Baryon Spectroscopy and Resonances

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Hadron 2011

Collaborators:
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Auspices of the Hadron Spectrum Collaboration
Where are the “Missing” Baryon Resonances?

- What are collective modes?
- Is there “freezing” of degrees of freedom?
- What is the structure of the states?
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Nucleon & Delta spectrum
PDG uncertainty on B-W mass
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**Nucleon & Delta spectrum**

PDG uncertainty on B-W mass

**QM predictions**

![Graphical representation of Nucleon & Delta spectrum with QM predictions and experimental data.](image-url)
Two-point correlator

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[
C_{ij}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle
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\[ Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle \]
Spectrum from variational method

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Matrix of correlators

\[ C(t) = \begin{pmatrix}
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“Rayleigh-Ritz method”

Diagonalize:
- eigenvalues → spectrum
- eigenvectors → spectral “overlaps” \( Z_i^n \)
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\[ \text{eigenvectors} \rightarrow \text{spectral “overlaps”} \quad Z_i^n \]

Each state optimal combination of \( \Phi_i \)

\[
\Omega^{(n)} = \sum_i v_i^{(n)} \Phi_i
\]

Thomas Jefferson National Accelerator Facility
Spectrum from variational method

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\[ \Omega^{(n)} = \sum_i \nu_i^{(n)} \Phi_i \]

Benefit: orthogonality for near degenerate states
Baryon operators

Construction: permutations of 3 objects
Baryon operators

Construction: permutations of 3 objects

- **Symmetric:**
  - e.g., $uud + udu + duu$

- **Antisymmetric:**
  - e.g., $uud - udu + duu - ...$

- **Mixed:** (antisymmetric & symmetric)
  - e.g., $udu - duu$ & $2duu - udu - uud$
Baryon operators

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- Symmetric × Antisymmetric → Antisymmetric
- Mixed × Mixed → Symmetric⊕Antisymmetric⊕Mixed
- ...)
Baryon operators

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Color antisymmetric → Require **Space x [Flavor x Spin]** symmetric
Baryon operators

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  - ...

Space: couple covariant derivatives onto single-site spinors - build any J,M

\[ \Phi^{JM} \leftarrow (CGC')_{i,j,k} \left[ \bar{D} \right]_i \left[ \bar{D} \right]_j \left[ \Psi \right]_k \]

\[ J \leftarrow 1 \otimes 1 \otimes S \]

1104.5152
Baryon operators

Construction: permutations of 3 objects

- **Symmetric:**
  - e.g., uud+udu+duu
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- **Mixed:** (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

Multiplication rules:
- Symmetric $\rightarrow$ Antisymmetric
- Mixed $\times$ Mixed $\rightarrow$ Symmetric $\oplus$ Antisymmetric $\oplus$ Mixed
- ... 

Classify operators by permutation symmetries:
- Leads to rich structure

Color antisymmetric $\rightarrow$ Require **Space x [Flavor x Spin]** symmetric

**Space:** couple covariant derivatives onto single-site spinors - build any J,M

\[ \Phi^{JM} \leftarrow (C^C')_{i,j,k} \left[ \tilde{D}_i \right] \left[ \tilde{D}_j \right] [\Psi]_k \]

\[ J \leftarrow 1 \otimes 1 \otimes S \]

1104.5152
Baryon operator basis

3-quark operators & up to two covariant derivatives: some $J^P$

\[
\left( \text{Flavor} \otimes \text{Dirac} \otimes \text{Space}_{\text{symmetry}} \right)^{J^P}
\]
Baryon operator basis

3-quark operators & up to two covariant derivatives: some $J^P$

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\left( \text{Flavor} \otimes \text{Dirac} \right) \otimes \text{Space}_{\text{symmetry}}
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Spatial symmetry classification:

e.g., Nucleons: $N^{2S+1L_{\pi}} J^P$

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>#ops</th>
<th>E.g., spatial symmetries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J=1/2^-$</td>
<td>24</td>
<td>$N^2P_M \frac{1}{2}^-$, $N^4P_M \frac{1}{2}^+$</td>
</tr>
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</tr>
<tr>
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<td>24</td>
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<tr>
<td>$J=7/2^+$</td>
<td>4</td>
<td>$N^4D_{M} 7/2^+$</td>
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</table>
Baryon operator basis

3-quark operators & up to two covariant derivatives: some \( J^P \)

\[
\left( \begin{array}{ccc}
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\end{array} \right)^{J^P}
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- \( J=7/2^+ \): 4 \( N^4D_M \frac{7}{2}^+ \)

By far the largest operator basis ever used for such calculations

E.g., Nucleons: \( N^{2S+1L_\pi} J^P \)
Spin identified Nucleon & Delta spectrum

$m_\pi \sim 520\text{MeV}$

Statistical errors < 2%

arXiv:1104.5152
Spin identified Nucleon & Delta spectrum

arXiv:1104.5152

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$\frac{m}{m_\omega}$

$N^*$

$\Delta^*$

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SU(6) x O(3) counting
No parity doubling

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Discern structure: spectral overlaps

arXiv:1104.5152

\( m_\pi \sim 520 \text{MeV} \)
$N=2$ J$^+$ Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in J$^+$
$N=2$   $J^+$  Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in $J^+$

No “freezing” of degrees of freedom
Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) O(3) counting, but heavily mixed

Discrepancies??

