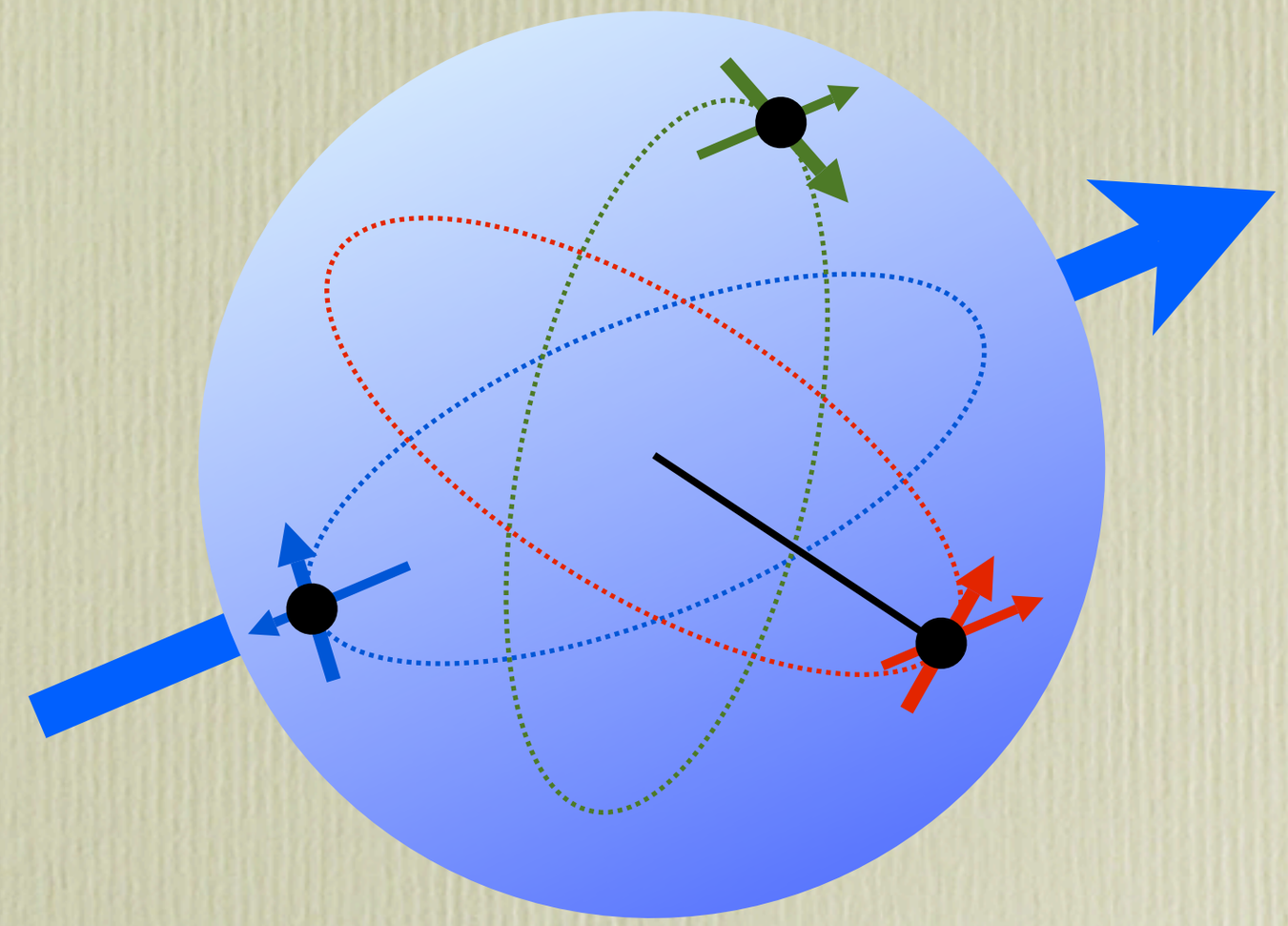
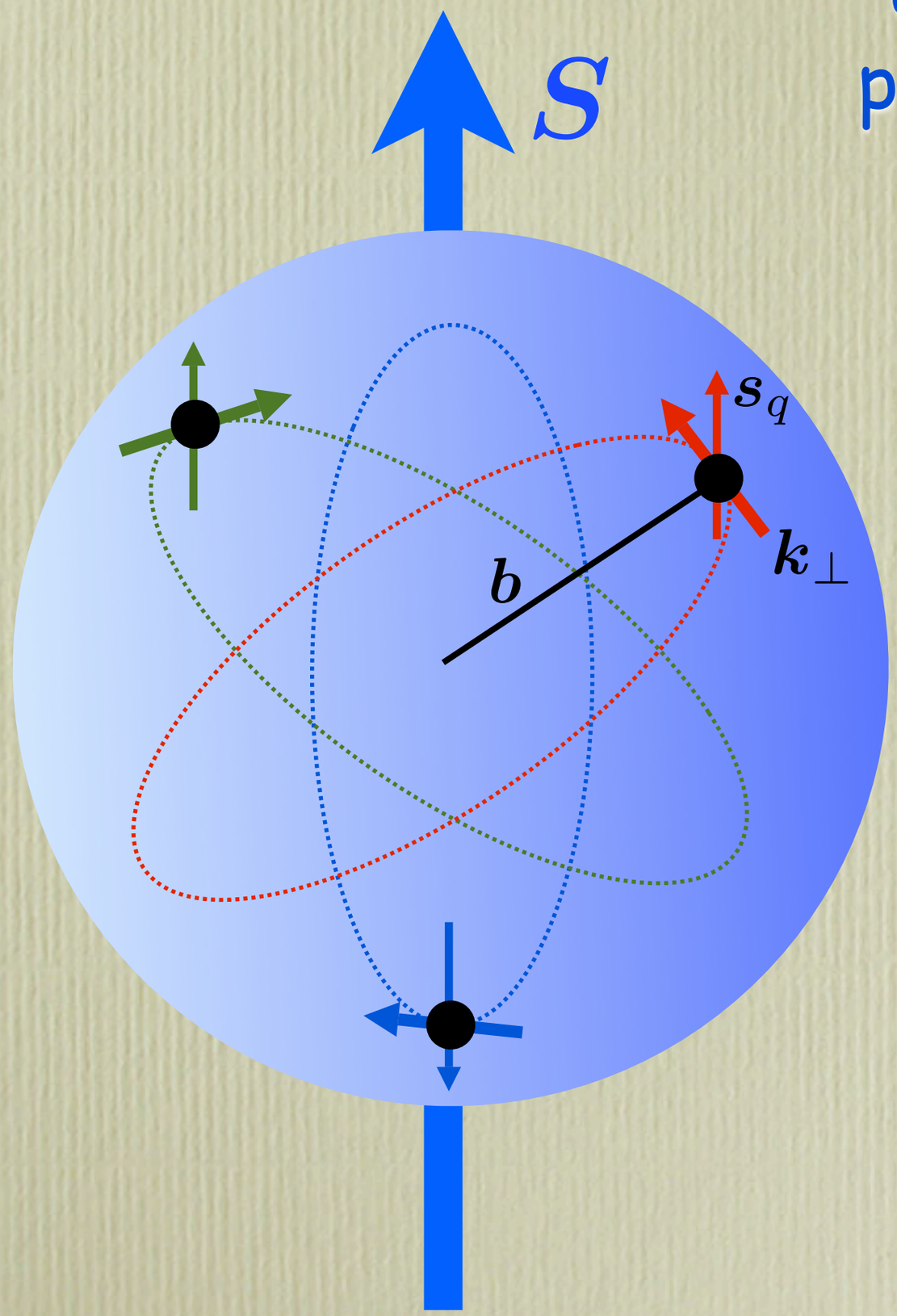
The 3-D momentum structure of the nucleon

