$\begin{aligned} r &\sim 1/mv \\ \mathbf{E}, \mathbf{B} &\sim (mv^2)^2 \\ \mathbf{E}^{em}, \mathbf{B}^{em} &\sim k_\gamma^2 \\ \boldsymbol{\nabla} = \partial/\partial \mathbf{R} &\sim mv^2\,,\, k_\gamma \end{aligned}$$

Quarkonium states and transitions

- quarkonium state (leading Fock space component):

$$|H(\mathbf{P},\lambda)\rangle = \int d^3R \int d^3r\, e^{i\mathbf{P}\cdot\mathbf{R}} \mathrm{Tr}\left\{\phi_{H(\lambda)}(\mathbf{r}) \mathrm{S}^\dagger(\mathbf{r},\mathbf{R})|0\rangle\right\} ,$$

- at leading order:

$$H_S^{(0)}\phi^{(0)}_{H(\lambda)} = \left(-\frac{\boldsymbol{\nabla}_r{}^2}{m} + V_S^{(0)}\right)\phi^{(0)}_{H(\lambda)} = E^{(0)}_{H(\lambda)}\phi^{(0)}_{H(\lambda)}$$

- at higher orders: wave-function corrections due to higher order potentials and singlet-octet transitions

$\rightarrow$ calculation of decay rates for $H \rightarrow H'\gamma$ in CM frame

E1 Transitions

Work in progress
Formalism as for M1 transitions in N. Brambilla et al. (2006)

General properties

- definition: $\Delta S = 0, |\Delta L| = 1$
- change in parity, no change in C parity

Examples

$$1^3P_J \to 1^3S_1 \quad (\chi_c \to J/\psi\gamma\,,\, \chi_b \to \Upsilon(1S)\gamma)$$
$$1^1P_1 \to 1^1S_0 \quad (h_c \to \eta_c\gamma\,,\, h_b \to \eta_b\gamma)$$

- for the considered transitions: $k_\gamma \sim mv^2$

Nonrelativistic limit

- leading order operator for E1 transitions

$$\mathcal{L}_{E1} = ee_Q \int d^3r \, \mathrm{Tr} \left\{ S^\dagger \mathbf{r} \cdot \mathbf{E}^{em} S \right\}$$

Nonrelativistic decay rate

$$\Gamma_{n^3P_{J=0,1,2} \to n'^3S_1 \gamma} = \frac{4}{9} \alpha_{em} e_Q^2 k_\gamma^3 I_3^2(n1 \to n'0) \sim \frac{k_\gamma^3}{m^2 v^2}$$

$$I_3(n1 \to n'0) = \int_0^\infty dr \, r^3 R_{n'0}(r) R_{n1}(r)$$

- differences to M1 transitions:
  $\rightarrow$ leading order amplitude depends on the wave-function
  $\rightarrow$ enhancement of E1 transitions by factor $1/v^2$
- now: relativistic corrections of $\mathcal{O}(v^2)$

Relevant pNRQCD Lagrangian for decays of order k_γ^3/m^2

$$
\begin{aligned}
\mathcal{L}^{E1}_{\gamma\mathrm{pNRQCD}} = ee_Q \int d^3r\, \mathrm{Tr}\Big\{ & V^{r\cdot E} S^\dagger \mathbf{r}\cdot\mathbf{E}^{em} S + V_O^{r\cdot E} O^\dagger \mathbf{r}\cdot\mathbf{E}^{em} O \\
& + \frac{1}{24} V^{(r\nabla)^2 r\cdot E} S^\dagger \mathbf{r}\cdot(\mathbf{r}\boldsymbol{\nabla})^2 \mathbf{E}^{em} S \\
& + i\frac{1}{4m} V^{\nabla\cdot(r\times B)} S^\dagger \{\boldsymbol{\nabla}\cdot, \mathbf{r}\times\mathbf{B}^{em}\} S \\
& + i\frac{1}{12m} V^{\nabla_r\cdot(r\times(r\nabla)B)} S^\dagger \{\boldsymbol{\nabla}_r\cdot, \mathbf{r}\times(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em}\} S \\
& + \frac{1}{4m} V^{(r\nabla)\sigma\cdot B} [S^\dagger, \boldsymbol{\sigma}]\cdot(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em} S \\
& + \frac{1}{mr} V^{r\cdot E/r} S^\dagger \mathbf{r}\cdot\mathbf{E}^{em} S \\
& - i\frac{1}{4m^2} V^{\sigma\cdot(E\times\nabla_r)} [S^\dagger, \boldsymbol{\sigma}]\cdot(\mathbf{E}^{em}\times\boldsymbol{\nabla}_r) S \Big\}
\end{aligned}
$$

