Baryon Spectroscopy and Resonances

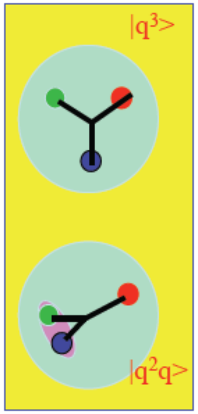
Robert Edwards
Jefferson Lab

Hadron 2011

Collaborators:
J. Dudek, B. Joo, D. Richards, S. Wallace
Auspices of the Hadron Spectrum Collaboration

Where are the "Missing" Baryon Resonances?

- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?



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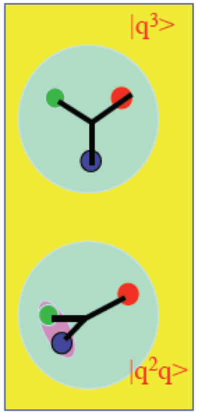
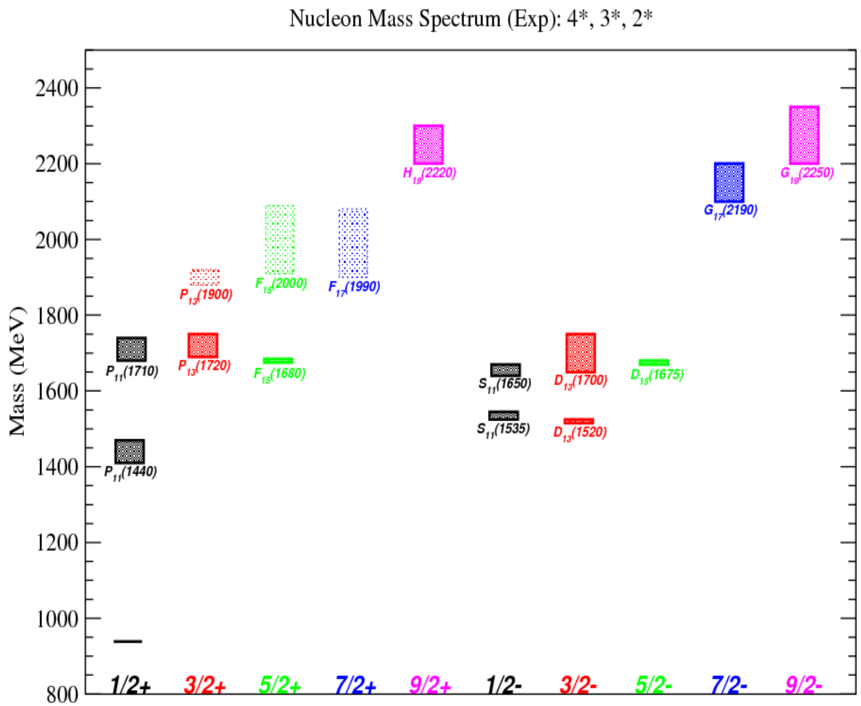
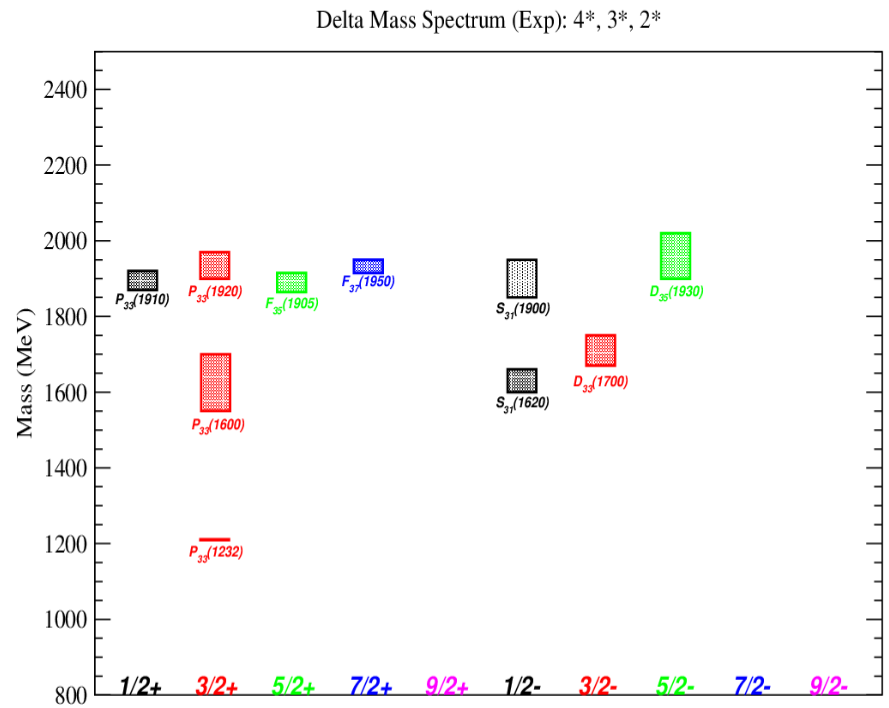


Nucleon & Delta spectrum
PDG uncertainty on
B-W mass





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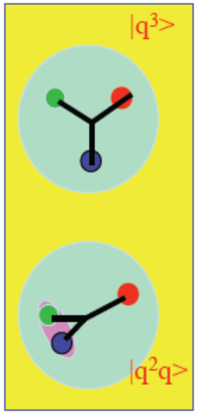

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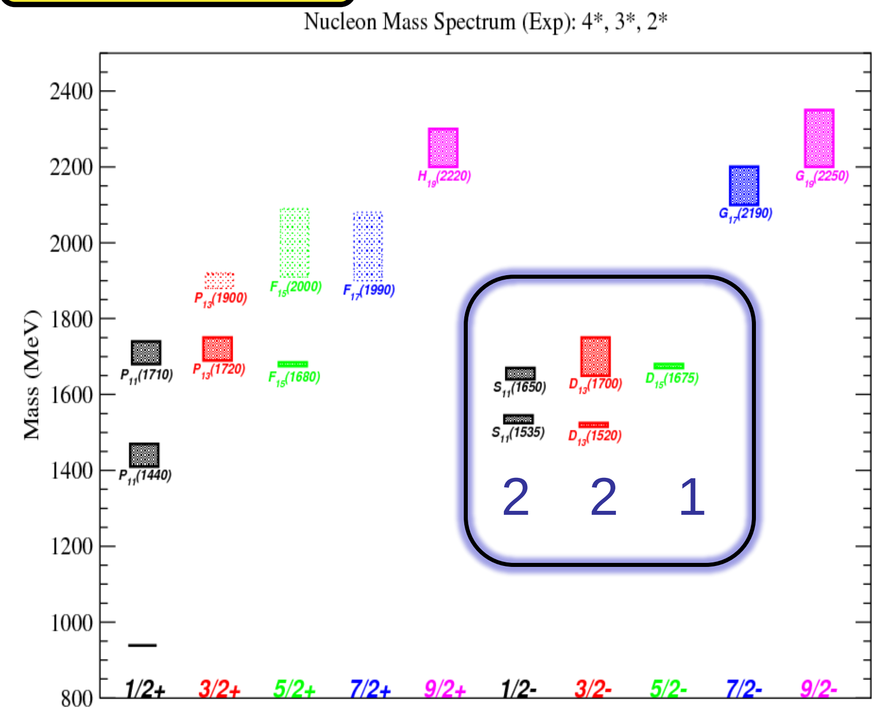

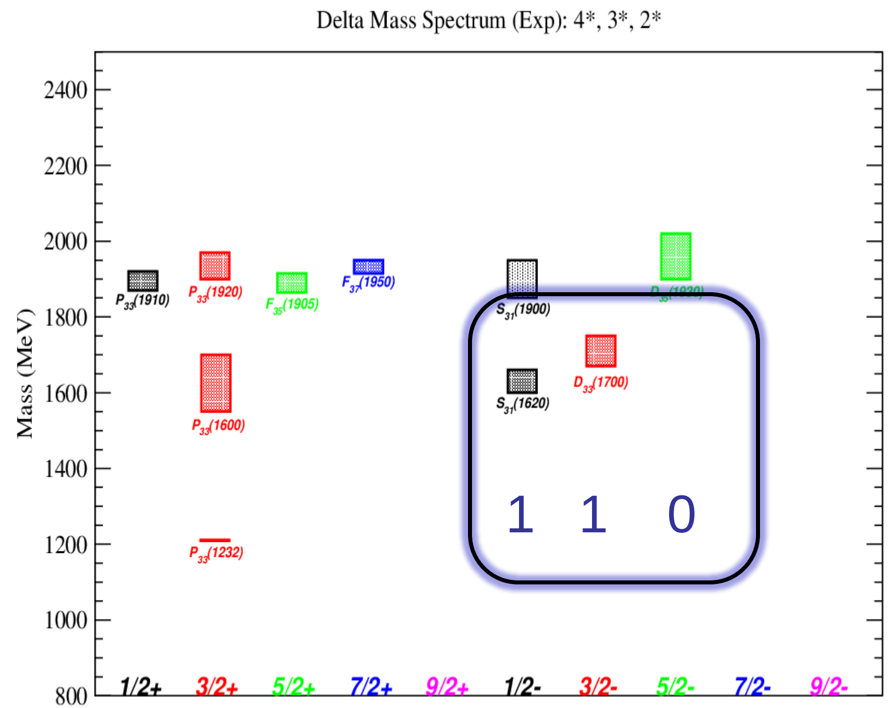

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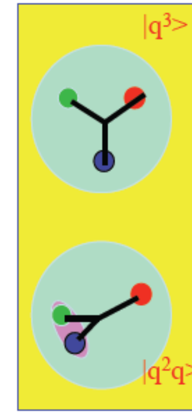

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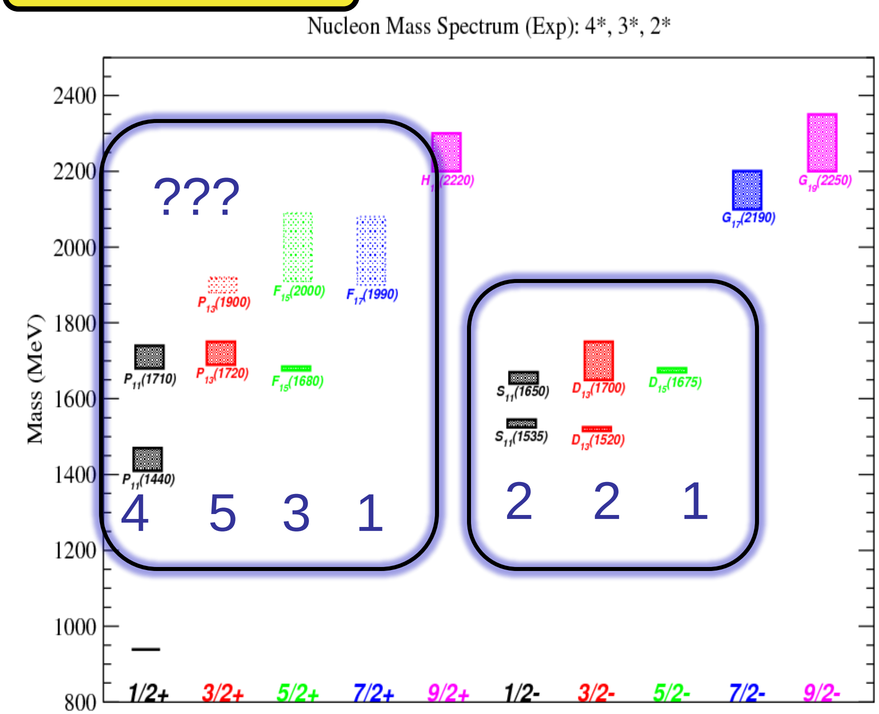

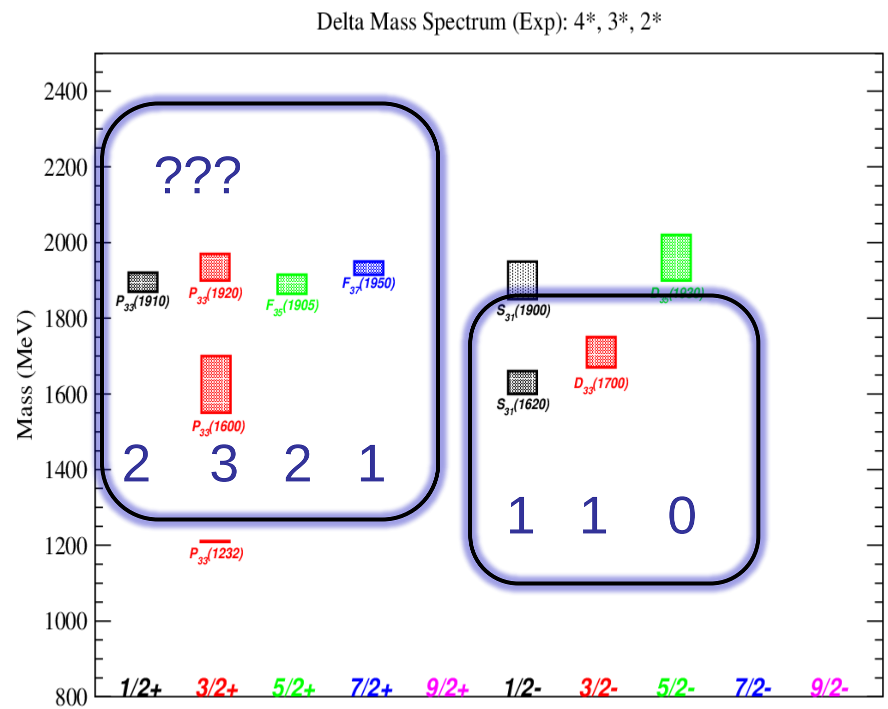

Spectrum from variational method

Two-point correlator

$$\boxed{C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle}$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}}t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle \qquad \boxed{Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\,\Phi_i^\dagger\,|0\rangle}$$

