The size of the proton

from the Lamb shift in muonic hydrogen

for the *CREMA* collaboration

Randolf Pohl

Max-Planck-Institut für Quantenoptik
Garching, Germany

Outline

- The problem:

  Proton rms charge radius r_{p} from muonic hydrogen $\mu\mathrm{p}$ is 4 % smaller than the values from elastic electron-proton scattering and hydrogen spectroscopy.

  That's 5σ ... 9.4σ.

  But the $\boldsymbol{\mu}$**p** result is 10 times more accurate than any other measurement.

- Introduction:

  Hydrogen, fundamental constants, QED tests and all that.

  How large is the proton?

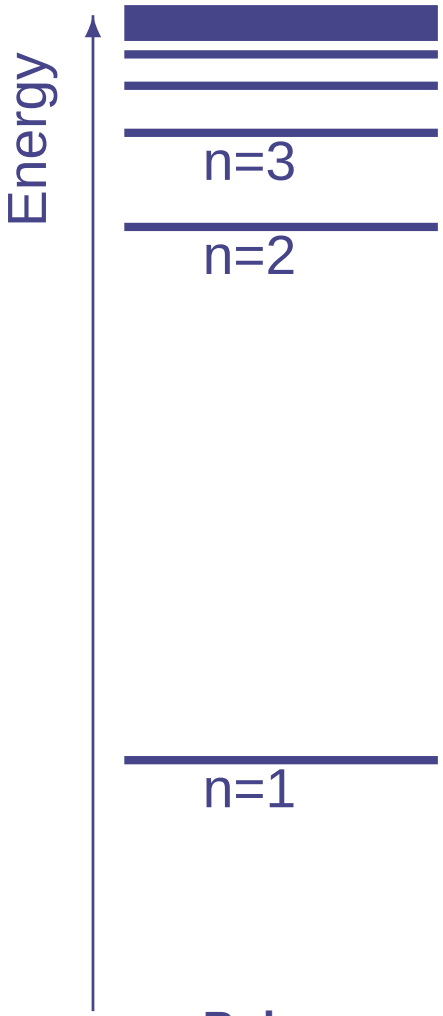
- Muonic hydrogen:

  (Finite) size does matter!

- Experiment:
  - Principle
  - Muon beam
  - Laser system

- A solution of the "proton size puzzle"

Hydrogen energy levels

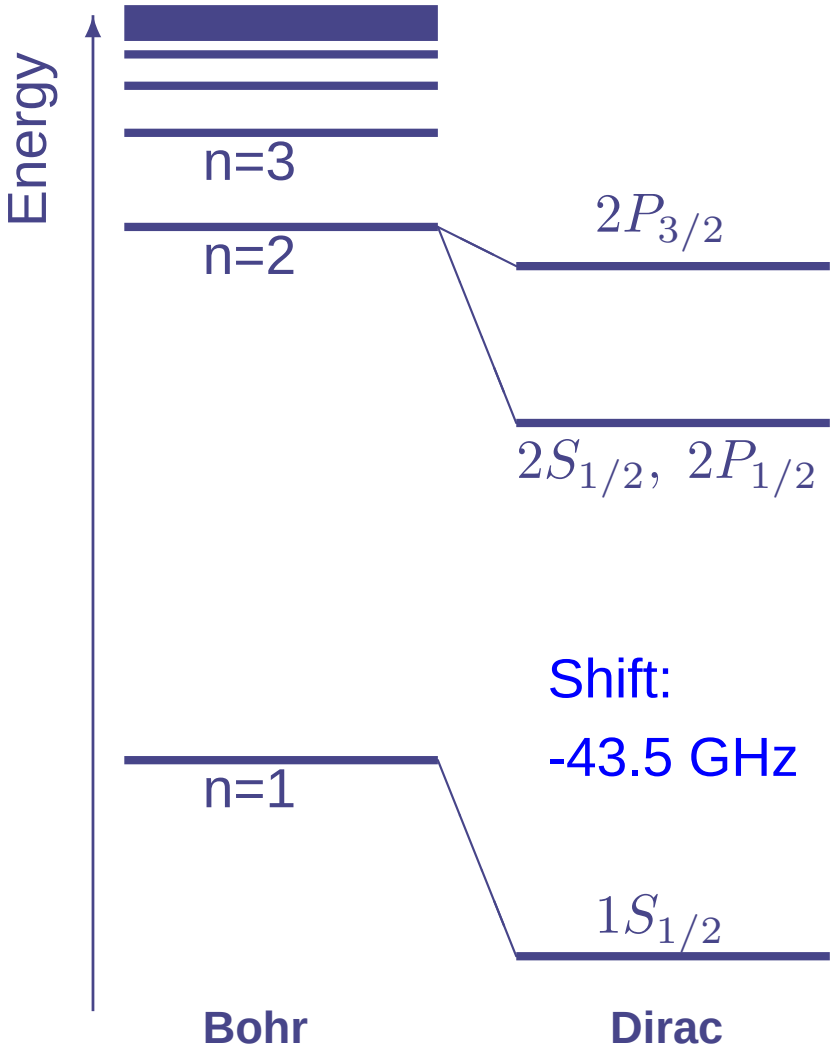


Bohr

$$E = R_\infty / n^2$$

$$V \sim 1/r$$

Hydrogen energy levels



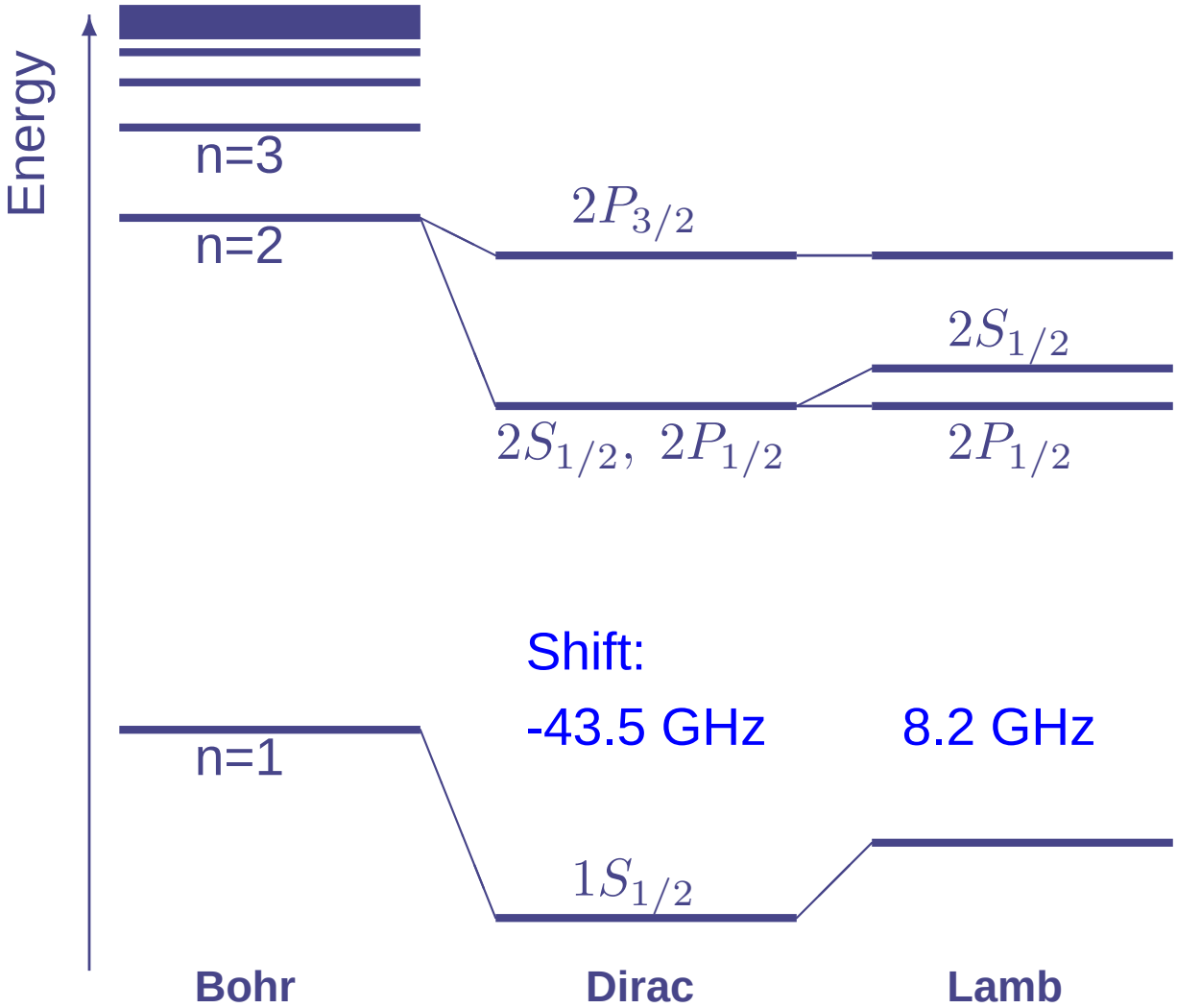
$E = R_\infty / n^2$

$V \sim 1/r$

e^- spin

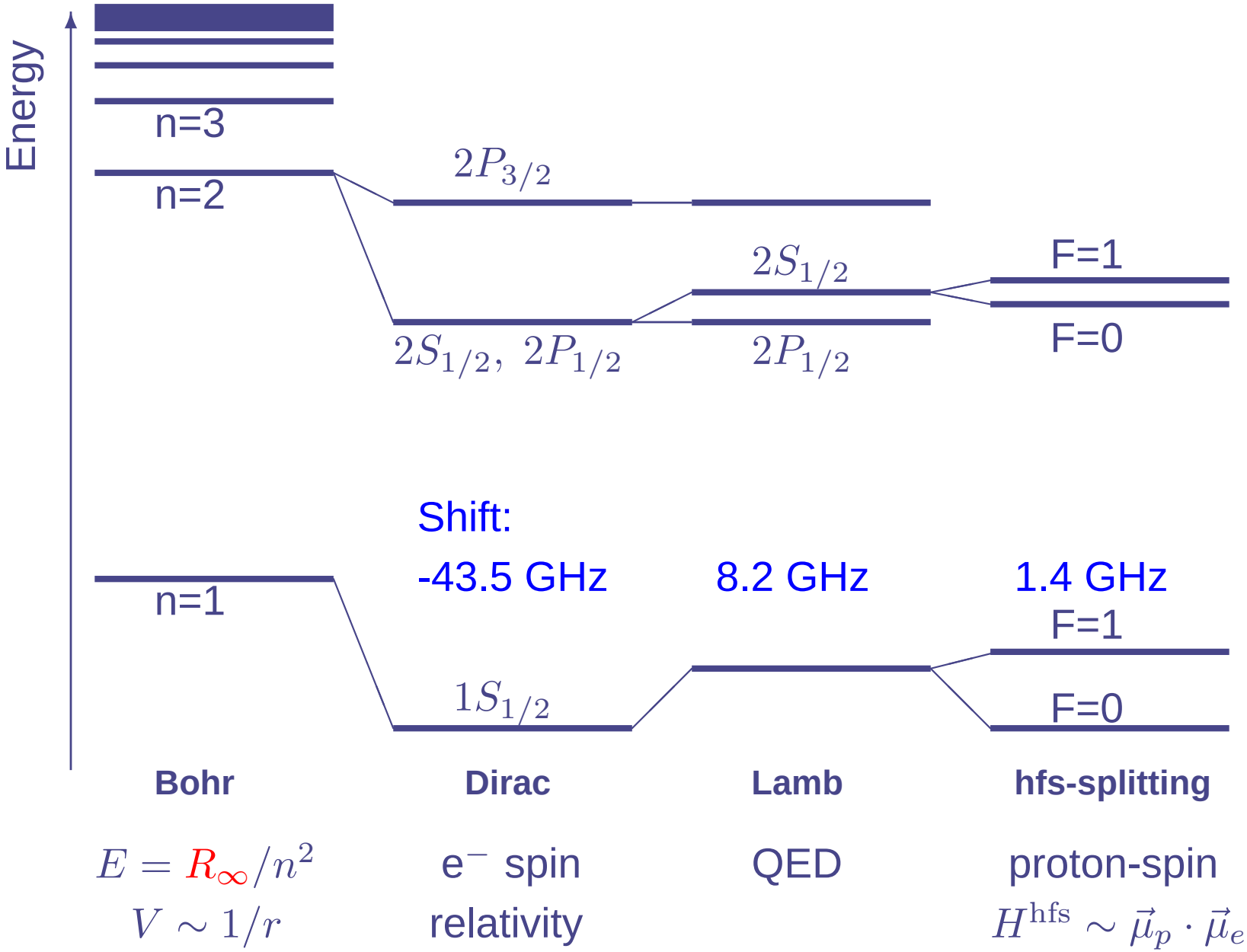
relativity

Hydrogen energy levels

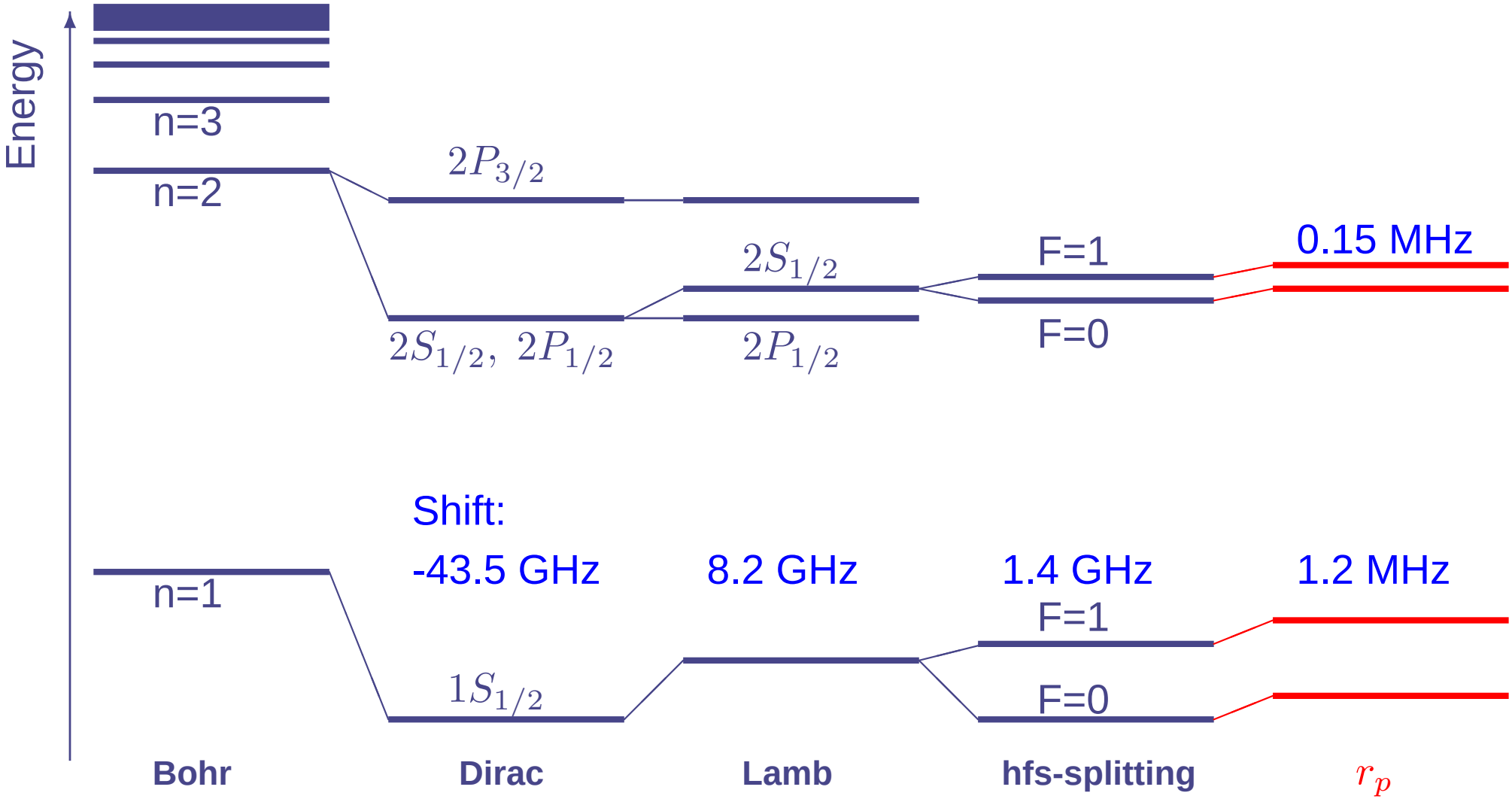


Bohr	Dirac	Lamb
$E = R_\infty/n^2$	e$^-$ spin	QED
$V \sim 1/r$	relativity	

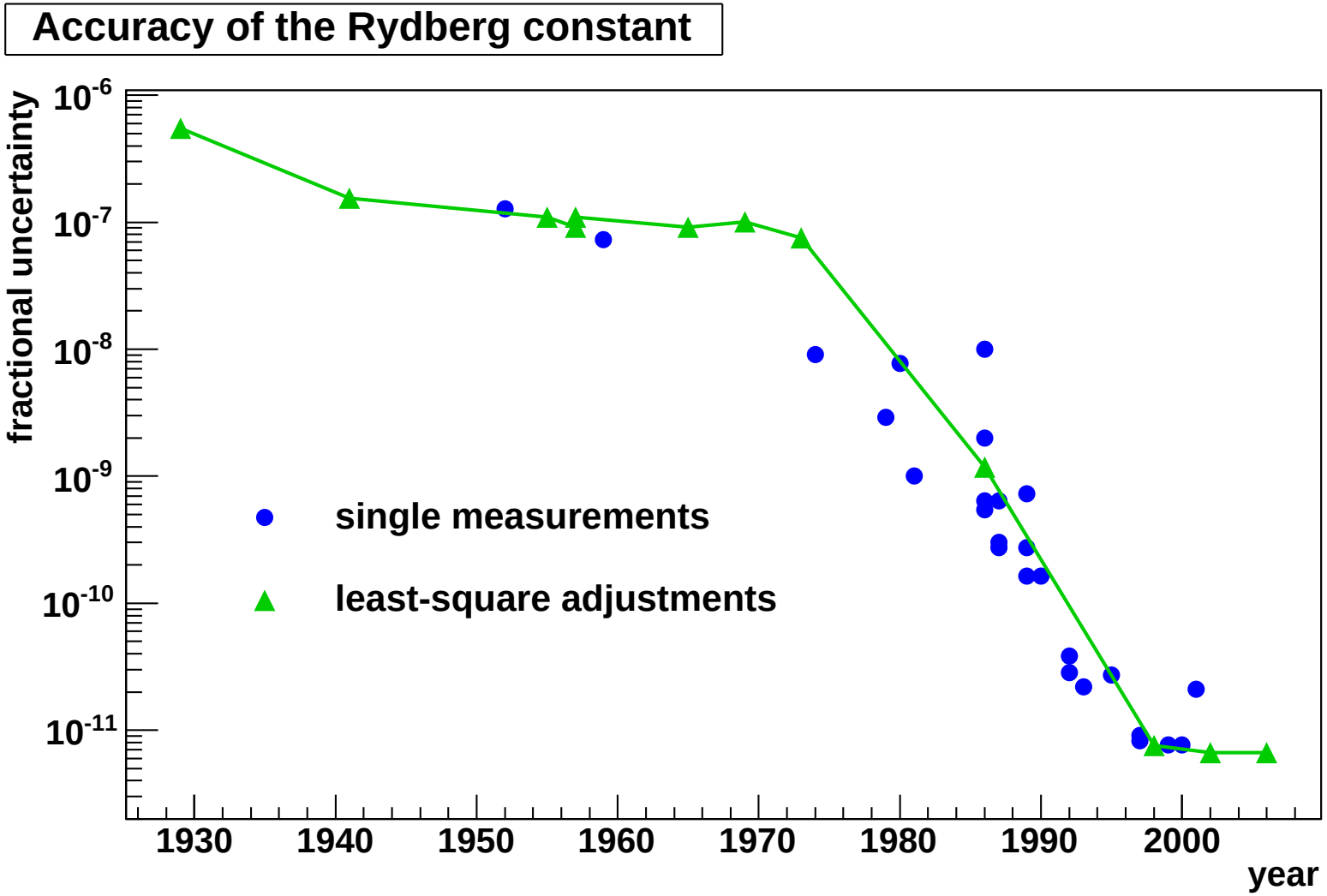
Hydrogen energy levels



Hydrogen energy levels



The Rydberg constant

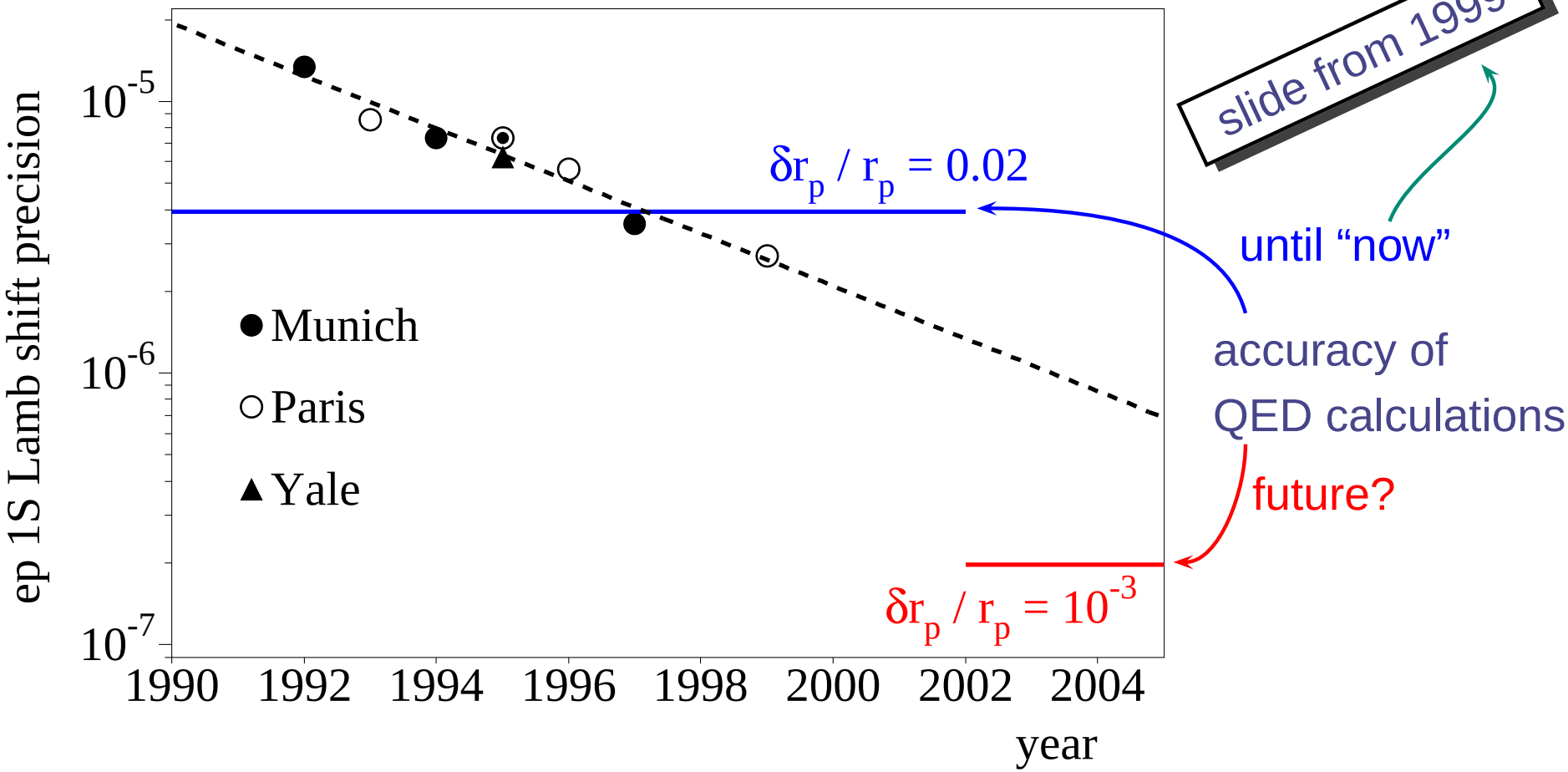


2006: R_∞ = $10\,973\,731.568\,525 \pm 0.000\,073\,\mathrm{m}^{-1}$ ($u_r = 6.6 \cdot 10^{-12}$)

is the 2nd most accurately determined fundamental constant.

Test of bound-state QED

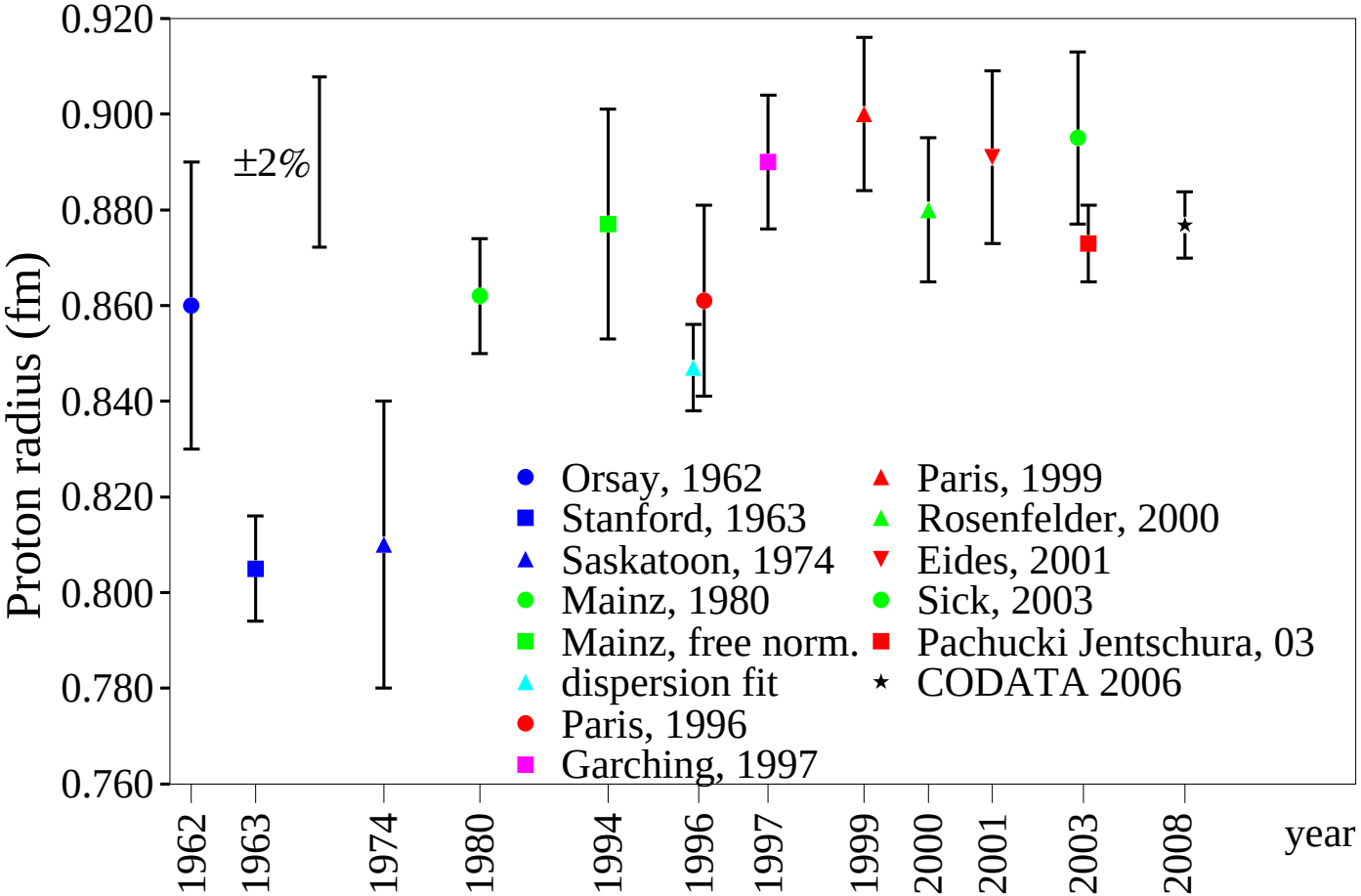
1S Lamb shift in hydrogen: $L_{1S}(r_p) = 8171.636(4) + 1.5645\,\langle r_p^2 \rangle$ MHz



QED-test is limited by the uncertainty of the proton rms charge radius.