what would we like to know?
what do we know and how?



XIV International Conference on Hadron Spectroscopy - HADRON 2011 - Munich, June 13-17, 2011

Mauro Anselmino, Torino University & INFN



Exploring the 3-dimensional phase-space structure of the nucleon

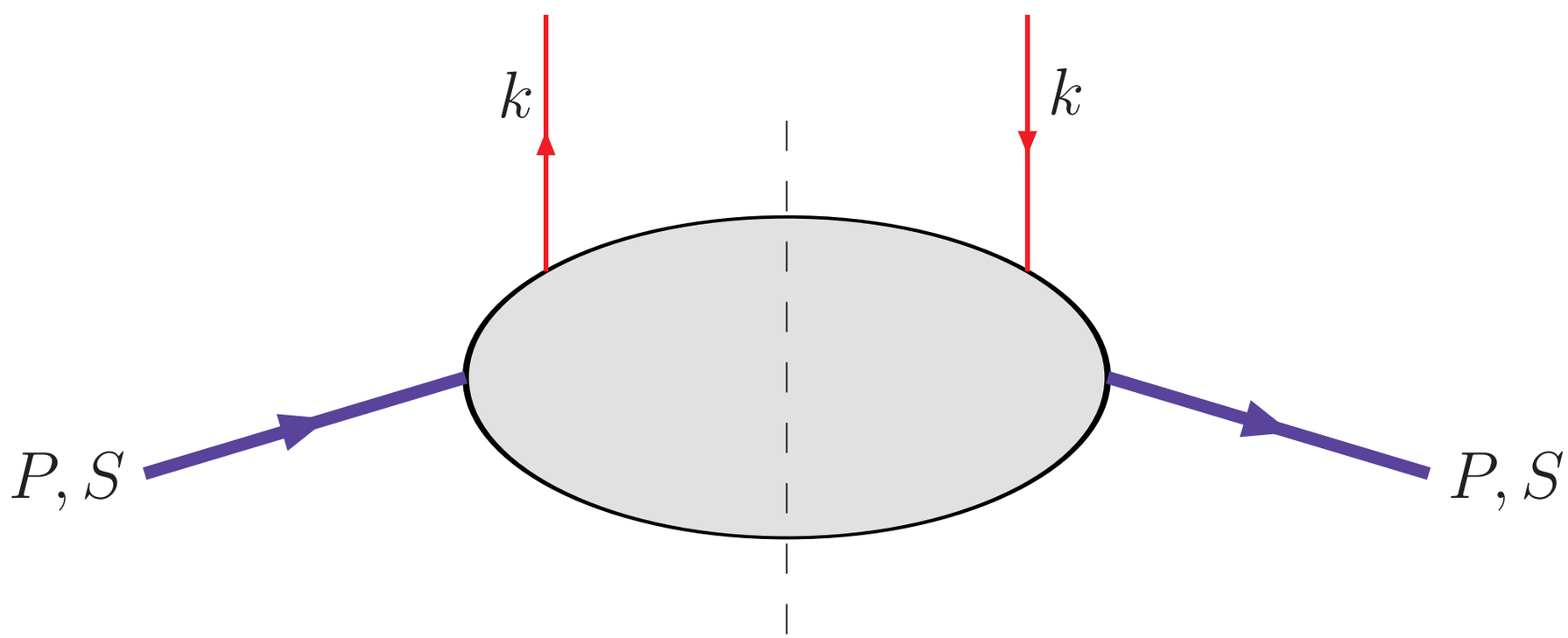
phase-space (**k**-**b**) distribution of partons in nucleons; parton intrinsic motion; spin-$k_\perp$ correlations? orbiting quarks?

information encoded in GPDs and TMDs (exclusive and inclusive processes)

new probes and concepts to explore the nucleon structure

TMDs - Transverse Momentum Dependent (distribution and fragmentation functions)