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Diagonalize:
  eigenvalues $\rightarrow$ spectrum
  eigenvectors $\rightarrow$ spectral "overlaps" Z_i^n

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Benefit: orthogonality for near degenerate states

Baryon operators

Construction : permutations of 3 objects

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
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  - e.g., udu - duu & 2duu - udu - uud

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Space: couple covariant derivatives onto single-site spinors - build any J,M

$$\Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

$$J \leftarrow \mathbf{1} \otimes \mathbf{1} \otimes S$$

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Classify operators by permutation symmetries:
- Leads to rich structure

1104.5152

Baryon operator basis

3-quark operators & up to two covariant derivatives: some J^P

$$\left(\left[\mathrm{Flavor}\otimes\mathrm{Dirac}\right]\otimes\mathrm{Space}_{\mathrm{symmetry}}\right)^{J^P}$$

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Spatial symmetry classification:

e.g., Nucleons: $N\ {}^{2S+1}L_{\pi}\ J^P$

J^P	#ops	E.g., spatial symmetries	
$J=1/2^-$	24	$N\ {}^2P_M\ \frac{1}{2}^-$	$N\ {}^4P_M\ \frac{1}{2}^-$
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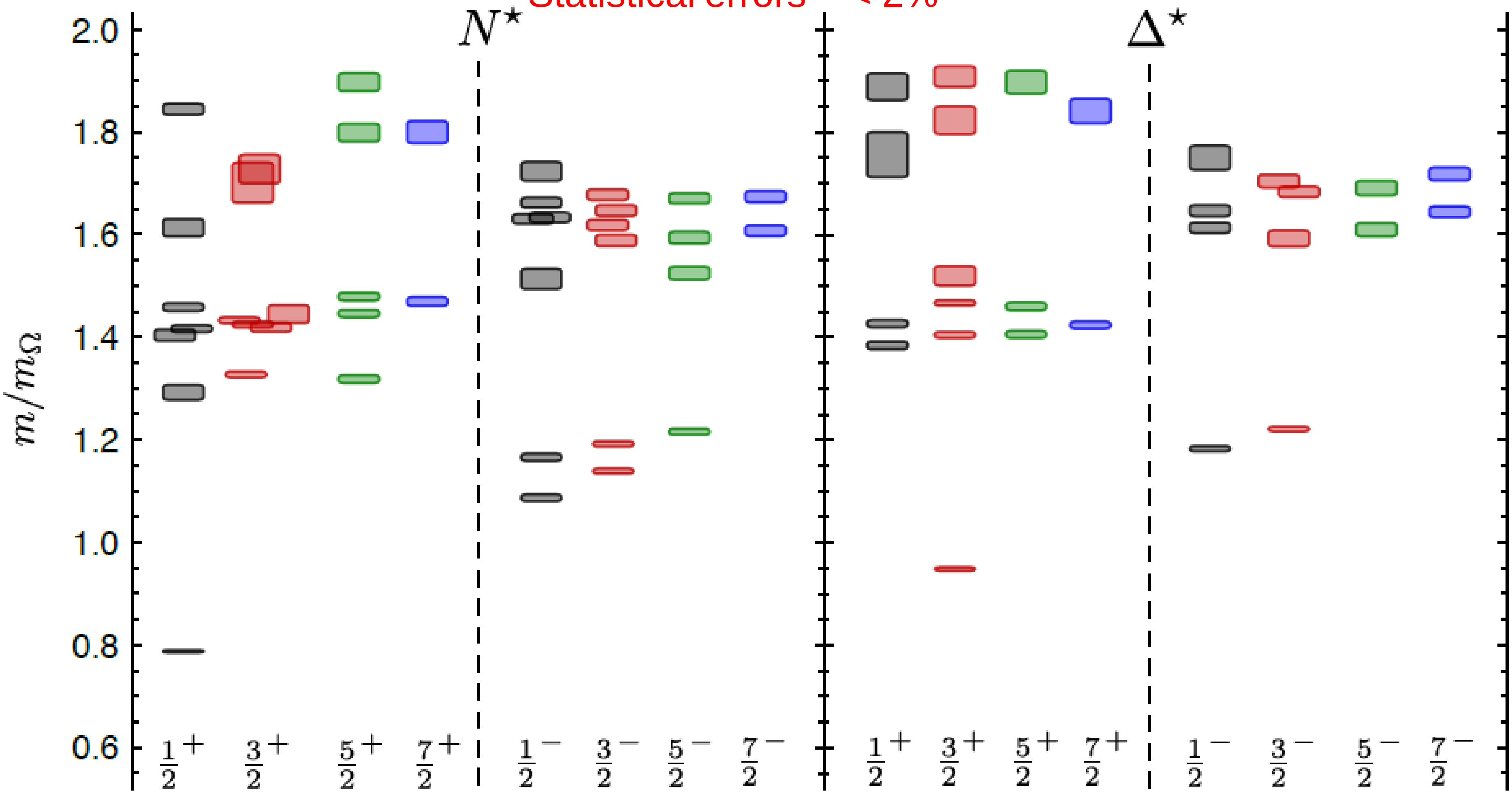
By far the largest operator basis ever used for such calculations

Spin identified Nucleon & Delta spectrum

arXiv:1104.5152

$m_\pi \sim 520$MeV

Statistical errors < 2%



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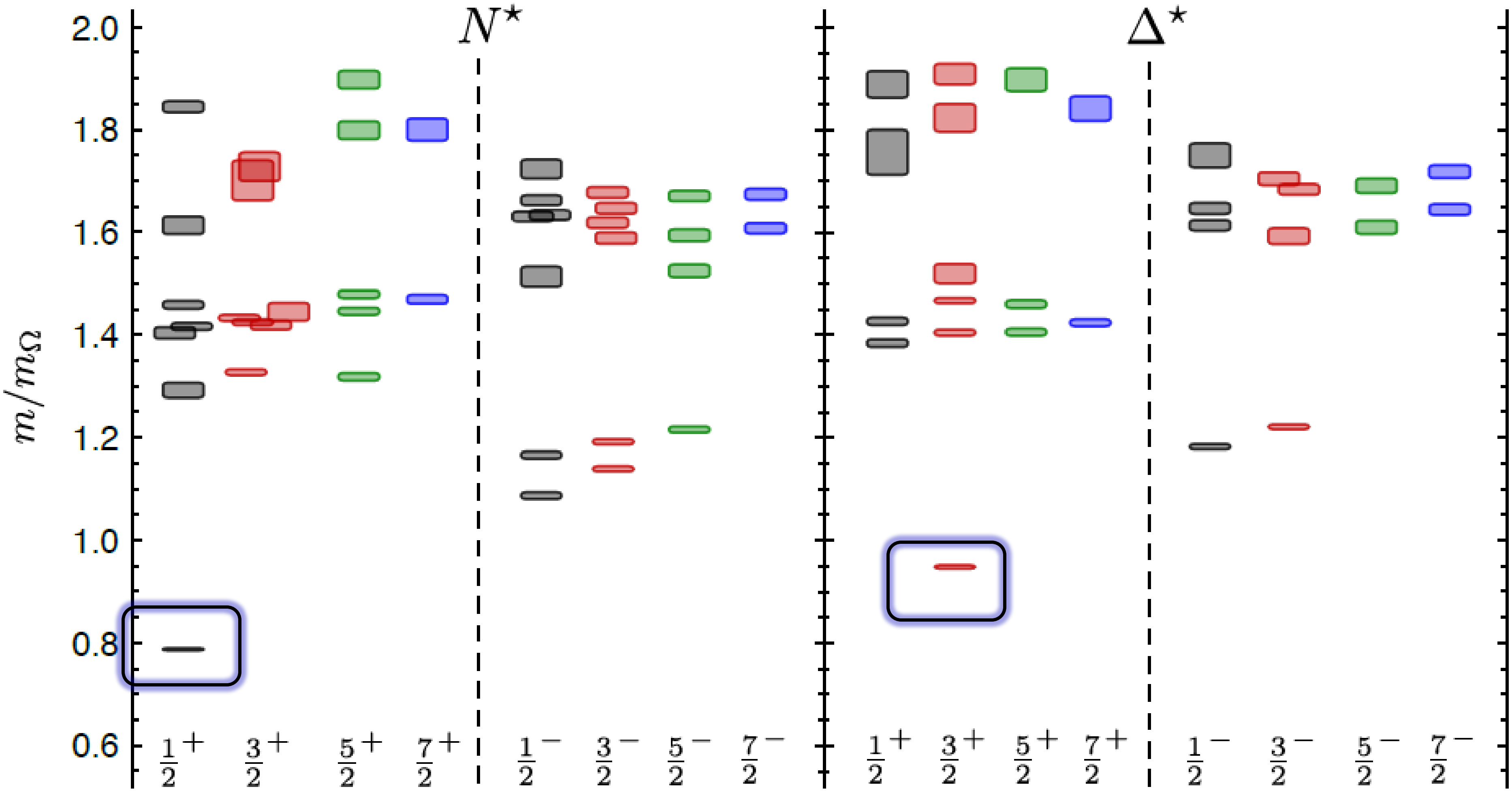

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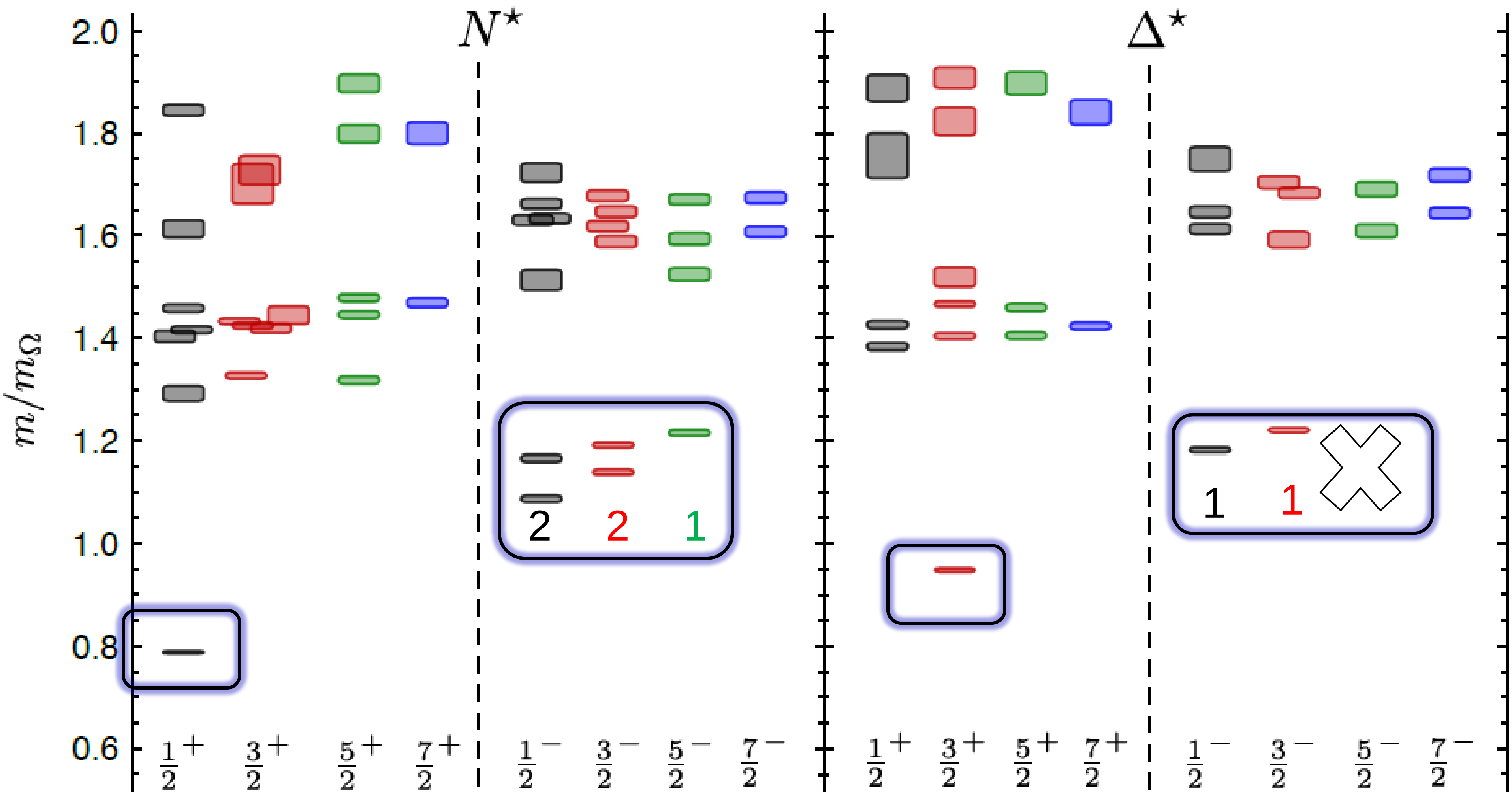