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



electron scattering

slope of G_E at $Q^2 = 0$

hydrogen spectr.

Lamb shift (S-states)

e-p scattering: r_p = 0.895(18) fm ($u_r = 2\,\%$)

CODATA: r_p = 0.8768(69) fm ($u_r = 0.8\,\%$)

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.

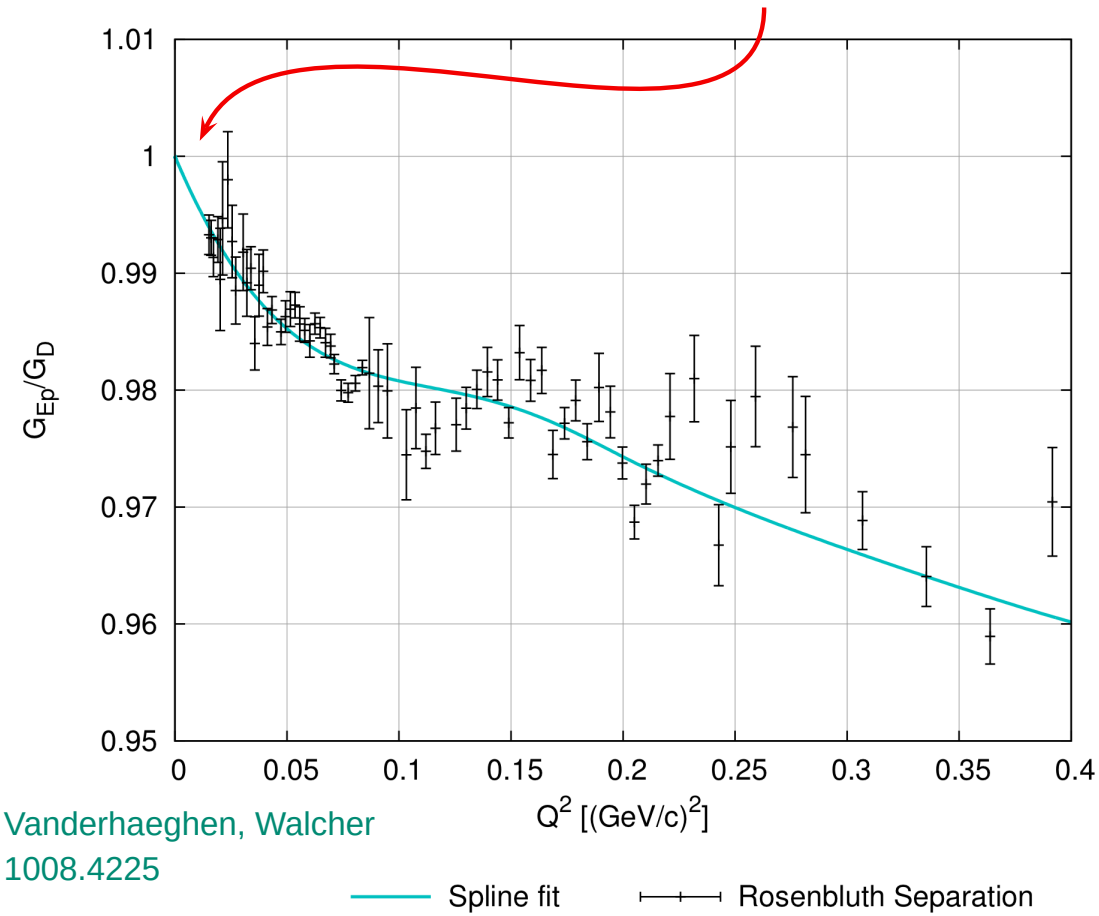
Proton radius (fm)

0.920
0.900
0.880
0.860
0.840
0.820
0.800
0.780
0.760

e-p
COD

Electron scattering:

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \Rightarrow \boxed{\text{slope of } G_E \text{ at } Q^2 = 0}$$



Vanderhaeghen, Walcher
1008.4225

- electron scattering
  slope of G_E at $Q^2 = 0$
- hydrogen spectr.
  Lamb shift (S-states)

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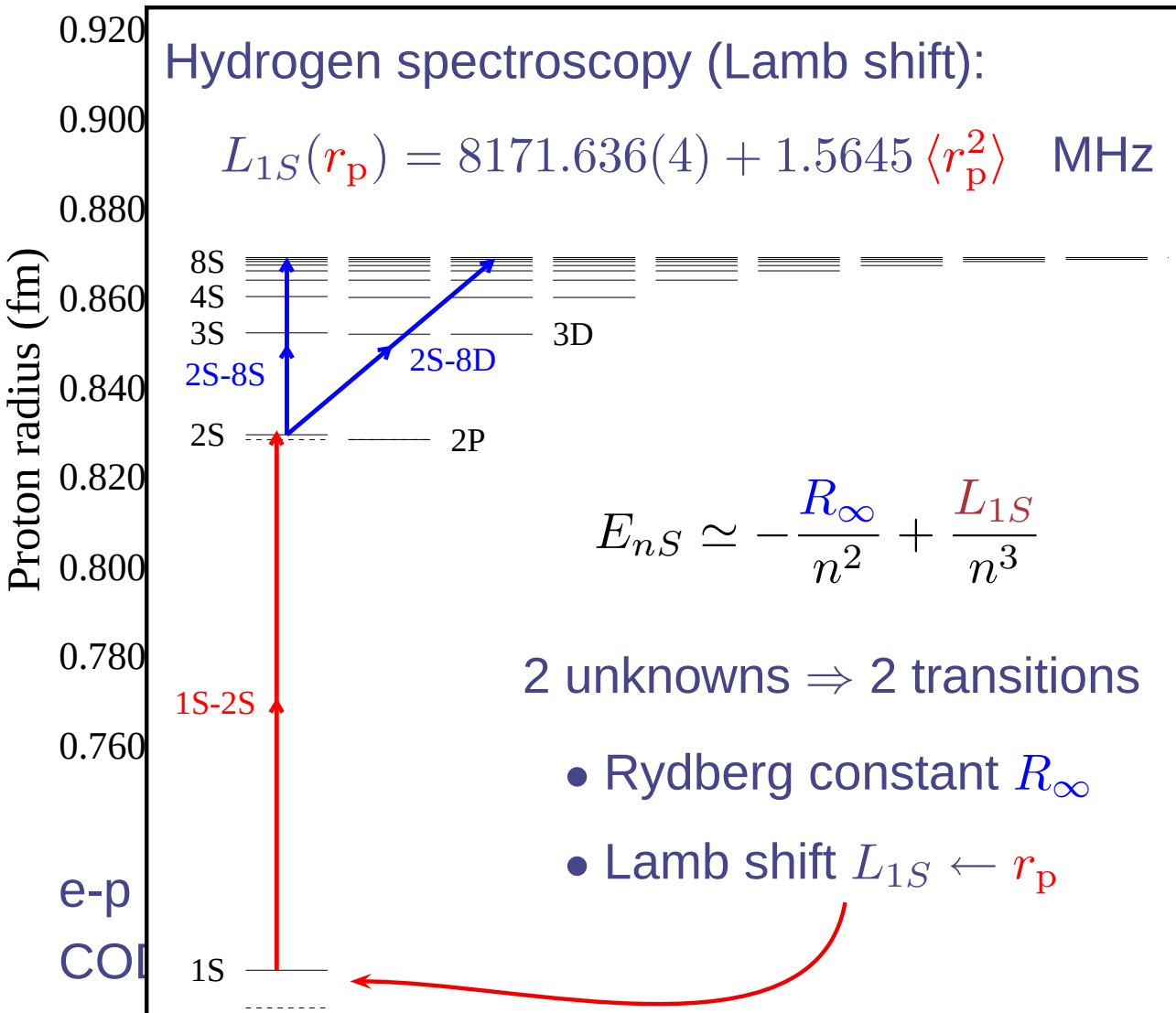
Hydrogen spectroscopy (Lamb shift):

$$L_{1S}(r_\mathrm{p}) = 8171.636(4) + 1.5645\,\langle r_\mathrm{p}^2\rangle \quad \text{MHz}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns $\Rightarrow$ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_\mathrm{p}$

e-p

COD

- electron scattering

  slope of G_E at $Q^2 = 0$

- hydrogen spectr.

  Lamb shift (S-states)

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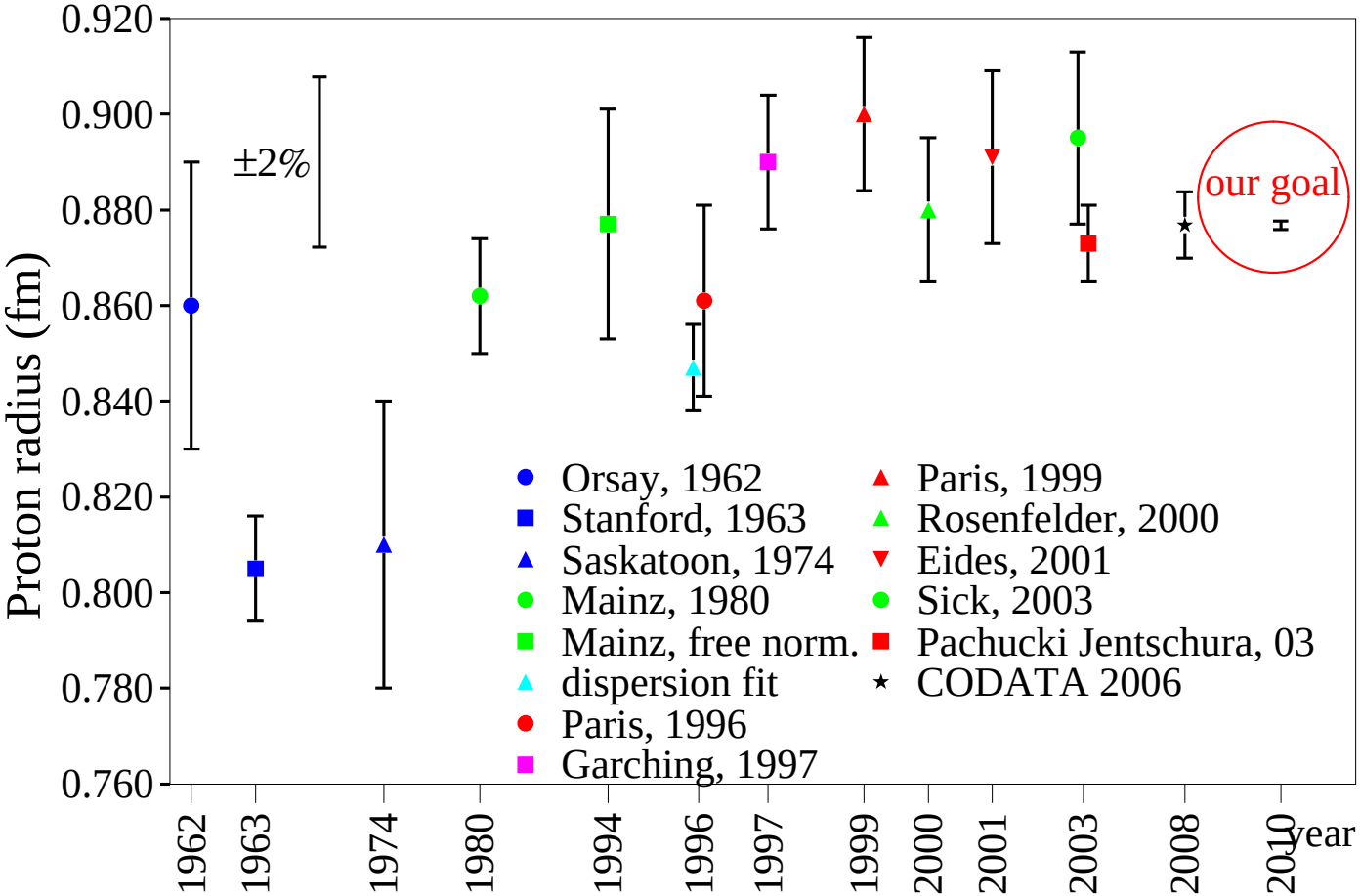

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CODATA: r_p = 0.8768(69) fm ($u_r = 0.8\,\%$)

muonic hydrogen goal (1998): $u_r = 0.1\,\%$

20x improvement

(aim: 10x better QED test in H)

Proton charge radius and muonic hydrogen

muonic hydrogen = μ^- p mass m_μ = 207 m_e

$\Rightarrow$ Bohr: $\langle r^{\rm orbit}\rangle \sim \dfrac{\hbar}{Z\alpha\, m_r\, c} n^2$

$\Delta E_{\rm finite\ size}(nl) \sim r_{\rm p}^2\, |\Psi(r=0)|^2$

$$\Rightarrow \boxed{\Delta E_{\rm finite\ size}(nl) = \frac{2(Z\alpha)^4 c^4}{3\hbar^2 n^3}\, m_r^3\, r_{\rm p}^2\, \delta_{l0}}$$

Lamb shift in μp: $\Delta E(2P^{\rm F=2}_{3/2} - 2S^{\rm F=1}_{1/2})$ =

$$\boxed{209.9779(49) - 5.2262\, r_{\rm p}^2 + 0.0347\, r_{\rm p}^3 \ \text{[meV]}}$$

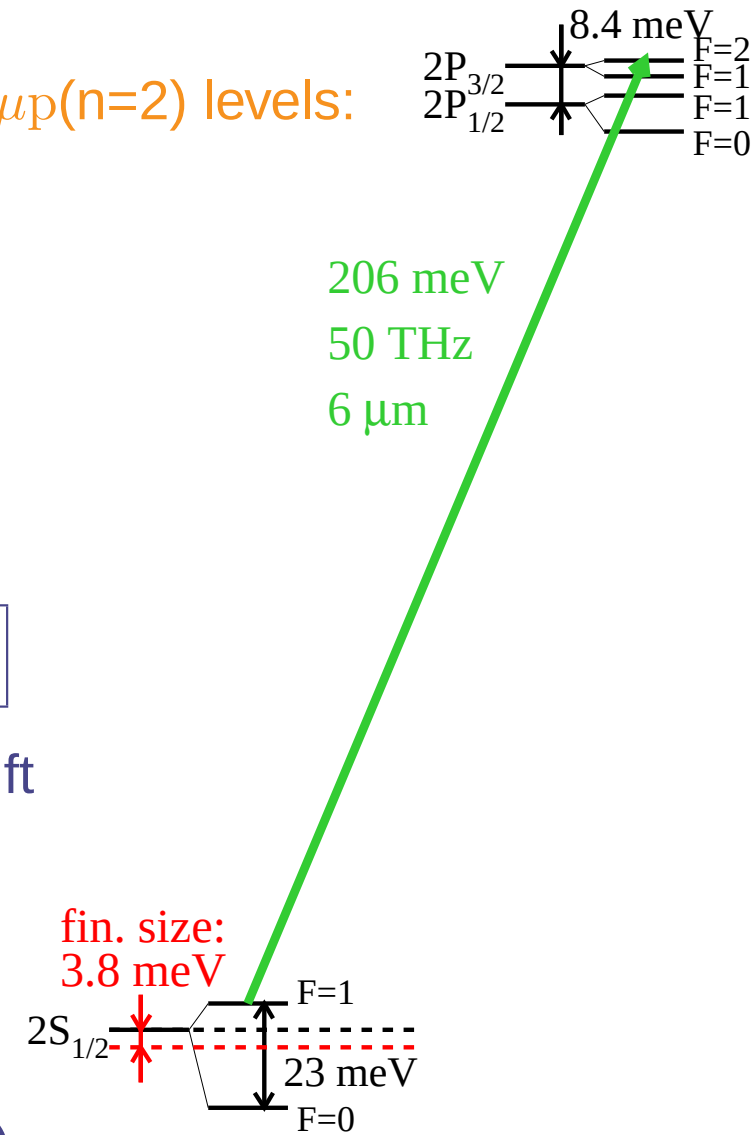
finite size contribution is 2% of the μp Lamb shift

measure ΔE(2S-2P) to 30 ppm = 1.5 GHz

$\boxed{\Rightarrow r_{\rm p} \text{ to } 10^{-3}}$

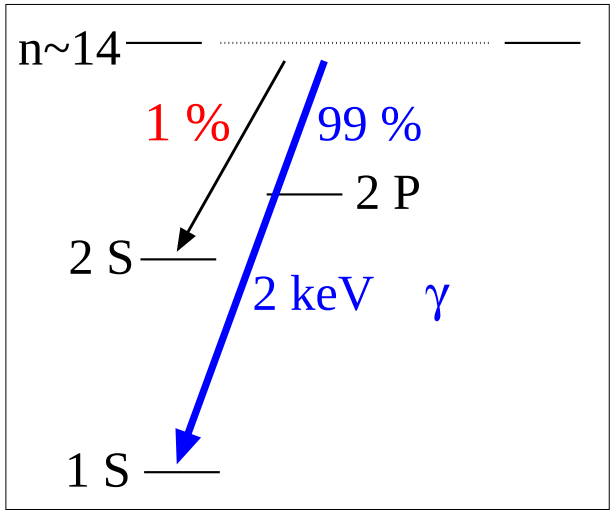
$\Gamma_{2P} = 18.6$ GHz $(\Gamma_{\rm rad.})$

μp(n=2) levels:

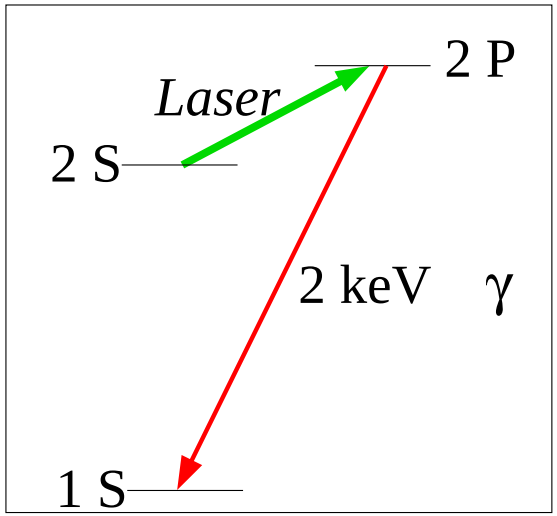


μp Lamb shift experiment: Principle

"prompt" ($t \sim 0$)



"delayed" ($t \sim 1\ \mu$s)



μ^- stop in H_2 gas
$\Rightarrow$ μp^* atoms formed ($n \sim 14$)

99%: cascade to μp(1S), emitting prompt K_α, K_β ...

1%: long-lived μp(2S) atoms

lifetime $\tau_{2S} \approx 1\,\mu$s at 1 mbar H_2

R. Pohl *et. al.*, Phys. Rev. Lett. 97, 193402 (2006).

fire laser ($\lambda \approx 6\,\mu$m, $\Delta E \approx 0.2$ eV)

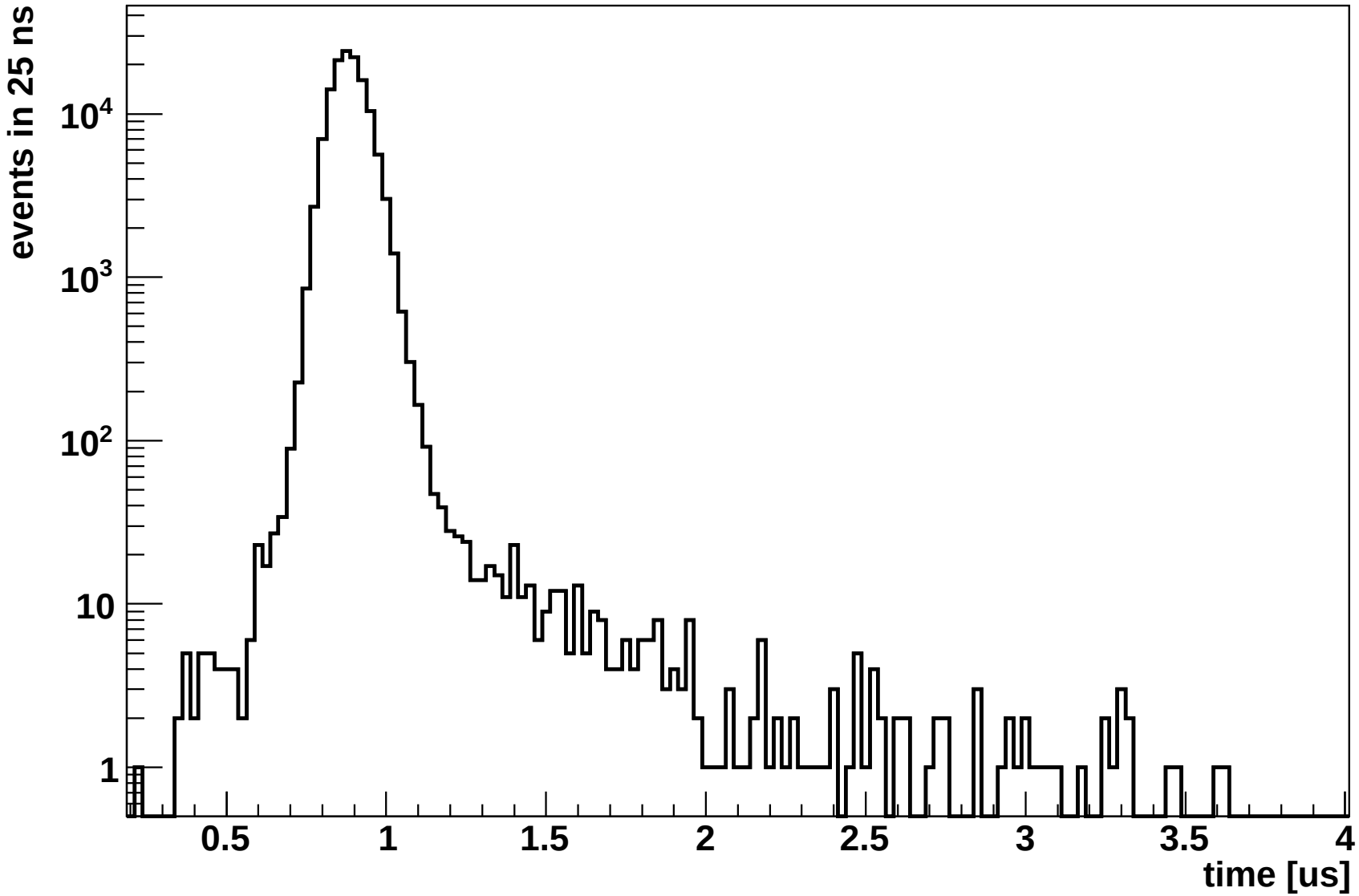
$\Rightarrow$ induce μp(2S) $\rightarrow$ μp(2P)

$\Rightarrow$ observe delayed K_α x-rays

$\Rightarrow$ normalize $\frac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha}$ x-rays

μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays (∼ 13 hours of data)