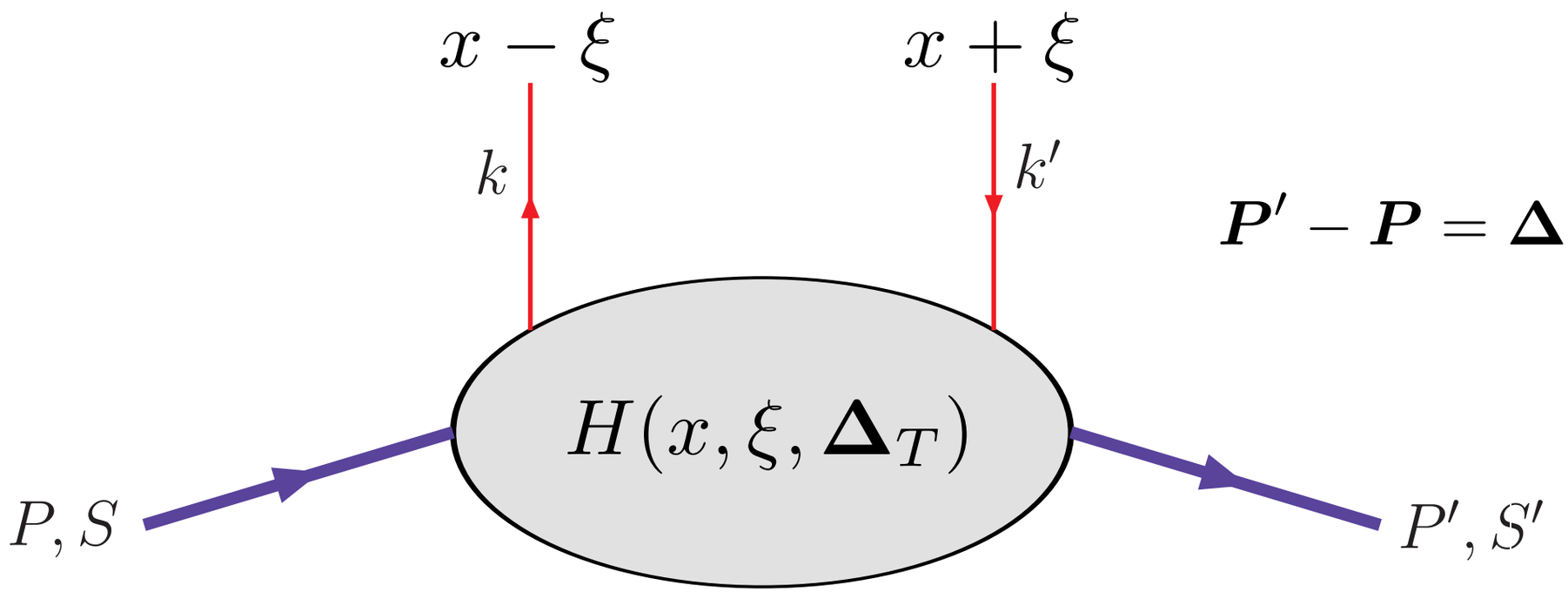
(polarized) SIDIS and Drell-Yan, spin asymmetries in inclusive (large P_T) NN processes



$$f_{a/p}(x, \boldsymbol{k}_\perp; \boldsymbol{s}_a, \boldsymbol{S})$$

GPDs - Generalized Partonic Distributions

exclusive processes in leptonic and hadronic interactions

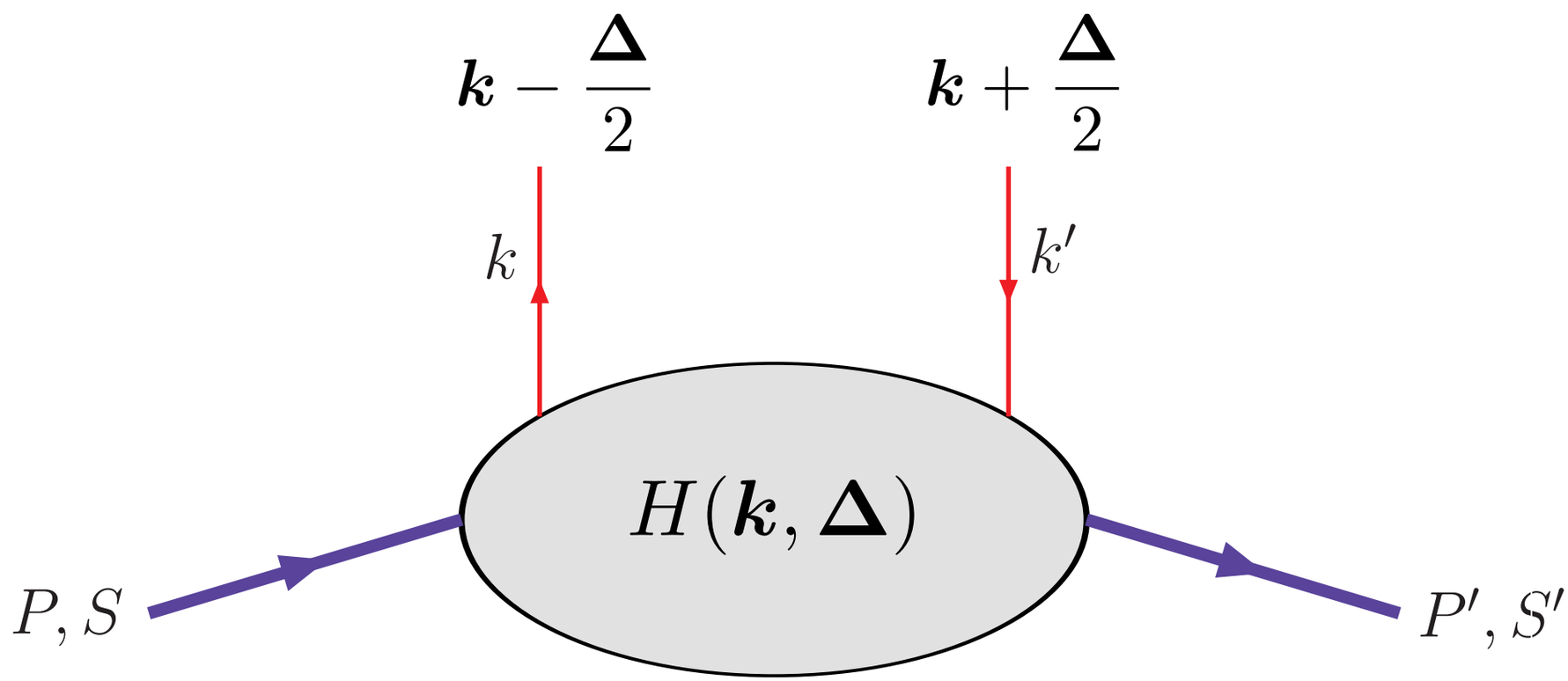


$$\boldsymbol{P}' - \boldsymbol{P} = \boldsymbol{\Delta}$$

$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} H_q(x, 0, -\boldsymbol{\Delta}_T^2) e^{-i\,\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T}$$