Tree level matching

- project NRQCD Hamiltonian onto the subspace spanned by $\psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t) \sim \varphi_\alpha(\mathbf{x}_1, t)\chi^\dagger_\beta(\mathbf{x}_2, t)$
- decompose $\psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t)$ into singlet and octet field components
- multipole expand in $r \ll 1/E$

Tree level results

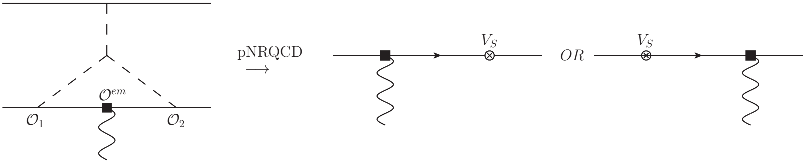
$$\begin{aligned} V_A = V^{r\cdot E} = V_O^{r\cdot E} = V^{(r\nabla)^2 r\cdot E} &= 1 \\ V^{\nabla\cdot(r\times B)} = V^{(r\nabla)\nabla_r\cdot(r\times B)} &= 1 \\ V^{(r\nabla)\sigma\cdot B} &= c_F^{em} \\ V^{\sigma\cdot(E\times\nabla_r)} &= c_S^{em} \\ V^{r\cdot E/r} &= 0\,. \end{aligned}$$

Beyond tree level

- matching of amplitudes order by order in $1/m$
- required for the perturbative matching:
  $\rightarrow \mathcal{O}(\alpha_s^2)$ corrections to $V^{r\cdot E}$
  $\rightarrow \mathcal{O}(\alpha_s)$ corrections to $V^{r\cdot E/r}$
- But: exact relations for all relevant coefficients can be obtained
- crucial argument: factorization of amplitudes into electromagnetic and gluonic terms

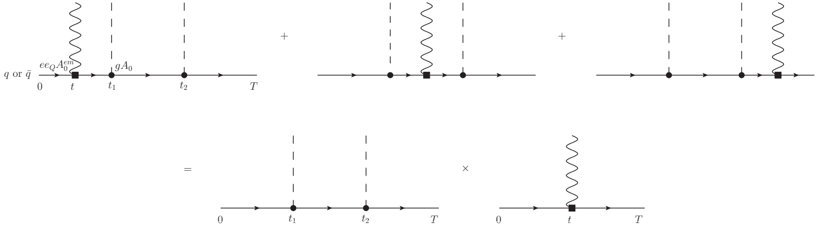
General factorization argument

$[\mathcal{O}^{em}, \mathcal{O}_1] = 0$ OR $[\mathcal{O}^{em}, \mathcal{O}_2] = 0$
$\Rightarrow$ the amplitude factorizes and gives no contribution to the matching of single operators



Matching of the electric dipole operator

Example: Exact matching of $V^{r\cdot E}$ possible (at order $1/m^0$)
Trivial factorization: $[A_0, A_0^{em}] = 0$



$\rightarrow$ $V^{r\cdot E} = 1$ to all orders in α_s

Similar arguments for all relevant operators
$\Rightarrow$ tree level results = exact results (for E1)