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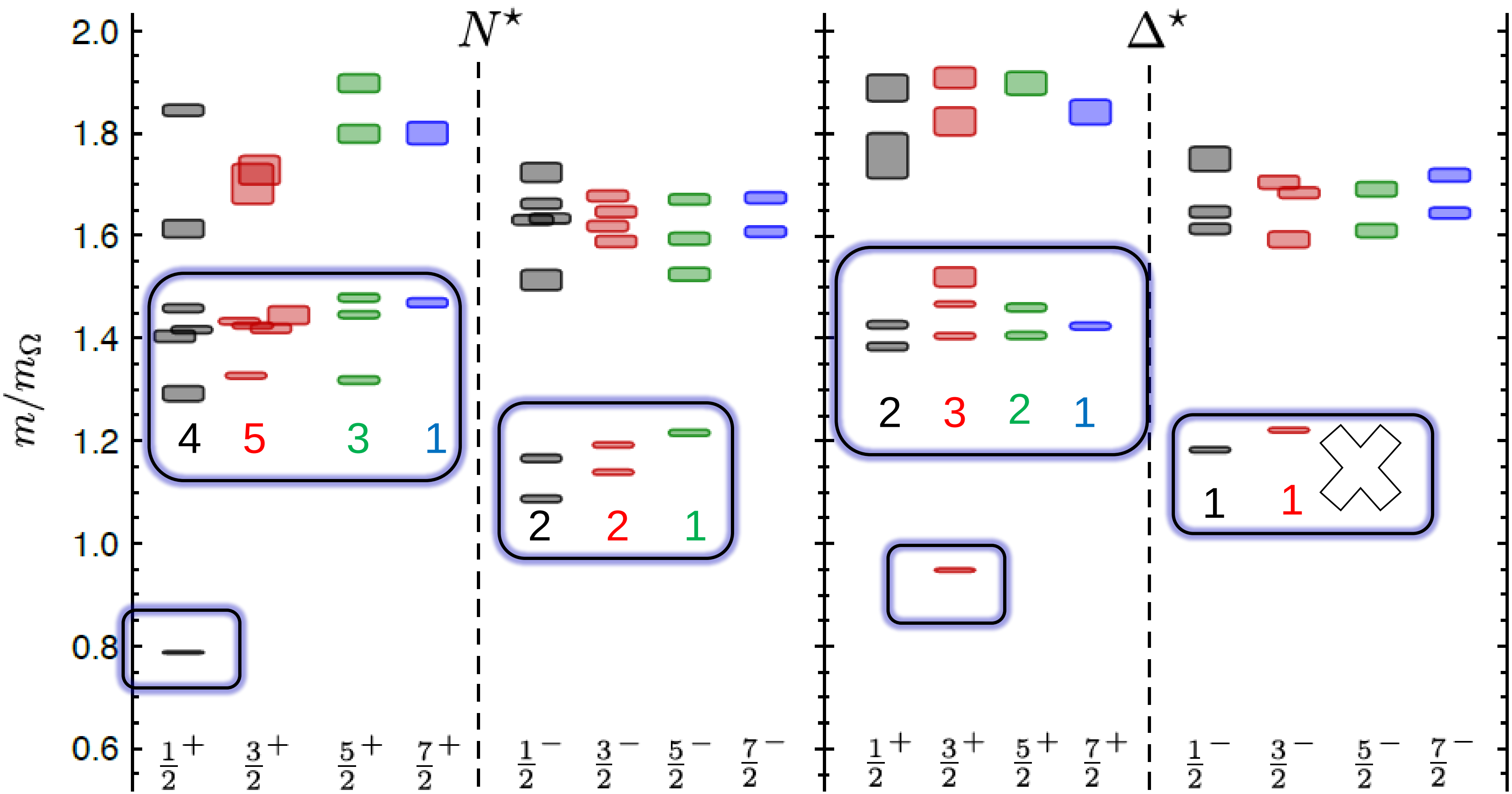

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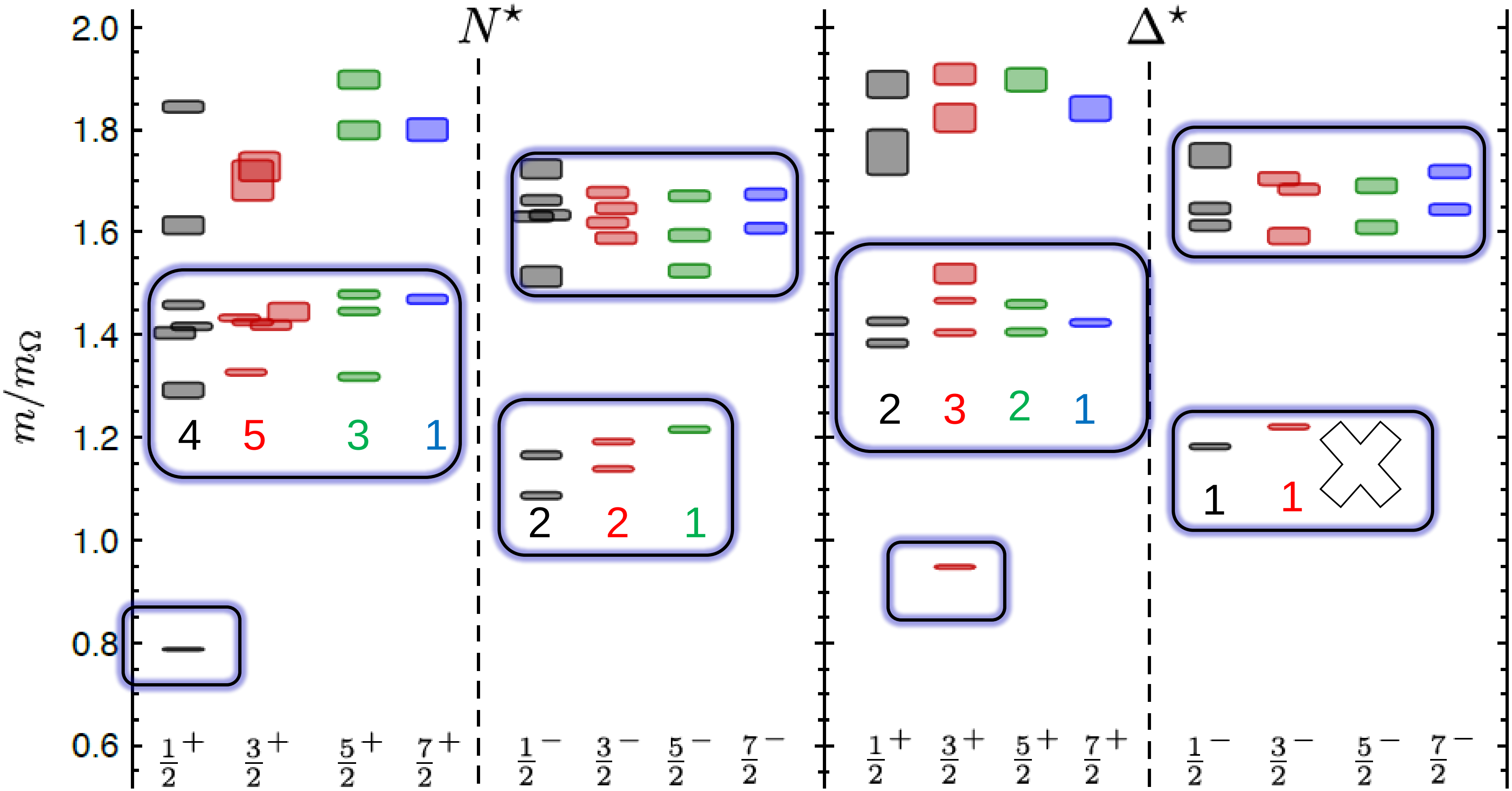

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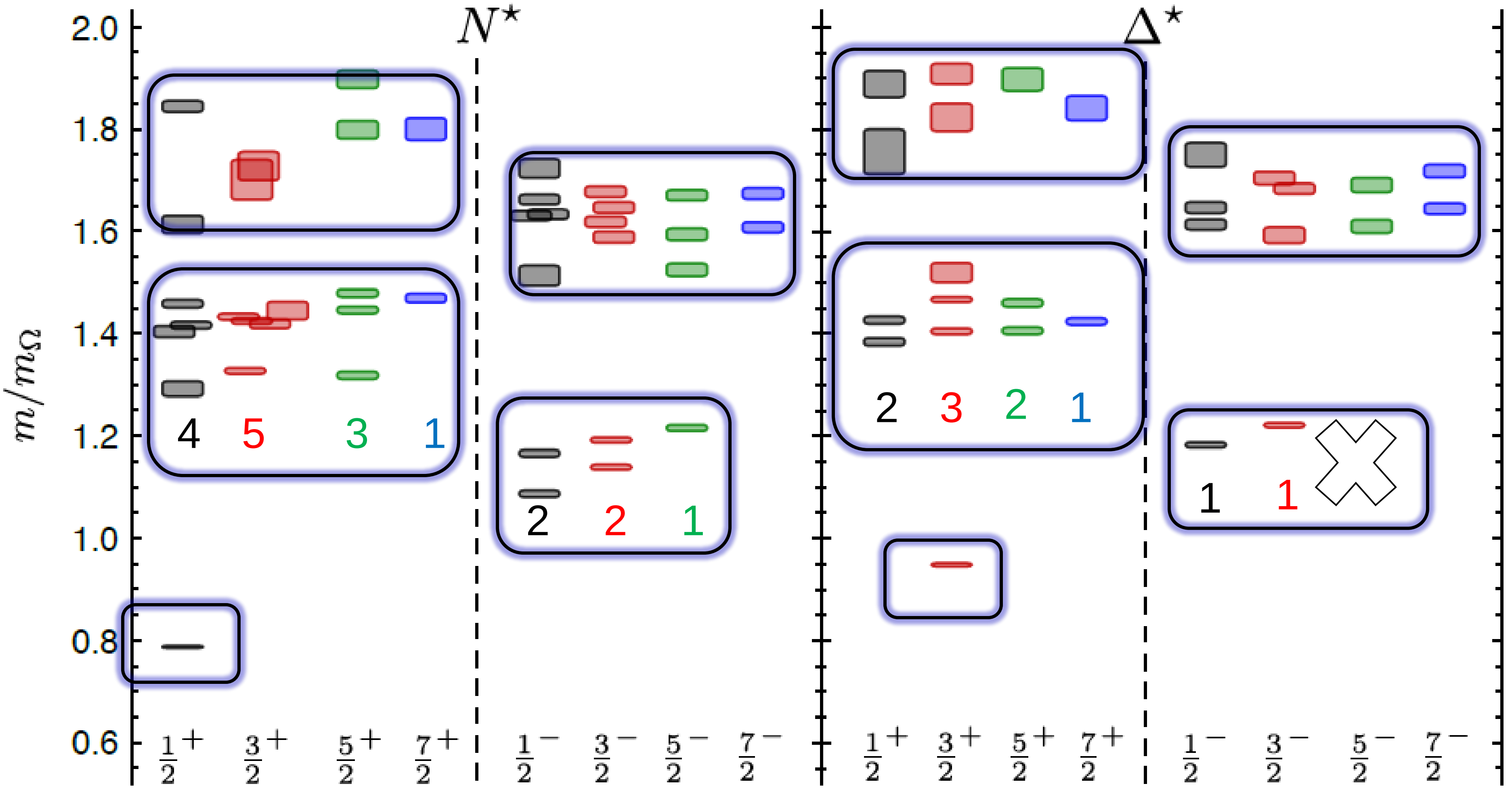

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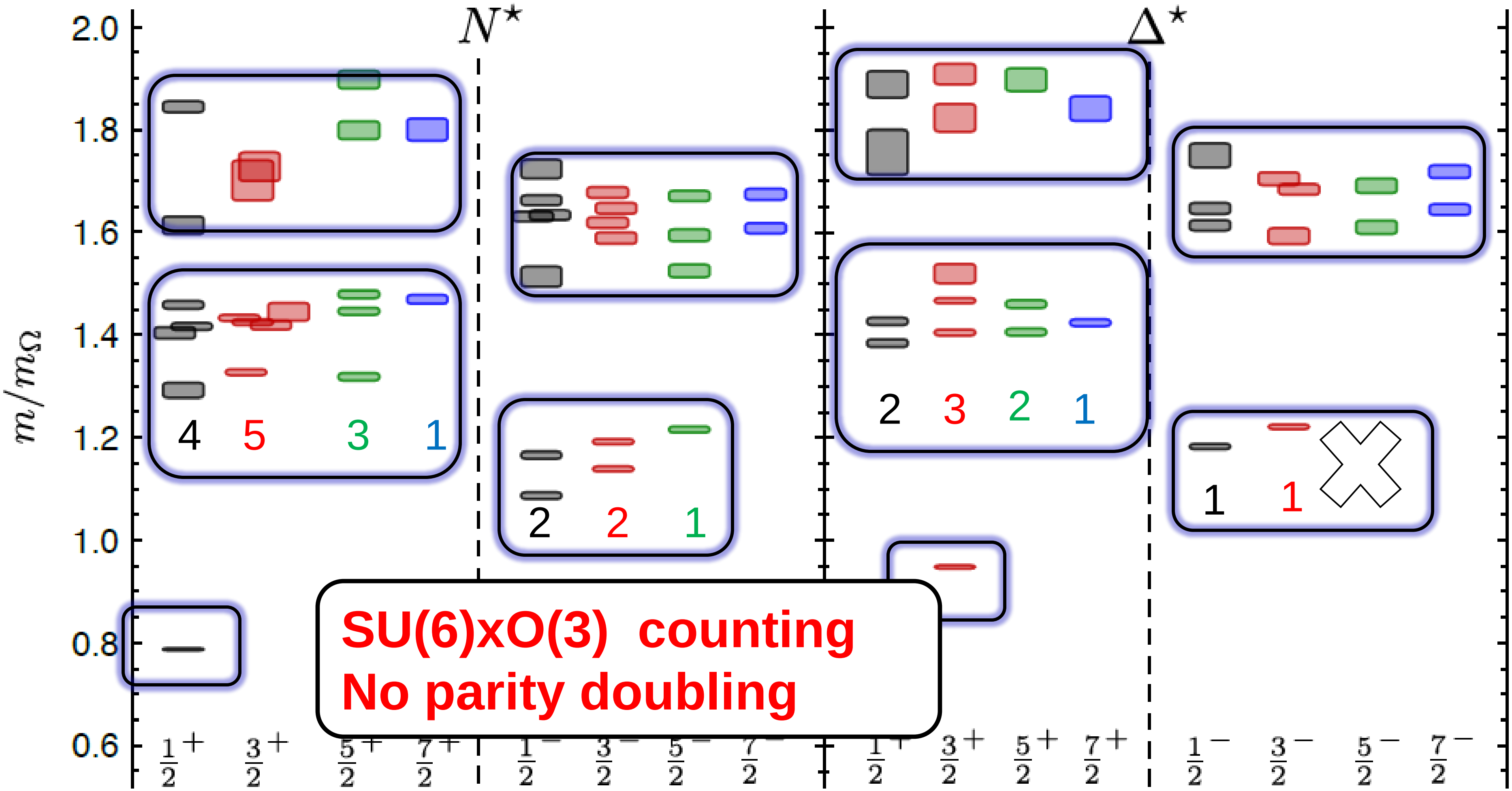