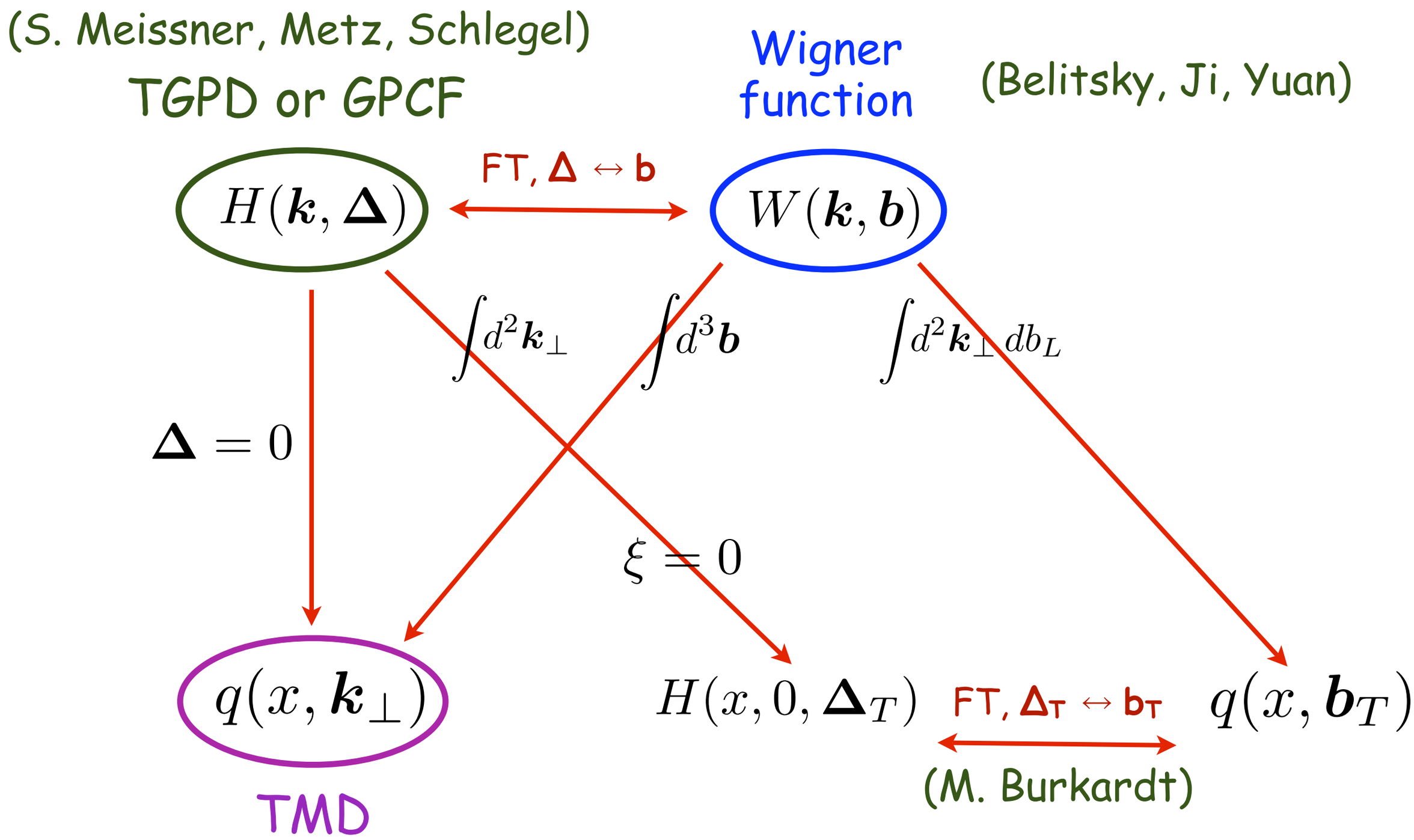
GTMDs - Generalized Transverse Momentum Dependent (partonic distributions)

exclusive processes in leptonic and hadronic interactions

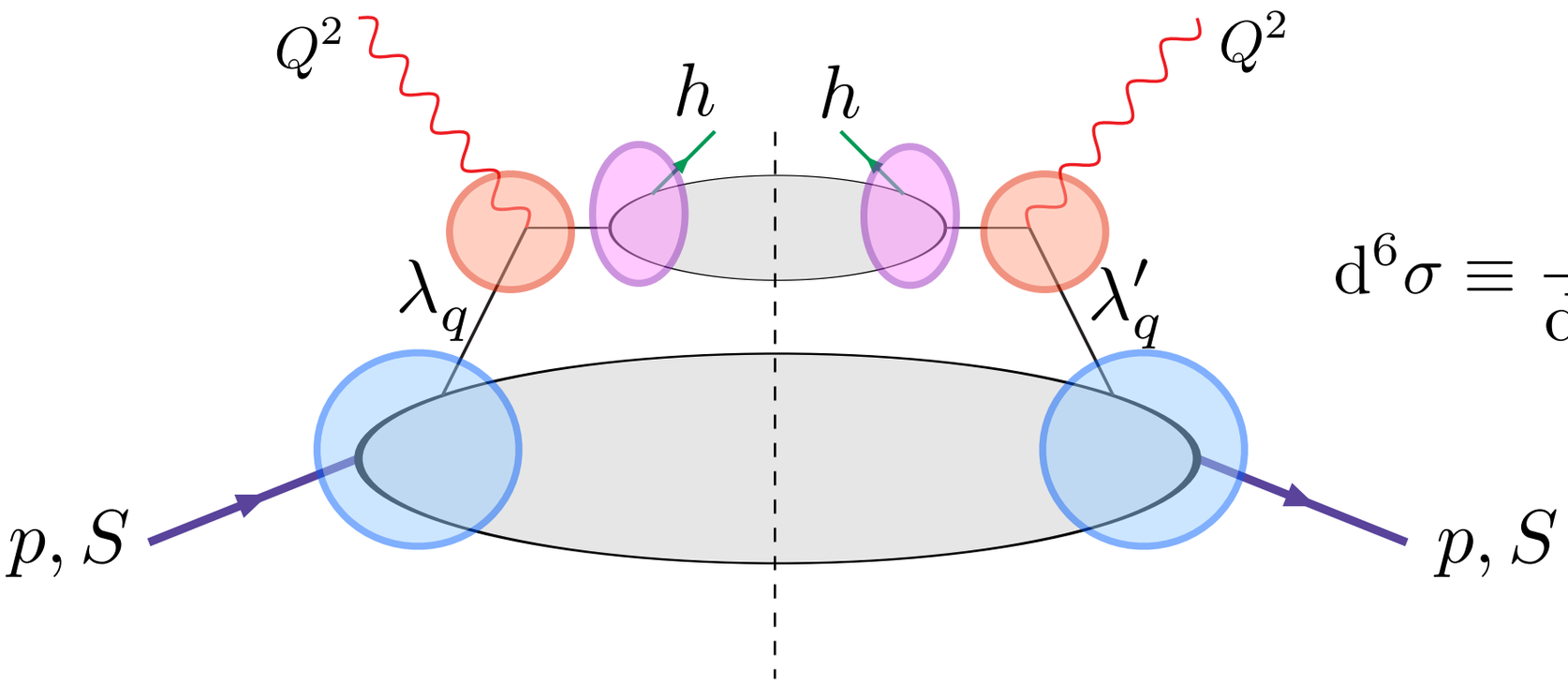


$$\int d^2\boldsymbol{k}_\perp \, H(\boldsymbol{k}, \boldsymbol{\Delta}) = H(x, \xi, \boldsymbol{\Delta}_T)$$

phase-space parton distribution, $W(\boldsymbol{k}, \boldsymbol{b})$



TMDs in SIDIS



$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^\uparrow \to \ell h X}}{\mathrm{d}x_B\, \mathrm{d}Q^2\, \mathrm{d}z_h\, \mathrm{d}^2\boldsymbol{P}_T\, \mathrm{d}\phi_S}$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\mathrm{QCD}}$

