πN scattering in relativistic BChPT revisited

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Part I

Introduction

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- There have been many attempts to study this process using ChPT, but every one has had their own problems:
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  - HBChPT **[Jenkins and Manohar, PLB 255 (1991) 558]** : Lorentz invariance is lost, does not converge in the subthreshold region **[Bernard, Kaiser, Meissner, Int.J.Mod.Phys.E4:193-346,1995]**, **[T. Becher and H. Leutwyler, JHEP 0106 (2001) 01]** $\Rightarrow$ We cannot check Chiral symmetry predictions for QCD.
- For our study we used Infrared Regularization scheme (IR). This scheme solves the power counting problem keeping manifest Lorentz invariance. **[Becher and Leutwyler, EPJC 9 (1999) 643]**

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    - The one-loop representation is not precise enough to allow a sufficiently accurate extrapolation of the physical data to the Cheng-Dashen point.
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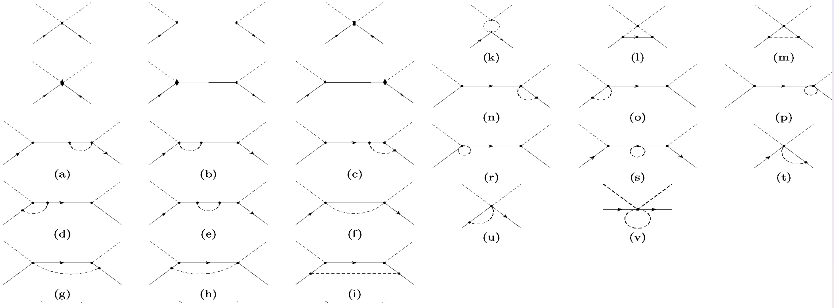
Part II

Perturbative Calculations

Perturbative Calculations

From the usual power counting, we have the following contributions:



In order to obtain the LECs we consider:

- PWA of the Karlsruhe group (KA85) **[Koch, NPA 448 (1986) 707]**
- Current PWA of the GWU group (WI08) **[R. A. Arndt et al., PRC 74 (2006) 045205. solution SM01].**

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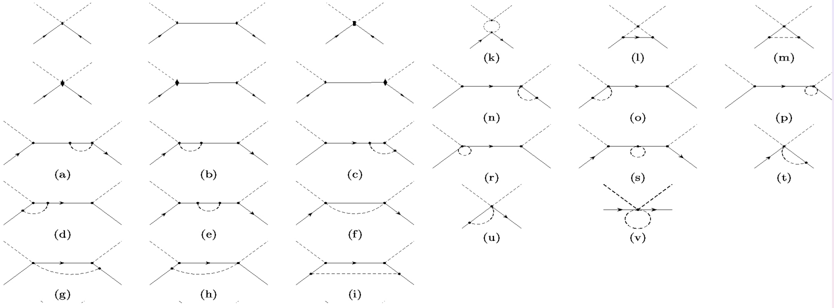

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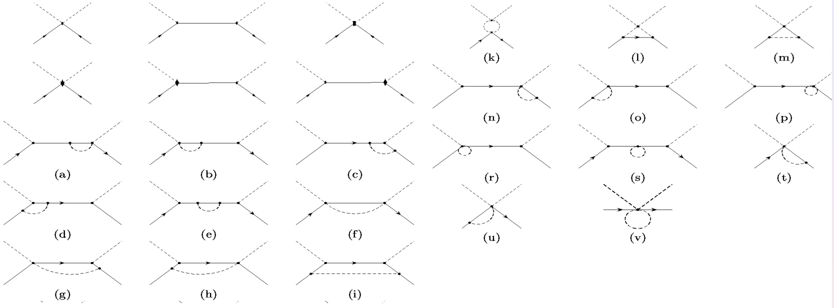

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Perturbative Fits

We consider two strategies to fit the KA85 and WI08 data.

