The size of the proton from the Lamb shift in muonic hydrogen

for the CREMA collaboration

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The problem:
Proton rms charge radius $r_p$ from muonic hydrogen $\mu p$ is 4% smaller than the values from elastic electron-proton scattering and hydrogen spectroscopy.
That’s $5\sigma \ldots 9.4\sigma$.
But the $\mu p$ result is 10 times more accurate than any other measurement.

Introduction:
Hydrogen, fundamental constants, QED tests and all that.
How large is the proton?
Muonic hydrogen:
(Finite) size does matter!

Experiment:
- Principle
- Muon beam
- Laser system

A solution of the “proton size puzzle”
Hydrogen energy levels

\[ E = \frac{R_{\infty}}{n^2} \]

\[ V \sim \frac{1}{r} \]
Hydrogen energy levels

Shift:
-43.5 GHz

Bohr

Dirac

\( E = \frac{R_\infty}{n^2} \)
\( V \sim \frac{1}{r} \)

e\(^{-}\) spin
relativity
Hydrogen energy levels

\[ E = \frac{R_{\infty}}{n^2} \]
\[ V \sim \frac{1}{r} \]

Bohr

Dirac

Lamb

\[ e^- \text{ spin relativity} \]

\[ QED \]

Shift:

-43.5 GHz

8.2 GHz
Hydrogen energy levels

Energy

\[ E = \frac{R_\infty}{n^2} \]

\[ V \sim \frac{1}{r} \]

\[ e^- \text{ spin} \]

\[ \text{relativity} \]

\[ \text{QED} \]

\[ \text{proton-spin} \]

\[ H^{\text{hfs}} \sim \vec{\mu}_p \cdot \vec{\mu}_e \]

\[ \text{Bohr} \]

\[ \text{Dirac} \]

\[ \text{Lamb} \]

\[ \text{hfs-splitting} \]

\[ \begin{align*}
    n=3 & & 2P_{3/2} \\
    n=2 & & 2S_{1/2}, 2P_{1/2} & & 2S_{1/2} \quad F=1 \\
    n=1 & & 1S_{1/2} & & 1S_{1/2} \quad F=0
\end{align*} \]

Shift:

\[ -43.5 \text{ GHz} \quad 8.2 \text{ GHz} \quad 1.4 \text{ GHz} \]

\[ F=1 \quad F=0 \]
Hydrogen energy levels

Energy

- n=3
- n=2
- n=1

Bohr

2P_{3/2}

2S_{1/2}, 2P_{1/2}

2S_{1/2}

2P_{1/2}

F=1

F=0

Shift:

-43.5 GHz

8.2 GHz

1.4 GHz

1.2 MHz

Shift:

e^- spin

QED

proton-spin

proton size

E = R_\infty / n^2

V \sim 1/r

V \sim 1/r

\mu_p \cdot \mu_e

V \sim 1/r

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The Rydberg constant

Accuracy of the Rydberg constant

2006: $R_{\infty} = 10\,973\,731.568\,525 \pm 0.000\,073\,m^{-1}$ ($\mu_r = 6.6 \cdot 10^{-12}$)
is the 2\textsuperscript{nd} most accurately determined fundamental constant.
Test of bound-state QED

1S Lamb shift in hydrogen: \( L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \) MHz

The QED-test is limited by the uncertainty of the proton rms charge radius.

\( \delta r_p / r_p = 0.02 \)

\( \delta r_p / r_p = 10^{-3} \)

until "now"

accuracy of QED calculations

future?
The proton rms charge radius is not the most accurate quantity in the universe.

\[ r_p = 0.895(18) \text{ fm (} u_r = 2\% \text{)} \]

CODATA: \[ r_p = 0.8768(69) \text{ fm (} u_r = 0.8\% \text{)} \]
Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.

Electron scattering:

\[
\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \Rightarrow \text{slope of } G_E \text{ at } Q^2 = 0
\]

- electron scattering
- hydrogen spectr.

Lamb shift (S-states)

Vanderhaegen, Walcher 1008.4225

Spline fit  Rosenbluth Separation
The proton rms charge radius is not the most accurate quantity in the universe.

Hydrogen spectroscopy (Lamb shift):

\[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

2 unknowns \( \Rightarrow \) 2 transitions

- Rydberg constant \( R_\infty \)
- Lamb shift \( L_{1S} \leftarrow r_p \)

slope of \( G_E \) at \( Q^2 = 0 \)

Lamb shift (S-states)
The proton rms charge radius is not the most accurate quantity in the universe.