Two scales: $P_T \ll Q^2$

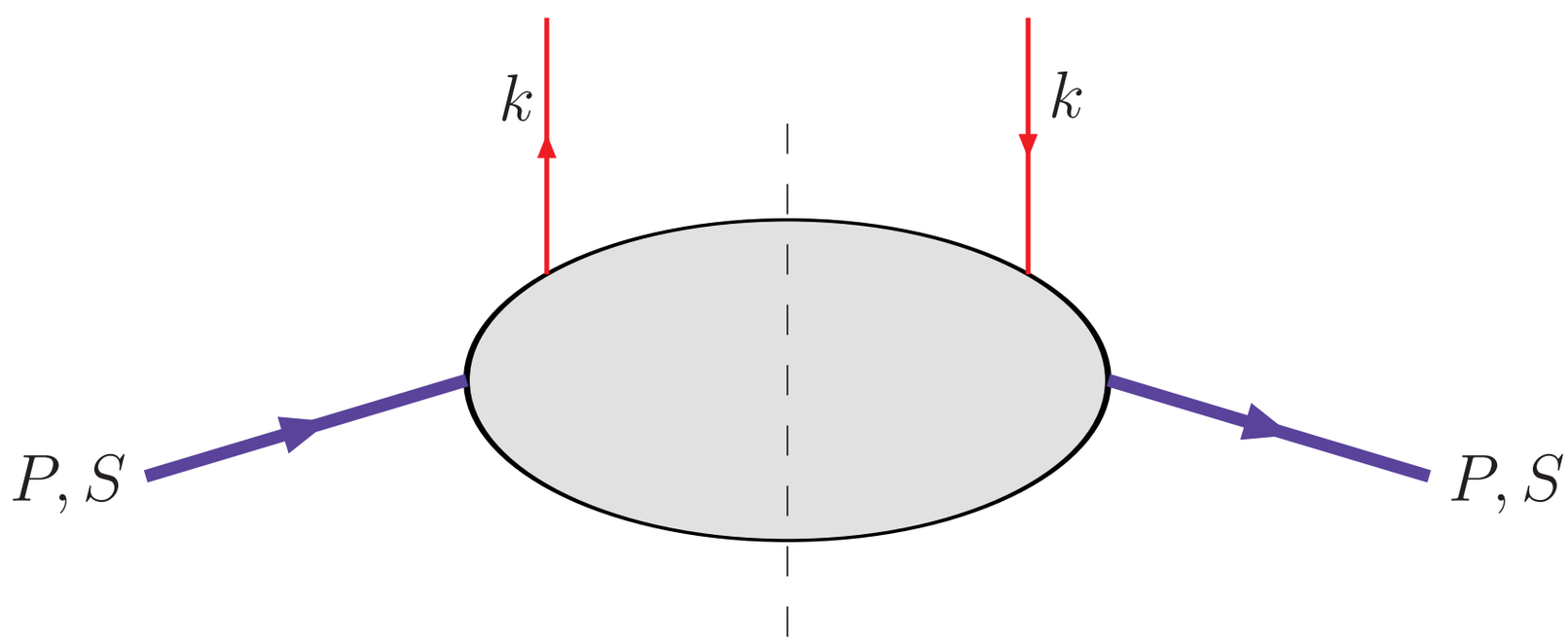
$$\boldsymbol{p}_\perp \simeq \boldsymbol{P}_T - z_h\, \boldsymbol{k}_\perp$$

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

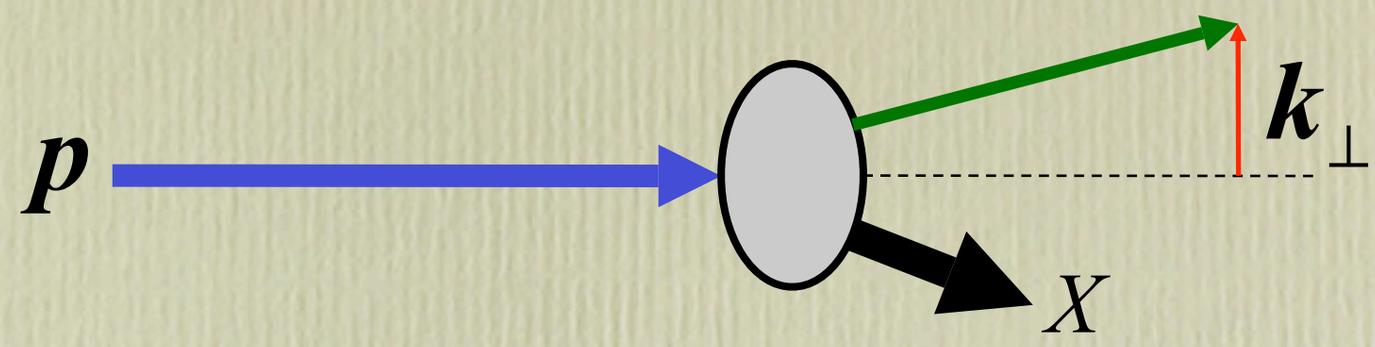
(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

TMDs: the leading-twist correlator, with intrinsic $k_\perp$, contains 8 independent functions

$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) \;=\; & \frac{1}{2}\Bigg[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\
& + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
& + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Bigg]
\end{aligned}
$$