μp Lamb shift experiment: Principle

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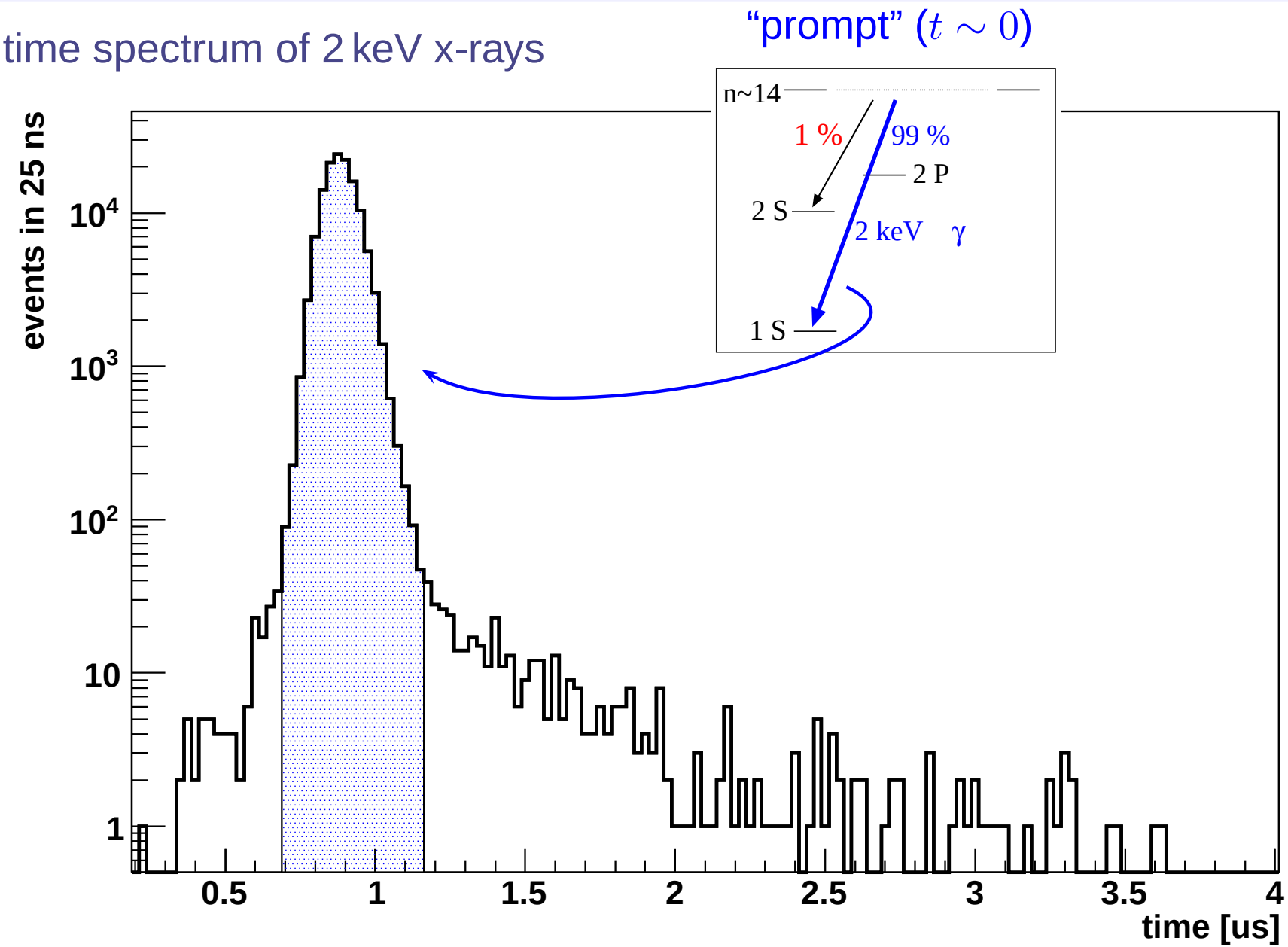
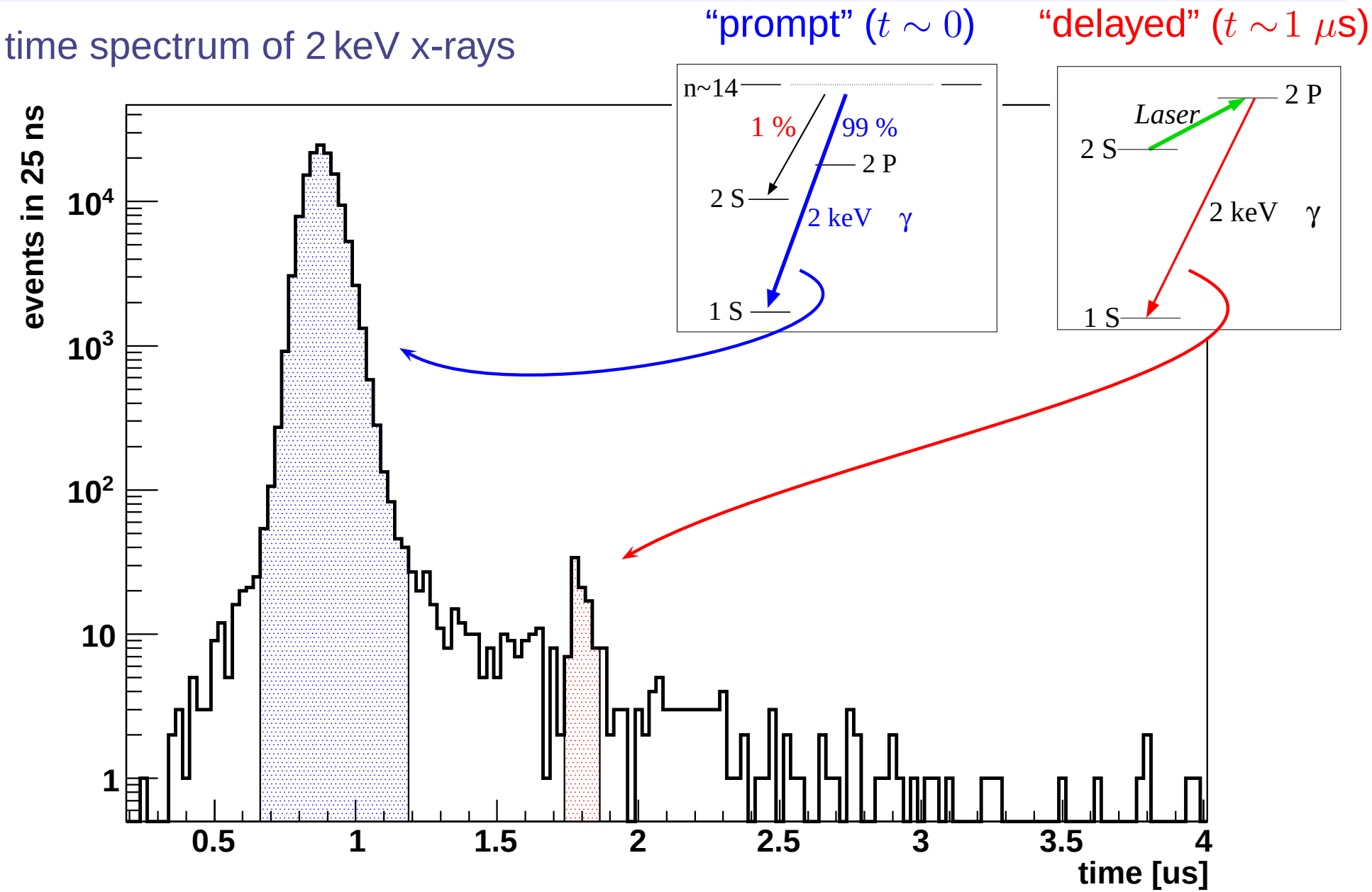


μp Lamb shift experiment: Principle



μp Lamb shift experiment: Principle

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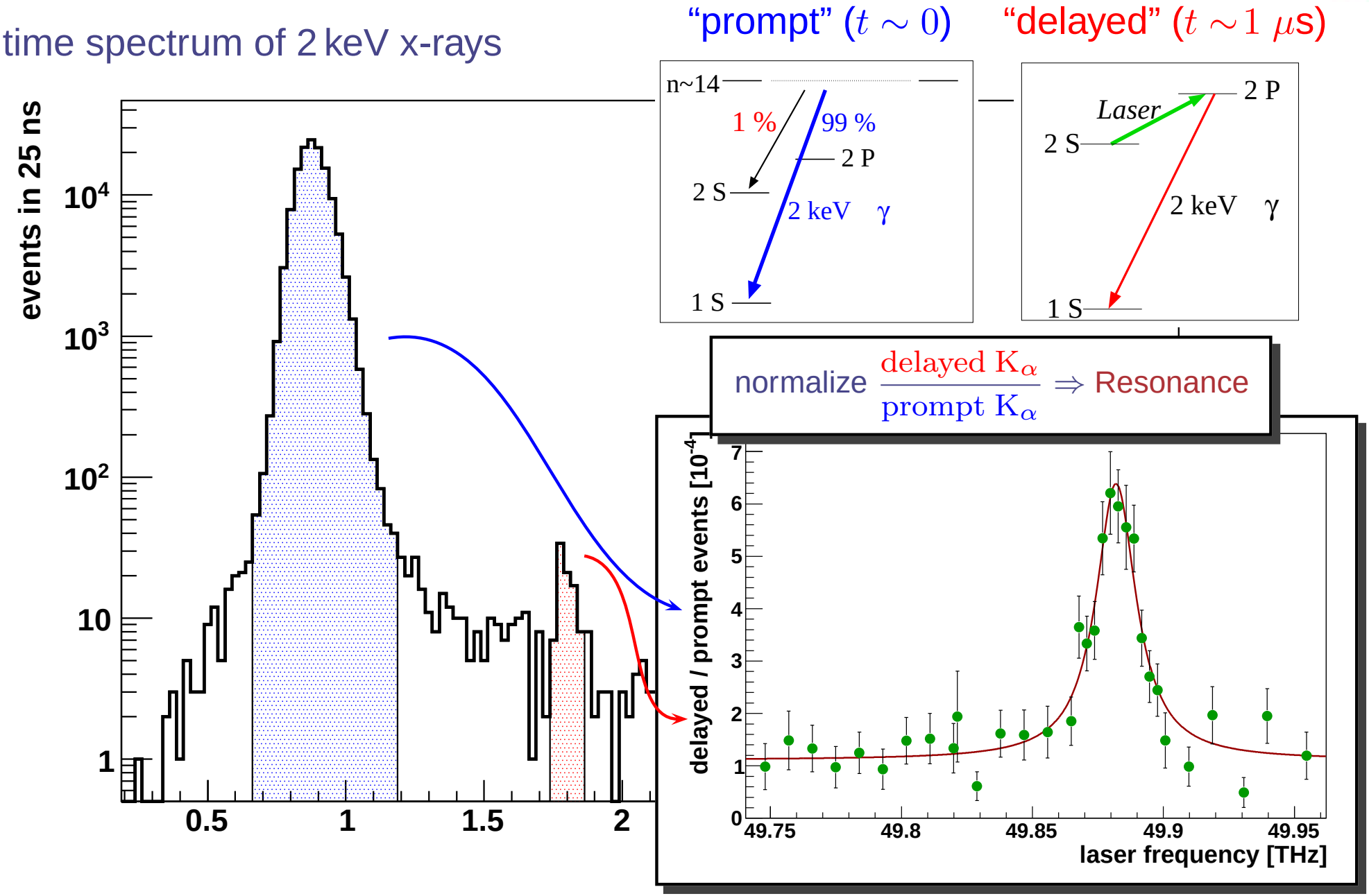
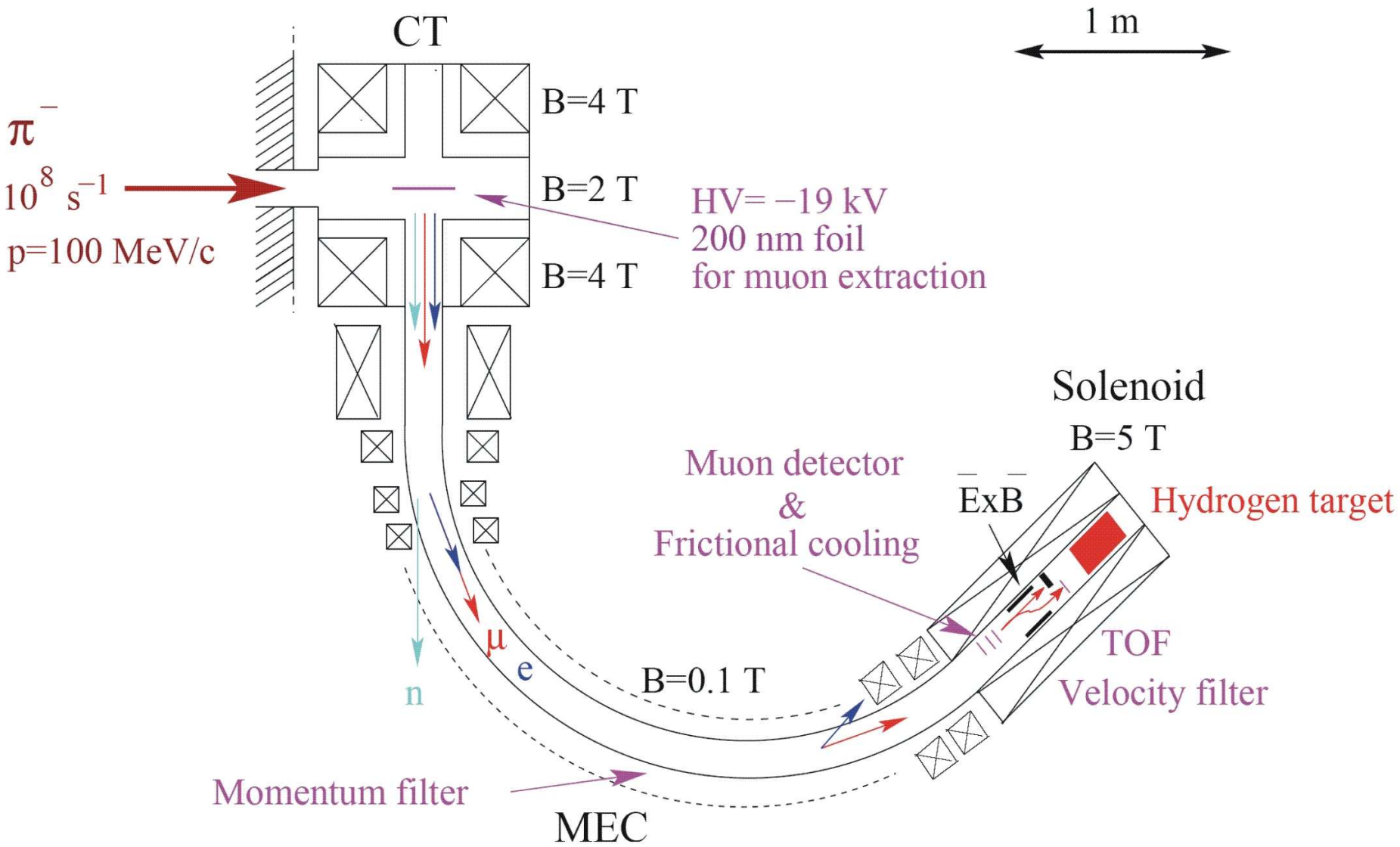
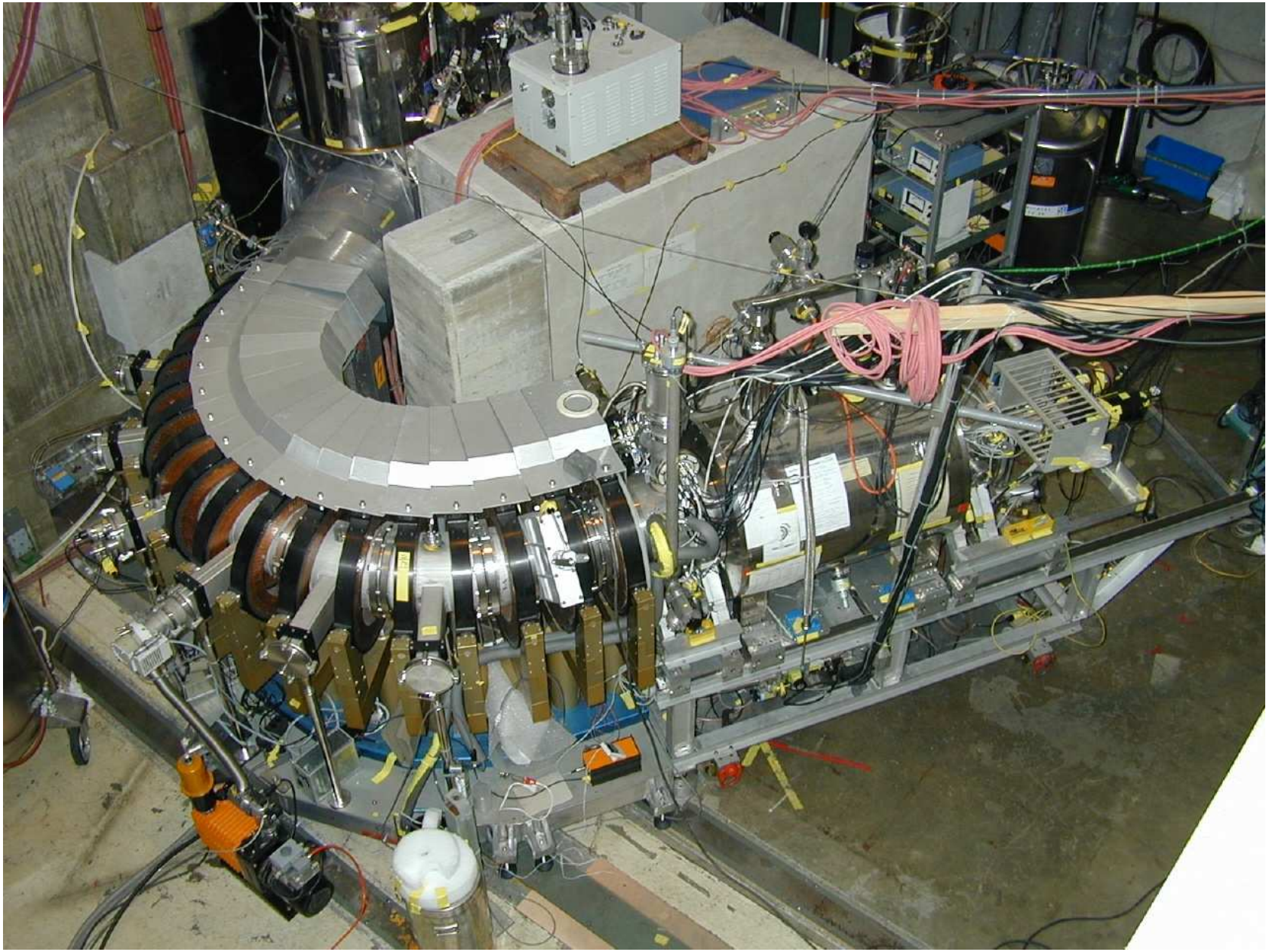


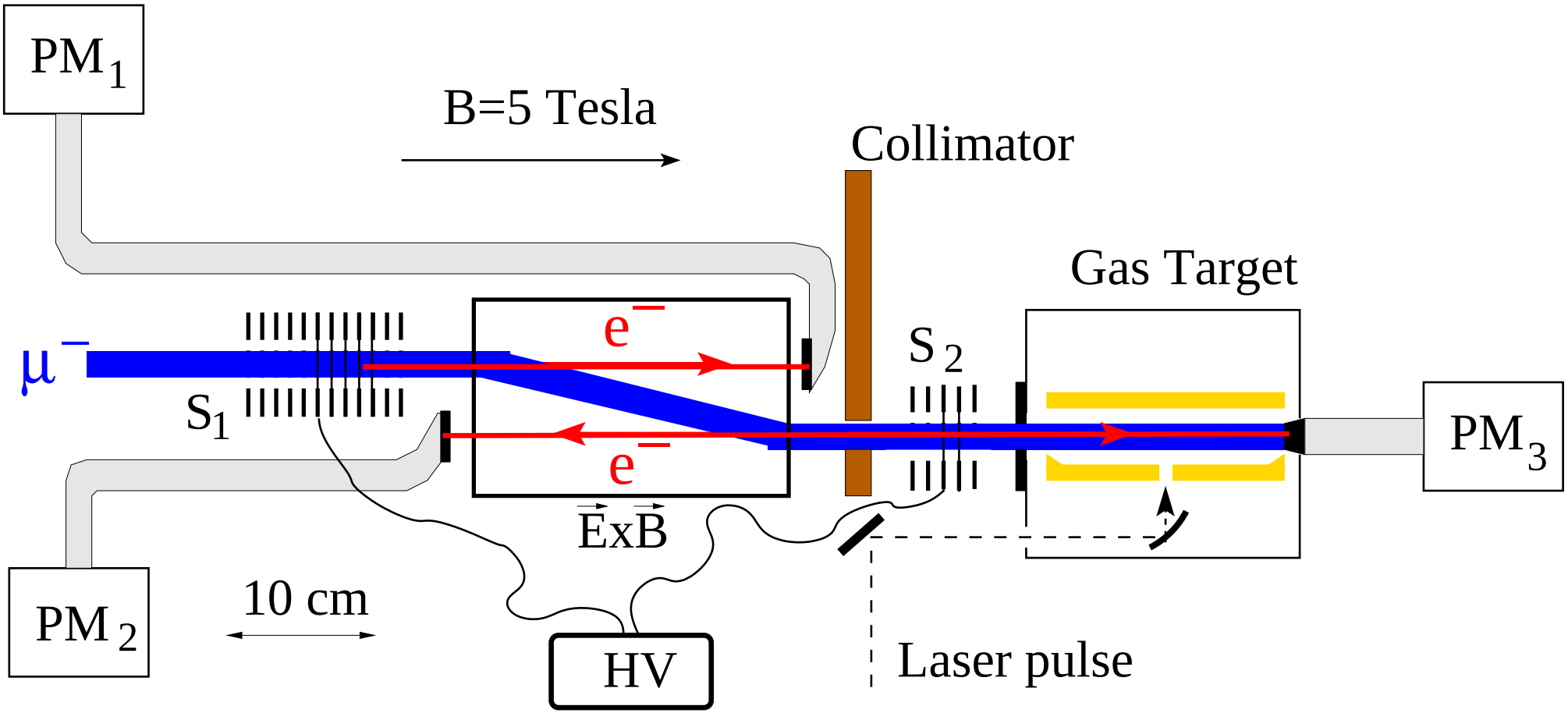
Muon beam line



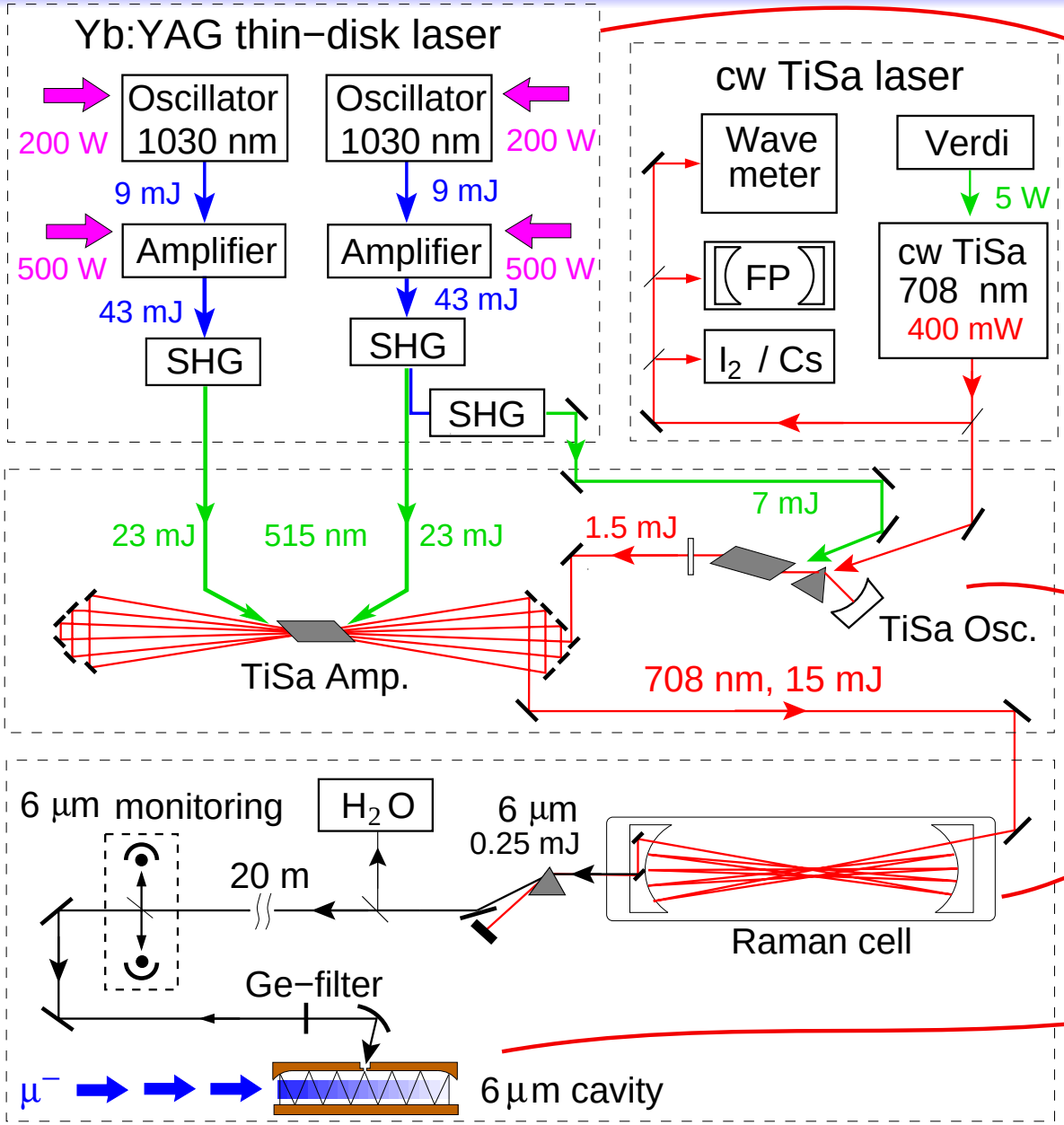
Muon beam line



Muon beam: inside 5 T solenoid



The laser system

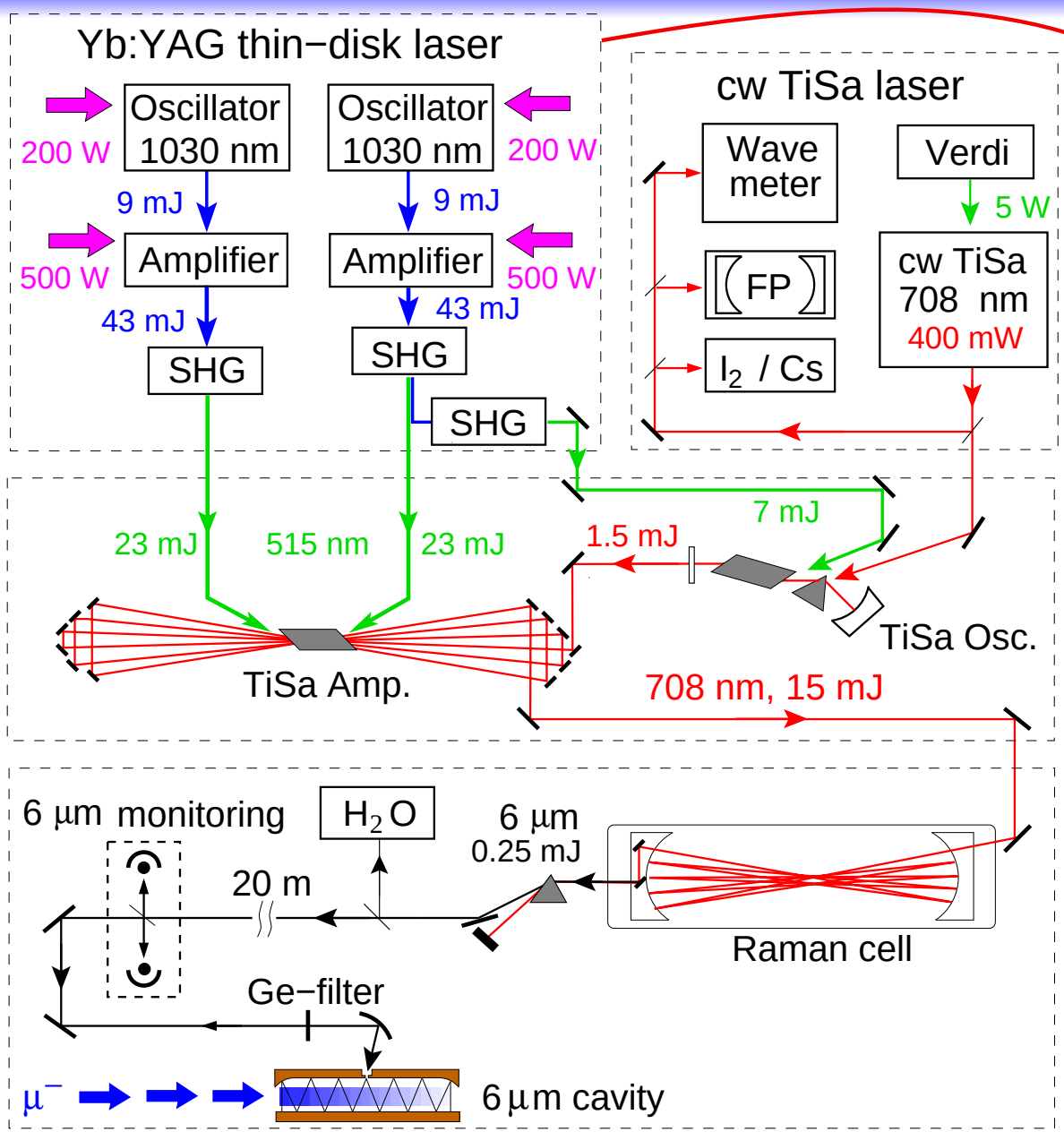


Main components:

- Thin-disk laser
  fast response to detected μ^-
- Frequency doubling
- TiSa laser:
  frequency stabilized cw laser
  injection seeded oscillator
  multipass amplifier
- Raman cell
  3 Stokes: 708 nm $\rightarrow$ 6 μm
  λ calibration @ 6 μm
- Target cavity