- First strategy (KA85-1, WI08-1):
  - Fit phase shifts up to $\sqrt{s}_{max} = 1.13$ GeV .
  - We use the standard χ^2
- Second strategy (KA85-2, WI08-2):
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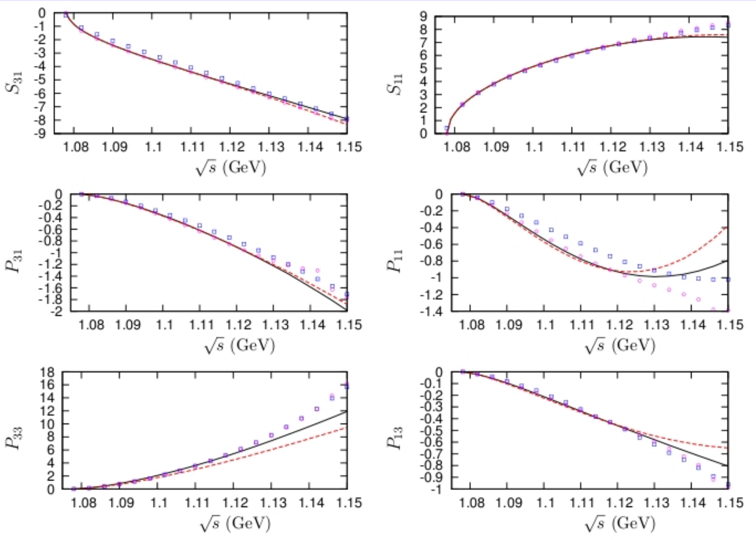
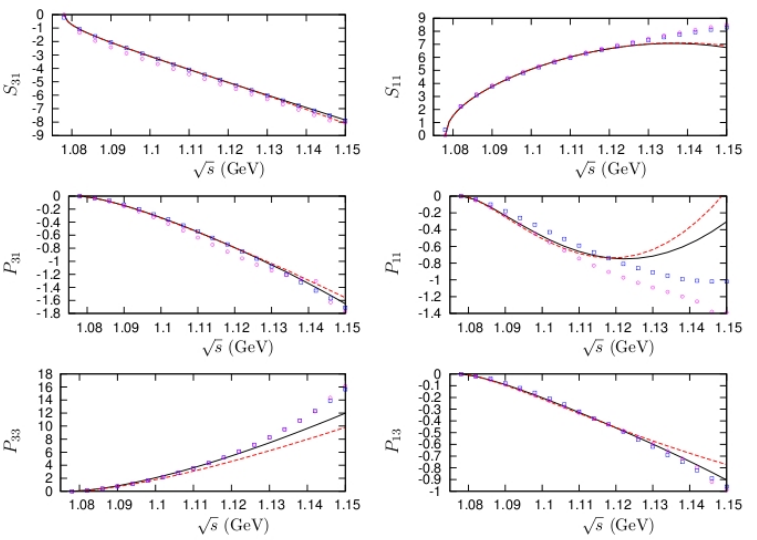
KA85 Fits



Solid line: KA85-1. Dashed line: KA85-2.

WI08 Fits



Solid line: WI08-1. Dashed line: WI08-2.

LECs Summary

Results for the LECs:

LEC	KA85-1	KA85-2	WI08-1	WI08-2	Average
c_1	-0.71 ± 0.49	-0.79 ± 0.51	-0.27 ± 0.51	-0.30 ± 0.48	-0.52 ± 0.60
c_2	4.32 ± 0.27	3.49 ± 0.25	4.28 ± 0.27	3.55 ± 0.30	3.91 ± 0.54
c_3	-6.53 ± 0.33	-5.40 ± 0.13	-6.76 ± 0.27	-5.77 ± 0.29	-6.12 ± 0.72
c_4	3.87 ± 0.15	3.32 ± 0.13	4.08 ± 0.13	3.60 ± 0.16	3.72 ± 0.37
$d_1 + d_2$	2.48 ± 0.59	0.94 ± 0.56	2.53 ± 0.60	1.16 ± 0.65	1.78 ± 1.1
d_3	-2.68 ± 1.02	-1.10 ± 1.16	-3.65 ± 1.01	-2.32 ± 1.04	-2.44 ± 1.6
d_5	2.69 ± 2.20	1.86 ± 2.28	5.38 ± 2.40	4.83 ± 2.18	3.69 ± 2.93
$d_{14} - d_{15}$	-1.71 ± 0.73	1.03 ± 0.71	-1.17 ± 1.00	1.27 ± 1.11	-0.145 ± 1.88
d_{18}	-0.26 ± 0.40	-0.07 ± 0.44	-0.86 ± 0.43	-0.72 ± 0.40	-0.48 ± 0.58