The figure shows the proton radius vs. time from 1962 to 2008, with various experiments noted:

- Orsay, 1962
- Stanford, 1963
- Saskatoon, 1974
- Mainz, 1980
- Mainz, free norm. dispersion fit
- Paris, 1996
- Garching, 1997
- Paris, 1999
- Rosenfelder, 2000
- Eides, 2001
- Sick, 2003
- Pachucki Jentschura, 03
- CODATA 2006

**e-p scattering**

\[ r_p = 0.895(18) \text{ fm} \quad (u_r = 2\%) \]

**CODATA**

\[ r_p = 0.8768(69) \text{ fm} \quad (u_r = 0.8\%) \]

- **20x improvement**
- **(aim: 10x better QED test in H)**

**muonic hydrogen goal (1998)**

\[ u_r = 0.1\% \]
muonic hydrogen = $\mu^- p$  \[\text{mass } m_\mu = 207 \text{ m}_e\]

\[\Rightarrow \text{Bohr: } \langle r^{\text{orbit}} \rangle \sim \frac{\hbar}{Z \alpha m_r c} n^2\]

\[\Delta E_{\text{finite size}}(nl) \sim r^2_p \mid \Psi(r = 0) \mid^2\]

\[\Rightarrow \Delta E_{\text{finite size}}(nl) = \frac{2(Z \alpha)^4 c^4}{3 \hbar^2 n^3} m^3_r r^2_p \delta_{l0}\]

Lamb shift in $\mu p$: \[\Delta E(2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}) = 209.9779(49) - 5.2262 r^2_p + 0.0347 r^3_p \text{ [meV]}\]

finite size contribution is 2% of the $\mu p$ Lamb shift measure $\Delta E(2S-2P)$ to 30 ppm = 1.5 GHz

\[\Rightarrow r_p \text{ to } 10^{-3}\]

\[\Gamma_{2P} = 18.6 \text{ GHz} \quad (\Gamma_{\text{rad.}})\]
**µp Lamb shift experiment: Principle**

### "prompt" ($t \sim 0$)

- µ⁻ stop in H₂ gas
- ⇒ µp* atoms formed ($n \sim 14$)

99%: cascade to µp(1S), emitting prompt $K_\alpha$, $K_\beta$ ...

1%: long-lived µp(2S) atoms
- lifetime $\tau_{2S} \approx 1 \mu s$ at 1 mbar H₂


### "delayed" ($t \sim 1 \mu s$)

- Fire laser ($\lambda \approx 6 \mu m$, $\Delta E \approx 0.2 eV$)
- ⇒ induce µp(2S) → µp(2P)
- ⇒ observe delayed $K_\alpha$ x-rays
- ⇒ normalize \( \frac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha} \) x-rays
µp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays \( (\sim 13 \text{ hours of data}) \)
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

```
<table>
<thead>
<tr>
<th>time [us]</th>
<th>events in 25 ns</th>
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<td>0.5</td>
<td>10^4</td>
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<tr>
<td>1</td>
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<td>3.5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
```

“prompt” ($t \sim 0$)

- $n \sim 14$
- 1 % 2 S
- 99 % 2 P
- 2 keV γ
- 1 S
µp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

"prompt" (t ∼ 0)  

"delayed" (t ∼ 1 µs)

Laser

2 S → 2 P

1 S → 2 S

2 keV γ

n~14

1 %

99 %

2 P

1 S

99 %

2 P

1 %

2 keV γ
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

“prompt” \((t \sim 0)\)  

“delayed” \((t \sim 1 \mu s)\)

normalize \(\frac{\text{delayed} K_\alpha}{\text{prompt} K_\alpha} \Rightarrow \text{Resonance}\)

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Muon beam line

$\pi^-$
$10^8 \text{ s}^{-1}$
$p=100 \text{ MeV/c}$

Solenoid
B=5 T
Hydrogen target

TOF
Velocity filter

Muon detector
&
Frictional cooling

ExB

HV= $-19 \text{ kV}$
200 nm foil
for muon extraction

CT
B=4 T

B=2 T

B=4 T

Momentum filter

MEC
B=0.1 T

$\mu$
$e$

n

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p. 9
Muon beam line
Muon beam: inside 5 T solenoid

- PM
- B=5 Tesla
- Collimator
- Gas Target
- Laser pulse
- PM₁
- S₁
- 10 cm
- PM₂
- HV
- PM₃
- ExB
The laser system

Main components:

- Thin-disk laser
  - fast response to detected $\mu^-$
- Frequency doubling
- TiSa laser:
  - frequency stabilized cw laser injection seeded oscillator
  - multipass amplifier
- Raman cell
  - 3 Stokes: 708 nm $\rightarrow$ 6 $\mu$m
  - $\lambda$ calibration @ 6 $\mu$m
- Target cavity

The laser system

Yb:YAG thin-disk laser

Oscillator 1030 nm

200 W

9 mJ

Amplifier

500 W

43 mJ

SHG

SHG

SHG

23 mJ

515 nm

23 mJ

708 nm, 15 mJ

TiSa Osc.