Wave-function corrections

- corrections due to higher order potentials to $\mathcal{O}(v^2)$

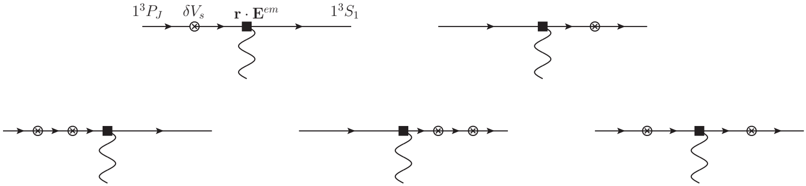
$$\begin{aligned}
\delta V_r^{(0)}(r) &= -\frac{C_F(\alpha_{V_s}(r) - \alpha_s(r))}{r} \\
V_r^{(1)}(r) &= -\frac{C_F C_A \alpha_s^2(r)}{2mr^2} \\
V_r^{(2)}(r) &= \frac{\pi C_F \alpha_s(r)}{m^2}\delta^{(3)}(\mathbf{r}) \\
V_{\mathbf{p}^2}^{(2)}(r) &= -\frac{C_F \alpha_s(r)}{2m^2}\left\{\frac{1}{r}, \mathbf{p}^2\right\} \\
V_{\mathbf{L}^2}^{(2)}(r) &= \frac{C_F \alpha_s(r)}{2m^2 r^3}\mathbf{L}^2 \\
V_{\mathbf{S}^2}^{(2)}(r) &= \frac{4\pi C_F \alpha_s(r)}{3m^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r}) \\
V_{\mathbf{LS}}^{(2)}(r) &= \frac{3C_F \alpha_s(r)}{2m^2 r^3}\mathbf{L}\cdot\mathbf{S} \\
V_{\mathbf{S}_{12}}^{(2)}(\hat{\mathbf{r}}) &= \frac{C_F \alpha_s(r)}{4m^2 r^3}\left[3(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1)(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]
\end{aligned}$$

Wave-function corrections

- relativistic kinetic energy correction

$$\delta H_s(r) = -\frac{\mathbf{p}^4}{4m^3}$$

- consider also running of α_s (as perturbation for fixed scale calculation)
- calculation with QM perturbation theory



Color-octet effects

- higher Fock space components via singlet-octet transitions

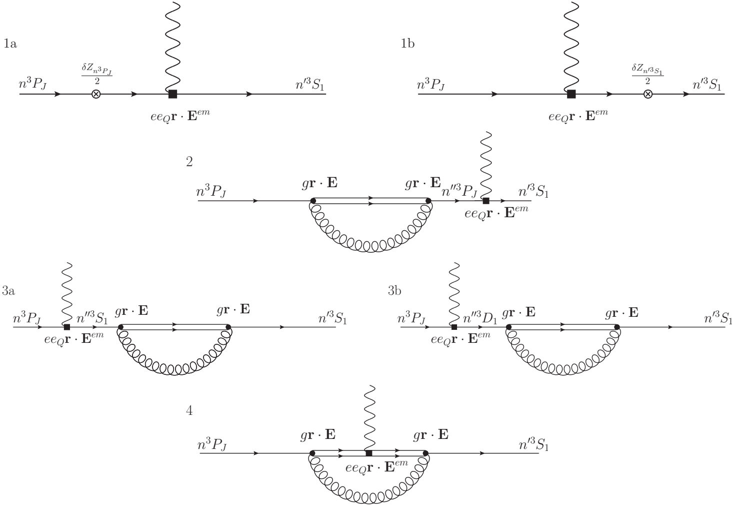
$$\mathcal{L} = \int d^3r \, \mathrm{Tr}\left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} S + S^\dagger \mathbf{r} \cdot g\mathbf{E} O \right\}$$

- not present in potential model approach
- no cancellation as for M1 transitions
- non-perturbative input (chromoelectric field correlators)

$$\langle 0 | \mathbf{E}^a(\mathbf{R}, t) \phi(t, 0)^{\mathrm{adj}}_{ab} \mathbf{E}^b(\mathbf{R}, 0) | 0 \rangle$$

Color-octet effects

Example: $n^3P_J \to n'^3S_1$



Strong coupling case

- strongly coupled quarkonia ($p \gtrsim \Lambda_{QCD}$)
  $\rightarrow$ nonperturbative treatment with confining potential at leading order (valid for excited states χ_c, χ_b,...)
- nonperturbative potentials taken from lattice simulations
- no octet fields
- matching for the relevant operators as before
- for $\Lambda_{QCD} \sim mv$ new operators become relevant