- Following a conservative procedure, the error given in the average is the sum in quarature of the largest statistical error and the one resulting from the dispersion in the central values.
- The average is compatible with those from $\mathcal{O}(p^3)$ HBChPT, except for the $d_{14} - d_{15}$ that differs by more than one standard deviation.

LECs Comparision

LEC	Average	HBCHPT $\mathcal{O}(p^3)$ **[1]**	HBCHPT Disp. **[2]**	HBCHPT $\mathcal{O}(p^3)$ **[3]**	RS **[3]**
c_1	-0.52 ± 0.60	$(-1.71, -1.07)$	-0.81 ± 0.12	-1.02 ± 0.06	
c_2	3.91 ± 0.54	$(3.0, 3.5)$	8.43 ± 56.9	3.32 ± 0.03	3.9
c_3	-6.12 ± 0.72	$(-6.3, -5.8)$	-4.70 ± 1.16	-5.57 ± 0.05	-5.3
c_4	3.72 ± 0.37	$(3.4, 3.6)$	3.40 ± 0.04		3.7
$d_1 + d_2$	1.78 ± 1.1	$(3.2, 4.1)$			
d_3	-2.44 ± 1.6	$(-4.3, -2.6)$			
d_5	3.69 ± 2.93	$(-1.1, 0.4)$			
$d_{14} - d_{15}$	-0.145 ± 1.88	$(-5.1, -4.3)$			
d_{18}	-0.48 ± 0.58	$(-1.6, -0.5)$			

[1] N. Fettes, U. G. Meißner and S. Steininger, Nucl. Phys. A 640 (1998) 199.

[2] P. Buettiker and U. G. Meißner, Nucl. Phys. A 668 (2000) 97.

[3] V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A 615 (1997) 483.

Threshold parameters summary

In order to obtain the scattering lengths and volumes we performed an effective range expansion (ERE) fit to our results in the low energy region, because numerical poblems prevent us to take directly the limit: $\lim_{|\vec{p}|\to 0}|\vec{p}|\frac{\mathrm{Re}\,T}{8\pi\sqrt{s}|\vec{p}|^{1+2\ell}}$

Partial Wave	KA85-1	KA85-2	WI08-1	WI08-2	Average
$a_{S_{31}}$	-0.100 ± 0.001	-0.103 ± 0.001	-0.081 ± 0.001	-0.082 ± 0.001	-0.092 ± 0.012
$a_{S_{11}}$	0.171 ± 0.001	0.172 ± 0.002	0.165 ± 0.002	0.167 ± 0.002	0.169 ± 0.004
a_{0+}^{+}	-0.010 ± 0.001	-0.011 ± 0.001	0.001 ± 0.001	0.001 ± 0.001	-0.005 ± 0.007
a_{0+}^{-}	0.090 ± 0.001	0.092 ± 0.001	0.082 ± 0.001	0.083 ± 0.001	0.087 ± 0.005
$a_{P_{31}}$	-0.052 ± 0.001	-0.051 ± 0.001	-0.048 ± 0.001	-0.051 ± 0.001	-0.051 ± 0.002
$a_{P_{11}}$	-0.078 ± 0.001	-0.088 ± 0.001	-0.073 ± 0.001	-0.080 ± 0.001	-0.080 ± 0.006
$a_{P_{33}}$	0.251 ± 0.002	0.214 ± 0002	0.252 ± 0.002	0.222 ± 0.002	0.232 ± 0.017
$a_{P_{13}}$	-0.034 ± 0.001	-0.035 ± 0.001	-0.032 ± 0.001	-0.035 ± 0.001	-0.034 ± 0.002