A. Antognini *et. al.*, Opt. Comm. 253, 362 (2005).

The laser system



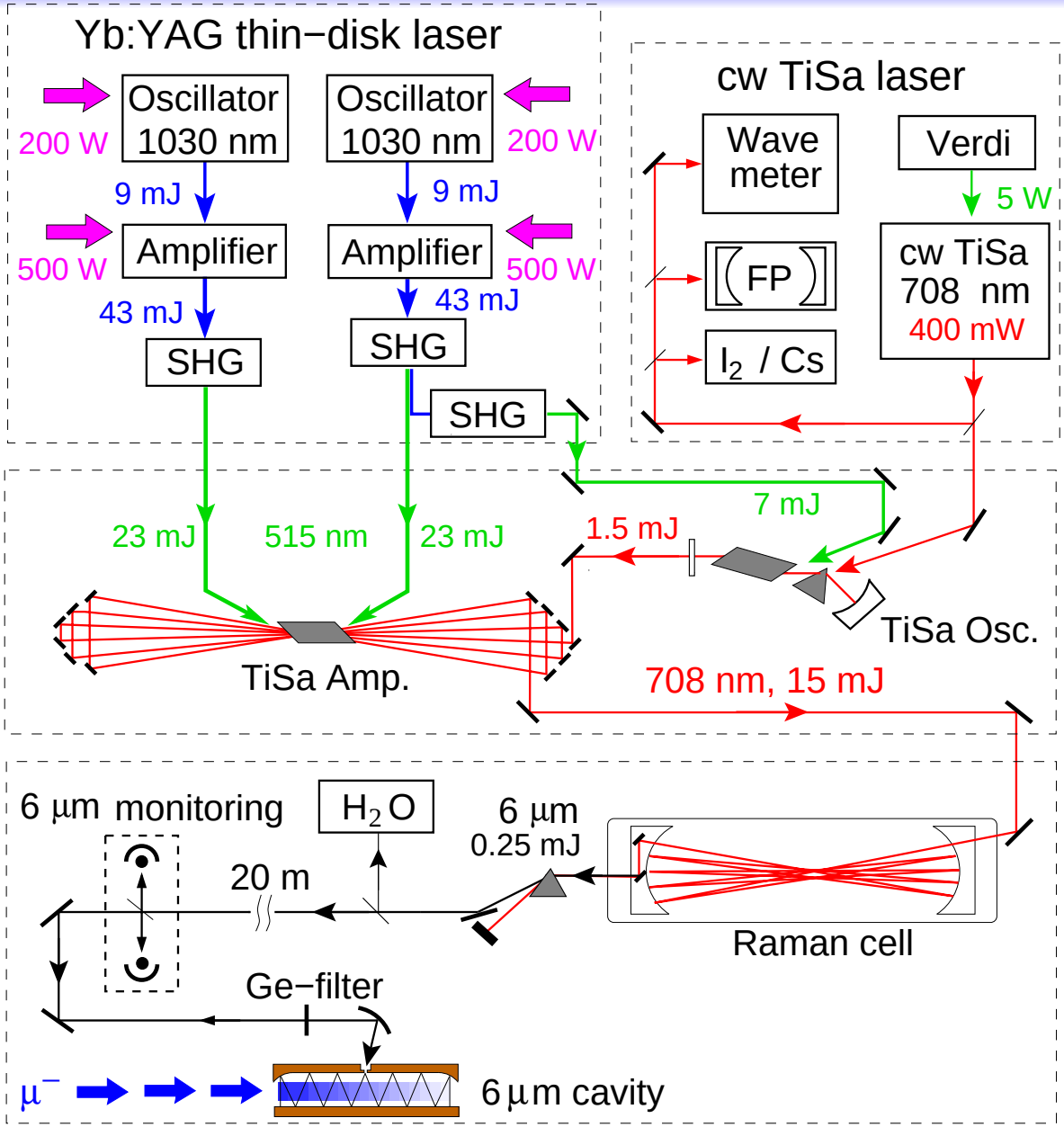
Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single μ^- triggers the laser system
- $2S$ lifetime $\approx 1\ \mu$s $\rightarrow$ short laser delay

A. Antognini *et. al.*, IEEE J. Quant. Electr. 45, 993 (2009).

The laser system



MOPA TiSa laser:

Cw frequency stabilized laser

- referenced to a stable FP cavity
- FP cavity calibrated with I_2, Rb, Cs lines

$$\nu_{\mathrm{FP}} = N \cdot FSR$$

$$FSR = 1497.344(6) \text{ MHz}$$

$\nu_{\mathrm{TiSa}}^{\mathrm{cw}}$ absolutely known to 30 MHz

$\Gamma_{2\mathrm{P}-2\mathrm{S}} = 18.6$ GHz

Seeded oscillator

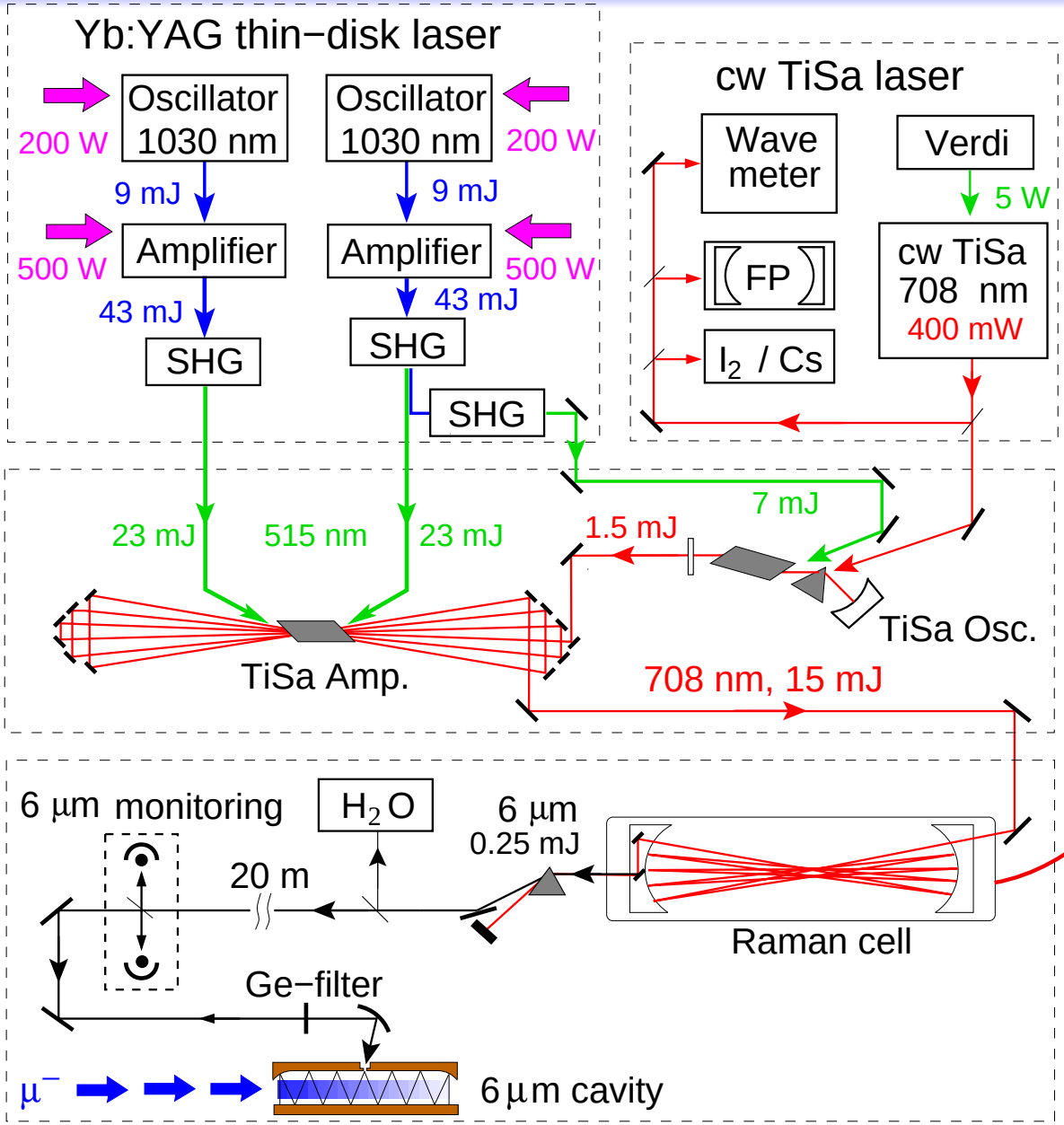
$$\rightarrow \nu_{\mathrm{TiSa}}^{\mathrm{pulsed}} = \nu_{\mathrm{TiSa}}^{\mathrm{cw}}$$

(frequency chirp ≤ 100 MHz)

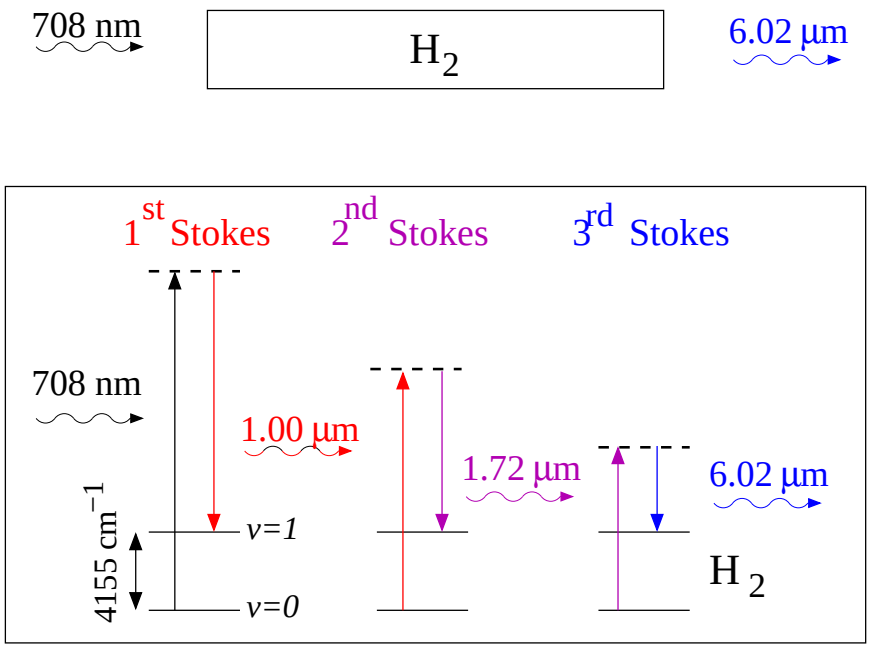
Multipass amplifier (2f- configuration)

gain=10

The laser system



Raman cell:

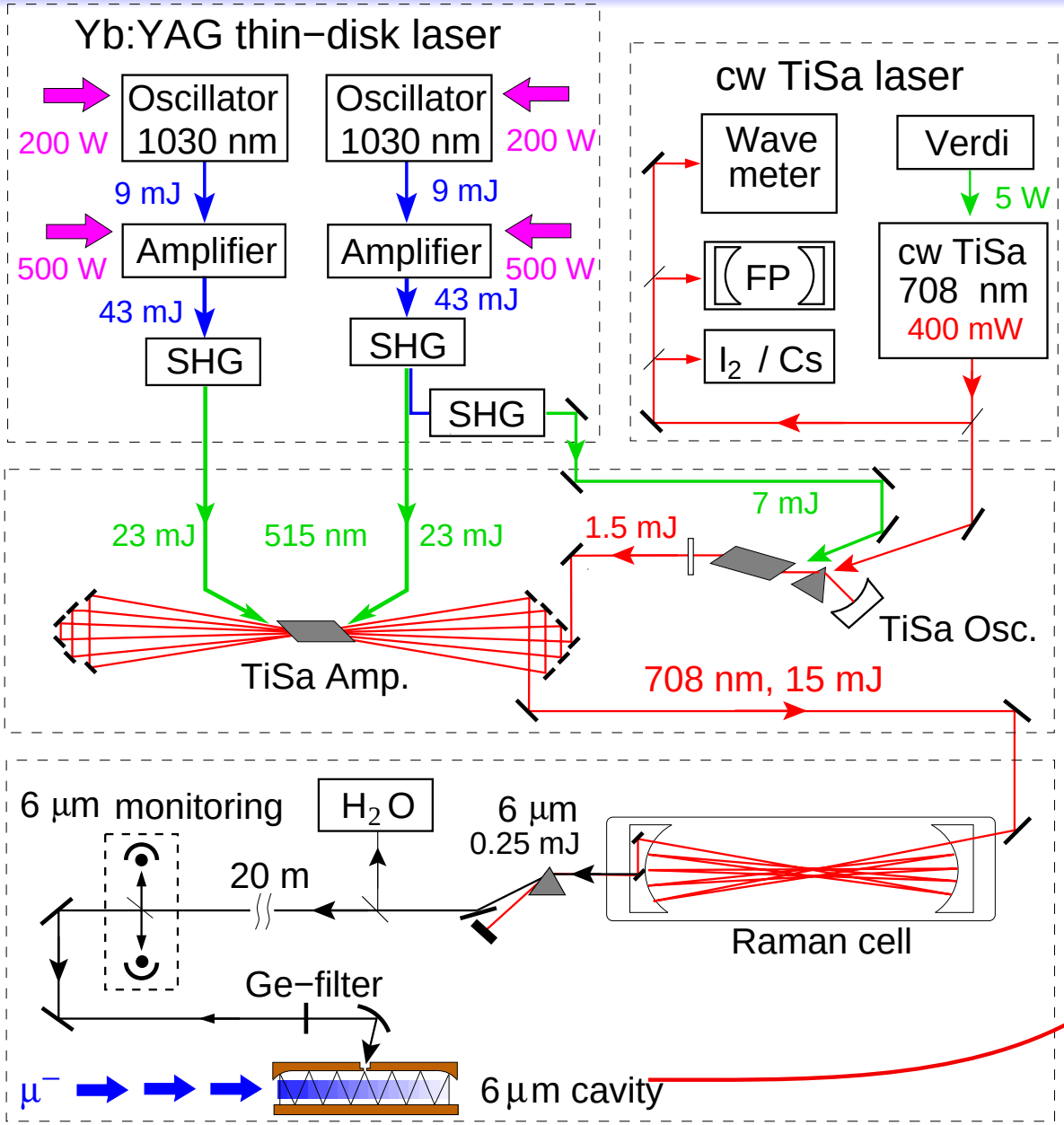


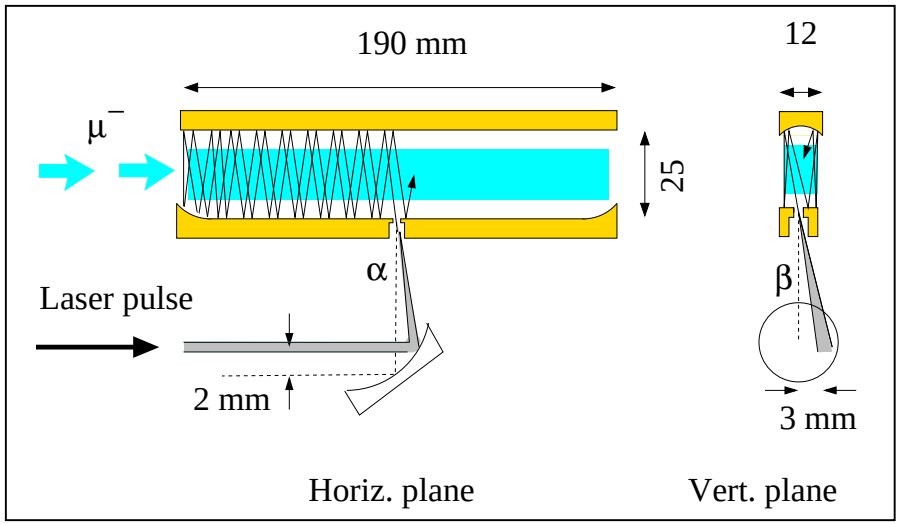
$$\nu^{6\mu m} = \nu^{708nm} - 3 \cdot \hbar\omega_{vib}$$

tunable $\omega_{vib}(p,T) = \text{const}$

P. Rabinowitz *et. al.*, IEEE J. QE **22**, 797 (1986)

The laser system





Design: insensitive to misalignment

Transverse illumination

Large volume

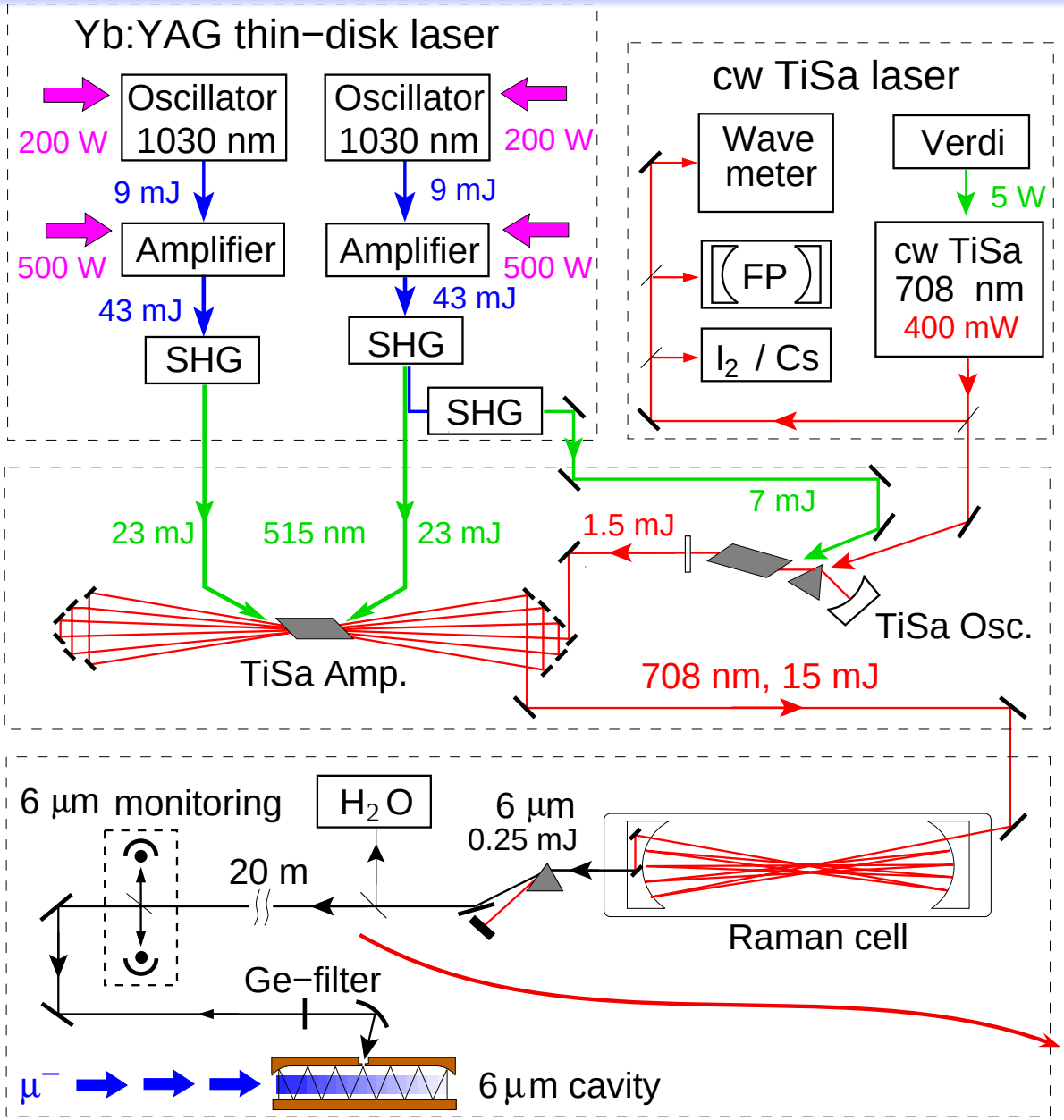
Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

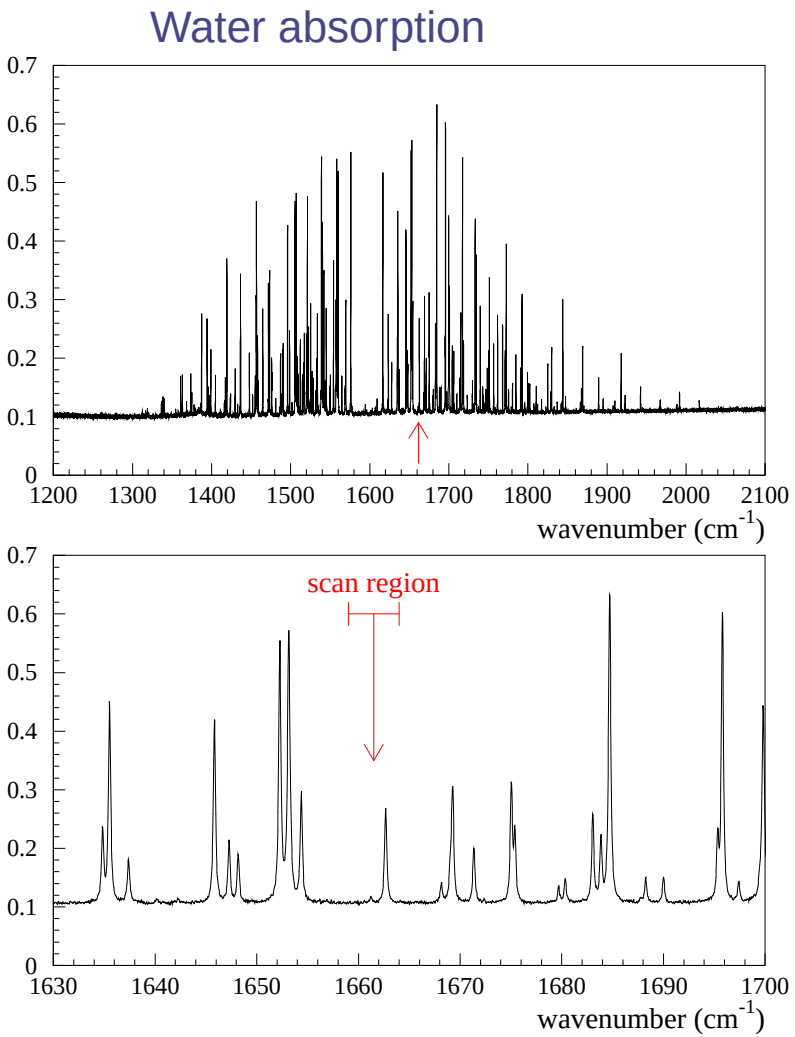
$\rightarrow$ Light makes 1000 reflections

$\rightarrow$ Light is confined for τ=50 ns

$\rightarrow$ 0.15 mJ saturates the $2S - 2P$ transition

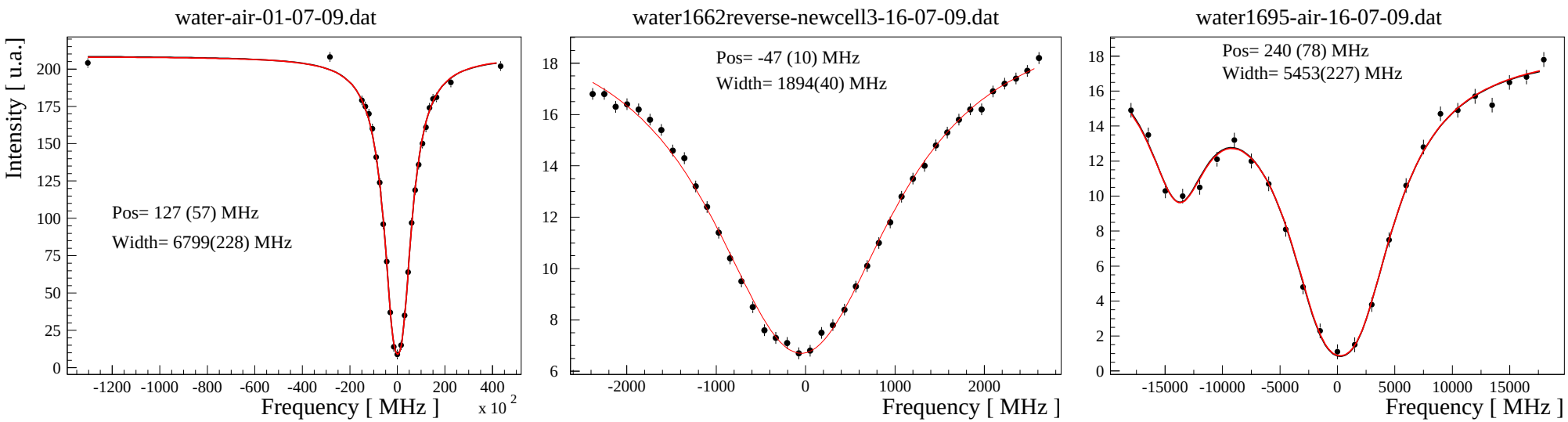
The laser system





- Vacuum tube for 6 μm beam transport.
- Direct frequency calibration at 6 μm.