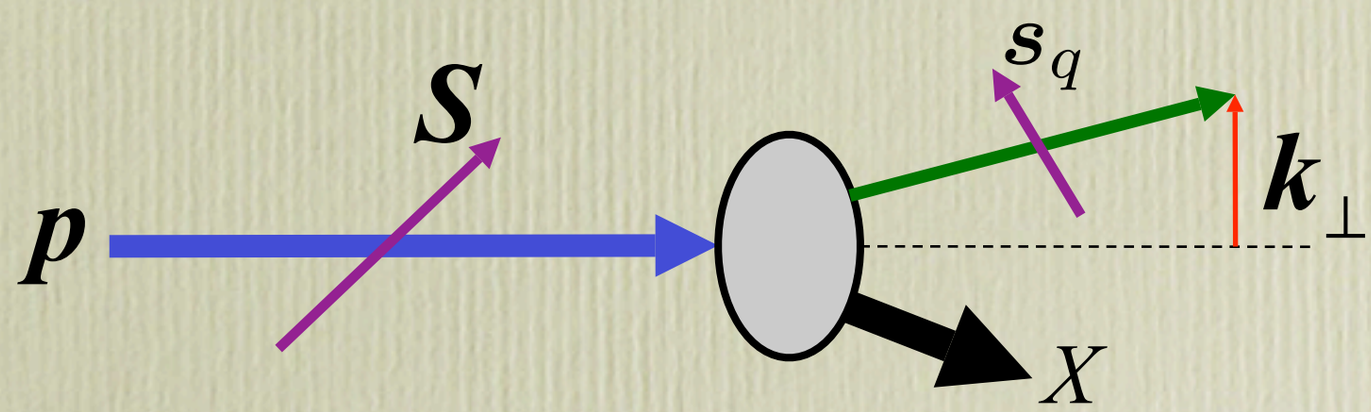


with partonic interpretation



$$f_1^q(x, k_\perp^2) \qquad\qquad q(x) = f_1^q(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp \, f_1^q(x, k_\perp^2)$$

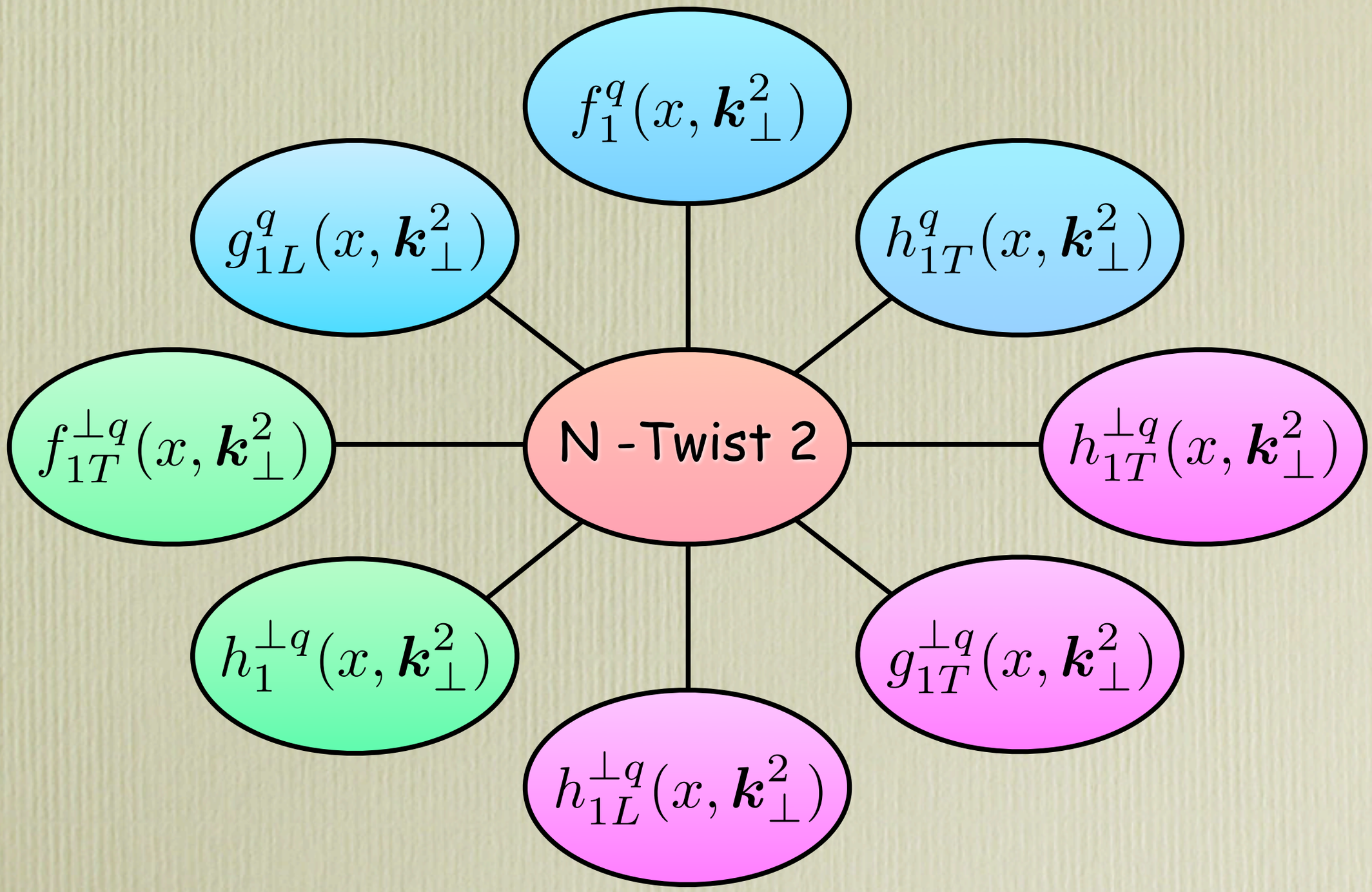
several spin-$\mathbf{k}_\perp$ correlations in TMDs



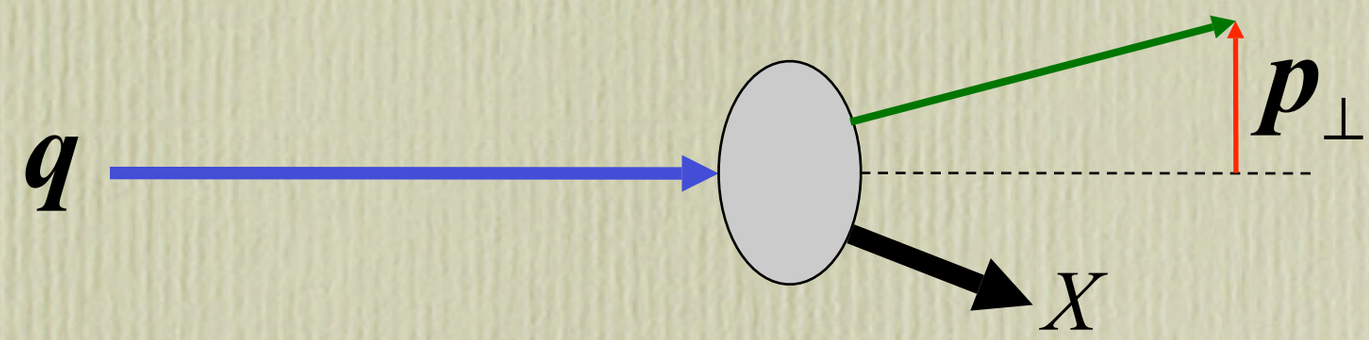
$$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp) \qquad \boldsymbol{s}_q \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp) \qquad (\boldsymbol{p} \cdot \boldsymbol{S})(\boldsymbol{p} \cdot \boldsymbol{s}_q) \qquad \cdots$$