SU(6)xO(3) counting
No parity doubling

Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

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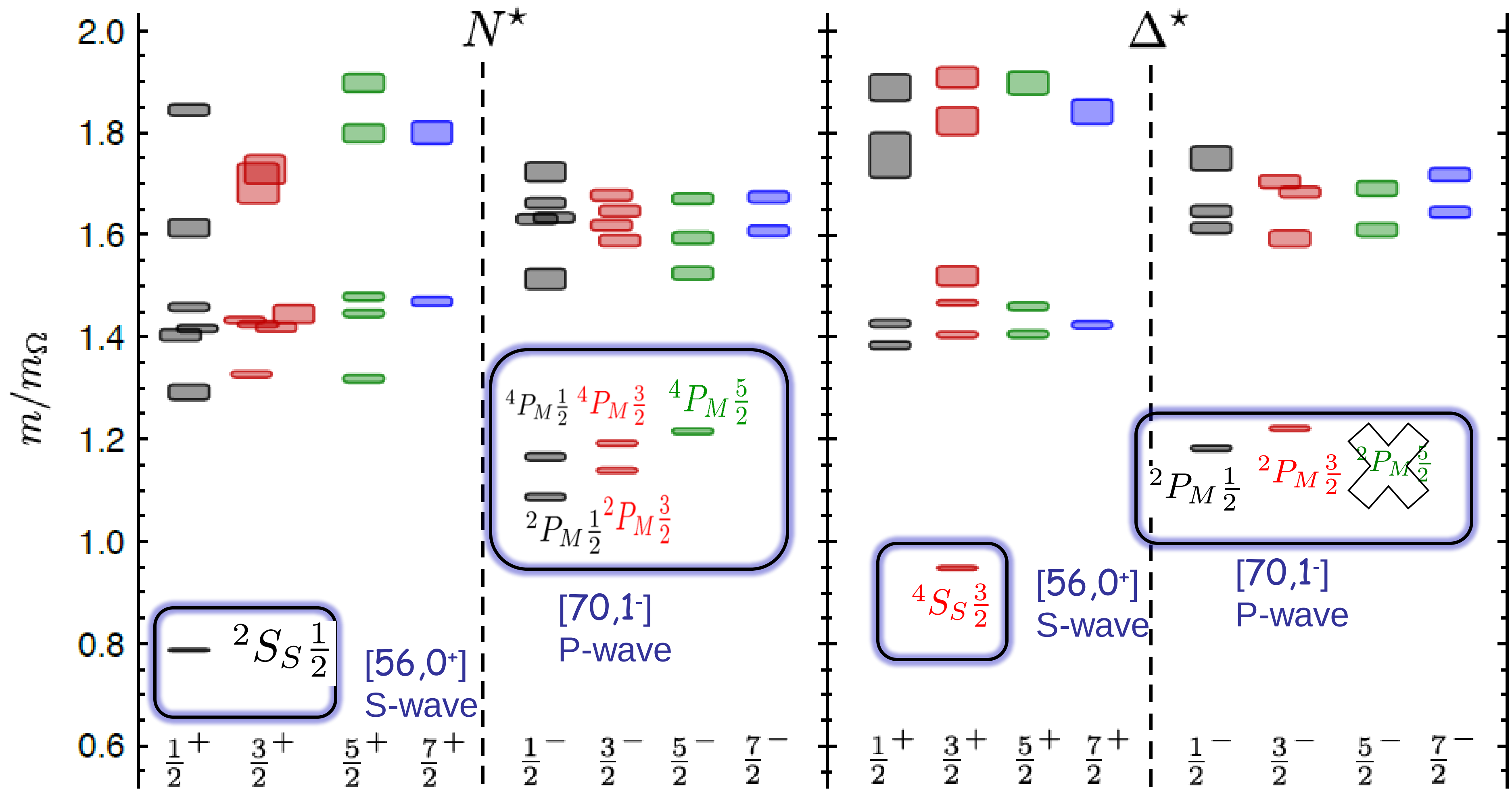
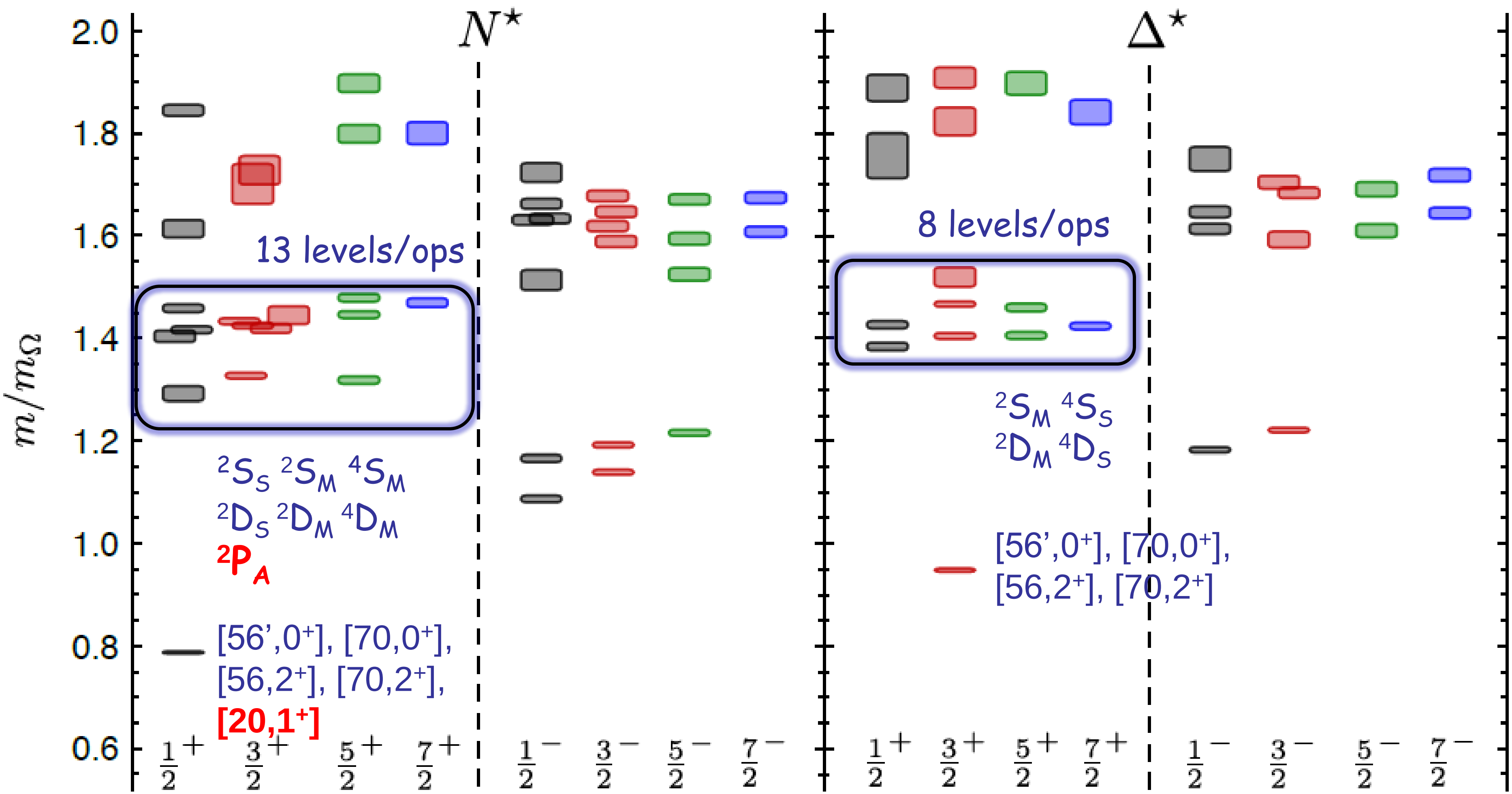


N=2 J^+ Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in J^+



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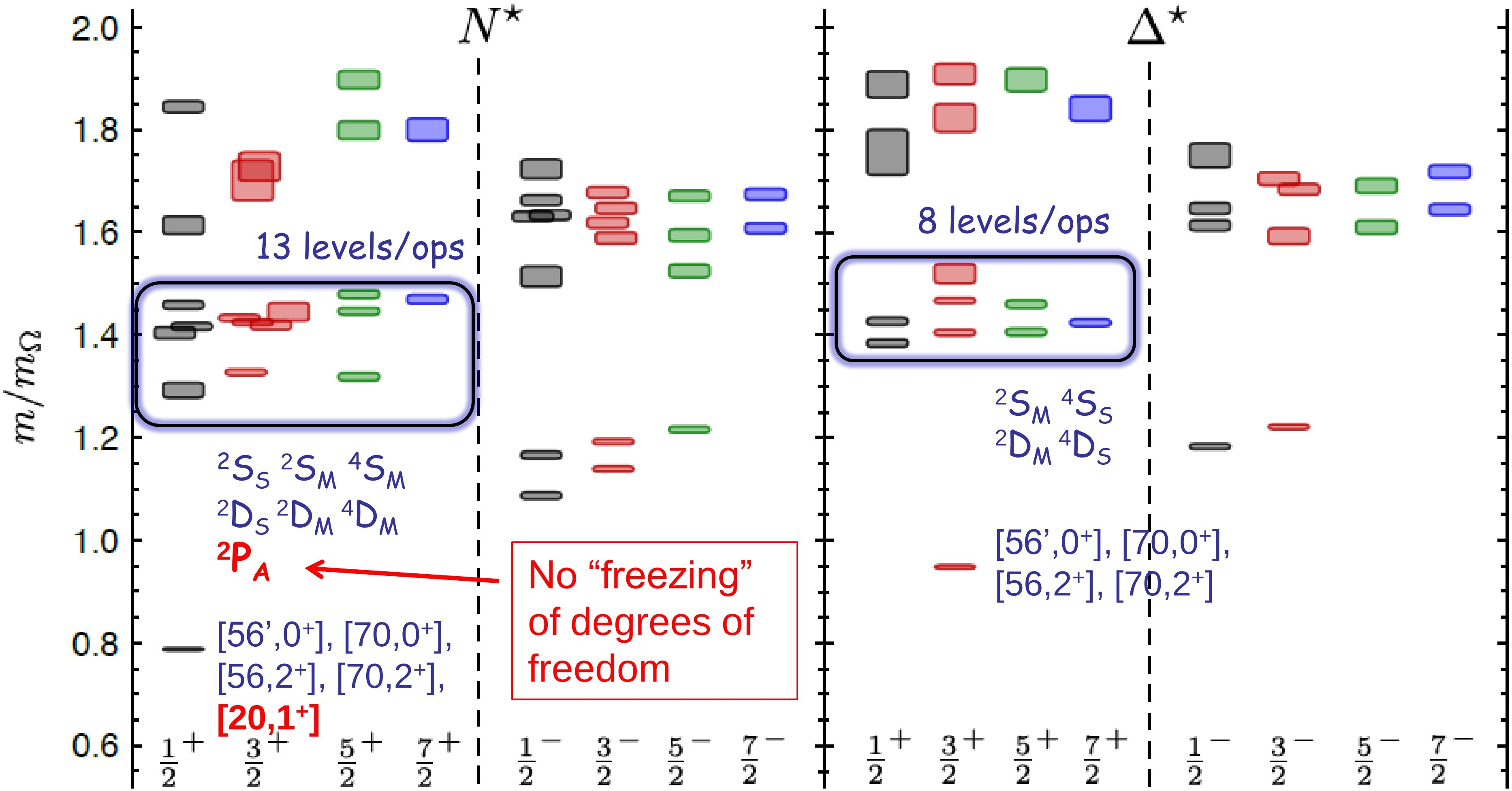
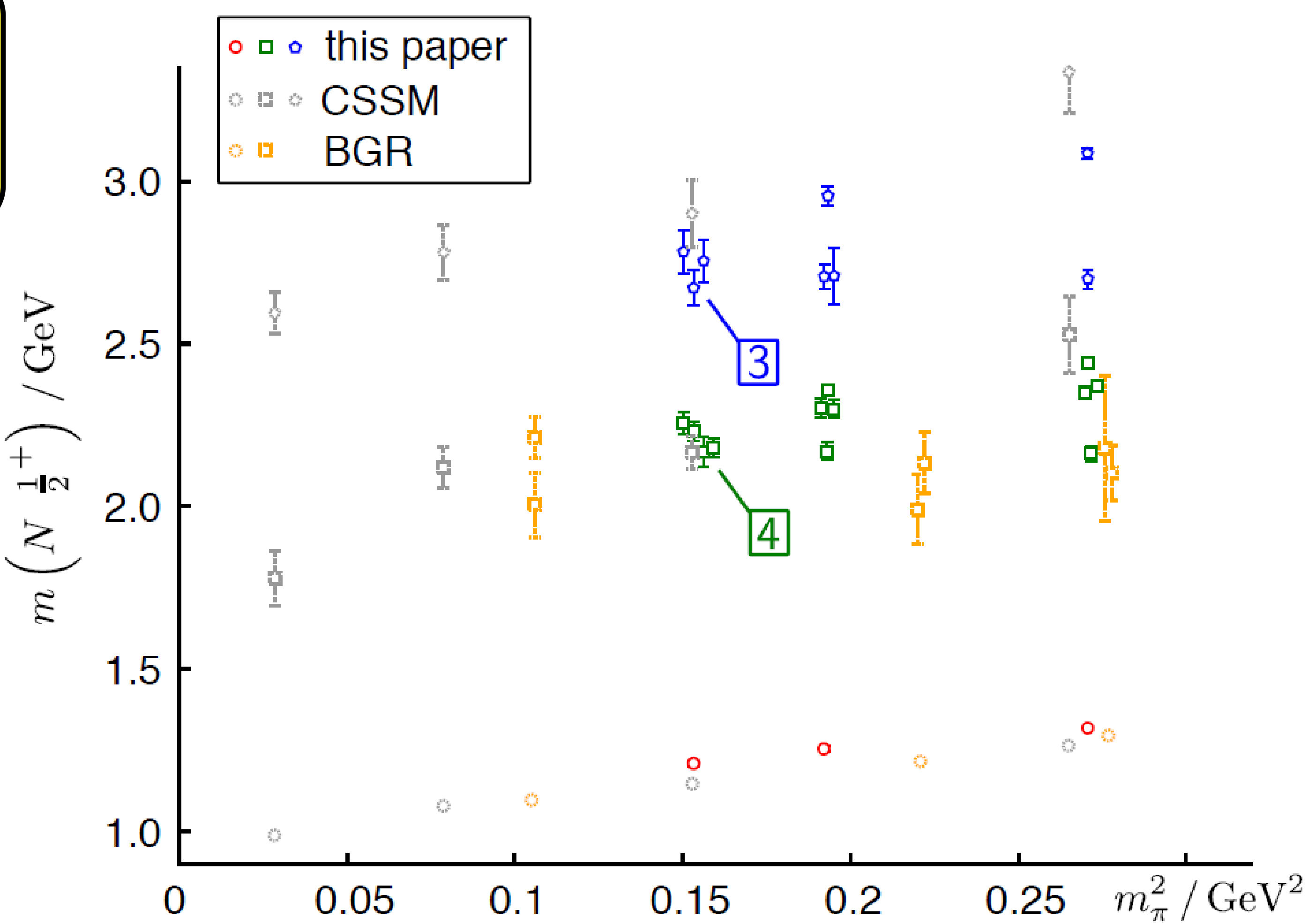