6 μm wavelength calibration



- $6\,\mu$m light calibration: H_2O vapor absoprtion measurement in air / cell

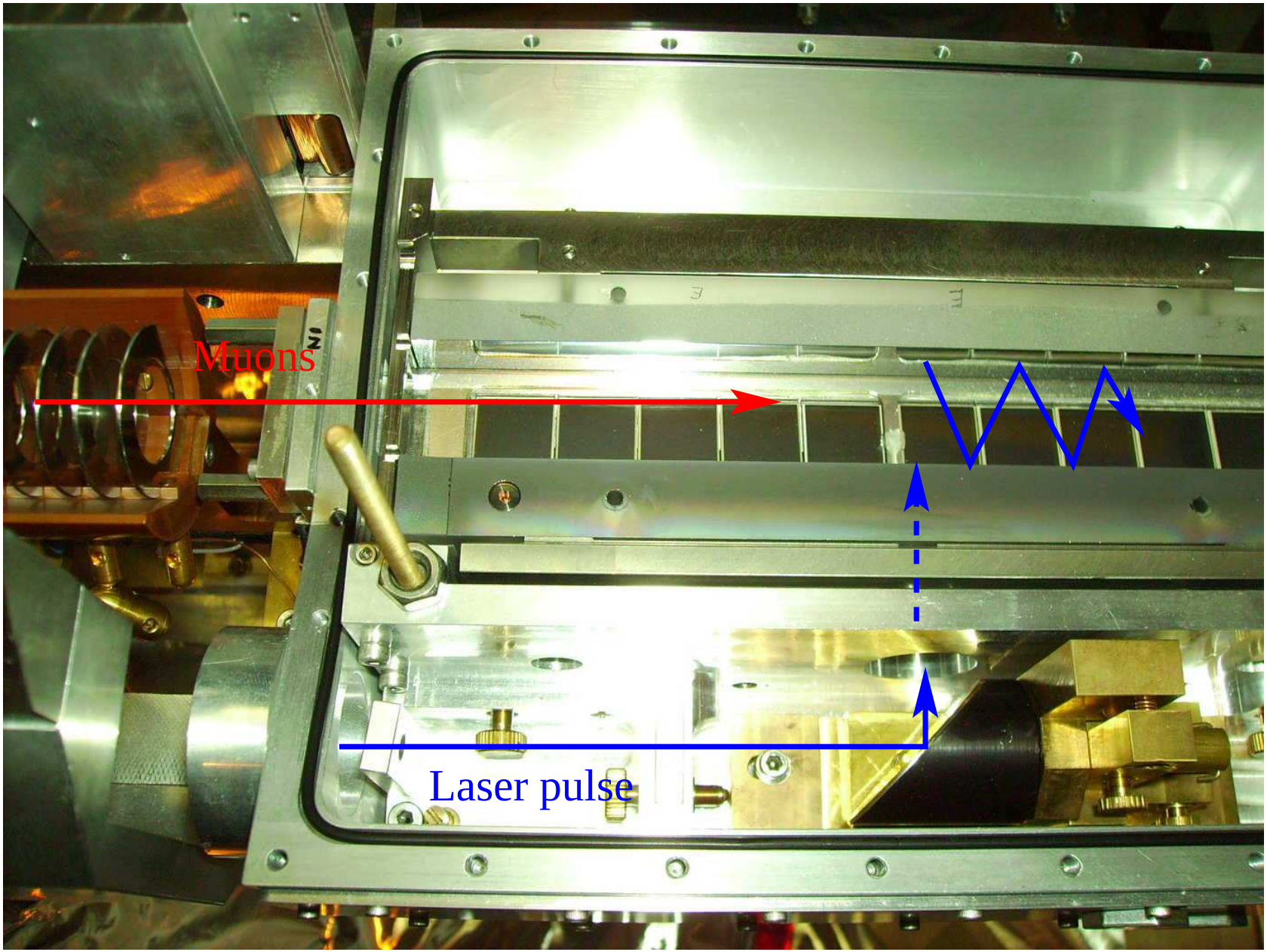
  H_2O absorption lines known to a few MHz (HITRAN)

  $\Rightarrow$ $\delta\nu \approx 300$ MHz uncertainty (6 ppm of ΔE_{2S-2P}) due to our calibration accuracy

  over the whole wavelength range λ = 5.5 ... 6.1 μm

- Laser frequency detuning is measured in number of Fabry-Perot cavity fringes
  - grid spacing of our measurement: FSR(FP) = 1497.344(6) MHz
  - all measured resonances are within ± 70 FP fringes of a H_2O line

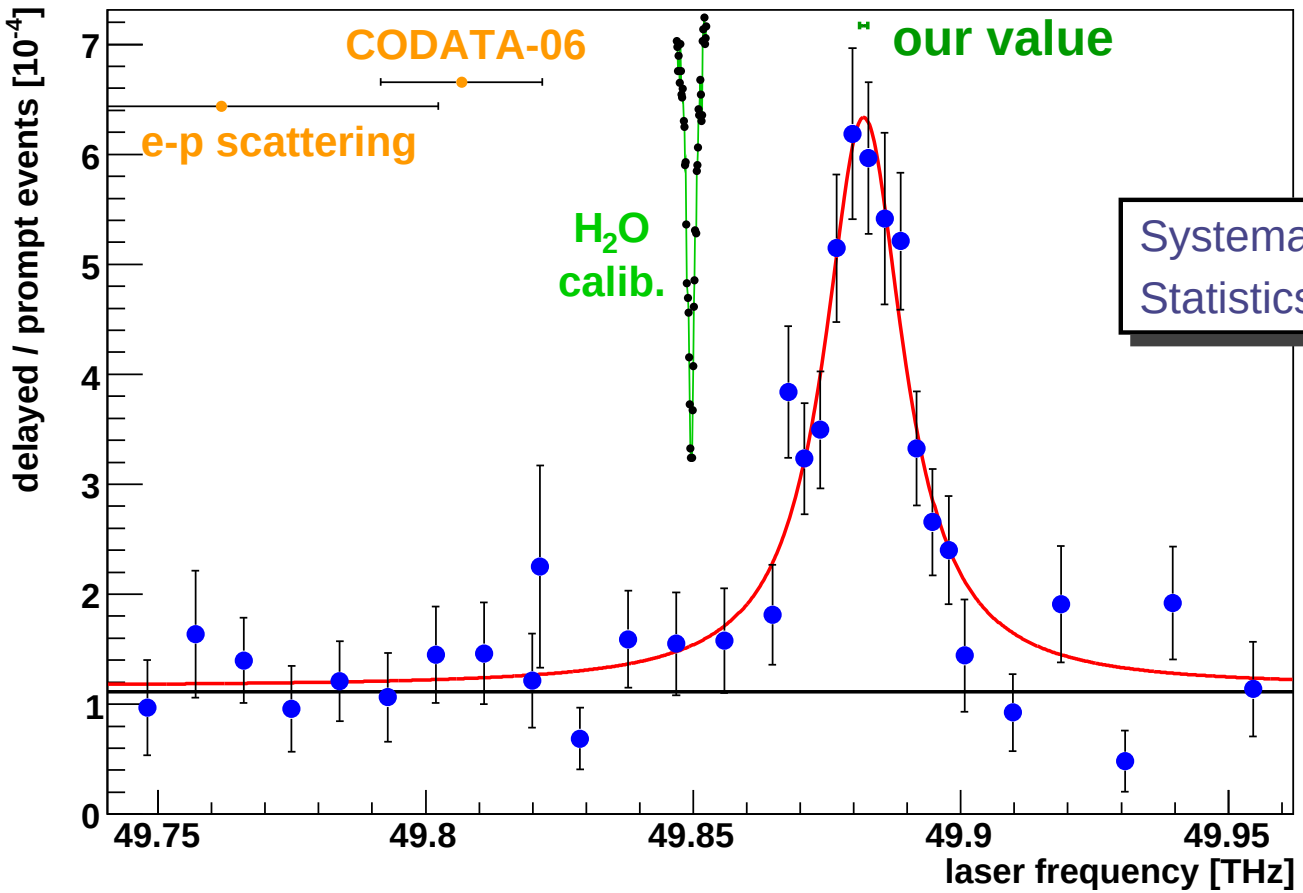
Target, cavity and detectors



The resonance: discrepancy, sys., stat.

Water-line/laser wavelength:
300 MHz uncertainty

$\Delta\nu$ water-line to resonance:
200 kHz uncertainty



Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy:
$5.0\,\sigma \;\leftrightarrow\; 75\ \mathrm{GHz} \;\leftrightarrow\; \delta\nu/\nu = 1.5 \times 10^{-3}$

R. Pohl *et al.*, Nature 466, 213 (2010).

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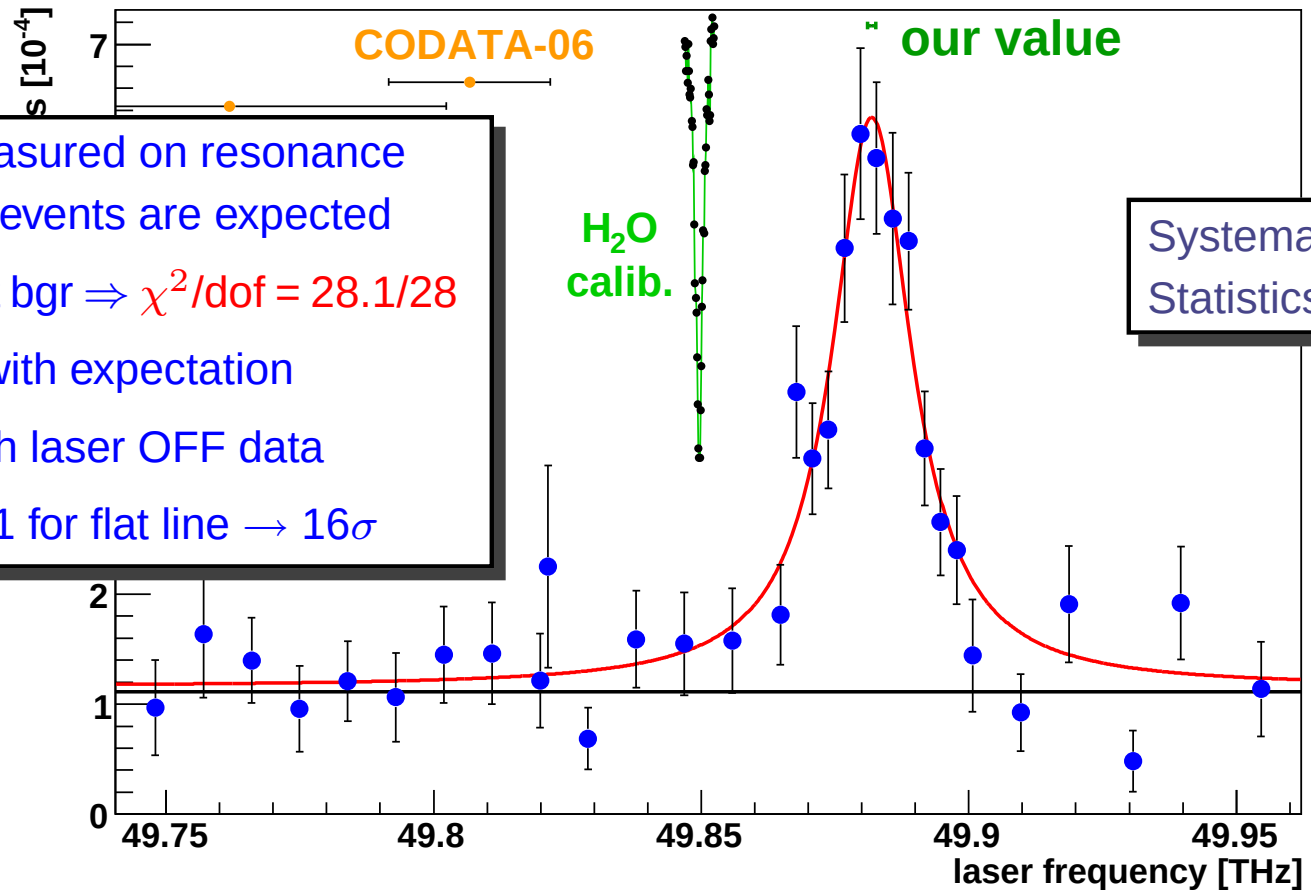

550 events measured on resonance
where 155 bgr events are expected

fit Lorentz + flat bgr $\Rightarrow$ χ^2/dof = 28.1/28

width agrees with expectation

bgr agrees with laser OFF data

χ^2/dof = 283/31 for flat line $\rightarrow$ 16σ

Systematics: 300 MHz
Statistics: 700 MHz

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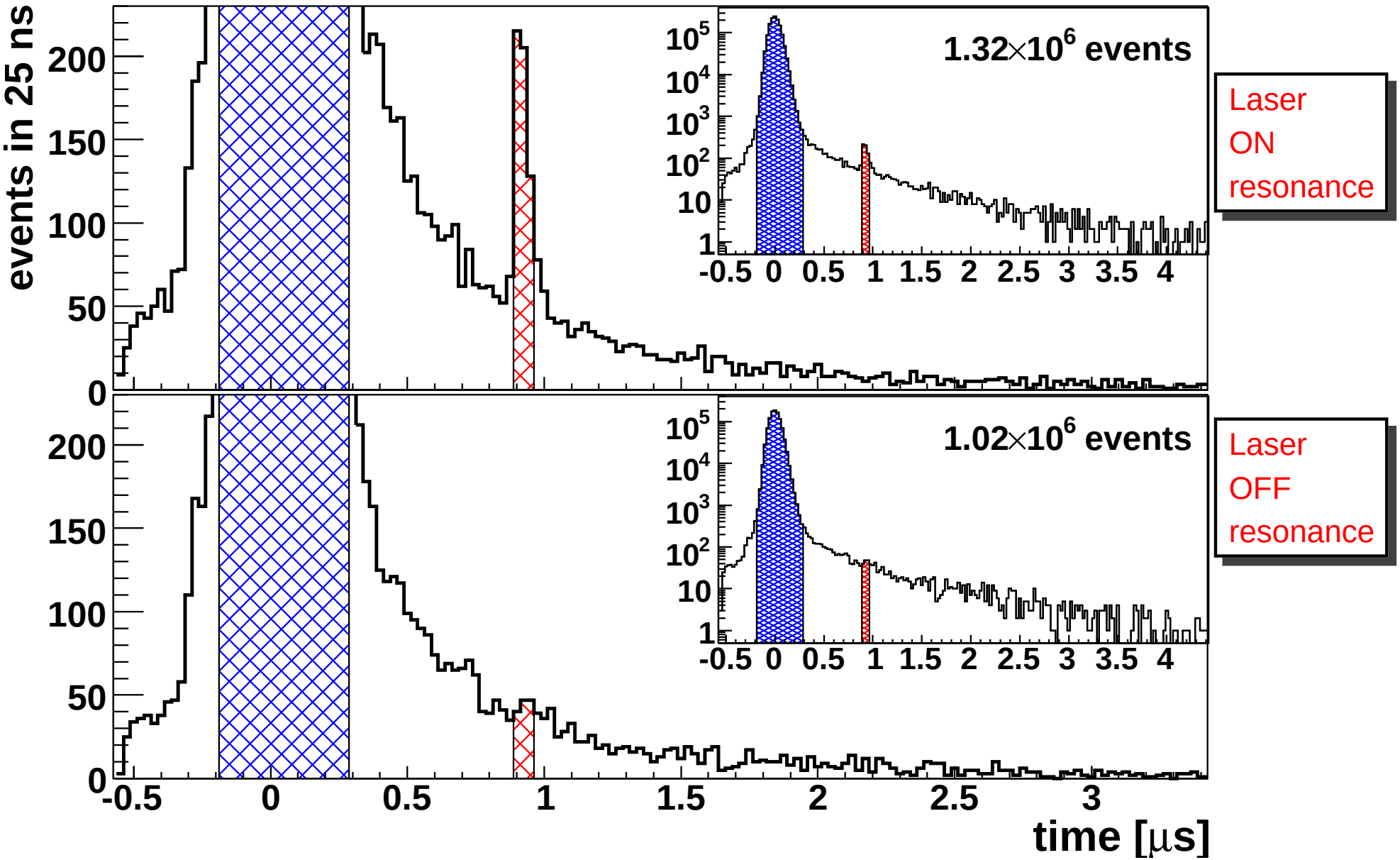
R. Pohl *et al.*, Nature 466, 213 (2010).

The time spectra



Uncertainty budget and sensitivity

- Statistics

Center position uncertainty ($\sim 4\%$ of Γ)	700 MHz

- Systematics

Laser frequency ($H_2$0 calibration)	300 MHz
AC and DC stark shift	< 1 MHz
Zeeman shift (5 Tesla)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz
Total uncertainty of the line determination	760 MHz

- Theory: proton polarizability 1200 MHz
- Discrepancy with CODATA prediction 75 300 MHz

Systematic effects are small since they scale like $\mathbf{1/m}$

Finite size effect scales like $\mathbf{m^3}$

Proton radius

$$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2}) = 49881.88(76)\ \text{GHz}.$$

$$\left.\begin{aligned} \widetilde{L}^{\text{exp.}} &= 206.2949(32)\ \text{meV} \\ \widetilde{L}^{\text{th.}} &= 209.9779(49) - 5.2262\, r_{\text{p}}^2 + 0.0347\, r_{\text{p}}^3\ \text{meV} \end{aligned}\right\} \Rightarrow \boxed{r_{\text{p}}\text{=0.84184(36)(56) fm}}$$

$$u_{\text{exp}} = 4.3 \times 10^{-4}$$
$$u_{\text{theo}} = 6.7 \times 10^{-4}$$

$$\boxed{r_{\text{p}}\text{=0.84184(67) fm} \qquad (u_r = 8 \times 10^{-4})}$$

CODATA 2006: $r_{\text{p}} = (0.8768 \pm 0.0069)$ fm
Hydrogen: $r_{\text{p}} = (0.876 \pm 0.008)$ fm
e-p scattering: $r_{\text{p}} = (0.895 \pm 0.018)$ fm (Sick 2005)

r_{p} is 4% smaller

5.0σ from CODATA-2006
4.3σ from H
3.1σ from e-p scatt.

R. Pohl *et al.*, Nature 466, 213 (2010).

Proton radius

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$$\left.\begin{aligned} \widetilde{L}^{\text{exp.}} &= 206.2949(32)\ \text{meV} \\ \widetilde{L}^{\text{th.}} &= 209.9779(49) - 5.2262\, r_{\text{p}}^2 + 0.0347\, r_{\text{p}}^3\ \text{meV} \end{aligned}\right\} \Rightarrow \boxed{r_{\text{p}}\text{=0.84184(36)(56) fm}}$$

$$u_{\text{exp}} = 4.3 \times 10^{-4}$$
$$u_{\text{theo}} = 6.7 \times 10^{-4}$$

$$\boxed{r_{\text{p}}\text{=0.84184(67) fm} \qquad (u_r = 8 \times 10^{-4})}$$

r_{p} is 4% smaller

5.0σ from CODATA-2006

4.3σ from H

8.4σ from scatt.

CODATA 2006: $r_{\text{p}} = (0.8768 \pm 0.0069)$ fm

Hydrogen: $r_{\text{p}} = (0.876 \pm 0.008)$ fm

e-p scattering: $r_{\text{p}} = (0.894 \pm 0.008)$ fm (Sick 2011)

$r_{\text{p}} = (0.879 \pm 0.008)$ fm (Mainz 2010)

$r_{\text{p}} = (0.875 \pm 0.010)$ fm (JLab Hall A 2011)

What is wrong?

μp experiment:	discrepancy 75 GHz
	100σ
	$\sim 4\,\Gamma_{\rm nat}$
	two independent wavelength calibrations
	one very significant line, no satellite
	fitted width = natural width
	another transition in μp confirms our $r_{\rm p}$
μp theory:	discrepancy 0.31 meV
	60σ
	0.15% of the total Lamb shift
	4th largest term
H theory:	L_{1S} off by 100 kHz
	25σ
	(almost) all terms only calculated by 2 groups + methods
	convergence?

$\Longrightarrow$ These are solid.

What is wrong?

μp experiment: discrepancy 75 GHz

100σ

$\sim 4\,\Gamma_{\rm nat}$

two independent wavelength calibrations

one very significant line, no satellite

fitted width = natural width

μp theory

Maybe both H spectroscopy **and** e-p scattering are off.

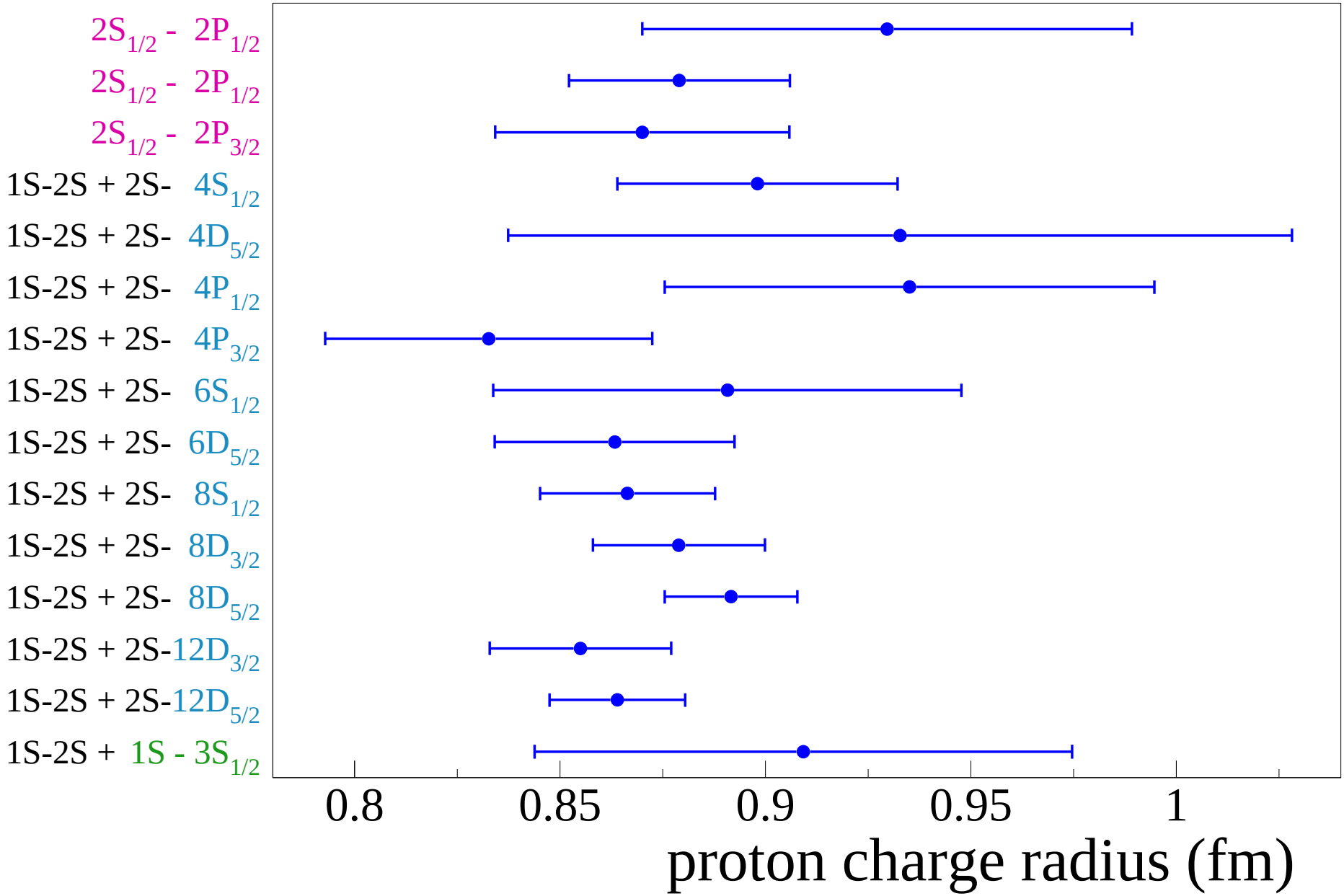
H theory:

(almost) all terms only calculated by 2 groups + methods

convergence?

$\Longrightarrow$ These are solid.

Hydrogen spectroscopy



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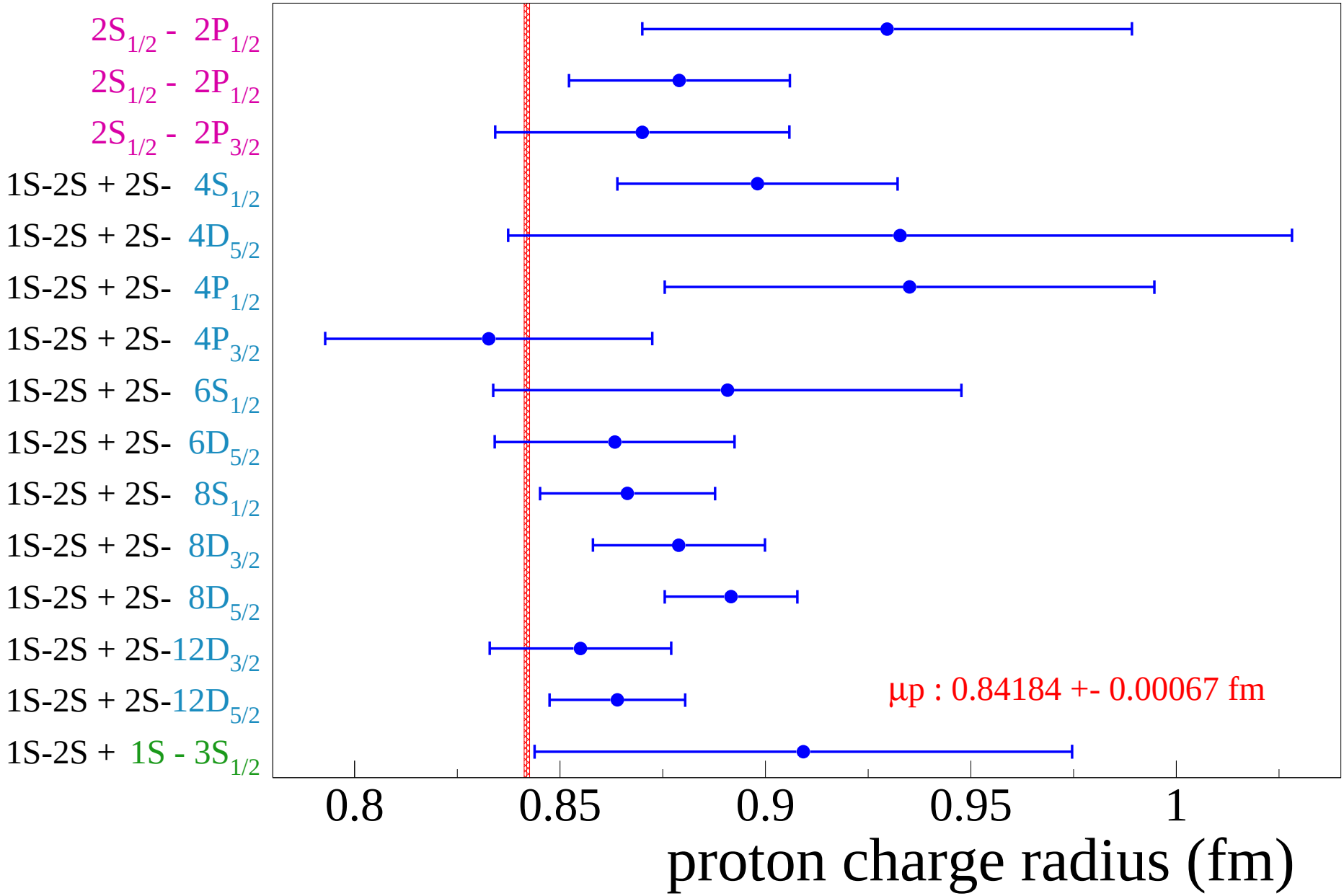