Threshold parameters comparision

Results for the threshold parameters:

Partial Wave	Average	KA85	WI08
$a_{S_{31}}$	-0.092 ± 0.012	-0.100 ± 0.004	-0.084
$a_{S_{11}}$	0.169 ± 0.004	0.175 ± 0.003	0.171
a_{0+}^{+}	-0.005 ± 0.007	-0.008	-0.0010 ± 0.0012
a_{0+}^{-}	0.087 ± 0.005	0.092	0.0883 ± 0.0005
$a_{P_{31}}$	-0.051 ± 0.002	-0.044 ± 0.002	-0.038
$a_{P_{11}}$	-0.080 ± 0.006	-0.078 ± 0.002	-0.058
$a_{P_{33}}$	0.232 ± 0.017	0.214 ± 0.002	0.194
$a_{P_{13}}$	-0.034 ± 0.002	-0.030 ± 0.002	-0.023

- None of our fits (KA85-1,KA85-2,WI08-1,WI08-2) is compatible with the value of $a_{P_{11}}$ given by WI08

Goldberger-Trieman relation

The value of d_{18} is important because is directly related to the violation of the Goldberger-Trieman (GT) relation. One has, up to $\mathcal{O}(M_\pi^3)$:

$$g_{\pi N} = \frac{g_A m}{F_\pi}\left(1 - \frac{2M_\pi^2 d_{18}}{g_A}\right)$$

We quantify the deviaton from the GT relation by:

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For our averaged value of d_{18} we have:

$$\Delta_{GT} = 0.015 \pm 0.018$$

Which is compatible with the values around $2-3\%$ obtained from πN and NN partial wave analyses **[Arndt, Workman and Pavan, PRC 49 (1994) 2729]**, **[Schröder et al],[Swart, Rentmeester and Timmermans, πN Newsletter 13 (1997)96]**.

This value of Δ_{GT} gives:

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- For the fit KA85-1 one has a 22% of violation for $\mu = 1$ GeV (scale) while for $\mu = 0.5$ GeV a 15% stems.

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Part III

Unitarized Calculations

Unitarized Calculations

In order to implement unitarity to the πN amplitude and take care of the analyticity properties associated with the right-hand cut we write our unitarized amplitude $T_{IJ\ell}$ by means of an interaction kernel $\mathcal{T}_{IJ\ell}$ and the unitary pion-nucleon loop function $g(s)$:

$$T_{IJ\ell} = \frac{1}{\mathcal{T}_{IJ\ell}^{-1} + g(s)}$$

- $T_{IJ\ell}$ satisfies unitarity exactly.
- The interaction kernel is determined order by order by matching with the perturbative ChPT result **[J. A. Oller and U. G. Meißner, PLB 500:263-272 (2001)]**.
- a_1 is fixed by requiring $g(m^2) = 0$ (in order to have the P_{11} nucleon pole in its right position).

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We introduce the contribution of the $\Delta(1232)$ in P_{33} through a CDD **[Castillejo, Dalitz and Dyson, PR 101 (1956) 453]**, **[Oller and Oset, PRD 60, 074023 (1999)]**:

- The CDD pole conserves the discontinuities of the partial wave amplitude across the cuts.
- The CDD pole corresponds to a zero of the partial wave amplitude along the real axis and hence to a pole in the inverse of the amplitude.