"Sivers effect" "Boer-Mulders effect"

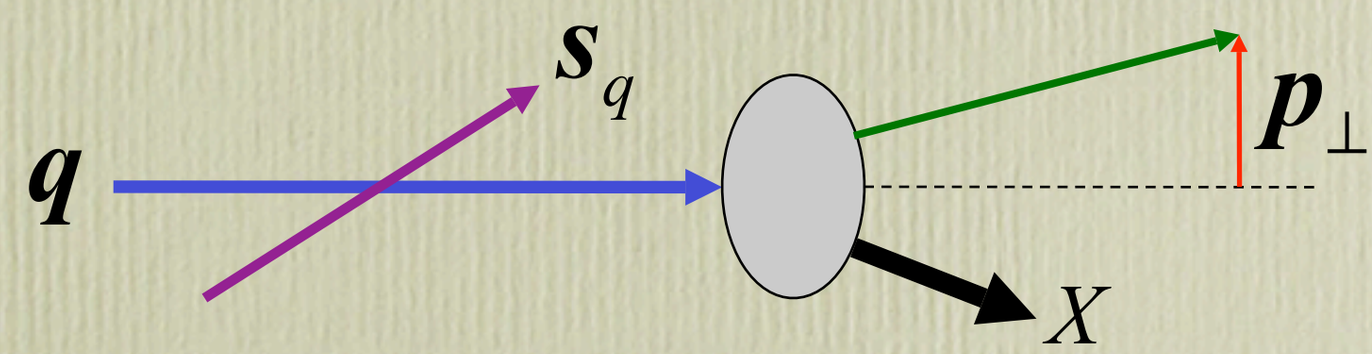
The nucleon at twist-2



similar spin-$\boldsymbol{p}_\perp$ correlations in fragmentation process (case of final spinless hadron)



$$D_1^q(x, \boldsymbol{p}_\perp^2)$$



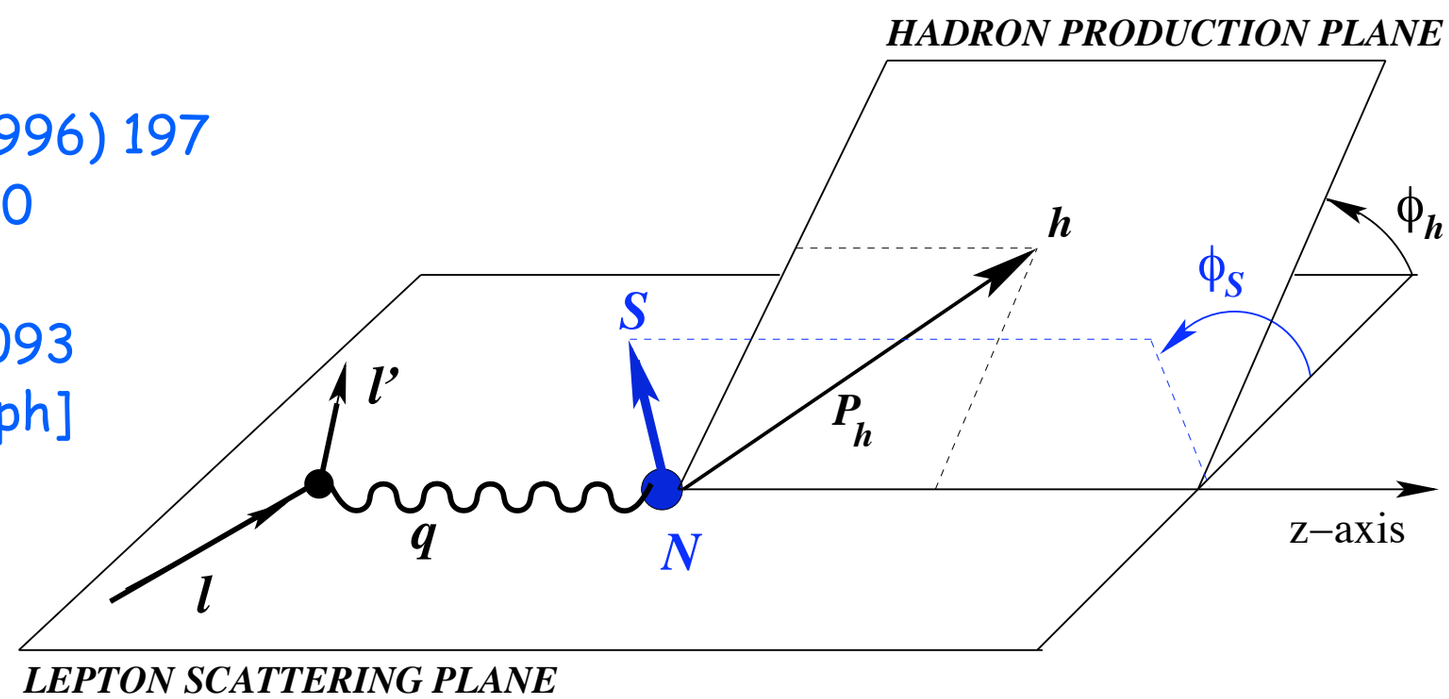
$$H_1^{\perp q}(x, \boldsymbol{p}_\perp^2)$$

$$\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{p}_\perp)$$

"Collins effect"

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} &= F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
&+ S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
&+ S_T \Big\{ \sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
&+ \frac{1}{Q} \left[\sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
&+ \lambda \left[\cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