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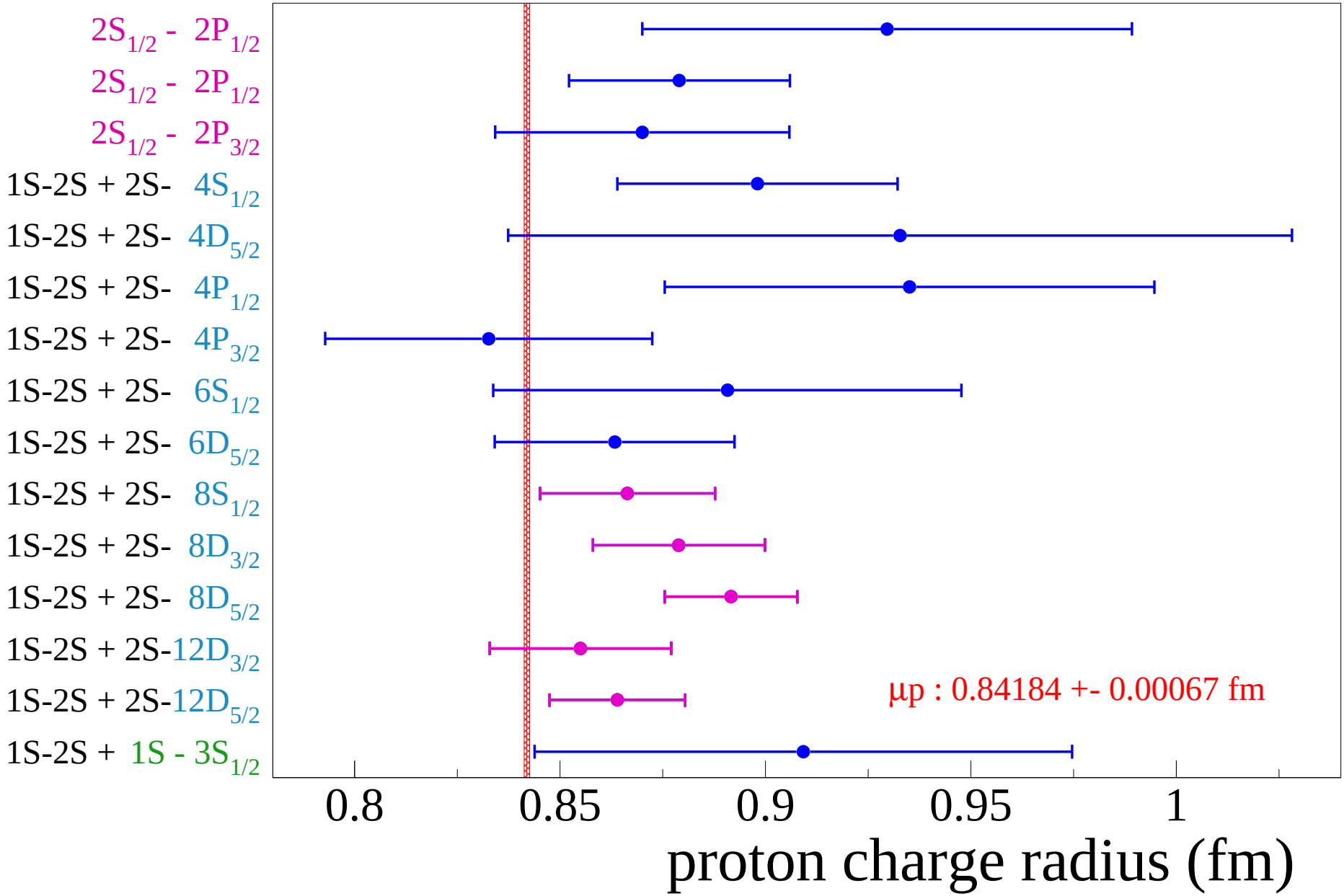
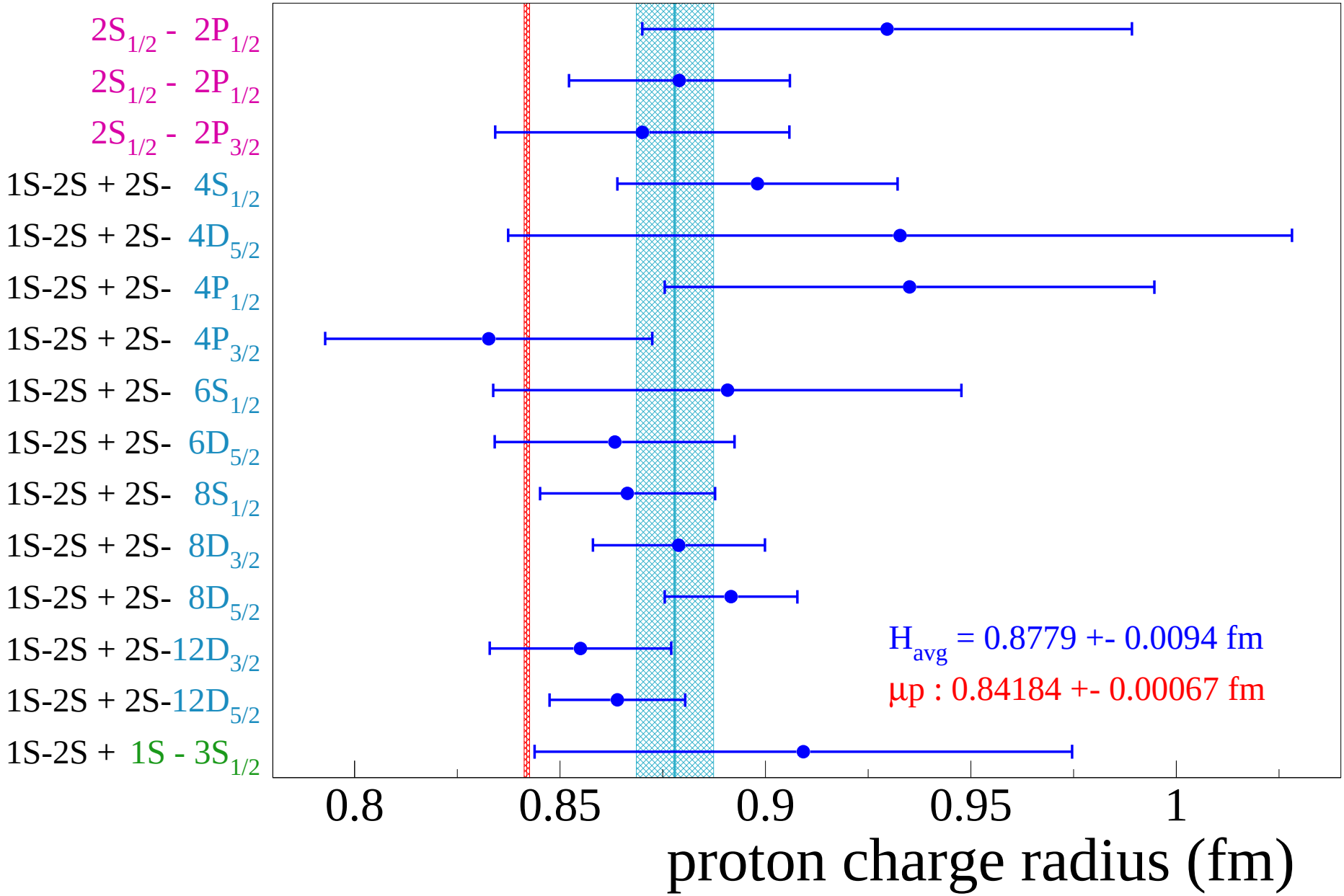
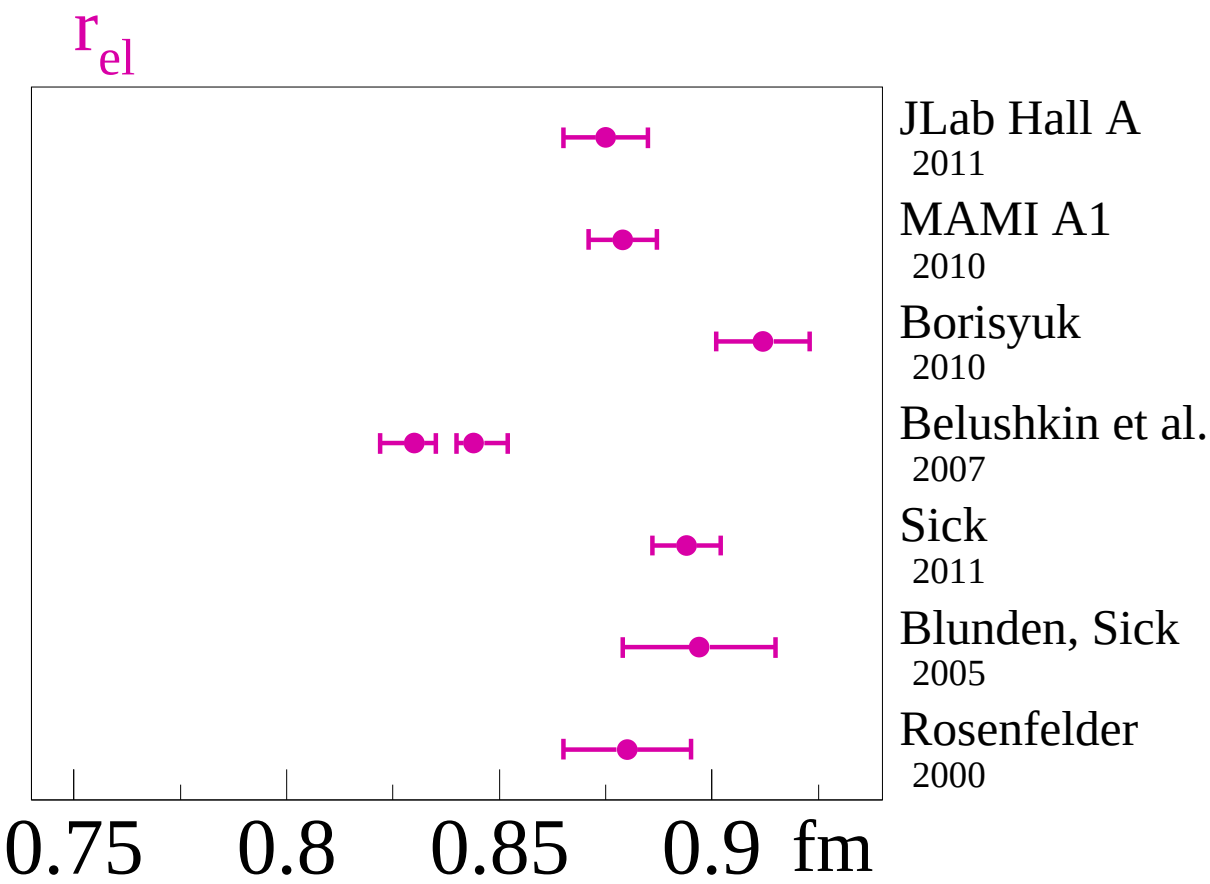


Hydrogen spectroscopy



Electron scattering



Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)

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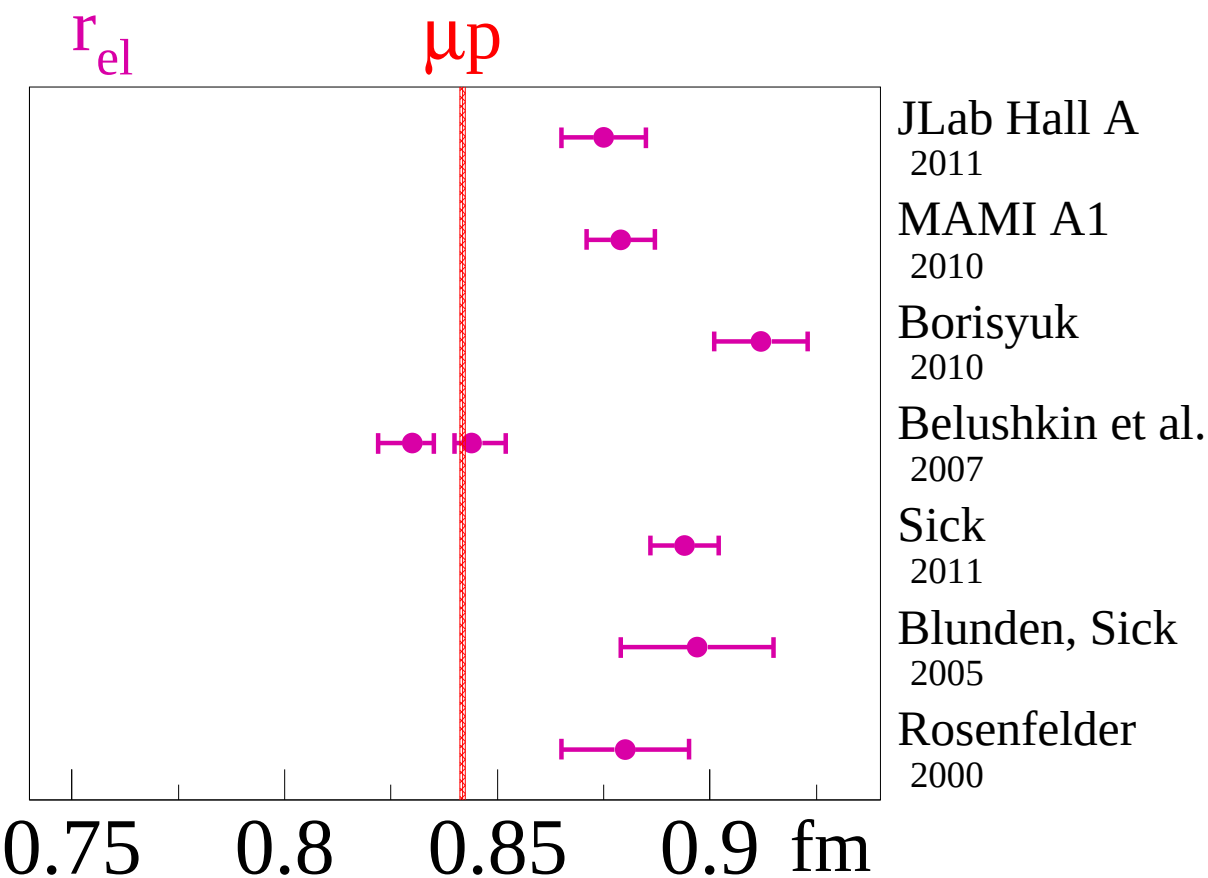

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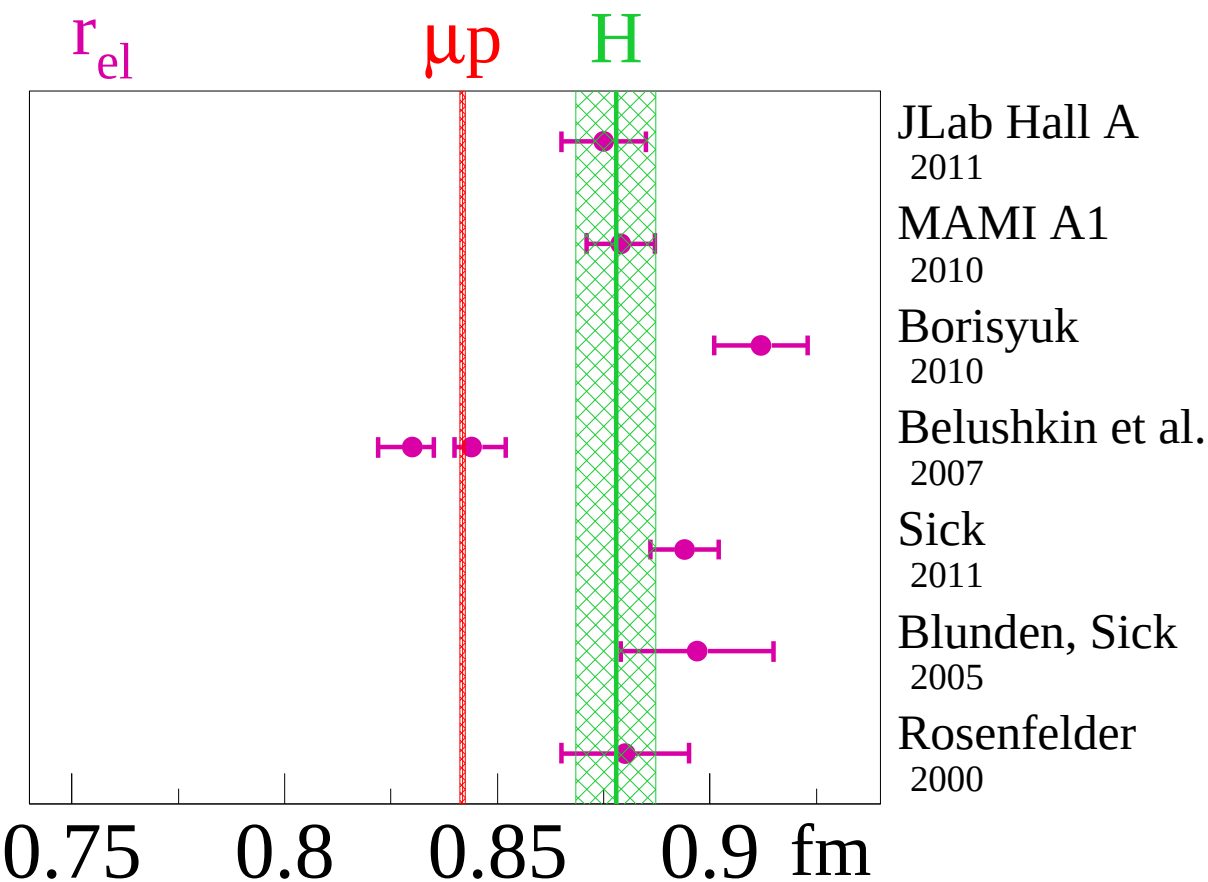

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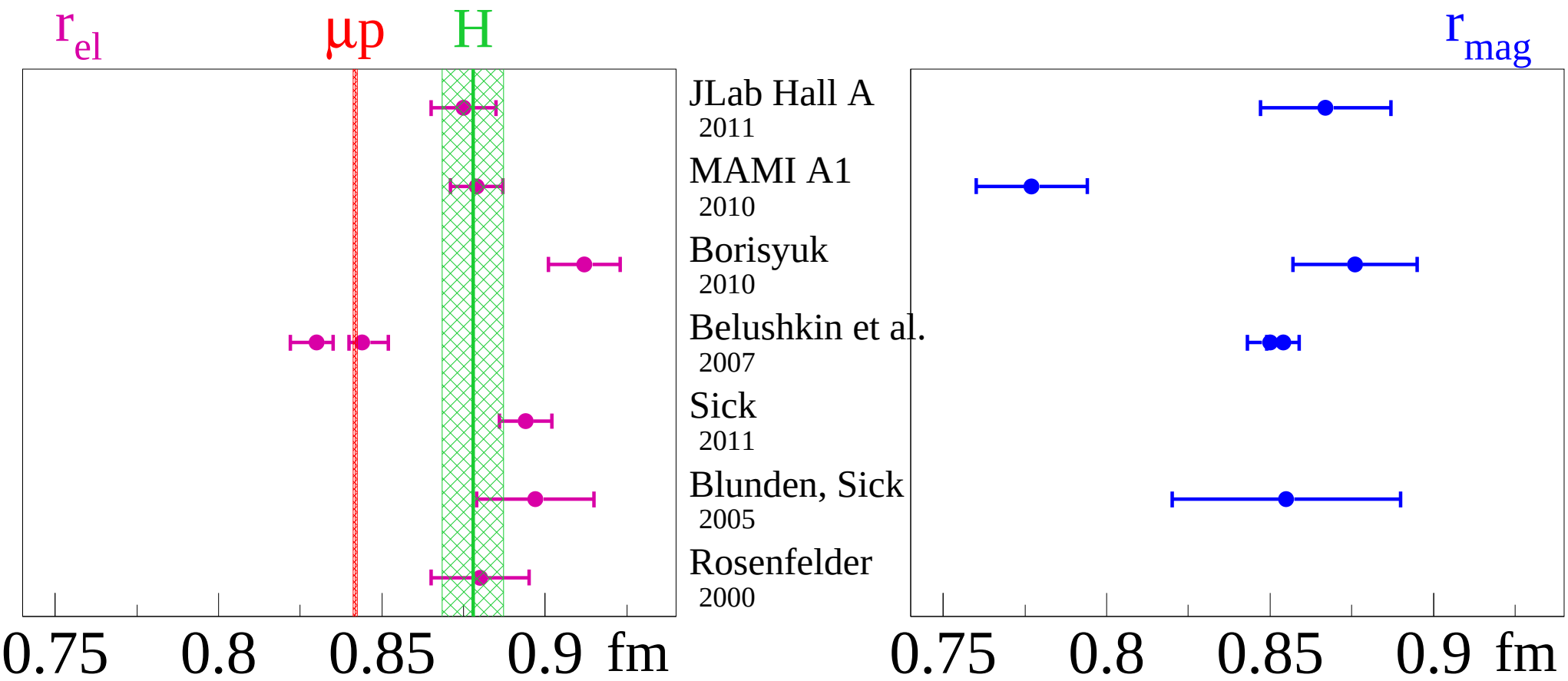

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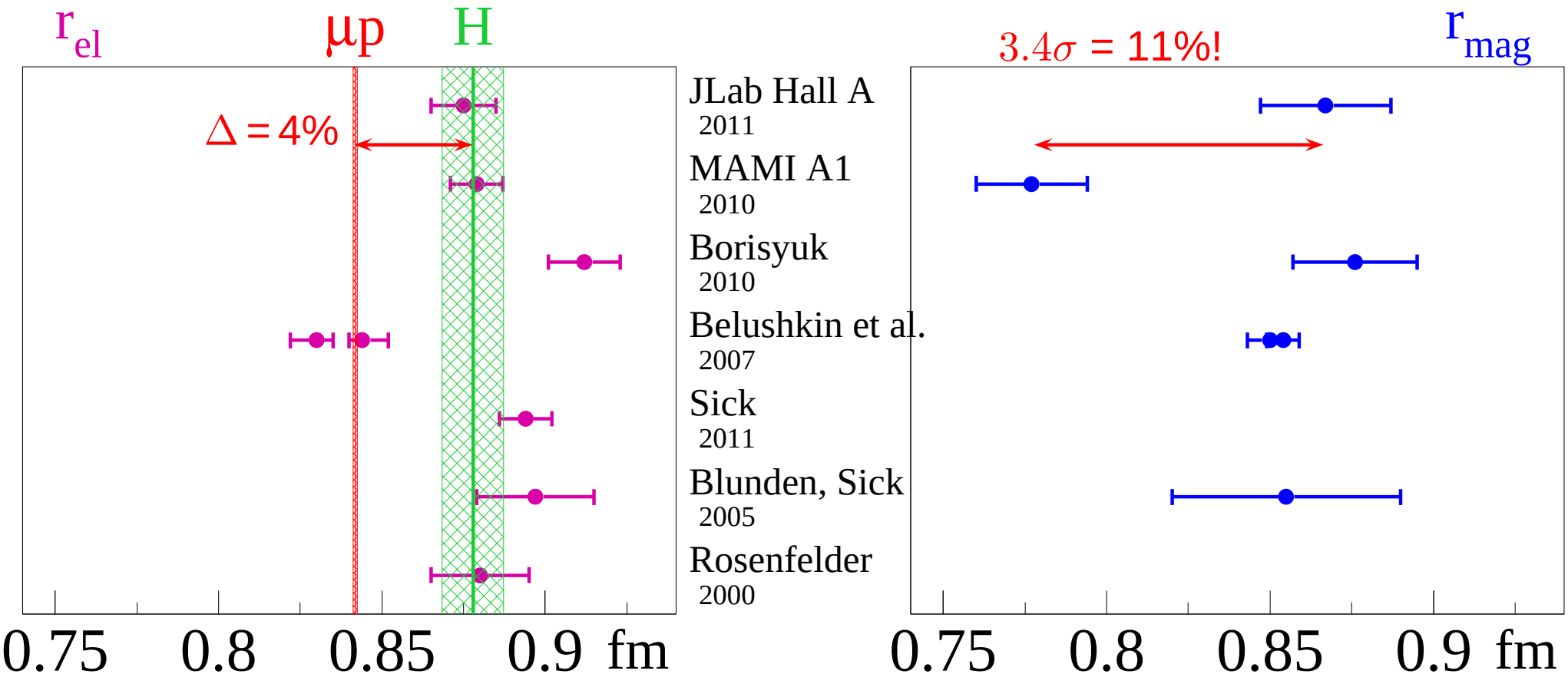
Electron scattering



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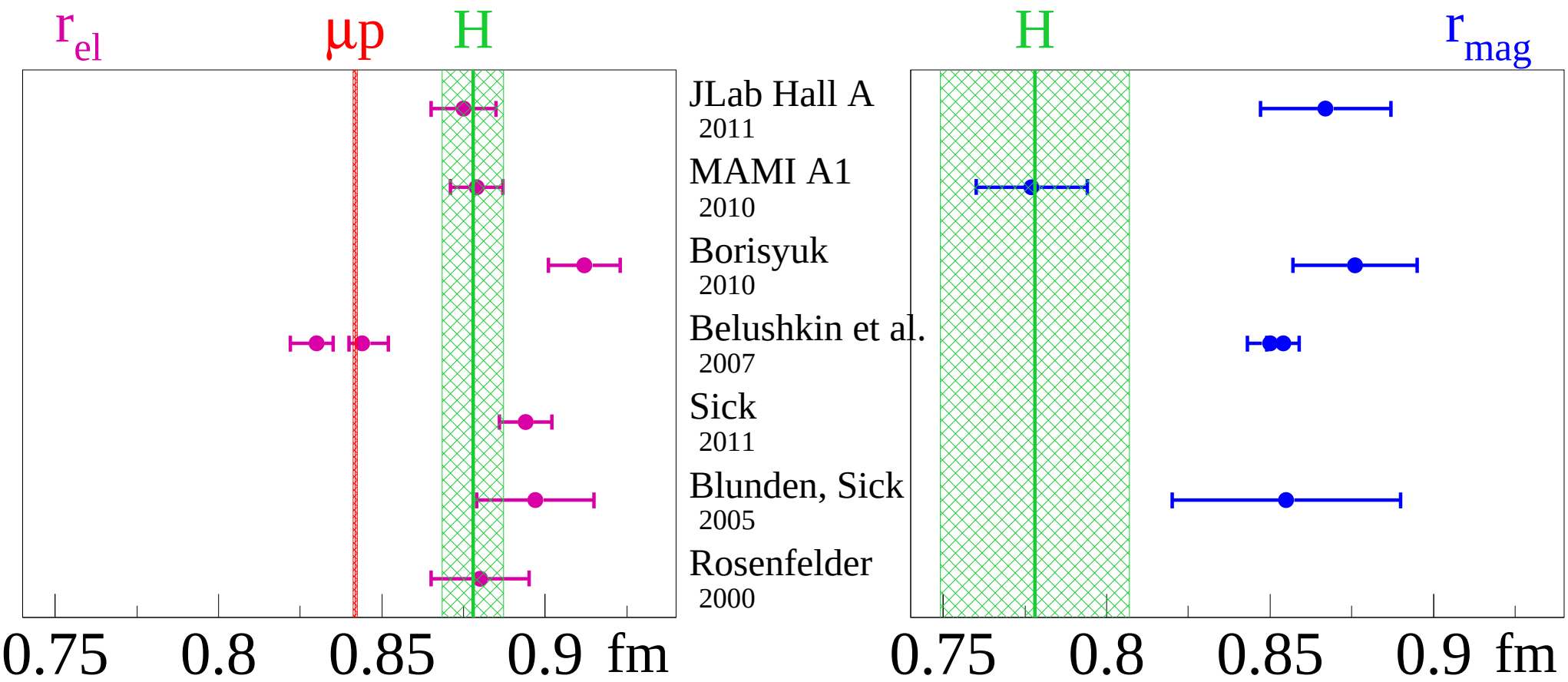