Operator basis – spatial structure
Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) $O(3)$ counting, but heavily mixed

Discrepancies??
Operator basis – spatial structure

What else?
Multi-particle operators
Spectrum of finite volume field

The idea: 1 dim quantum mechanics

Two spin-less bosons: $\psi(x,y) = f(x-y) \rightarrow f(z)$

\[ -\frac{1}{m} \frac{d^2}{dz^2} + V(z) \] 
\[ f(z) = E f(z) \]

Solutions

\[ f(z) \rightarrow \cos [k|z| + \delta(k)] , \quad E = \frac{k^2}{m} \]

Quantization condition when $-L/2 < z < L/2$

\[ kL + 2\delta(k) = 0 \mod 2\pi \]

Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)
Finite volume scattering

Scattering in a periodic cubic box (length $L$)
- Discrete energy levels in finite volume

E.g. just a single elastic resonance

At some $L$, have discrete excited energies

$E \rightarrow k; \quad kL + 2\delta(k) = 0 \mod 2\pi$

- $T$-matrix amplitudes $\rightarrow$ partial waves
- Finite volume energy levels $E(L) \leftrightarrow \delta(E)$
Scattering of composite objects in non-perturbative field theory

Resonances

\[ \delta_0(E) \]

\[ \delta_2(E) \]

isospin = 2 \( \pi \pi \)

Feng, et al., 1011.5288

1011.6352
Resonances

Scattering of composite objects in non-perturbative field theory

\[ m_{\pi} = 481 \text{ MeV} \]
\[ m_{\pi} = 421 \text{ MeV} \]
\[ m_{\pi} = 330 \text{ MeV} \]
\[ m_{\pi} = 290 \text{ MeV} \]

\[ \delta_0(E) \]

\[ \delta_2(E) \]

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Scattering of composite objects in non-perturbative field theory

Manifestation of “decay” in Euclidean space

Can extract pole position

Feng, et.al, 1011.5288
Resonances

Scattering of composite objects in non-perturbative field theory

\[ m_\rho = 481 \text{ MeV} \]
\[ m_\pi = 421 \text{ MeV} \]
\[ m_\eta = 330 \text{ MeV} \]
\[ m_\phi = 290 \text{ MeV} \]

Extracted coupling: stable in pion mass

Stability a generic feature of couplings??

Feng, et.al, 1011.5288
Hadronic Decays

Some candidates: determine phase shift
Somewhat elastic

$S_{11} \rightarrow [N\pi]_S$
$+[N\eta]_S$

$\Delta \rightarrow [N\pi]_P$

$m_\pi \sim 400$ MeV
Isoscalar & isovector meson spectrum

Isoscalars: flavor mixing determined

\[ m_{\pi} = 396 \text{ MeV} \]

- isoscalar
- isovector
- YM glueball

1102.4299
Isoscalar & isovector meson spectrum

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Exotics

\( m_\pi = 396 \text{ MeV} \)

- Iso scalar
- Isovector
- YM glue ball

1102.4299
Isoscalars: flavor mixing determined

Will need to build PWA within mesons

1102.4299
Summary & prospects

Results for baryon excited state spectrum:
• No “freezing” of degrees of freedom nor parity doubling
• Broadly consistent with non-relativistic quark model
• Add multi-particles → baryon spectrum becomes denser
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• Lighter pion masses (230MeV available)
• Extract couplings in multi-channel systems (with $\pi$, $\eta$, $K$…)
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Yes! [with caveats]
- Also complicated
- But all Minkowski information is there
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Optimistic: see confluence of methods (an “amplitude analysis”)
- Develop techniques concurrently with decreasing pion mass
• The end
Goal: resolve highly excited states

$N_f = 2 + 1 \ (u,d + s)$

Anisotropic lattices:

$(a_s)^{-1} \sim 1.6 \ \text{GeV}, \ (a_t)^{-1} \sim 5.6 \ \text{GeV}$
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Operators are not states

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Full basis of operators: many operators can create same state

Spectral “overlaps”

\[ \langle n; J^P | \Phi_i | 0 \rangle = Z_i^n \]

States may have subset of allowed symmetries
Overlaps

\[ Z_{i}^{n} = \langle J^{-} | \Phi_{i} | 0 \rangle \]

Little mixing in each \( J^{-} \)

Nearly “pure” [S= 1/2 & 3/2] \( 1^{-} \)
Spectrum of finite volume field theory

**Missing states:** “continuum” of multi-particle scattering states

- **Infinite volume:** continuous spectrum
  \[ E(p) = 2 \sqrt{m^2 + p^2} \]

- **Finite volume:** discrete spectrum

**Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift**

\[ \Delta E(L) \leftrightarrow \delta(E) : \text{Lüscher method} \]
\[ I=1 \quad \pi\pi : \text{the } \rho \]

Extract \( \delta_1(E) \) at discrete \( E \)

Extracted coupling: stable in pion mass

Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288
Form Factors

What is a form-factor off of a resonance?

What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

\[
\langle N | J_\mu | N^* \rangle_\infty (Q^2, E) \leftarrow [\delta'(E) + \Phi'(E)] \langle N | J_\mu | N^* \rangle_{\text{volume}}
\]

Requires excited level transition FF’s: some experience
- Charmonium E&M transition FF’s \(1004.4930\)
- Nucleon 1\textsuperscript{st} attempt: “Roper”\(\rightarrow N\) \(0803.3020\)

Range: few \(\text{GeV}^2\)
Limitation: spatial lattice spacing