Roper??

Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) O(3) counting, but heavily mixed

Discrepancies??

Operator basis – spatial structure



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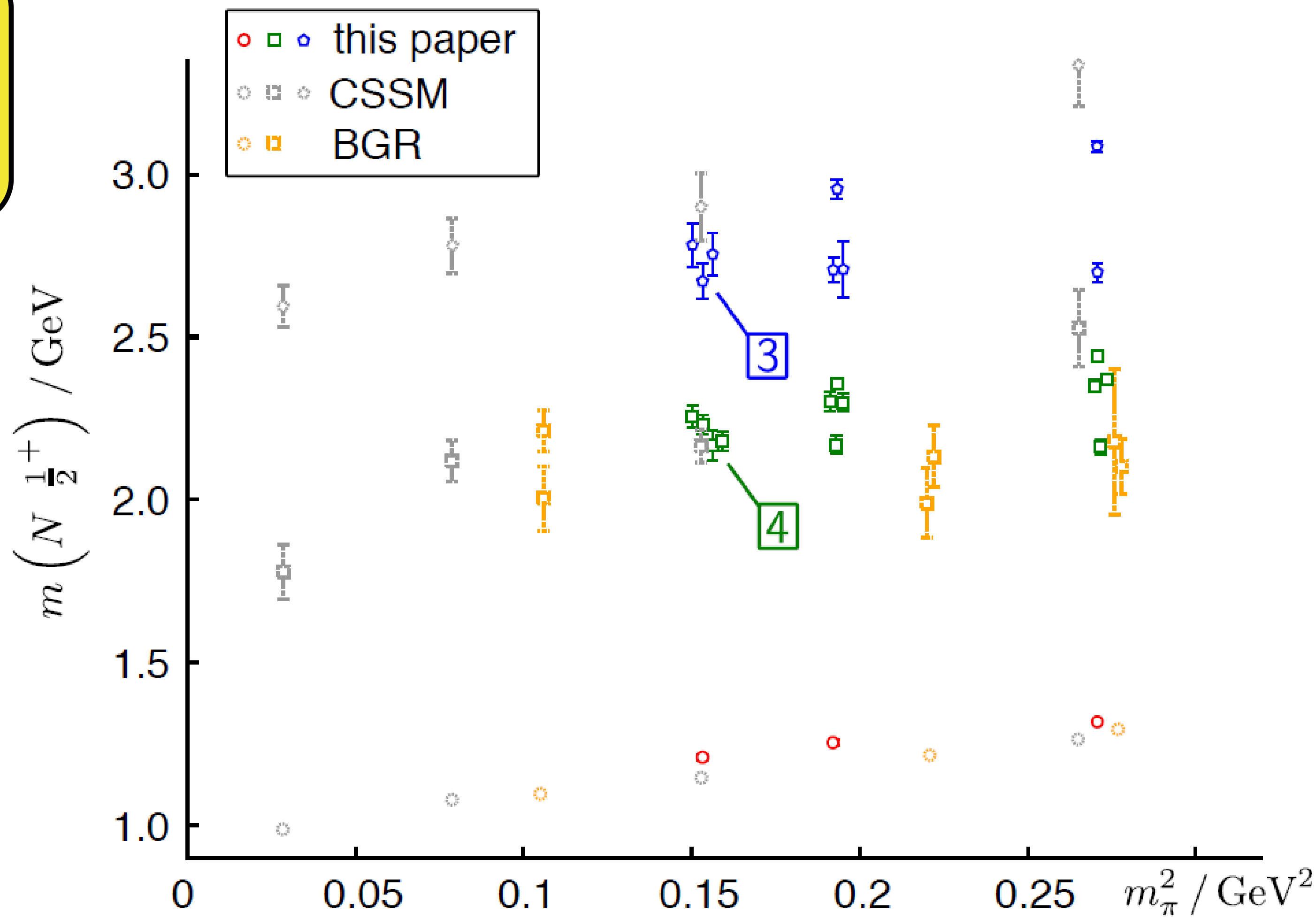
What else? Multi-particle operators



Spectrum of finite volume field

The idea: 1 dim quantum mechanics

Two spin-less bosons: ψ(x,y) = f(x-y) $\rightarrow$ f(z)

$$\left[-\frac{1}{m}\frac{d^2}{dz^2} + V(z) \right] f(z) = E\, f(z)$$

Solutions

$$f(z) \rightarrow \cos\left[k|z| + \delta(k)\right], \qquad E = k^2/m$$

Quantization condition when -L/2 < z < L/2

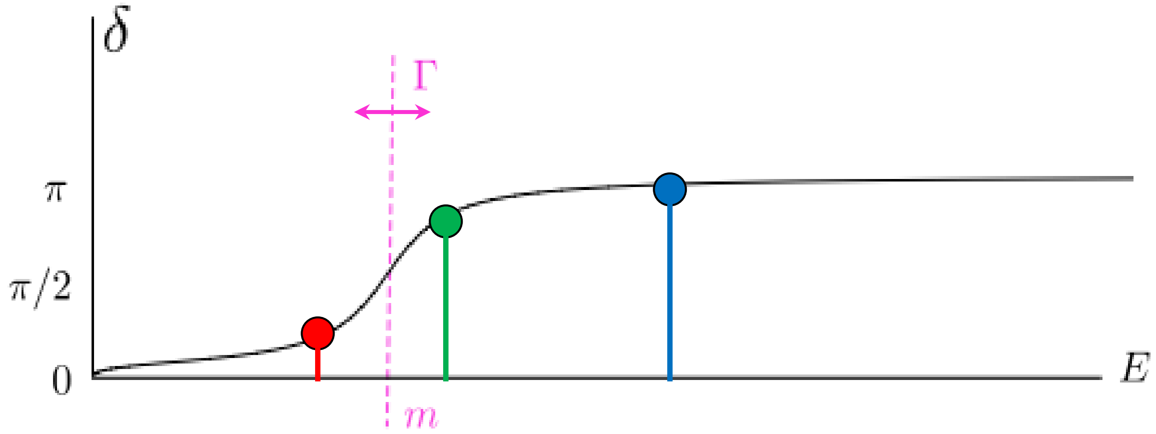
$$kL + 2\delta(k) = 0 \mod 2\pi$$

Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)

Finite volume scattering

Scattering in a periodic cubic box (length **L**)
- Discrete energy levels in finite volume

E.g. just a single elastic resonance



e.g.

$$\pi\pi \to \rho \to \pi\pi$$

$$\pi N \to \Delta \to \pi N$$

At some **L ,** have discrete excited energies

$$E \to k; \quad kL + 2\delta(k) = 0 \mod 2\pi$$

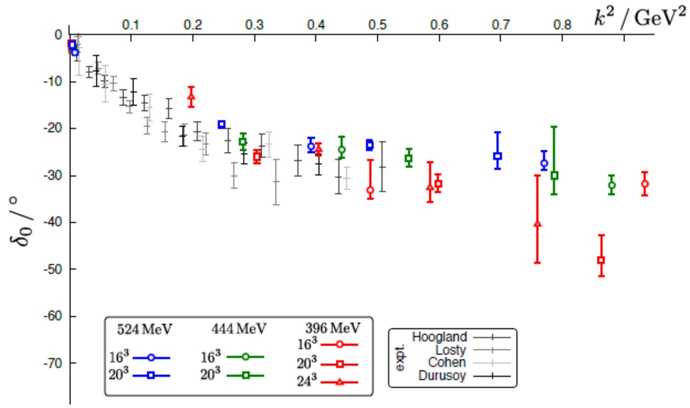
- T-matrix amplitudes $\to$ partial waves
- Finite volume energy levels **E(L)** $\leftrightarrow$ **δ(E)**

Resonances

Scattering of composite objects in non-perturbative field theory

isospin=2 $\pi\pi$

$\delta_0(E)$



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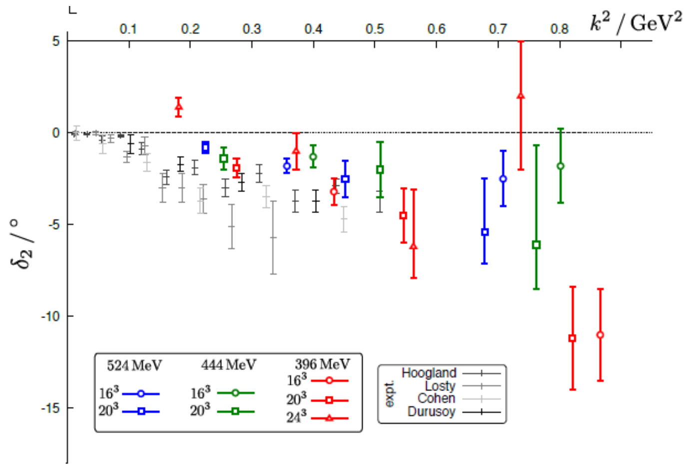

1011.6352

Feng, et.al, 1011.5288

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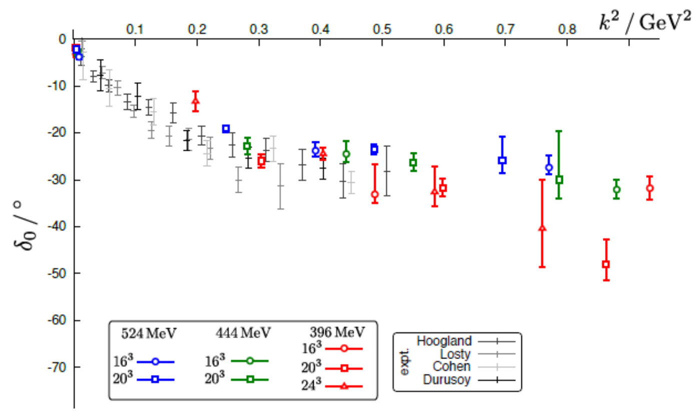

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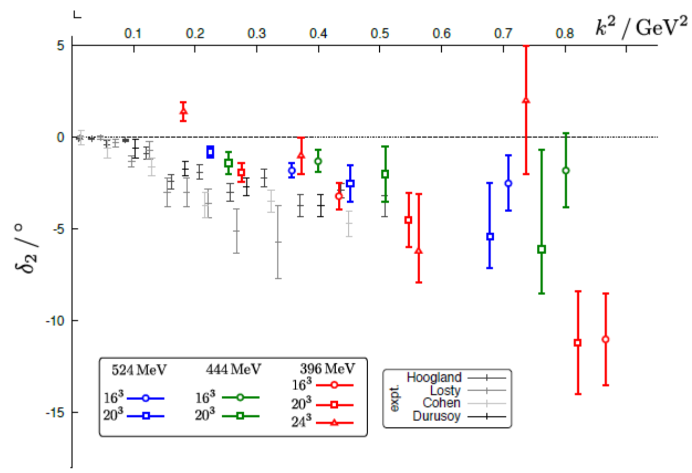