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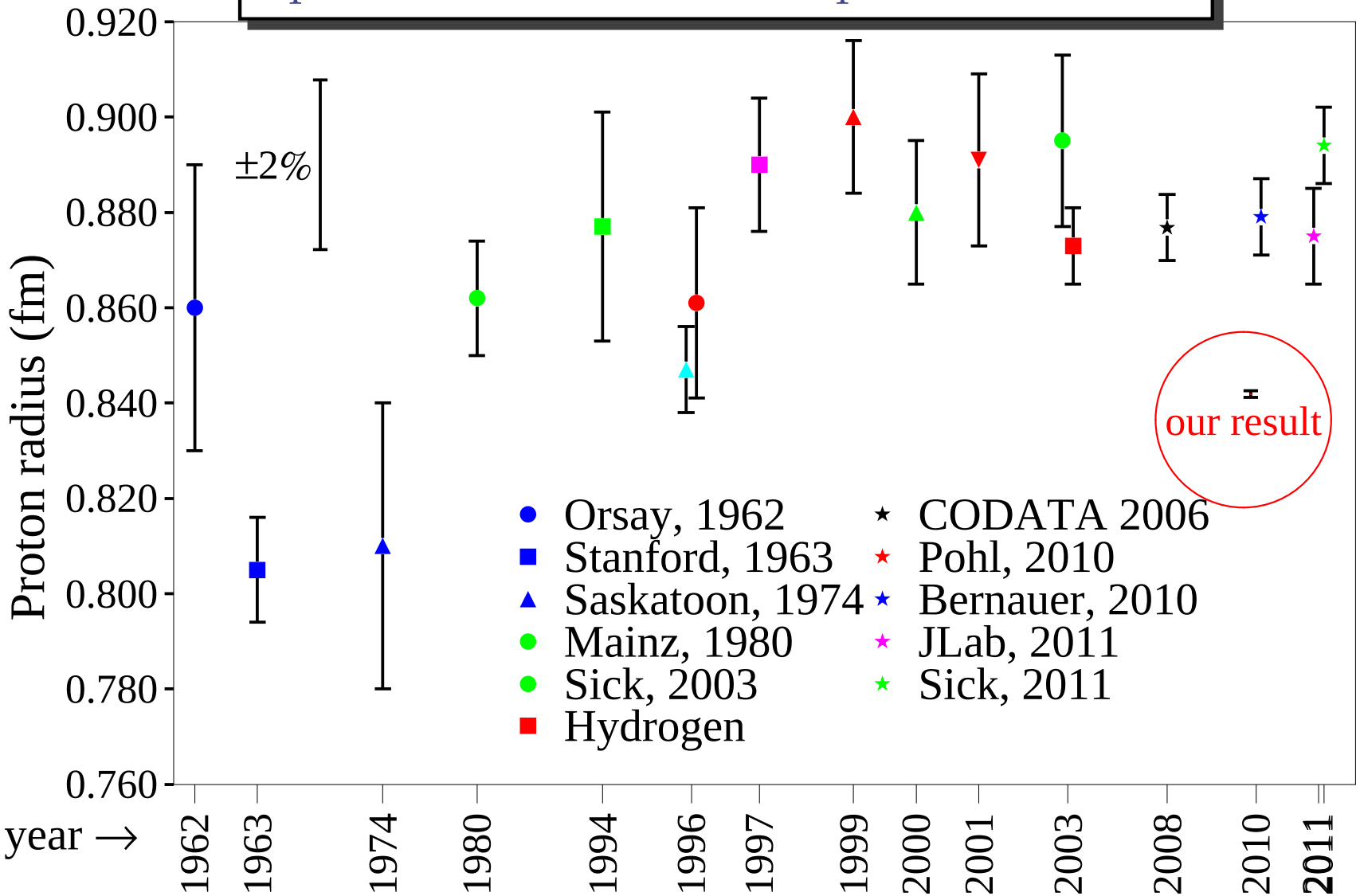
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)
r_{mag}(H) Volotka et al., Eur Phys J D33, 23 (2005)

Proton radius 2011

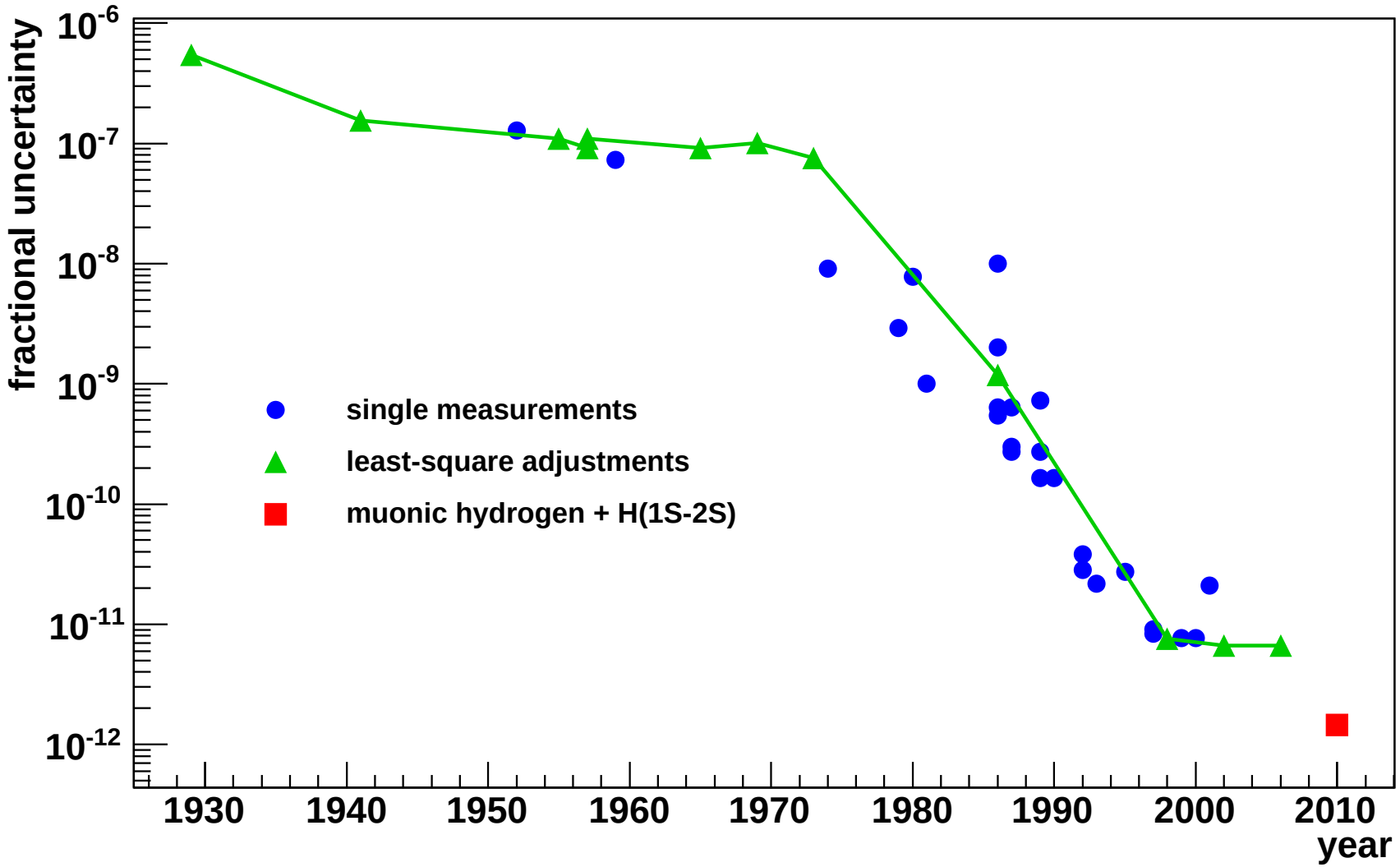
r_p=0.84184 (36)$_{exp}$(56)$_{theo}$ fm



R. Pohl *et al.*, Nature 466, 213 (2010).

Rydberg constant 2011

R_∞ = 10973731.568160(16) m^{-1} [1.5 parts in 10^{12}]



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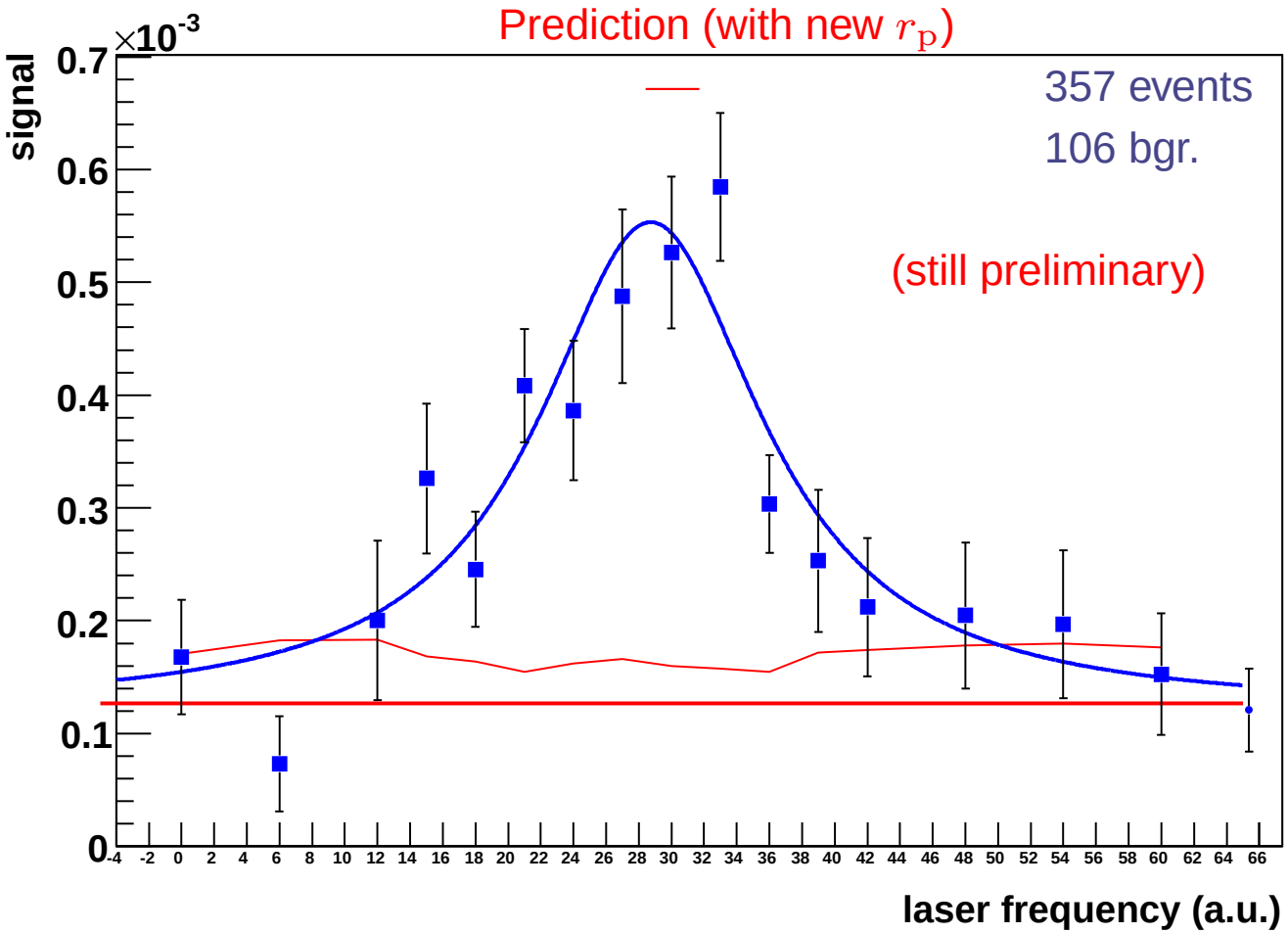
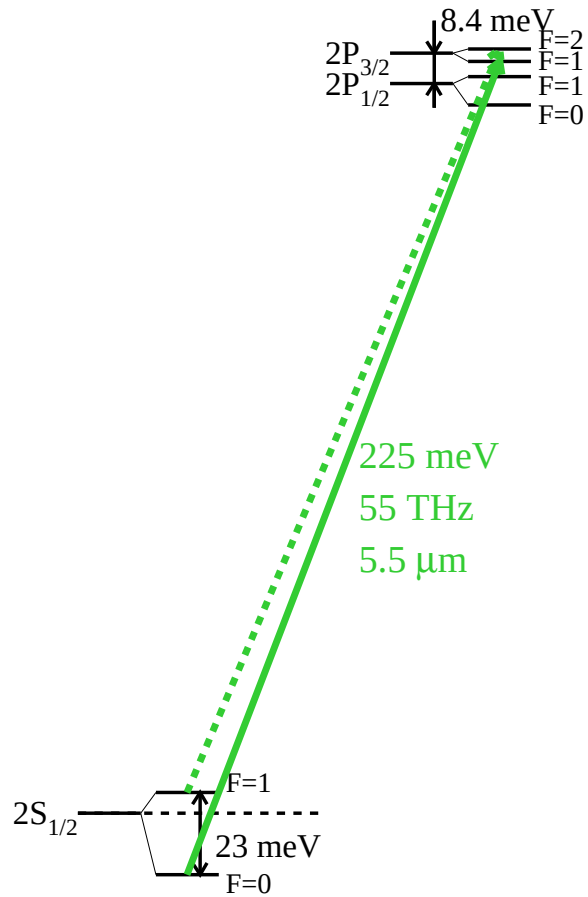
More measurements

2nd line in muonic hydrogen

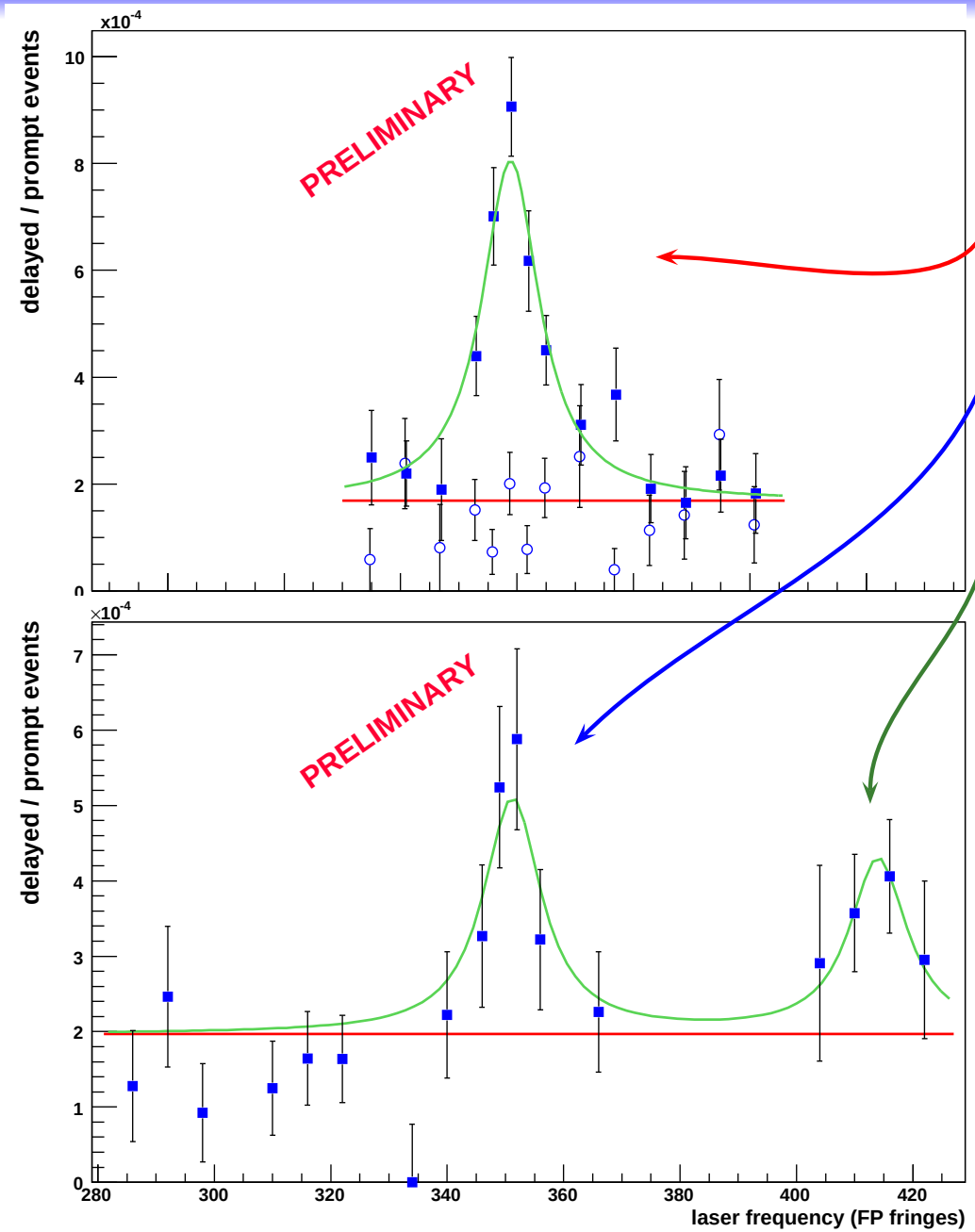




- $\sigma_{\text{position}} = 1.1$ GHz $\Longleftrightarrow$ 25 ppm $(\Gamma = 18.6$ GHz$)$
- Position fits perfectly with theory using new r_p

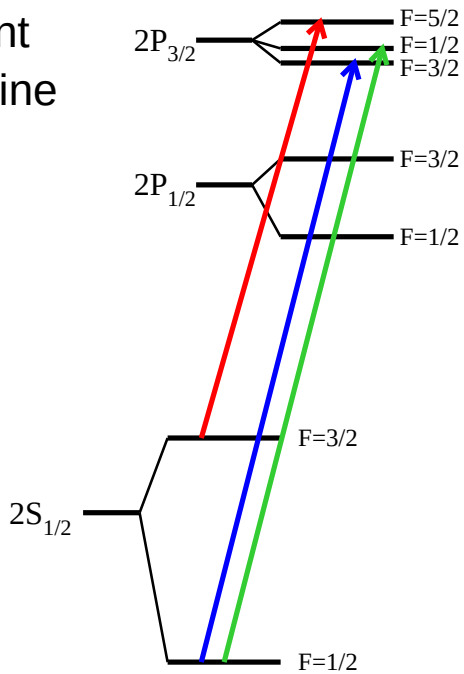
Extract HFS and r_{Zemach}

Muonic DEUTERIUM



2.5 resonances in muonic **deuterium**

- μd [$2S_{1/2}$(F=3/2) $\longrightarrow$ $2P_{3/2}$(F=5/2)]
  20 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\longrightarrow$ $2P_{3/2}$(F=3/2)]
  45 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\longrightarrow$ $2P_{3/2}$(F=1/2)]
  70 ppm (stat., online)
  only 5σ significant
  identifies F=3/2 line



Summary

- muonic **hydrogen** $2S_{1/2}$(F=1) $\rightarrow$ $2P_{3/2}$(F=2) to 15 ppm (stat.+syst.)
  $\rightarrow$ r_p to 8×10^{-4} (experimental precision 4×10^{-4})
  $r_p = 0.84184 \pm 0.00067$ fm is 5σ away from CODATA-2006

  The proton is 4% smaller, and the Rydberg constant R_∞ is 4.9 sigma off

- muonic **hydrogen** $2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1) to 25 ppm (stat., online)
  exactly at the position deduced with our new r_p
  $\rightarrow$ 2S hyperfine splitting to $\sim$ 200 ppm
  $\rightarrow$ Zemach radius to a few % (radius of the magnetic moment distribution)

- muonic **deuterium** $2S_{1/2}$(F=3/2) $\rightarrow$ $2P_{3/2}$(F=5/2) to 20 ppm (stat., online)
  Theory: missing QED and nuclear structure corrections
  $\rightarrow$ deuteron charge radius and polarizability

- muonic **deuterium** $2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=3/2 and F=1/2)
  $\rightarrow$ check calculations in μd

http://muhy.web.psi.ch

μp Lamb shift collaboration in 2009

F. KOTTMANN	ETH Zürich, Switzerland
A. ANTOGNINI, T.W. HÄNSCH, T. NEBEL, R. POHL	MPQ, Garching, Germany
D. TAQQU	PSI, Switzerland
E.-O. Le BIGOT, F. BIRABEN, P. INDELICATO, L. JULIEN, F. NEZ	Laboratoire Kastler Brossel, Paris, France
F.D. AMARO, J.M.R. CARDOSO, D.S. COVITA, L.M.P. FERNANDES, J.A.M. LOPEZ, C.M.B. MONTEIRO, J.M.F. DOS SANTOS, J.F.C.A. VELOSO	Department of Physics, Coimbra, Portugal
A. GIESEN, K. SCHUHMANN	Dausinger + Giesen, Stuttgart, Germany
T. GRAF	Institut für Strahlwerkzeuge, Stuttgart, Germany
C.-Y. KAO, Y.-W. LIU	National Tsing Hua University, Hsinchu, Taiwan
P. RABINOWITZ	Department of Chemistry, Princeton, USA
A. DAX, P. KNOWLES, L. LUDHOVA, F. MULHAUSER, L. SCHALLER	former members, spent holidays at run 2009

Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)

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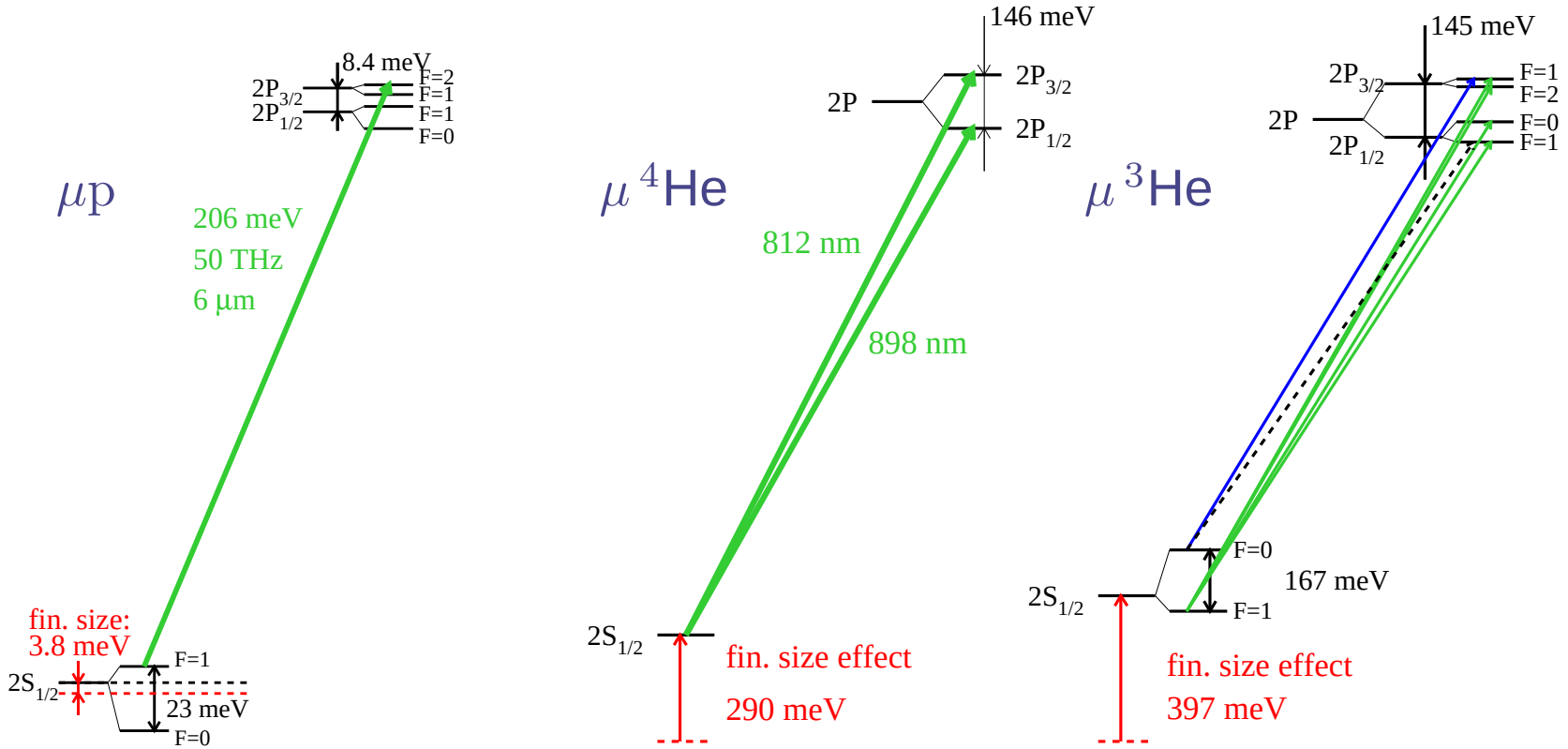

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- aims:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: ^{1}H, ^{2}D, ^{3}He, ^{4}He
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  - QED test with He$^+$(1S-2S) [Udem @ MPQ, Eikema @ Amsterdam]
- identical muon beam
- similar laser, no Raman cell ($\rightarrow$ more pulse energy)
- similar, maybe better x-ray detectors (8.2 keV)
- event rate: 16-48 events per hour (not 6 per hour, μp)
- line with 300 GHz (1 nm!)

Proton Size Investigators thank you for your attention

Contributions to the μp Lamb shift

#	Contribution	Value	Unc.
3	Relativistic one loop VP	205.0282	
4	NR two-loop electron VP	1.5081	
5	Polarization insertion in two Coulomb lines	0.1509	
6	NR three-loop electron VP	0.00529	
7	Polarisation insertion in two and three Coulomb lines (corrected)	0.00223	
8	Three-loop VP (total, uncorrected)		
9	Wichmann-Kroll	−0.00103	
10	Light by light electron loop ((Virtual Delbrück)	0.00135	0.00135
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	−0.00500	0.0010
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	−0.00150	
13	Mixed electron and muon loops	0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	0.01077	0.00038
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	0.000047	
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	−0.000015	
17	Recoil contribution	0.05750	
18	Recoil finite size	0.01300	0.001
19	Recoil correction to VP	−0.00410	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	−0.66770	
21	Muon Lamb shift 4th order	−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	−0.04497	
23	Recoil of order α^6	0.00030	
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	−0.00960	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	0.00019	
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	−0.00001	
	Sum	206.0573	0.0045

Contributions to the μp Lamb shift

Contribution	our selection		Pachucki	Borie
Leading nuclear size contribution	-5.19745	$< r_{\rm p}^2 >$	-5.1974	-5.1971
Radiative corrections to nuclear finite size effect	-0.0275	$< r_{\rm p}^2 >$	-0.0282	-0.0273
Nuclear size correction of order $(Z\alpha)^6 < r_{\rm p}^2 >$	-0.001243	$< r_{\rm p}^2 >$		
Total $< r_{\rm p}^2 >$ contribution	-5.22619	$< r_{\rm p}^2 >$	-5.2256	-5.2244
Nuclear size correction of order $(Z\alpha)^5$	0.0347	$< r_{\rm p}^3 >$	0.0363	0.0347
Nuclear size correction of order $(Z\alpha)^6 < r_{\rm p}^4 >$	-0.000043	$< r_{\rm p}^2 >^2$		

Contributions to the μp Lamb shift

Lamb shift: $\Delta E_{LS} = 206.0573(45) - 5.2262\, r_{\rm p}^2 + 0.0347\, r_{\rm p}^3$ meV

$u = 0.0045$ meV dominated by proton polarizability

$2S$ Hyperfine structure: $\Delta E_{HFS}^{2S} = 22.8148\,(78)$ meV

using $R_Z = 1.022$ fm and scatter.