Results

Final formula for $n^3P_J \to n'^3S_1$

$$\Gamma_{E1} = \Gamma_{E1}^{(0)} \left(1 + R - \frac{k_\gamma^2}{60}\frac{I_5}{I_3} - \frac{k_\gamma}{6m} + \frac{k_\gamma(c_F^{em} - 1)}{2m}\left[\frac{J(J+1)}{2} - 2\right]\right)$$

$$I_N(n1 \to n'0) = \int_0^\infty dr\, r^N R_{n'0}(r) R_{n1}(r)$$

$R \to$ wave function corrections

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$$I_N(n1 \to n'0) = \int_0^\infty dr\, r^N R_{n'0}(r) R_{n1}(r)$$

$R \to$ wave function corrections

- comparison with potential models (Grotch): equivalence to the given order, but:
  $\to$ range of validity ($E \gtrsim \Lambda_{QCD}$)
  $\to$ systematic inclusion of relativistic corrections (including $V_r^{(1)}$)
  $\to$ color-octet effects included for weak coupling
- similar for $n^1P_1 \to n^1S_0$ (without spin-dependent terms)

Conclusion and Outlook

- Summary:
  EFT treatment for E1 transitions up to $\mathcal{O}(v^2)$-corrections
  $\longrightarrow$ relevant Lagrangian: exact matching for all operators
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  $\longrightarrow$ relevant Lagrangian: exact matching for all operators
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- Outlook:
  $\longrightarrow$ evaluation of octet effects
  $\longrightarrow$ numerical calculation with perturbative potentials for short and nonperturbative ones for long distances
  $\longrightarrow$ full strong coupling analysis for higher excited states

Thank you for your attention!

Wave-functions

- S-wave states

$$\phi^{(0)}_{n^1S_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)$$

$$\phi^{(0)}_{n^3S_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)\,\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}_{n^3S_1}(\lambda)$$

- P-wave states

$$\phi^{(0)}_{n^1P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\,\hat{\mathbf{e}}_{n^1P_1}(\lambda)\cdot\hat{\mathbf{r}}$$

$$\phi^{(0)}_{n^3P_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n1}(r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}$$

$$\phi^{(0)}_{n^3P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{16\pi}} R_{n1}(r)\,\boldsymbol{\sigma}\cdot\hat{(}\mathbf{r}\times\hat{\mathbf{e}}_{n^3P_1}(\lambda))$$

$$\phi^{(0)}_{n^3P_2(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\,\boldsymbol{\sigma}^i h^{ij}_{n^3P_2}(\lambda)\hat{\mathbf{r}}^j\,.$$

General non-relativistic formula

$$\Gamma^{(0)}_{n^{2s+1}L_J \to n'^{2s+1}L'_{J'},\gamma} = \frac{4}{3}\,\alpha_{em} e_Q^2 (2J'+1) S^{E1} k_\gamma^3 I_3^2(nl \to n'l')$$

$$S^{E1} = \max(l,l') \left\{ \begin{array}{ccc} J & 1 & J' \\ l' & s & l \end{array} \right\}^2$$

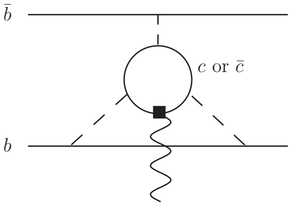
$$I_3(nl \to n'l') = \int_0^\infty dr\, r^3 R_{n'l'}(r) R_{n1}(r)$$

Light quark effects

- Loop effects with electromagnetic coupling to u, d and s cancel

$$q_u + q_d + q_s = 0$$

- charm quark effects for bottomonium
  $\longrightarrow$ leading order diagram highly suppressed



$\longrightarrow$ furthermore: decoupling at typical momentum scale
`Brambilla, N. et al., Phys.Rev. D65 (2002), 034001`

Lineshape of the h_b

Decay $h_b \to \eta_b\gamma \to X\gamma$, resonance in the photon spectrum observable

Lineshape from pNRQCD calculation:

$$\frac{d\Gamma_{h_b}}{dE_\gamma} = \frac{4\alpha_{em}}{81\pi} I_3^2(11 \to 10) E_\gamma^3 \frac{\Gamma_{\eta_b}/2}{(E_\gamma^{\text{peak}} - E_\gamma)^2 + \Gamma_{\eta_b}^2/4} .$$

with $E_\gamma^{\text{peak}} \approx E_{h_b} - E_{\eta_b}$

$\to$ modified Breit-Wigner curve