TiSa Amp.

43 mJ

1.5 mJ

7 mJ

1030 nm

FP

I$_2$ / Cs

Verdi

5 W

cw TiSa laser

Wave meter

Raman cell

7 mJ

H$_2$O

6 μm monitoring

20 m

0.25 mJ

Ge-filter

μ$^-$

6 μm cavity

Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single $\mu^-$ triggers the laser system
- $2S$ lifetime $\approx 1 \mu$s $\rightarrow$ short laser delay

A. Antognini et. al.,
The laser system

**Yb:YAG thin–disk laser**

- Oscillator 1030 nm
- Amplifier 43 mJ
- SHG

**cw TiSa laser**

- Wave meter
- Verdi 5 W
- Oscillator 1030 nm
- Amplifier 43 mJ
- SHG

**Oscillator**

- 200 W
- 500 W

**Amplifier**

- 9 mJ
- 500 W

**SHG**

- 23 mJ
- 515 nm

**Verdi**

- 5 W

**Multipass amplifier (2f- configuration)**

- Gain=10

**MOPA TiSa laser:**

- Cw frequency stabilized laser
  - referenced to a stable FP cavity
  - FP cavity calibrated with $I_2$, Rb, Cs lines
  
  \[ \nu_{FP} = N \cdot FSR \]
  
  \[ FSR = 1497.344(6) \text{ MHz} \]
  
  \[ \nu_{\text{cw TiSa}} \text{ absolutely known to 30 MHz} \]
  
  \[ \Gamma_{2P-2S} = 18.6 \text{ GHz} \]

**Seeded oscillator**

\[ \rightarrow \nu_{\text{pulsed TiSa}} = \nu_{\text{cw TiSa}} \]

(frequency chirp \( \leq 100 \text{ MHz} \))

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The laser system

Yb:YAG thin-disk laser

- Oscillator 1030 nm
  - 9 mJ
- Amplifier 1030 nm
  - 43 mJ
- SHG
- 1.5 mJ
- TiSa Osc.

200 W

- 500 W

- 708 nm, 15 mJ

- 6 μm monitoring
- H₂O
  - 20 m
  - 6 μm
- 0.25 mJ

SHG

Verdi

Wave meter

I₂ / Cs

SHG

Raman cell:

- 708 nm
- H₂
- 6.02 μm

1st Stokes

2nd Stokes

3rd Stokes

6 μm cavity

μ⁻

P. Rabinowitz et. al., IEEE J. QE 22, 797 (1986)
The laser system

Yb:YAG thin-disk laser

- Oscillator 1030 nm
- Amplifier 1030 nm
- SHG
- SHG
- SHG
- 9 mJ
- 200 W
- 43 mJ
- 43 mJ
- 200 W
- 43 mJ
- 500 W
- 43 mJ
- 500 W
- 43 mJ

 cw TiSa laser

- Wave meter
- Verdi
- 5 W
- 5 W
- cw TiSa 708 nm
- 400 mW
- I_2 / Cs

Design: insensitive to misalignment
Transverse illumination
Large volume

Dielectric coating with \( R \geq 99.9\% \) (at 6 \( \mu m \))

→ Light makes 1000 reflections
→ Light is confined for \( \tau=50 \) ns
→ 0.15 mJ saturates the \( 2S \to 2P \) transition
The laser system

Yb:YAG thin-disk laser

Oscillator 1030 nm
Amplifier 1030 nm
Amplifier 1030 nm
SHG

Oscillator 1030 nm
Amplifier 1030 nm
SHG

SHG

SHG

Oscillator 200 W
Amplifier 500 W

Verdi

Wave meter

7 mJ

SHG

I$_2$ / Cs

SHG

708 nm

I$_2$ / Cs

200 W

500 W

43 mJ

7 mJ

1.5 mJ

23 mJ

515 nm

23 mJ

1200 1300 1400 1500 1600 1700 1800 1900 2000 2100

wavenumber (cm$^{-1}$)

Water absorption

Verdi

SHG

H$_2$O

20 m

Ge-filter

μ$^-$

6 μm cavity

· Vacuum tube for 6 μm beam transport.