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- Strong violation of unitarity.
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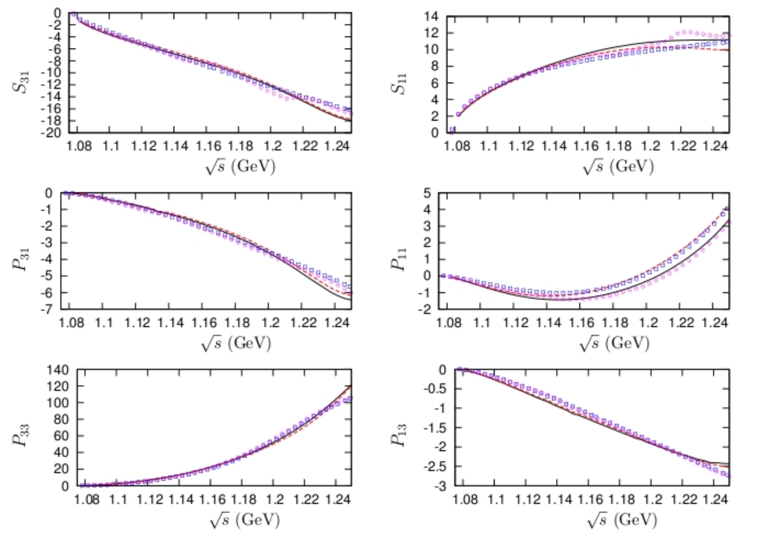

Solid line: Fit to KA85 data. Dashed line: Fit to WI08 data.

Unitarized Calculations

LEC	Fit KA85	Fit WI08	Average (Perturbative)	Partial Wave	Fit KA85	Fit WI08	Average (Perturbative)
c_1	-0.48 ± 0.51	-0.52 ± 0.60	-0.53 ± 0.48	$a_{S_{31}}$	-0.115	-0.104	-0.092 ± 0.012
c_2	4.62 ± 0.27	4.73 ± 0.30	3.91 ± 0.54	$a_{S_{11}}$	0.152	0.150	0.169 ± 0.004
c_3	-6.16 ± 0.27	-6.41 ± 0.29	-6.12 ± 0.72	a^+_{0+}	-0.026	-0.020	-0.005 ± 0.007
c_4	3.68 ± 0.13	3.81 ± 0.16	3.72 ± 0.37	a^-_{0+}	0.089	0.085	0.087 ± 0.005
$d_1 + d_2$	2.55 ± 0.60	2.70 ± 0.65	1.78 ± 1.1	$a_{P_{31}}$	-0.050	-0.048	-0.051 ± 0.002
d_3	-1.61 ± 1.01	-1.73 ± 1.04	-2.44 ± 1.6	$a_{P_{11}}$	-0.080	-0.075	-0.080 ± 0.006
d_5	0.93 ± 2.40	1.13 ± 2.18	3.69 ± 2.93	$a_{P_{33}}$	0.245	0.250	0.232 ± 0.017
$d_{14} - d_{15}$	-0.46 ± 1.00	-0.61 ± 1.11	-0.145 ± 1.88	$a_{P_{13}}$	-0.41	-0.039	-0.034 ± 0.002
d_{18}	0.01 ± 0.21	-0.03 ± 0.20	-0.48 ± 0.58				

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- The values of these LECs do not constitute an alternative determination to the perturbative results.
- These values only should be employed within UChPT studies.
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Part IV

Summary and Conclusions

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- Perturbative calculations:
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  - An accurate reproduction of the phase-shifts was obtained up to 1.14 GeV, similar in quality to that obtained previously with $\mathcal{O}(p^3)$ HBChPT $\Rightarrow$ **Improvement** compared with previous works.
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Part V