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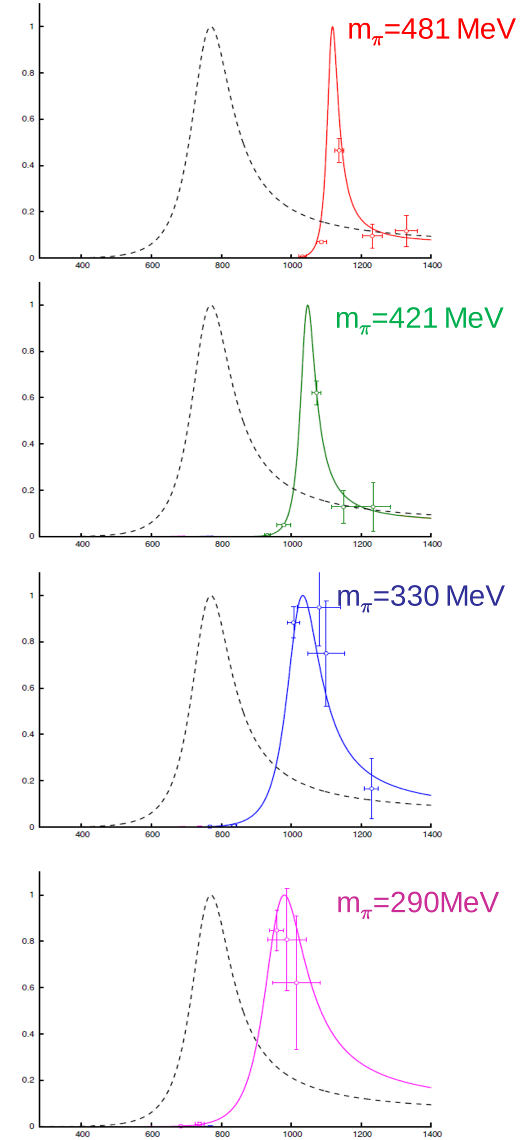


Feng, et.al, 1011.5288

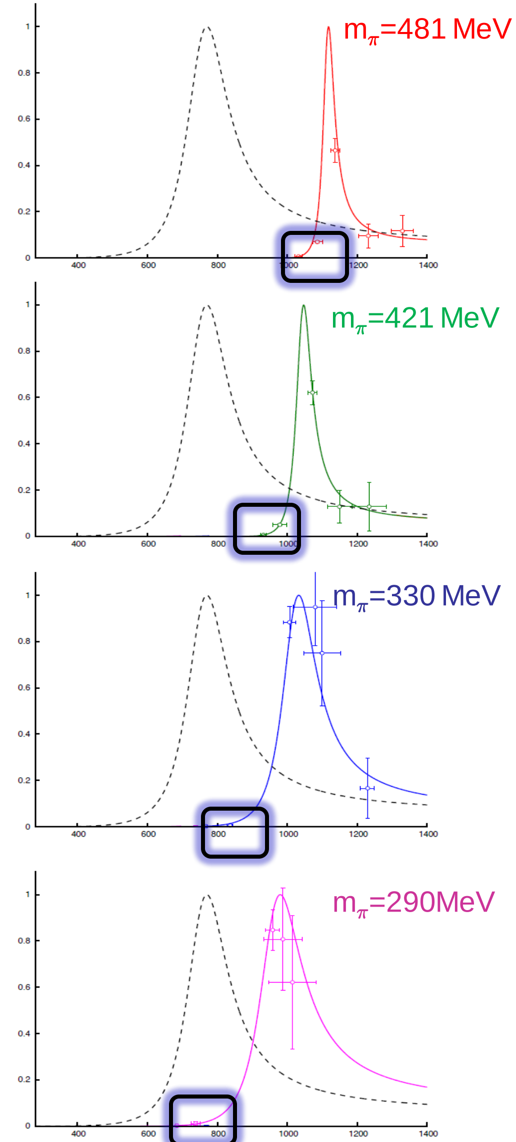
Resonances

isospin=1 $\pi\pi$

Scattering of composite objects in non-perturbative field theory

Manifestation of "decay" in Euclidean space

Can extract pole position



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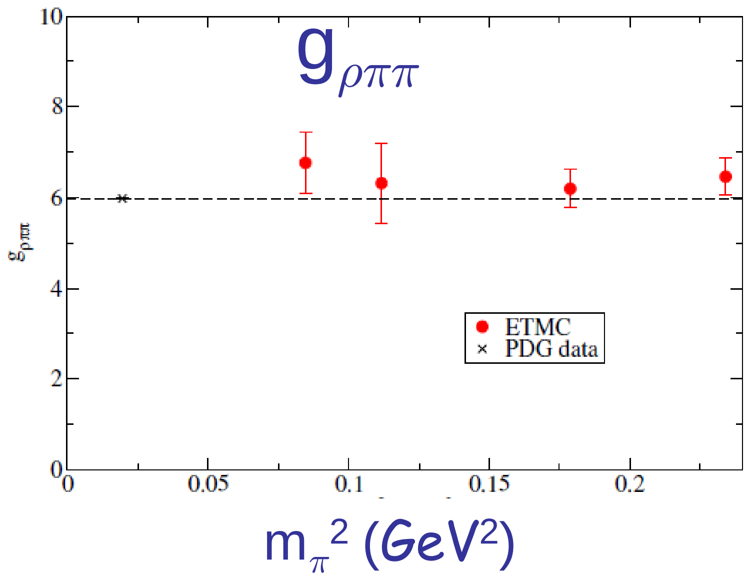
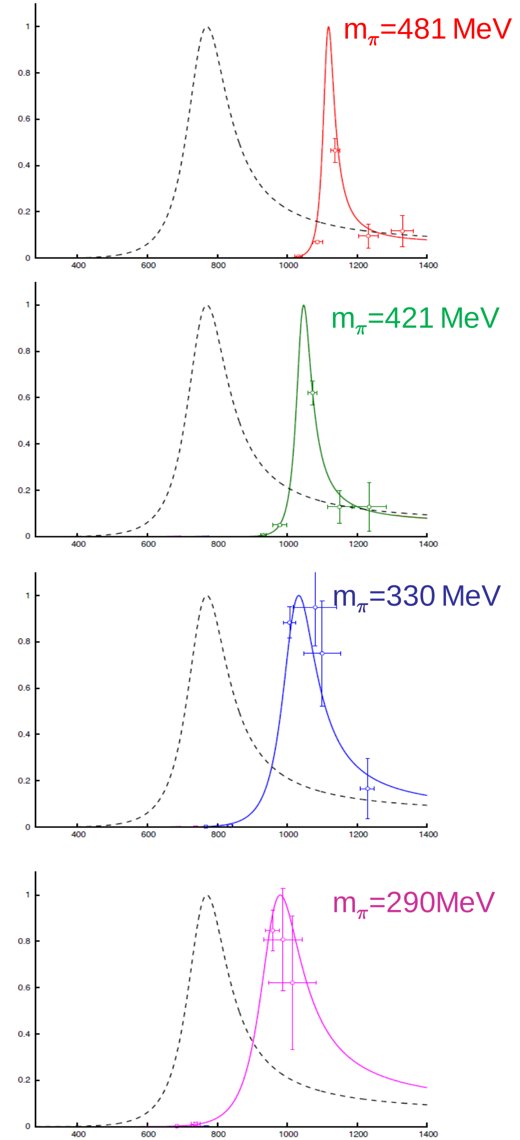
Extracted coupling: stable in pion mass





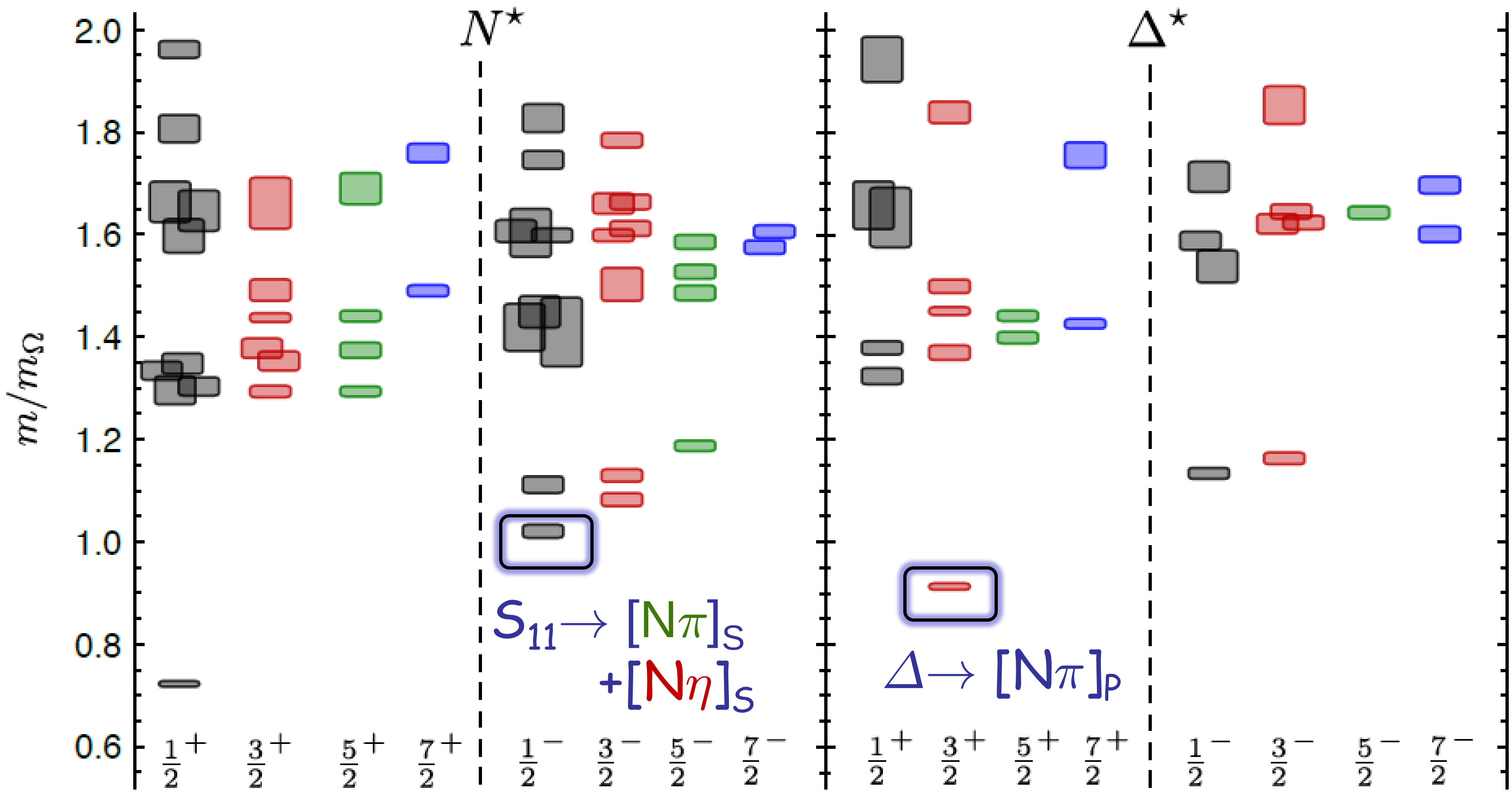
Stability a generic feature of couplings??

Feng, et.al, 1011.5288

Hadronic Decays

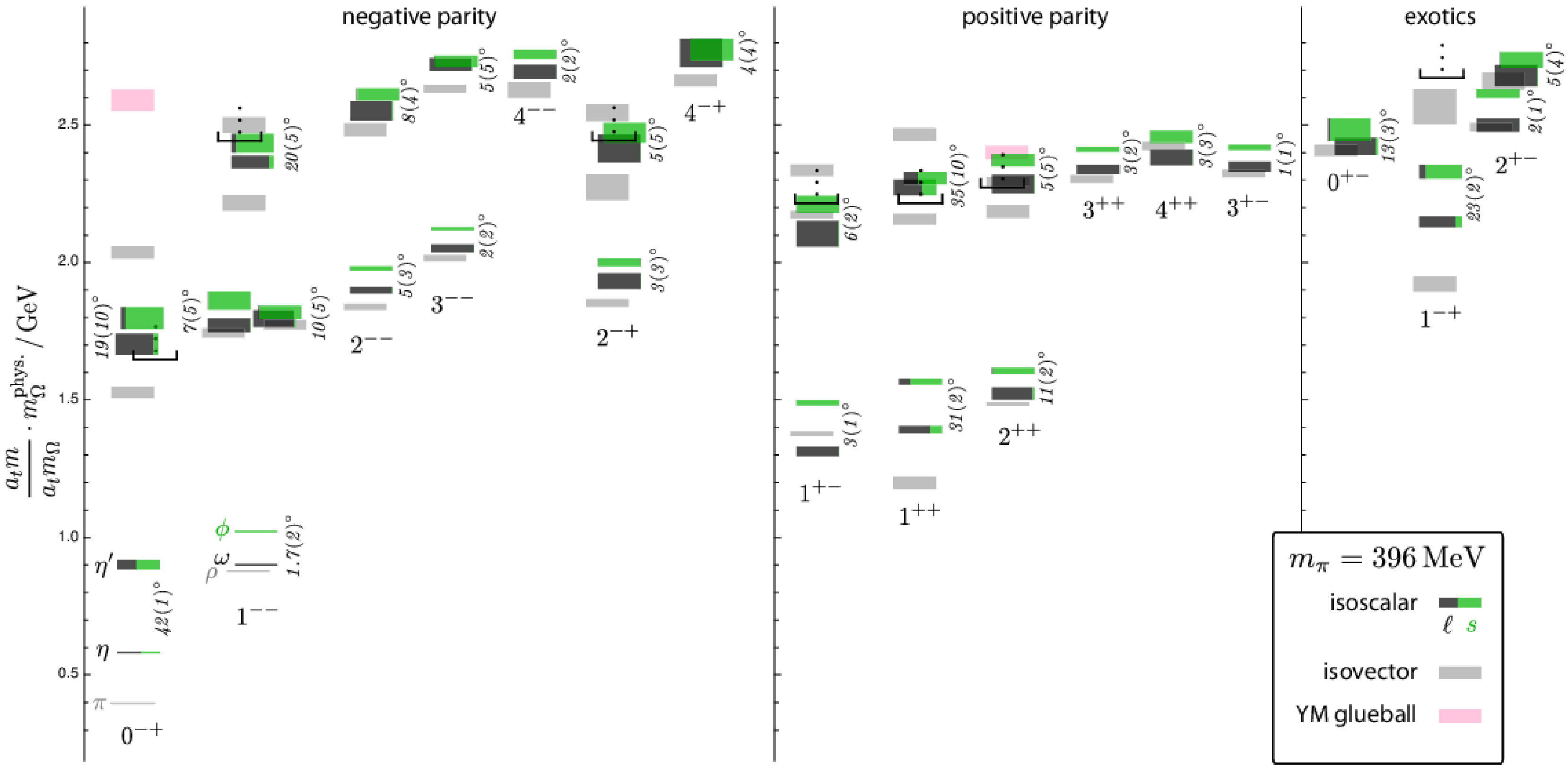
Some candidates: determine phase shift
Somewhat elastic