· Direct frequency calibration at 6 μm.
$6 \mu m$ wavelength calibration

- $6 \mu m$ light calibration: $H_2O$ vapor absorption measurement in air / cell

$H_2O$ absorption lines known to a few MHz (HITRAN)

$\Rightarrow \delta \nu \approx 300$ MHz uncertainty (6 ppm of $\Delta E_{2S-2P}$) due to our calibration accuracy over the whole wavelength range $\lambda = 5.5 \ldots 6.1 \mu m$

- Laser frequency detuning is measured in number of Fabry-Perot cavity fringes
  - grid spacing of our measurement: $FSR(FP) = 1497.344(6)$ MHz
  - all measured resonances are within $\pm 70$ FP fringes of a $H_2O$ line
Target, cavity and detectors
The resonance: discrepancy, sys., stat.

Water-line/laser wavelength: 300 MHz uncertainty

Δν water-line to resonance: 200 kHz uncertainty

CODATA-06

our value

H₂O calib.

e-p scattering

Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy:
5.0σ ↔ 75 GHz ↔ δν/ν = 1.5 × 10⁻³

The resonance: discrepancy, sys., stat.

Water-line/laser wavelength: 300 MHz uncertainty

$\Delta \nu$ water-line to resonance: 200 kHz uncertainty

CODATA-06

our value

Systematics: 300 MHz
Statistics: 700 MHz

550 events measured on resonance where 155 bgr events are expected
fit Lorentz + flat bgr $\Rightarrow \chi^2$/dof = 28.1/28
width agrees with expectation
bgr agrees with laser OFF data
$\chi^2$/dof = 283/31 for flat line $\rightarrow$ 16$\sigma$

Discrepancy:
$5.0 \sigma \leftrightarrow 75 \text{ GHz} \leftrightarrow \delta \nu / \nu = 1.5 \times 10^{-3}$

The time spectra

![Graph showing time spectra with two peaks labeled Laser ON resonance and Laser OFF resonance.]
Uncertainty budget and sensitivity

- **Statistics**
  - Center position uncertainty ($\sim 4\%$ of $\Gamma$) 700 MHz

- **Systematics**
  - Laser frequency ($\text{H}_2\text{O}$ calibration) 300 MHz
  - AC and DC stark shift < 1 MHz
  - Zeeman shift (5 Tesla) < 30 MHz
  - Doppler shift < 1 MHz
  - Collisional shift 2 MHz

- Total uncertainty of the line determination 760 MHz

- **Theory:** proton polarizability 1200 MHz

- **Discrepancy** with CODATA prediction 75 300 MHz

Systematic effects are small since they scale like $1/m$

Finite size effect scales like $m^3$
Proton radius

\[ \nu \left( 2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2} \right) = 49881.88(76) \text{ GHz} \]

\[ \tilde{L}^{\text{exp.}} = 206.2949(32) \text{ meV} \]
\[ \tilde{L}^{\text{th.}} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV} \]

\( r_p = 0.84184(36)(56) \text{ fm} \)

\( u_{\text{exp}} = 4.3 \times 10^{-4} \)
\( u_{\text{theo}} = 6.7 \times 10^{-4} \)

CODATA 2006: \( r_p = (0.8768 \pm 0.0069) \text{ fm} \)
Hydrogen: \( r_p = (0.876 \pm 0.008) \text{ fm} \)
e-p scattering: \( r_p = (0.895 \pm 0.018) \text{ fm} \) (Sick 2005)

\( r_p \) is 4% smaller
5.0\( \sigma \) from CODATA-2006
4.3\( \sigma \) from H
3.1\( \sigma \) from e-p scatt.

Proton radius

\[ \nu \left(2S_{1/2}^F \rightarrow 2P_{3/2}^F\right) = 49881.88(76) \text{ GHz}. \]

\[ \tilde{L}^{\text{exp.}} = 206.2949(32) \text{ meV} \]

\[ \tilde{L}^{\text{th.}} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV} \]

\[ r_p = 0.84184(67) \text{ fm} \quad (u_r = 8 \times 10^{-4}) \]

CODATA 2006: \( r_p = (0.8768 \pm 0.0069) \text{ fm} \)
Hydrogen: \( r_p = (0.876 \pm 0.008) \text{ fm} \)
e-p scattering: \( r_p = (0.894 \pm 0.008) \text{ fm} \) (Sick 2011)
\( r_p = (0.879 \pm 0.008) \text{ fm} \) (Mainz 2010)
\( r_p = (0.875 \pm 0.010) \text{ fm} \) (JLab Hall A 2011)

\( r_p \) is 4% smaller

5.0\( \sigma \) from CODATA-2006
4.3\( \sigma \) from H
8.4\( \sigma \) from scatt.
What is wrong?