EOMS

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- This scheme removes explicity the power counting breaking terms appearing in the loop integrals in dimensional regularization.
- These PCBT terms are absorbed in the LECs (IR result).
- Preliminar results:
  - Better description of phase shifts.
  - Scale independence.
  - Small GT violation $\rightarrow$ Compatible with PWA determinations **[Arndt, Workman and Pavan, PRC 49 (1994) 2729]**, **[Schröder et al]**, **[Swart, Rentmeester and Timmermans, πN Newsletter 13 (1997)96]**.
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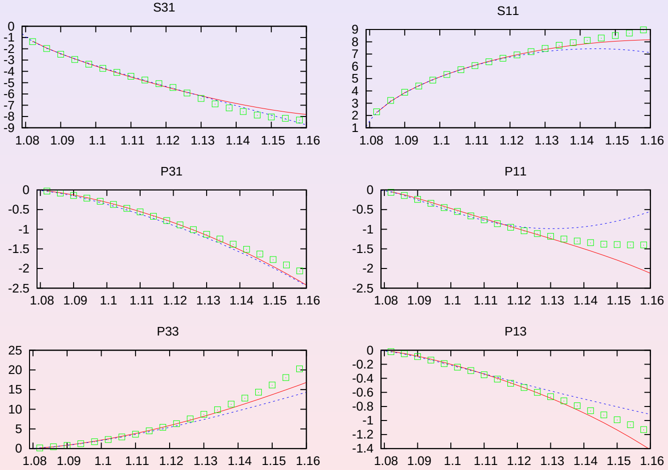

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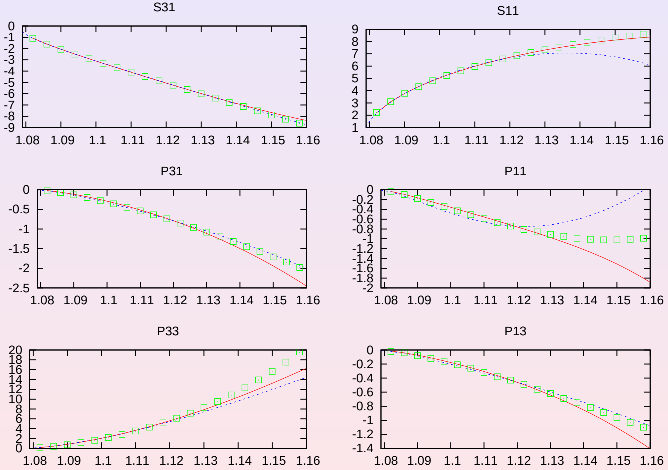
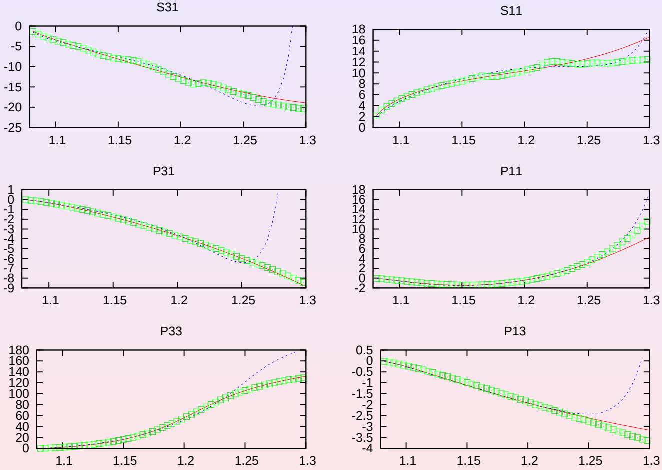


EOMS-KA85 (unitarized)



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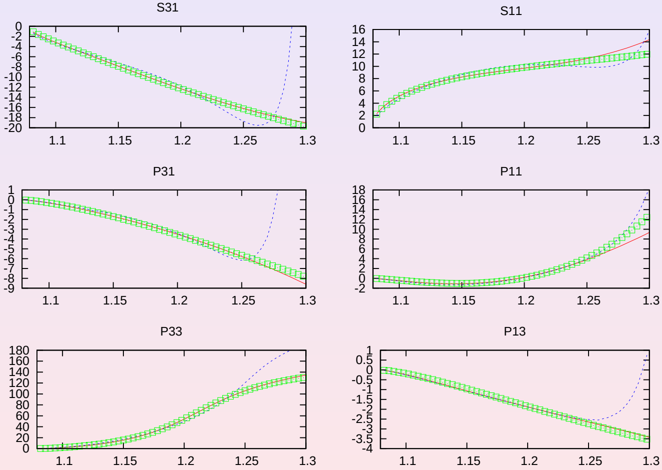

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