$m_\pi \sim 400$ MeV



Isoscalar & isovector meson spectrum

Isoscalars: flavor mixing determined



1102.4299

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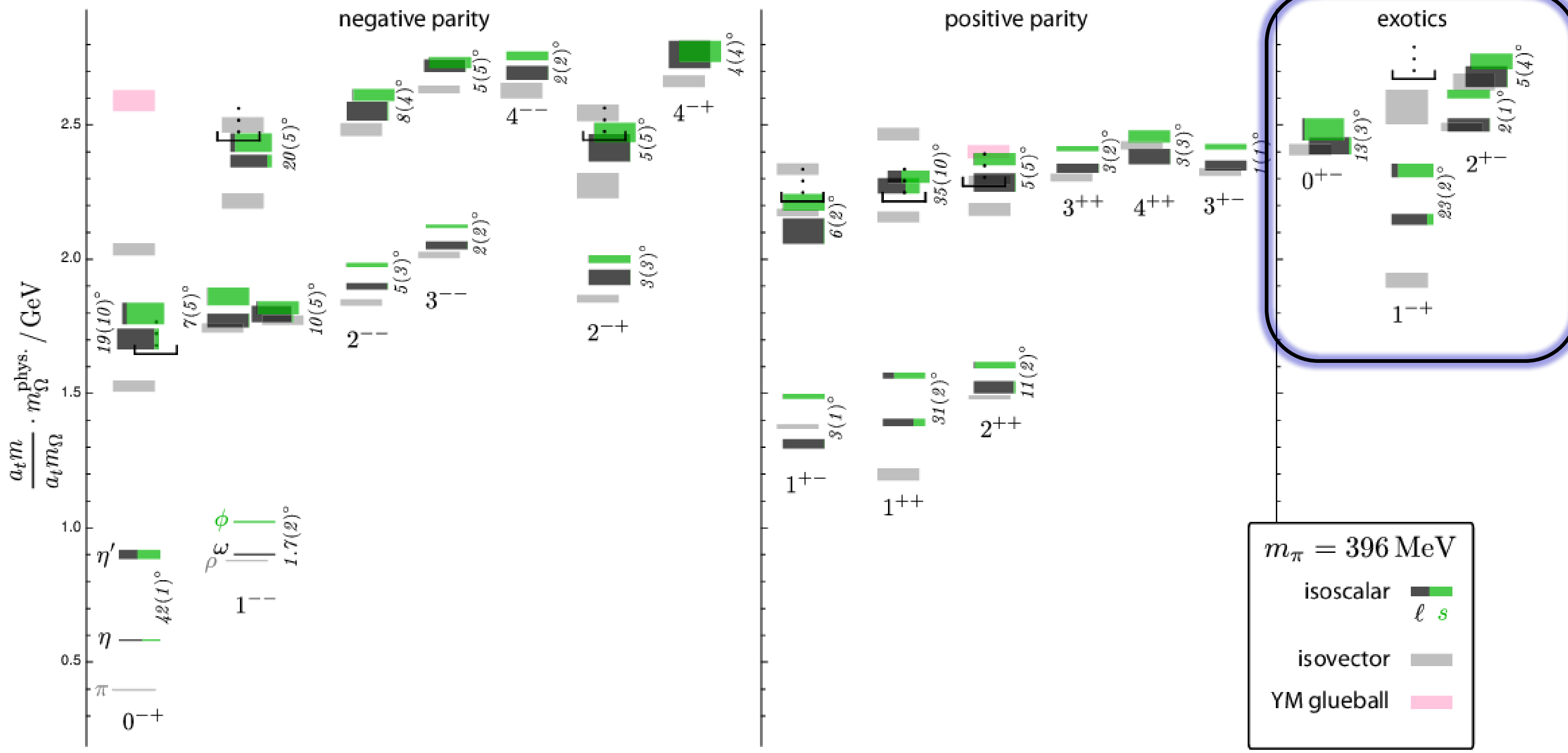
Exotics



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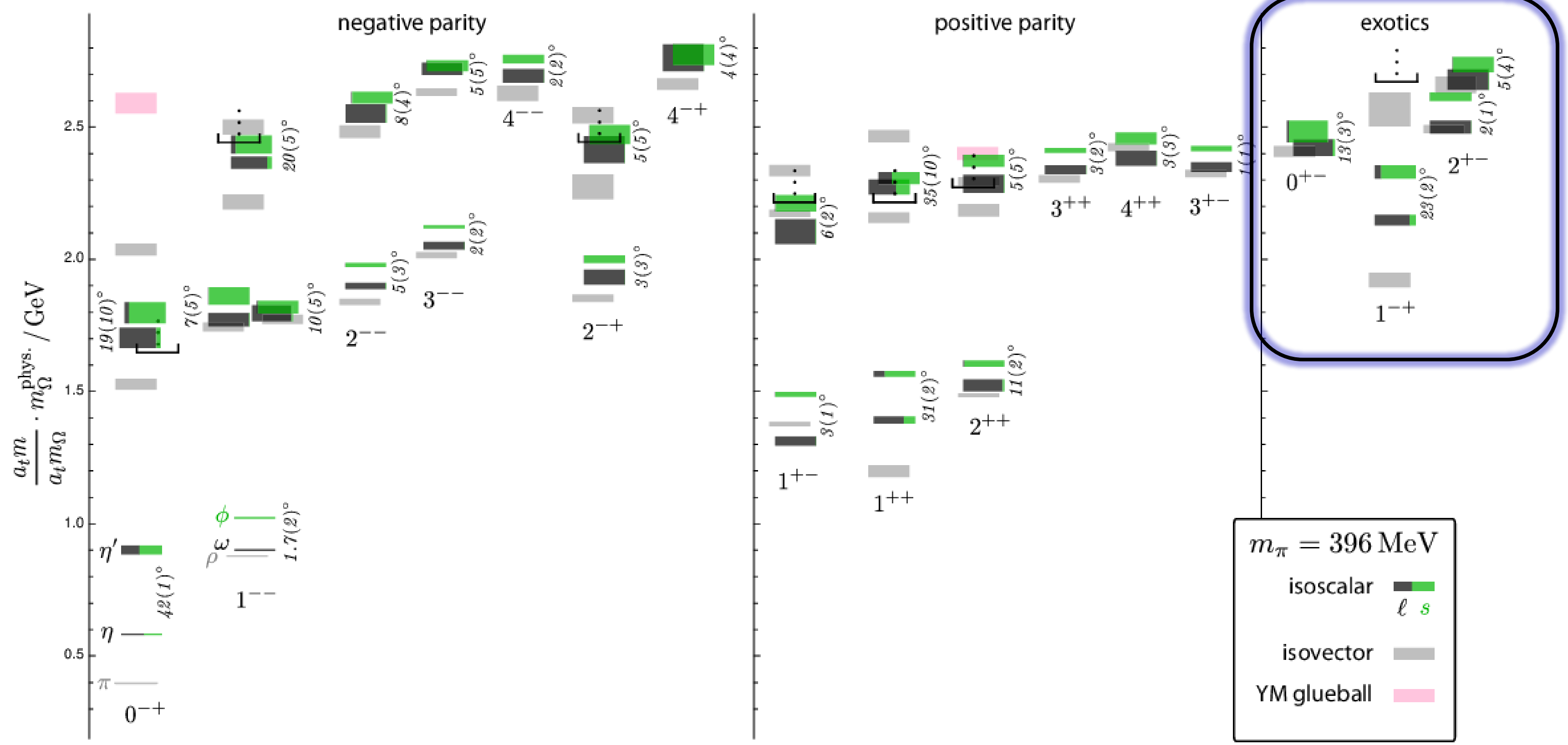

Will need to build PWA within mesons

1102.4299

Summary & prospects

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- No “freezing” of degrees of freedom nor parity doubling
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Optimistic: see confluence of methods (an "amplitude analysis")
- Develop techniques concurrently with decreasing pion mass

Backup slides

- The end

Lattice QCD

Goal: resolve highly excited states

$N_f = 2 + 1$ (u,d + s)

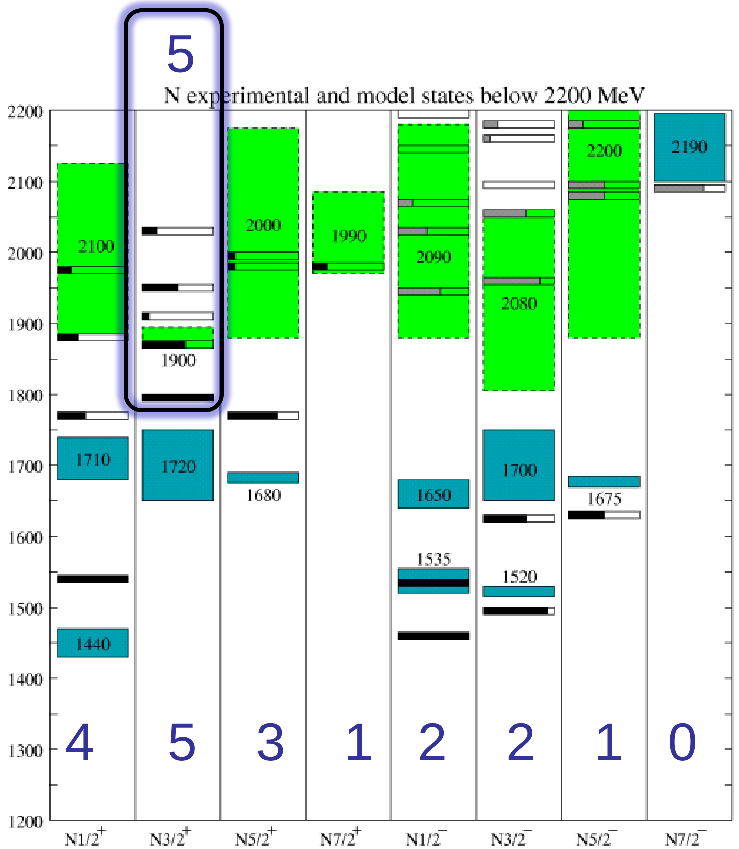
Anisotropic lattices:

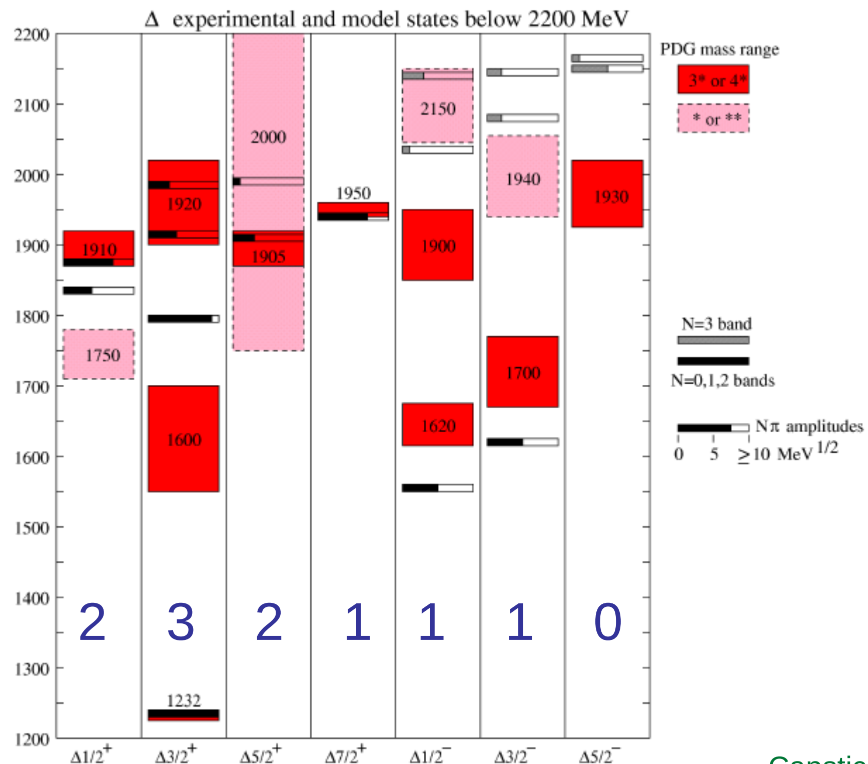
$(a_s)^{-1} \sim 1.6$ GeV, $(a_t)^{-1} \sim 5.6$ GeV

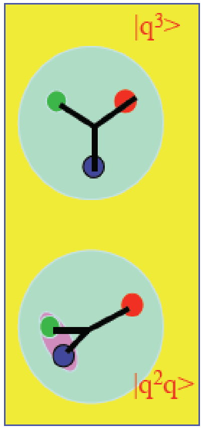
0810.3588, 0909.0200, 1004.4930

Where are the "Missing" Baryon Resonances?

- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?







Capstick, Isgur;
Capstick, Roberts

Operators are not states

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

Full basis of operators: many operators can create same state

Spectral “overlaps”

$$\langle \mathfrak{n};\, J^P \mid \Phi_i \mid 0 \rangle \;=\; Z_i^{\mathfrak{n}}$$

States may have subset of allowed symmetries

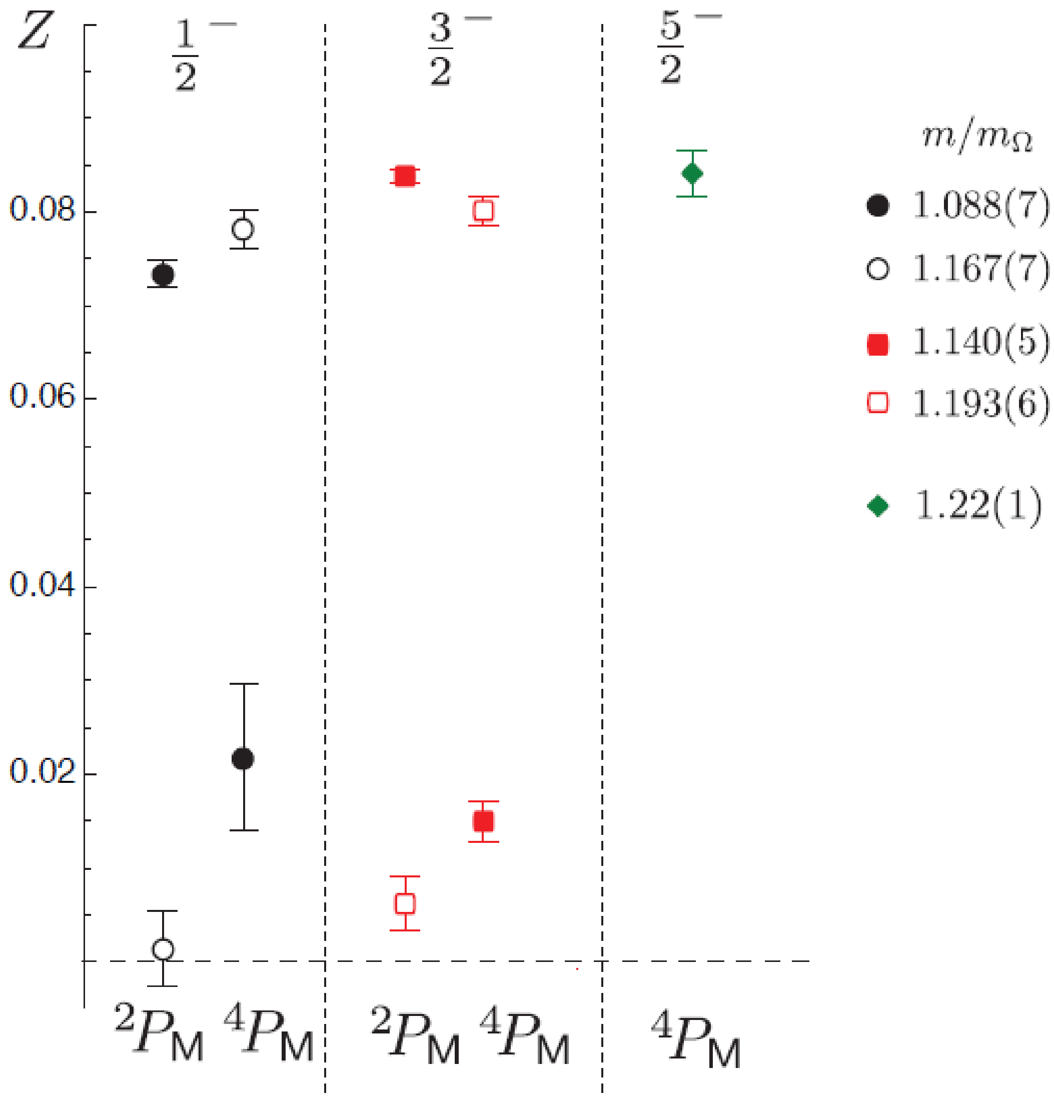
Nucleon J^-

Overlaps

$$Z_i^{\mathfrak{n}} = \langle J^- \mid \Phi_i \mid 0 \rangle$$

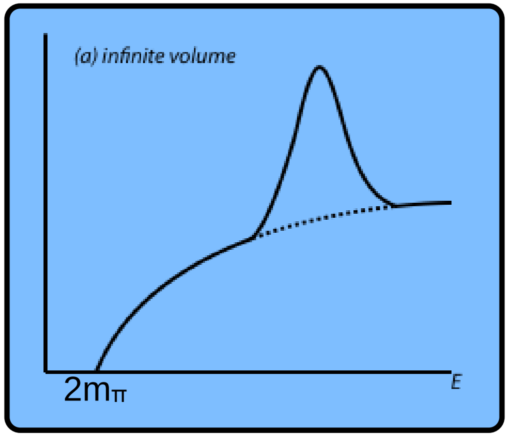
Little mixing in each J^-

Nearly "pure" [S= 1/2 & 3/2] 1^-



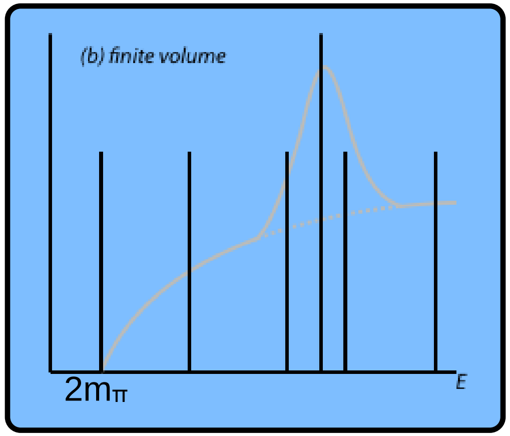
Spectrum of finite volume field theory

Missing states: "continuum" of multi-particle scattering states

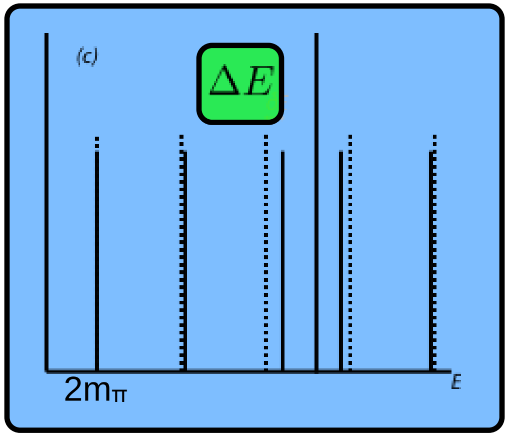


Infinite volume: continuous spectrum

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$



Finite volume: discrete spectrum

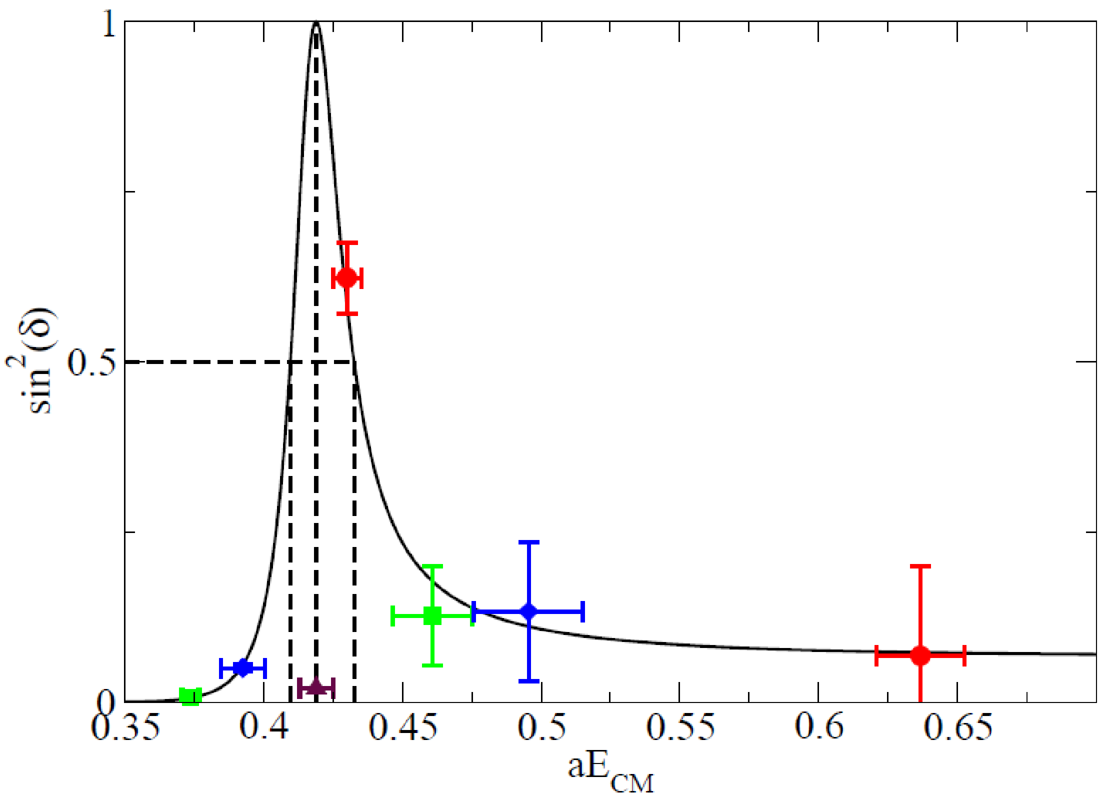


Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift

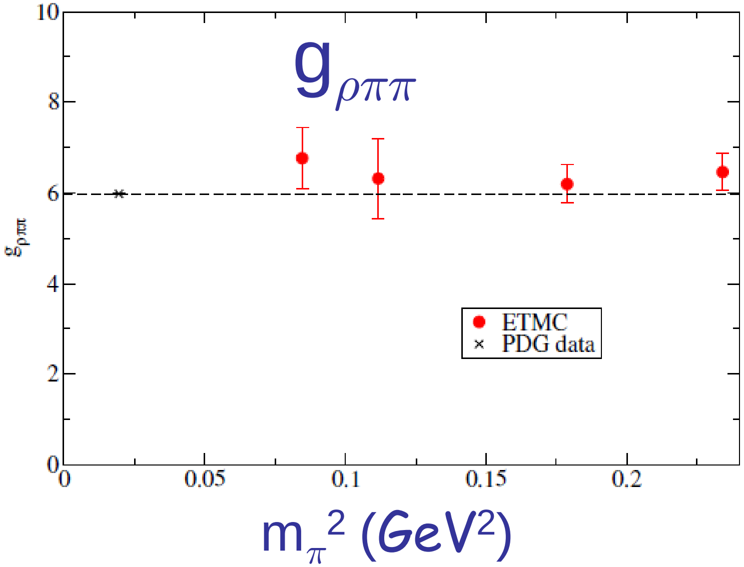
$\Delta E(L) \leftrightarrow \delta(E)$: Lüscher method

I=1 $\pi\pi$: the "ρ"

Extract $\delta_1(E)$ at discrete E



Extracted coupling: stable in pion mass



Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288

Form Factors

What is a form-factor off of a resonance?

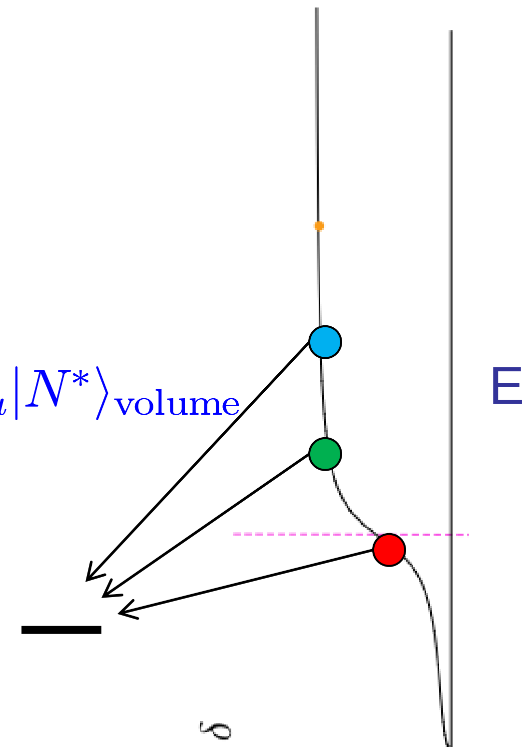
What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

$$\langle N|J_\mu|N^*\rangle_\infty(Q^2,E) \leftarrow [\delta'(E)+\Phi'(E)]\,\langle N|J_\mu|N^*\rangle_{\text{volume}}$$

Phase shift

Kinematic factor



Requires excited level transition FF's: some experience
- Charmonium E&M transition FF's (1004.4930)
- Nucleon 1st attempt: "Roper"->N (0803.3020)

Range: few GeV^2
Limitation: spatial lattice spacing