$\mu_p$ experiment: discrepancy 75 GHz

$100\sigma$

$\sim 4 \Gamma_{\text{nat}}$

two independent wavelength calibrations

one very significant line, no satellite

fitted width = natural width

another transition in $\mu_p$ confirms our $r_p$

$\mu_p$ theory: discrepancy 0.31 meV

$60\sigma$

0.15% of the total Lamb shift

4th largest term

H theory: $L_{1S}$ off by 100 kHz

$25\sigma$

(almost) all terms only calculated by 2 groups + methods

convergence?

$\Rightarrow$ These are solid.
**What is wrong?**

**μp experiment:** discrepancy 75 GHz

100σ

\( \sim 4 \Gamma_{\text{nat}} \)

two independent wavelength calibrations

one very significant line, no satellite

fitted width = natural width

**μp theory:**

Maybe both H spectroscopy and e-p scattering are off.

**H theory:**

(almost) all terms only calculated by 2 groups + methods

convergence?

⇒ These are solid.
Hydrogen spectroscopy

proton charge radius (fm)
Hydrogen spectroscopy

\[
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
2S_{1/2} - 2P_{3/2} \\
1S-2S + 2S- 4S_{1/2} \\
1S-2S + 2S- 4D_{5/2} \\
1S-2S + 2S- 4P_{1/2} \\
1S-2S + 2S- 4P_{3/2} \\
1S-2S + 2S- 6S_{1/2} \\
1S-2S + 2S- 6D_{5/2} \\
1S-2S + 2S- 8S_{1/2} \\
1S-2S + 2S- 8D_{3/2} \\
1S-2S + 2S- 8D_{5/2} \\
1S-2S + 2S- 12D_{3/2} \\
1S-2S + 2S- 12D_{5/2} \\
1S-2S + 1S - 3S_{1/2}
\]

\[ \mu_p : 0.84184 \pm 0.00067 \text{ fm} \]
Hydrogen spectroscopy

\[ \text{m}_p : 0.84184 \pm 0.00067 \text{ fm} \]

Proton charge radius (fm)

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HADRON München, 13. June 2011 p. 20
Hydrogen spectroscopy

\[
\begin{align*}
2S_{1/2} & \rightarrow 2P_{1/2} \\
2S_{1/2} & \rightarrow 2P_{1/2} \\
2S_{1/2} & \rightarrow 2P_{3/2} \\
1S-2S + 2S & \rightarrow 4S_{1/2} \\
1S-2S + 2S & \rightarrow 4D_{5/2} \\
1S-2S + 2S & \rightarrow 4P_{1/2} \\
1S-2S + 2S & \rightarrow 4P_{3/2} \\
1S-2S + 2S & \rightarrow 6S_{1/2} \\
1S-2S + 2S & \rightarrow 6D_{5/2} \\
1S-2S + 2S & \rightarrow 8S_{1/2} \\
1S-2S + 2S & \rightarrow 8D_{3/2} \\
1S-2S + 2S & \rightarrow 8D_{5/2} \\
1S-2S + 2S & \rightarrow 12D_{3/2} \\
1S-2S + 2S & \rightarrow 12D_{5/2} \\
1S-2S + 1S - 3S & \rightarrow \text{H}
\end{align*}
\]

\[H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]

\[\mu_p : 0.84184 \pm 0.00067 \text{ fm} \]
Electron scattering

\[ r_{\text{el}} \]

- Blunden, Sick, PRC 72, 057601 (2005)
- Sick, Few Body Syst. (2011)
- Belushkin et al., PRC 75, 035202 (2007)
- MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
- JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)
Electron scattering

$r_{el}$  $\mu p$

0.75 0.8 0.85 0.9 fm

Blunden, Sick, PRC 72, 057601 (2005)
Sick, Few Body Syst. (2011)
Belushkin et al., PRC 75, 035202 (2007)

MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)
Electron scattering

\[ r_{el} \quad \mu p \quad H \]

0.75 0.8 0.85 0.9 fm

Blunden, Sick, PRC 72, 057601 (2005)
Sick, Few Body Syst. (2011)
Belushkin et al., PRC 75, 035202 (2007)

JLab Hall A
2011
MAMI A1
2010
Borisyuk
2010
Belushkin et al.
2007
Sick
2011
Blunden, Sick
2005
Rosenfelder
2000

MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
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Electron scattering

\[ r_{el} \quad \mu p \quad H \]

\[ \Delta = 4\% \]

JLab Hall A 2011
MAMI A1 2010
Borisyuk 2010
Belushkin et al. 2007
Sick 2011
Blunden, Sick 2005
Rosenfelder 2000

3.4\sigma = 11\%!

MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

Blunden, Sick, PRC 72, 057601 (2005)
Sick, Few Body Syst. (2011)
Belushkin et al., PRC 75, 035202 (2007)
Electron scattering

\[ r_{el} \quad \mu p \quad H \]

- Blunden, Sick, PRC 72, 057601 (2005)
- Sick, Few Body Syst. (2011)
- Belushkin et al., PRC 75, 035202 (2007)

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\[ H \]

- MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
- JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)
- \( r_{mag}(H) \) Volotka et al., Eur Phys J D33, 23 (2005)
Proton radius 2011

\[ r_p = 0.84184 (36) \text{exp}(56) \text{theo fm} \]

- Orsay, 1962
- Stanford, 1963
- Saskatoon, 1974
- Mainz, 1980
- Sick, 2003
- Hydrogen
- CODATA 2006
- Pohl, 2010
- Bernauer, 2010
- JLab, 2011
- Sick, 2011

\[ \pm 2\% \]

Rydberg constant 2011

\[ R_{\infty} = 10973731.568160(16) \text{ m}^{-1} \quad [1.5 \text{ parts in } 10^{12}] \]

More measurements
2nd line in muonic hydrogen

**Prediction (with new \( r_p \))**

- 357 events
- 106 bgr.

(still preliminary)

\[ \begin{align*}
\sigma_{\text{position}} &= 1.1 \ \text{GHz} \iff 25 \text{ ppm (} \Gamma = 18.6 \ \text{GHz)} \\
\text{Position fits perfectly with theory using new } r_p
\end{align*} \]

Extract HFS and \( r_{\text{Zemach}} \)

Randolf Pohl
p. 25
2.5 resonances in muonic deuterium

- $\mu d \left[ 2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2) \right]$  
  20 ppm (stat., online)

- $\mu d \left[ 2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2) \right]$  
  45 ppm (stat., online)

- $\mu d \left[ 2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=1/2) \right]$  
  70 ppm (stat., online)  
  only $5\sigma$ significant  
  identifies $F=3/2$ line
Summary

- **muonic hydrogen** $2S_{1/2}(F=1) \rightarrow 2P_{3/2}(F=2)$ to 15 ppm (stat.+syst.)
  $\rightarrow r_p$ to $8 \times 10^{-4}$ (experimental precision $4 \times 10^{-4}$)
  $r_p = 0.84184 \pm 0.00067$ fm is $5\sigma$ away from CODATA-2006

  - The proton is 4% smaller, and the Rydberg constant $R_\infty$ is 4.9 sigma off.

- **muonic hydrogen** $2S_{1/2}(F=0) \rightarrow 2P_{3/2}(F=1)$ to 25 ppm (stat., online)
  exactly at the position deduced with our new $r_p$
  $\rightarrow$ 2S hyperfine splitting to $\sim 200$ ppm
  $\rightarrow$ Zemach radius to a few % (radius of the magnetic moment distribution)

- **muonic deuterium** $2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2)$ to 20 ppm (stat., online)
  Theory: missing QED and nuclear structure corrections
  $\rightarrow$ deuteron charge radius and polarizability

- **muonic deuterium** $2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2 \text{ and } F=1/2)$
  $\rightarrow$ check calculations in $\mu d$

http://muhy.web.psi.ch
μp Lamb shift collaboration in 2009

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D. TAQQU
E.-O. Le BIGOT, F. BIRABEN, P. INDELICATO, L. JULIEN, F. NEZ
A. GIESEN, K. SCHUHMANN T. GRAF
C.-Y. KAO, Y.-W. LIU
P. RABINOWITZ
A. DAX, P. KNOWLES, L. LUDHOVA, F. MULHAUSER, L. SCHALLER