Fine structure: $\Delta E_{FS} = 8.352082$ meV

$2P_{3/2}$ Hyperfine structure: $\Delta E_{HFS}^{2P_{3/2}} = 3.392588$ meV

Mainz scattering data at lowest Q^2

PhD thesis J.C. Bernauer

- Rosenbluth cross section $\rightarrow$ Sachs form factor $\rightarrow r_{\mathrm{p}}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon {G_E}^2 + \tau {G_M}^2}{\varepsilon(1+\tau)} \qquad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1} \quad ; \quad \tau = \frac{Q^2}{4m_p^2}$$

G_E and G_M are the Fourier transforms of the charge and magnetization distributions

$G_E(0) = 1$ (charge), and $G_M(0) = \mu_p$ (magnetic moment)

$$\langle r_{\mathrm{p}}^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{\mathrm{Q}^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \boxed{\text{slope of } G_E \text{ at } Q^2 = 0}$$

$\boxed{\text{extrapolation to } Q^2 \rightarrow 0 \text{ required}}$

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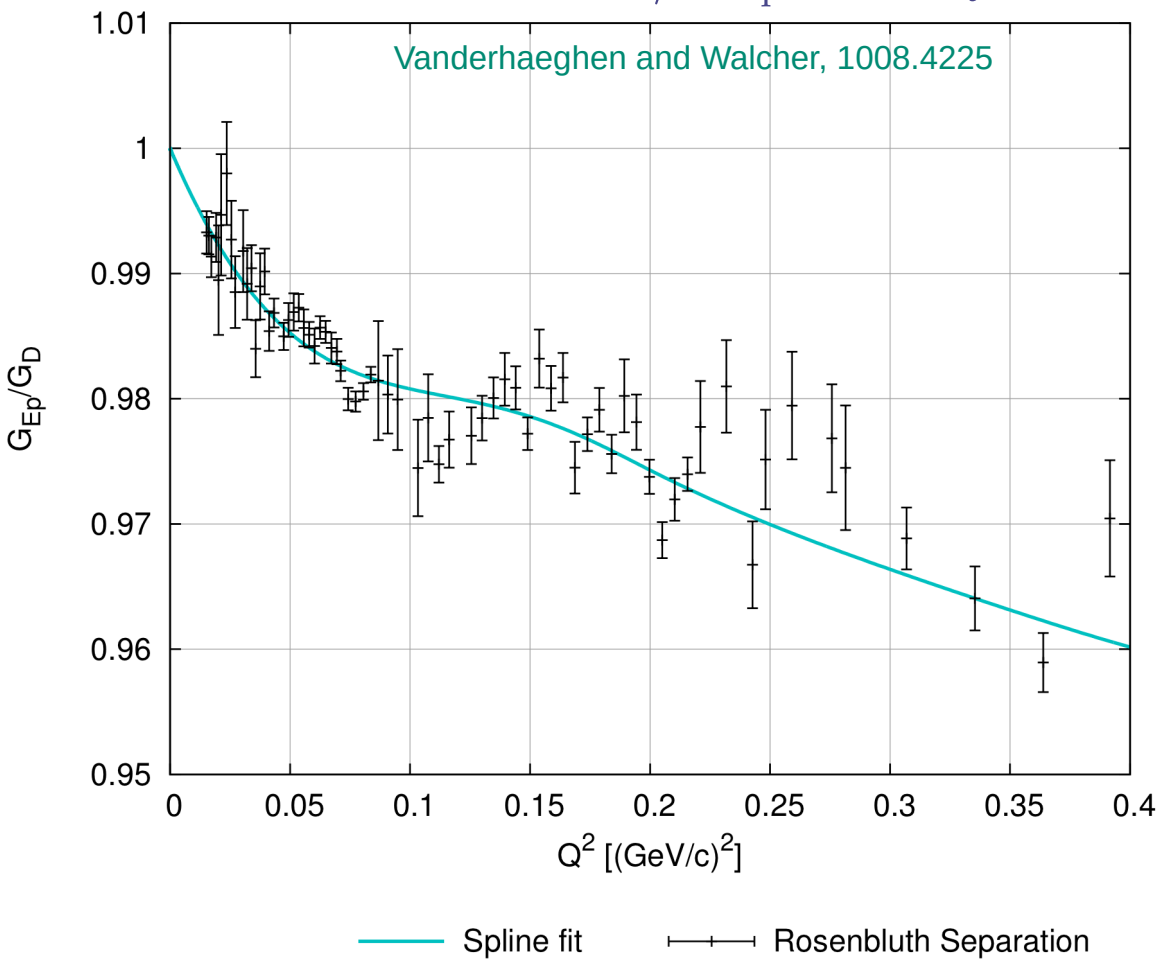
New Mainz data: $G_E/G_{\rm dipole}$ vs. Q^2



extrapolation to $Q^2 \to 0$ required

Mainz scattering data at lowest Q^2

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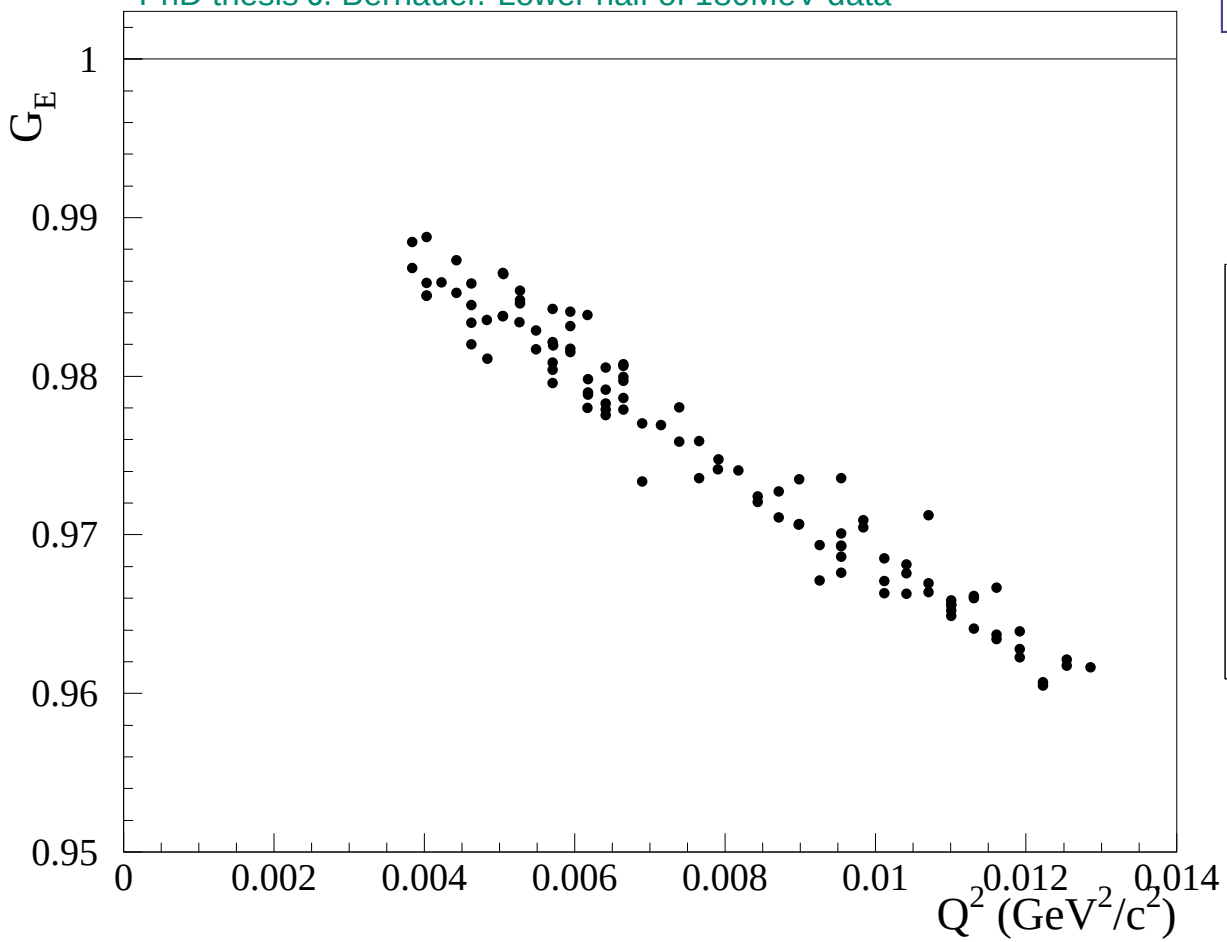
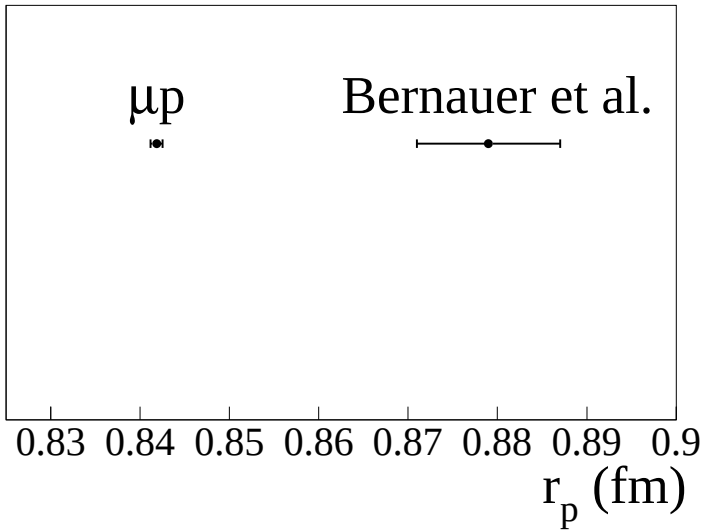
PhD thesis J. Bernauer. Lower half of 180MeV data



extrapolation to $Q^2 \to 0$ required



Mainz scattering data at lowest Q^2

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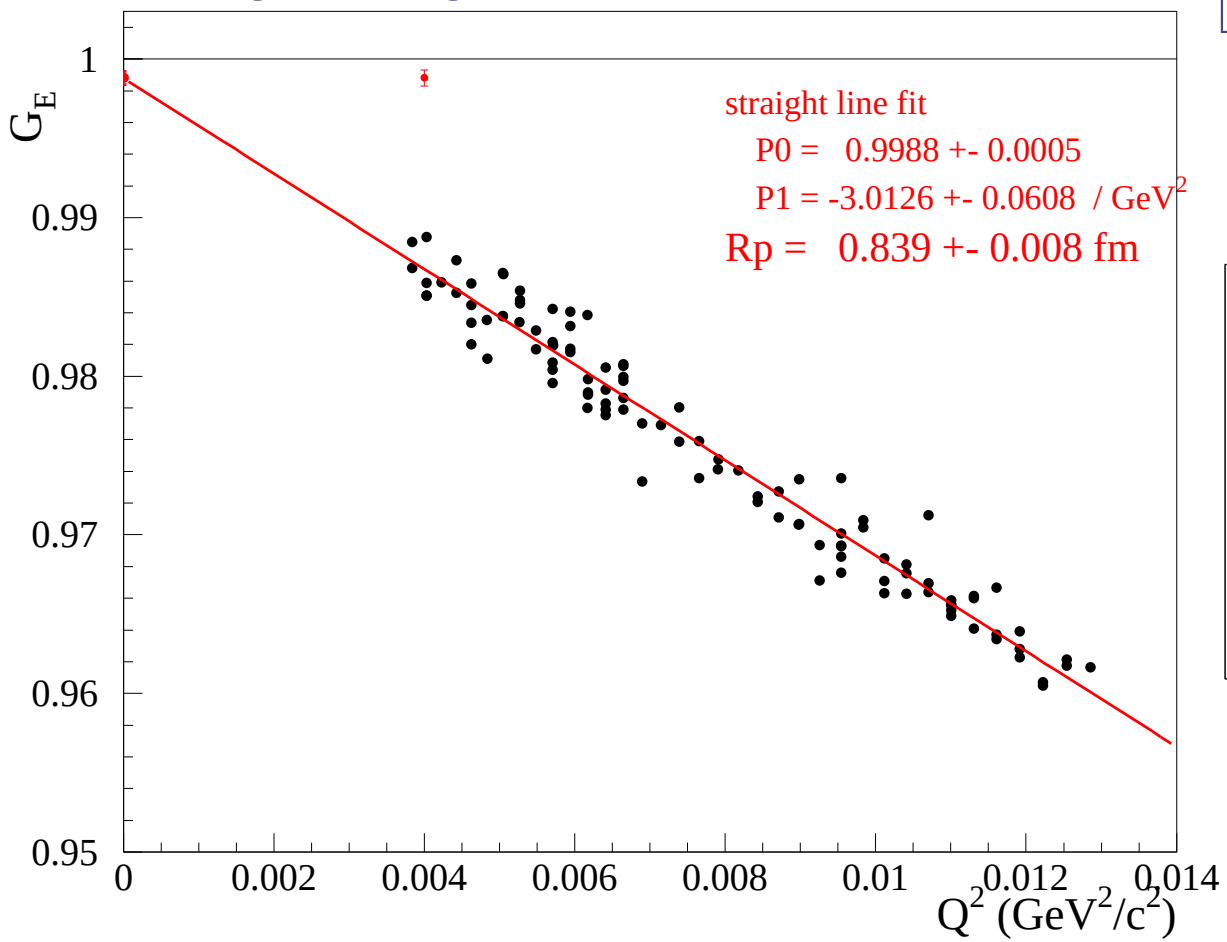
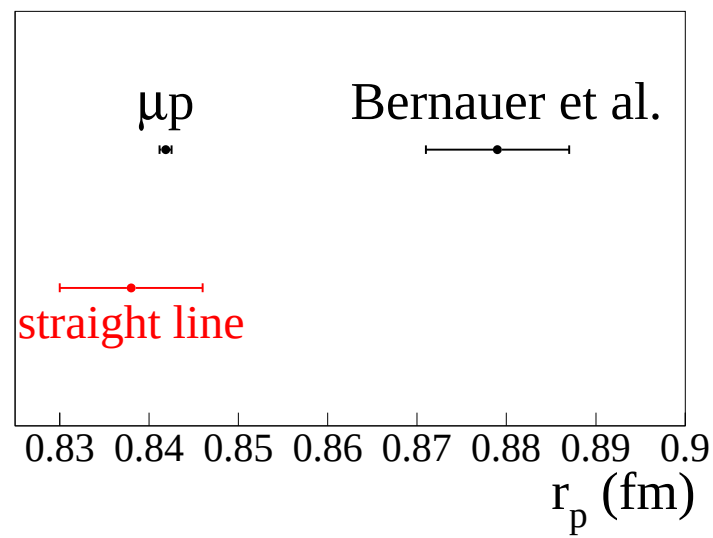
Fitting a straight line

extrapolation to $Q^2 \to 0$ required





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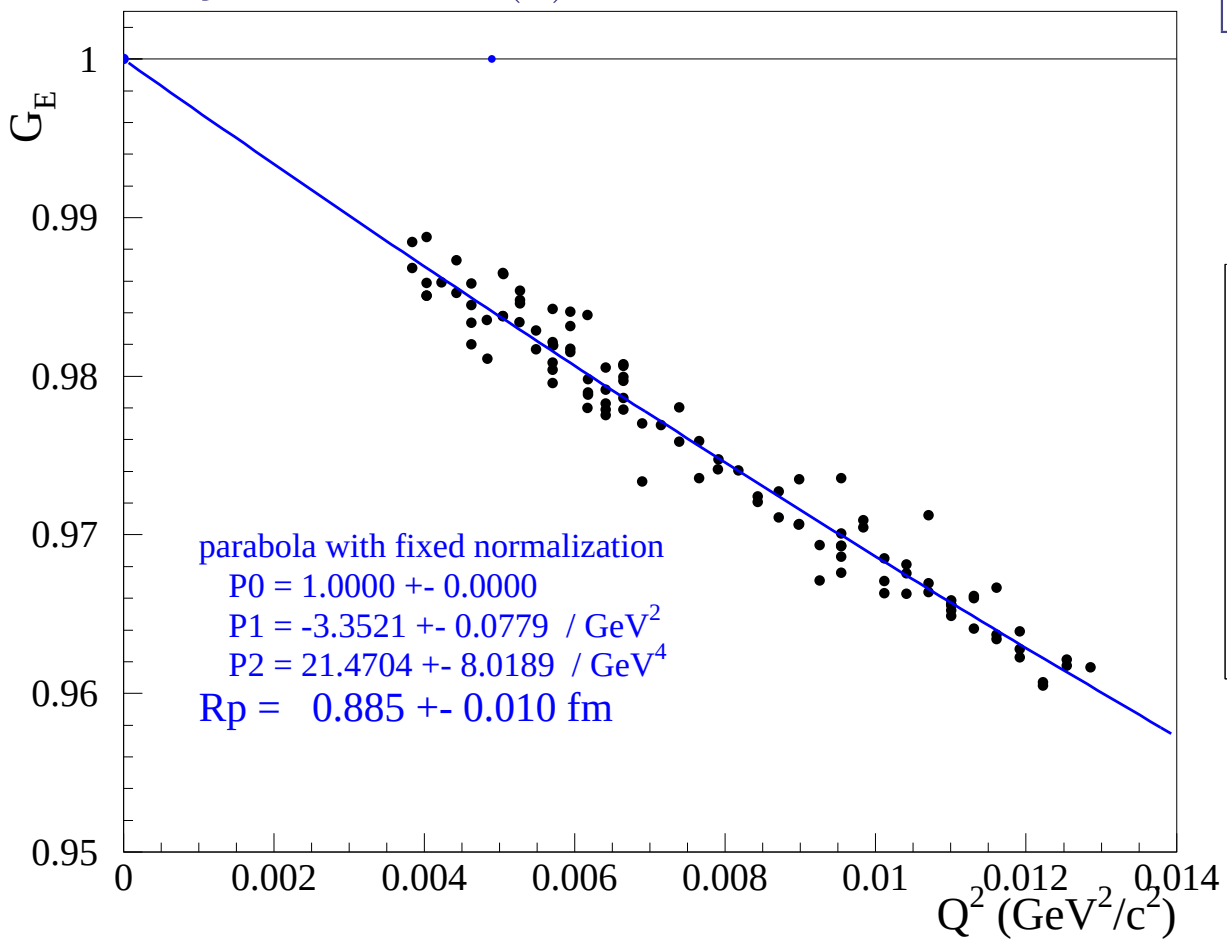
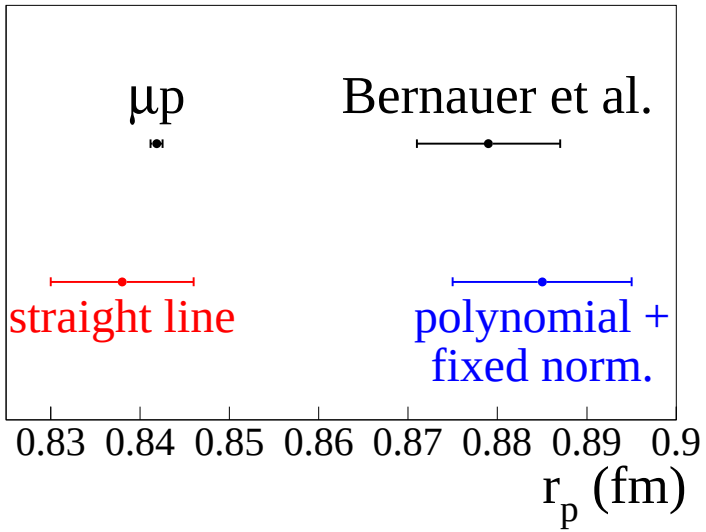
Polynomial, $G_E(0) = 1$

extrapolation to $Q^2 \to 0$ required



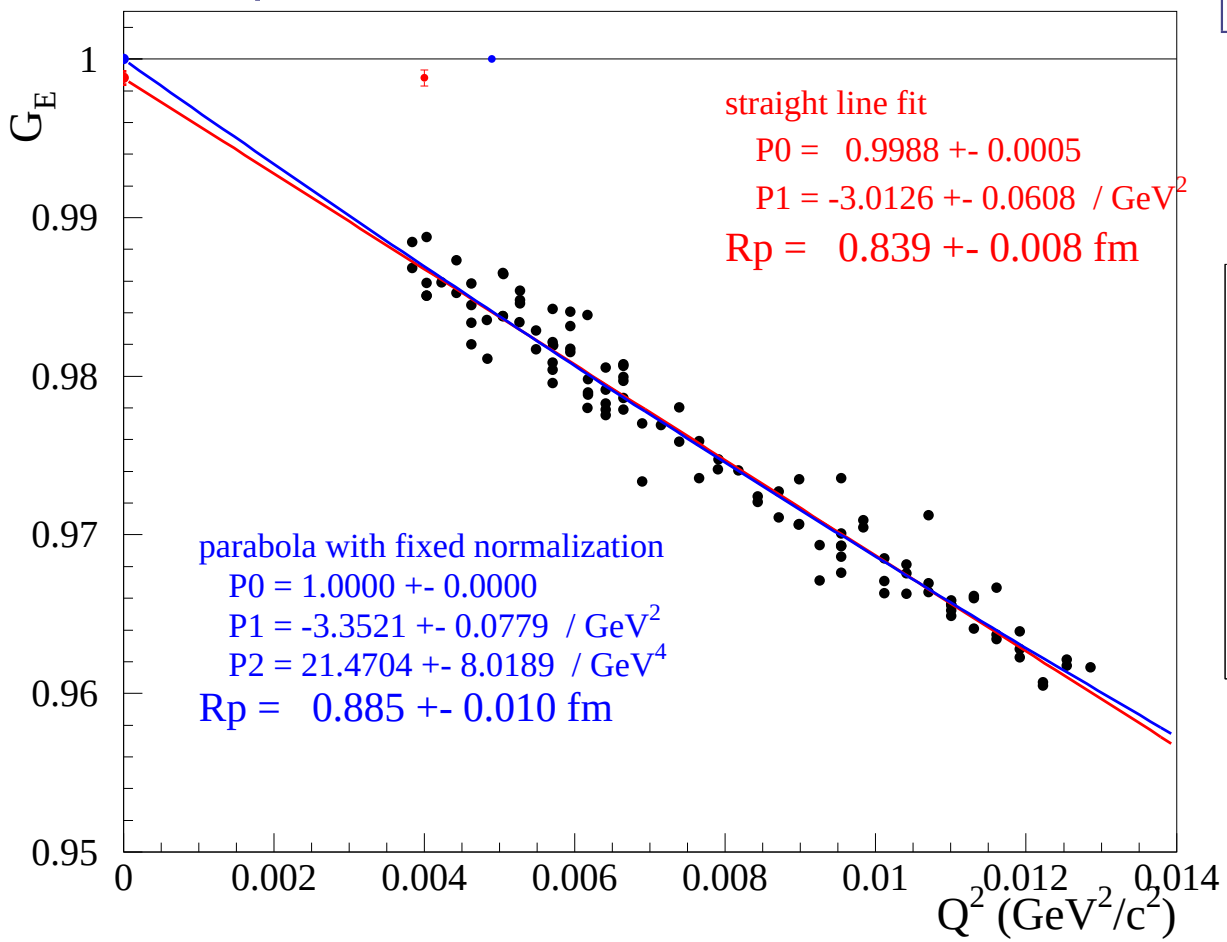


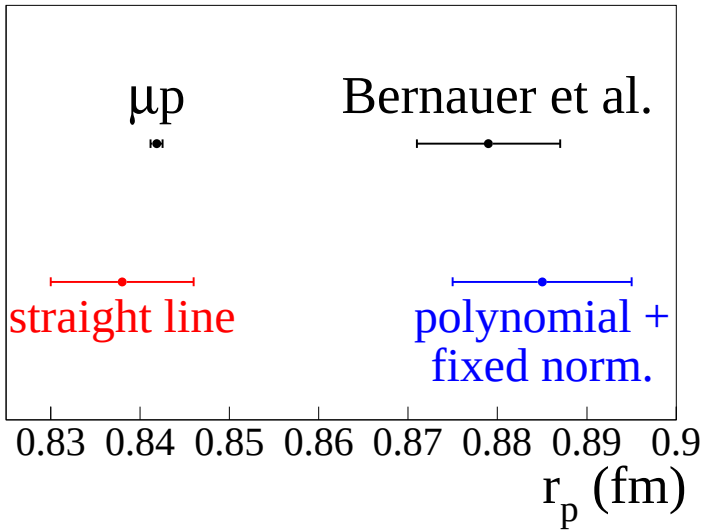
Mainz scattering data at lowest Q^2

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \boxed{\text{slope of } G_E \text{ at } Q^2 = 0}$$

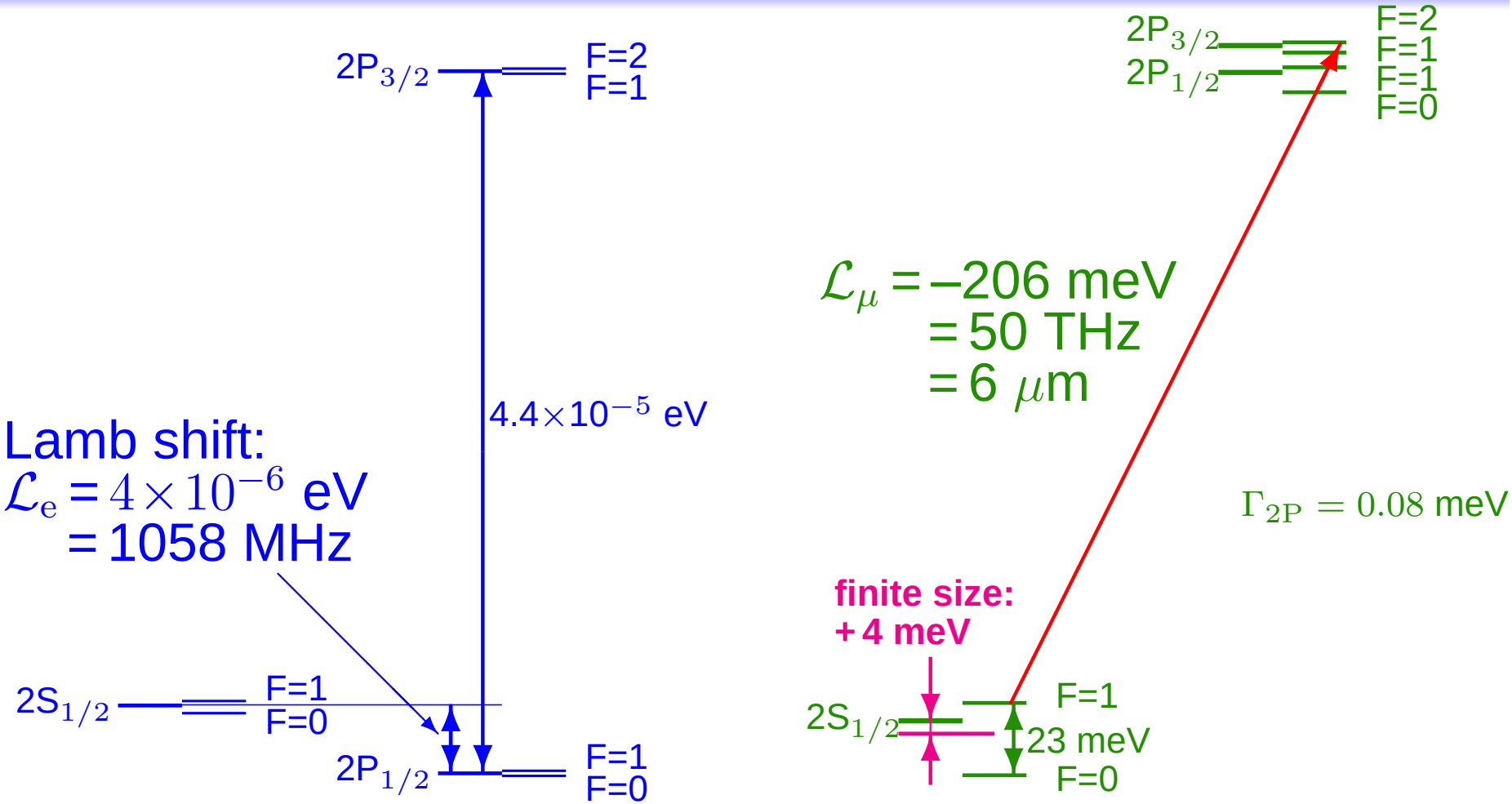
Extrapolation is subtle

extrapolation to $Q^2 \to 0$ required





(n=2) - states of ep and μp



$a_o = 5\times10^{-11}$ m

ΔE_{2P-1S} = 10 eV

self energy = +1086 MHz

vac. pol. = – 27 MHz

$a_o^\mu = 3\times10^{-13}$ m

ΔE_{2P-1S} = 1900 eV

self energy = + 0.6 meV

vac. pol. = – 206 meV