ETH Zürich, Switzerland
MPQ, Garching, Germany
PSI, Switzerland
Laboratoire Kastler Brossel, Paris, France
Department of Physics, Coimbra, Portugal
Dausinger + Giesen, Stuttgart, Germany
Institut für Strahlwerkzeuge, Stuttgart, Germany
National Tsing Hua University, Hsinchu, Taiwan
Department of Chemistry, Princeton, USA
former members, spent holidays at run 2009
Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure $\Delta E(2S-2P)$ in $\mu^4$He, $\mu^3$He
- $\Rightarrow$ alpha particle and helion charge radius to $3 \times 10^{-4}$ (0.0005 fm)
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- **Aims**:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: $^1\text{H}$, $^2\text{D}$, $^3\text{He}$, $^4\text{He}$
  - QED test with $\text{He}^+(1S-2S)$ [Udem @ MPQ, Eikema @ Amsterdam]
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  - low-energy effective nuclear models: $^1$H, $^2$D, $^3$He, $^4$He
  - QED test with He$^+(1S-2S)$ [Udem @ MPQ, Eikema @ Amsterdam]
- identical muon beam
- similar laser, no Raman cell ($\rightarrow$ more pulse energy)
- similar, maybe better x-ray detectors (8.2 keV)
- event rate: 16-48 events per hour (not 6 per hour, $\mu_p$)
- line with 300 GHz (1 nm!)
Proton Size Investigators thank you for your attention
Contributions to the $\mu p$ Lamb shift

<table>
<thead>
<tr>
<th>#</th>
<th>Contribution</th>
<th>Value</th>
<th>Unc.</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>Relativistic one loop VP</td>
<td>205.0282</td>
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<tr>
<td>4</td>
<td>NR two-loop electron VP</td>
<td>1.5081</td>
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<tr>
<td>5</td>
<td>Polarization insertion in two Coulomb lines</td>
<td>0.1509</td>
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<td>6</td>
<td>NR three-loop electron VP</td>
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<td>7</td>
<td>Polarisation insertion in two and three Coulomb lines (corrected)</td>
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<tr>
<td>8</td>
<td>Three-loop VP (total, uncorrected)</td>
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<tr>
<td>9</td>
<td>Wichmann-Kroll</td>
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<td>−0.00103</td>
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<tr>
<td>10</td>
<td>Light by light electron loop ((Virtual Delbrück)</td>
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<td>−0.00135</td>
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<tr>
<td>11</td>
<td>Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$</td>
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<td>−0.00500</td>
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<tr>
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<td>Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$</td>
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<td>−0.00150</td>
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<tr>
<td>13</td>
<td>Mixed electron and muon loops</td>
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<td>0.00007</td>
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<tr>
<td>14</td>
<td>Hadronic polarization $\alpha(Z\alpha)^4 m_r$</td>
<td>0.01077</td>
<td>0.00038</td>
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<tr>
<td>15</td>
<td>Hadronic polarization $\alpha(Z\alpha)^5 m_r$</td>
<td>0.00047</td>
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<td>16</td>
<td>Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$</td>
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<td>−0.000015</td>
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<tr>
<td>17</td>
<td>Recoil contribution</td>
<td>0.05750</td>
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<tr>
<td>18</td>
<td>Recoil finite size</td>
<td>0.01300</td>
<td>0.001</td>
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<td>19</td>
<td>Recoil correction to VP</td>
<td>−0.00410</td>
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<td>20</td>
<td>Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$</td>
<td>−0.66770</td>
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<tr>
<td>21</td>
<td>Muon Lamb shift 4th order</td>
<td>−0.00169</td>
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<tr>
<td>22</td>
<td>Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$</td>
<td>−0.04497</td>
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<tr>
<td>23</td>
<td>Recoil of order $\alpha^6$</td>
<td>0.00030</td>
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<td>24</td>
<td>Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$</td>
<td>−0.00960</td>
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<tr>
<td>25</td>
<td>Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)</td>
<td>0.015</td>
<td>0.004</td>
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<tr>
<td>26</td>
<td>Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$</td>
<td>0.00019</td>
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<td>27</td>
<td>Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$</td>
<td>−0.00001</td>
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<tr>
<td></td>
<td>Sum</td>
<td>206.0573</td>
<td>0.0045</td>
</tr>
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</table>
## Contributions to the $\mu p$ Lamb shift

<table>
<thead>
<tr>
<th>Contribution</th>
<th>our selection</th>
<th>Pachucki</th>
<th>Borie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leading nuclear size contribution</td>
<td>$-5.19745$</td>
<td>$-5.1974$</td>
<td>$-5.1971$</td>
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<tr>
<td>Radiative corrections to nuclear finite size effect</td>
<td>$-0.0275$</td>
<td>$-0.0282$</td>
<td>$-0.0273$</td>
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<tr>
<td>Nuclear size correction of order $(Z\alpha)^6 &lt; r_p^2$</td>
<td>$-0.001243$</td>
<td>$-0.001243$</td>
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<tr>
<td>Total $&lt; r_p^2$ contribution</td>
<td>$-5.22619$</td>
<td>$-5.2256$</td>
<td>$-5.2244$</td>
</tr>
<tr>
<td>Nuclear size correction of order $(Z\alpha)^5$</td>
<td>$0.0347$</td>
<td>$0.0363$</td>
<td>$0.0347$</td>
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<tr>
<td>Nuclear size correction of order $(Z\alpha)^6 &lt; r_p^4$</td>
<td>$-0.000043$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Contributions to the $\mu p$ Lamb shift

Lamb shift: $\Delta E_{LS} = 206.0573(45) - 5.2262 r_p^2 + 0.0347 r_p^3$ meV

$u = 0.0045$ meV dominated by proton polarizability

$2S$ Hyperfine structure: $\Delta E_{HFS}^{2S} = 22.8148 (78)$ meV

using $R_Z = 1.022$ fm and scatter.

Fine structure: $\Delta E_{FS} = 8.352082$ meV

$2P_{3/2}$ Hyperfine structure: $\Delta E_{HFS}^{2P_{3/2}} = 3.392588$ meV
Mainz scattering data at lowest $Q^2$

- Rosenbluth cross section → Sachs form factor → $r_p$

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Ros.}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon(1 + \tau)}
\]

\[
\varepsilon = \left( 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right)^{-1} \quad ; \quad \tau = \frac{Q^2}{4m_p^2}
\]

$G_E$ and $G_M$ are the Fourier transforms of the charge and magnetization distributions

$G_E(0) = 1$ (charge), and $G_M(0) = \mu_p$ (magnetic moment)

\[
\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \text{slope of } G_E \text{ at } Q^2 = 0
\]

extrapolation to $Q^2 \rightarrow 0$ required
Mainz scattering data at lowest $Q^2$

\[ \langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \text{slope of } G_E \text{ at } Q^2 = 0 \]

New Mainz data: $G_E/G_{\text{dipole}}$ vs. $Q^2$

extrapolation to $Q^2 \to 0$ required
Mainz scattering data at lowest $Q^2$

\[ \langle r_p^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \Rightarrow \text{rms charge radius} = \text{slope of } G_E \text{ at } Q^2 = 0 \]

extrapolation to $Q^2 \to 0$ required

PhD thesis J. Bernauer. Lower half of 180MeV data
Mainz scattering data at lowest $Q^2$

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \text{slope of } G_E \text{ at } Q^2 = 0$$

Fitting a straight line

extrapolation to $Q^2 \to 0$ required

straight line fit

$P_0 = 0.9988 \pm 0.0005$

$P_1 = -3.0126 \pm 0.0608 \text{ / GeV}^2$

$R_p = 0.839 \pm 0.008 \text{ fm}$
Mainz scattering data at lowest $Q^2$

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Polynomial, $G_E(0) = 1$

extrapolation to $Q^2 \to 0$ required

$G_E$ parabola with fixed normalization
- $P_0 = 1.0000 \pm 0.0000$
- $P_1 = -3.3521 \pm 0.0779$ / GeV$^2$
- $P_2 = 21.4704 \pm 8.0189$ / GeV$^4$
- $r_p = 0.885 \pm 0.010$ fm

Bernauer et al.

$\mu p$ straight line

polynomial + fixed norm.
Mainz scattering data at lowest $Q^2$

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \Rightarrow \text{rms charge radius} = \text{slope of } G_E \text{ at } Q^2 = 0$$

Extrapolation is subtle

extrapolation to $Q^2 \rightarrow 0$ required

straight line fit

$P_0 = 0.9988 \pm 0.0005$
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parabola with fixed normalization

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$R_p = 0.885 \pm 0.010 \text{ fm}$
(n=2) - states of ep and $\mu p$

Lamb shift:
$\mathcal{L}_e = 4 \times 10^{-6}$ eV
$= 1058$ MHz

$\Delta E_{2P - 1S} = 10$ eV
self energy = $+1086$ MHz
vac. pol. = $-27$ MHz

$a_o = 5 \times 10^{-11}$ m

$\mathcal{L}_\mu = -206$ meV
$= 50$ THz
$= 6$ $\mu$m

finite size:
$+4$ meV

$\Delta E_{2P - 1S} = 1900$ eV
self energy = $+0.6$ meV
vac. pol. = $-206$ meV

$a_\mu^o = 3 \times 10^{-13}$ m

$\Gamma_{2P} = 0.08$ meV