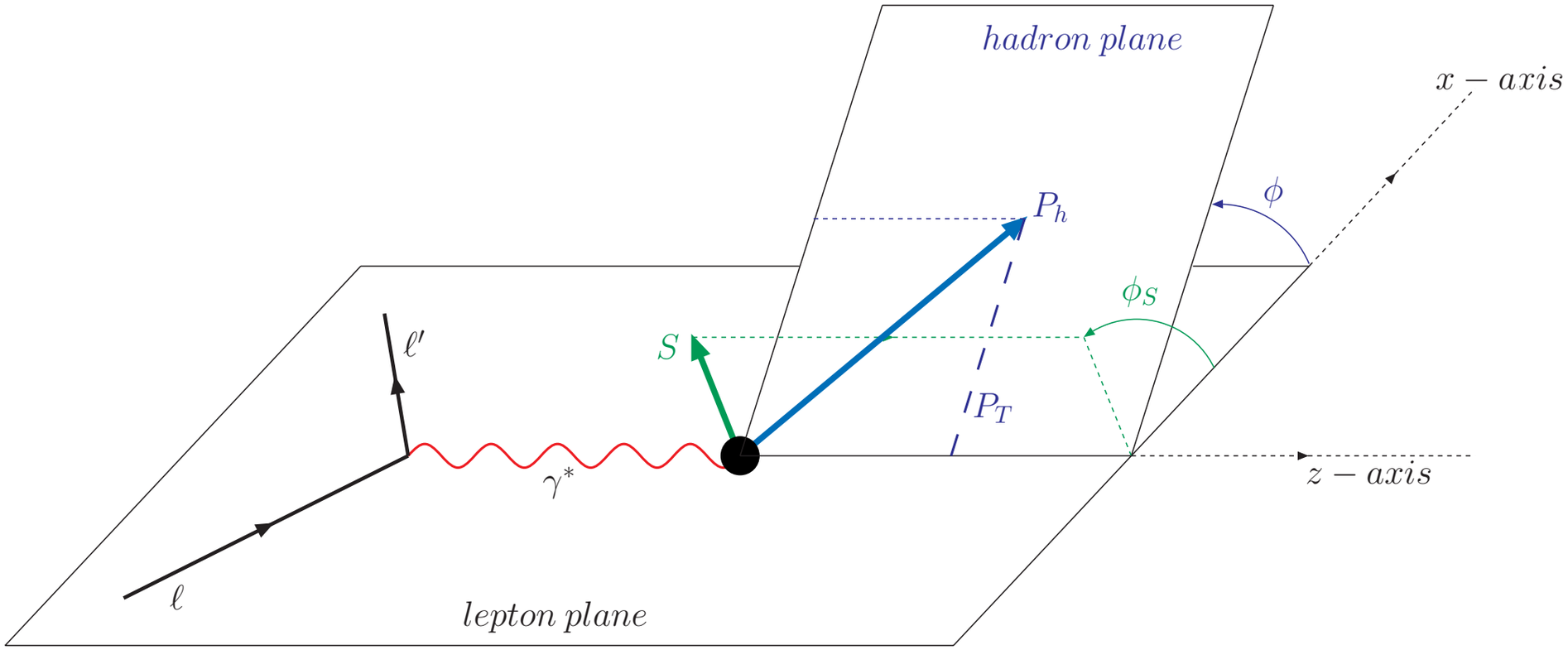
Kotzinian, **NP B441** (1995) 234
Mulders and Tangermann, **NP B461** (1996) 197
Boer and Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093
Anselmino et al., arXiv:1101.1011 [hep-ph]



many spin asymmetries

$$\mathrm{d}\sigma(\boldsymbol{S}) \neq \mathrm{d}\sigma(-\boldsymbol{S})$$

$F_{S_B S_T}^{(\ldots)}$ contain the TMDs



$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

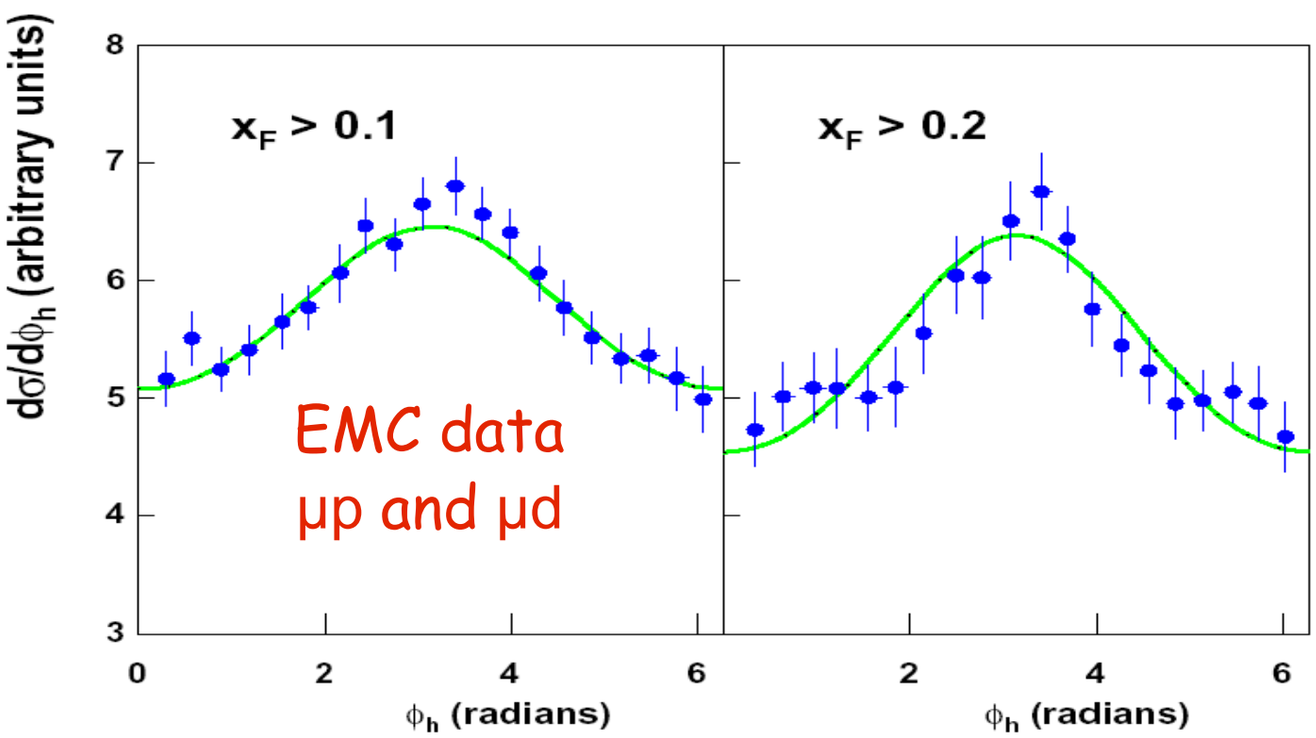
chiral-odd TMDs

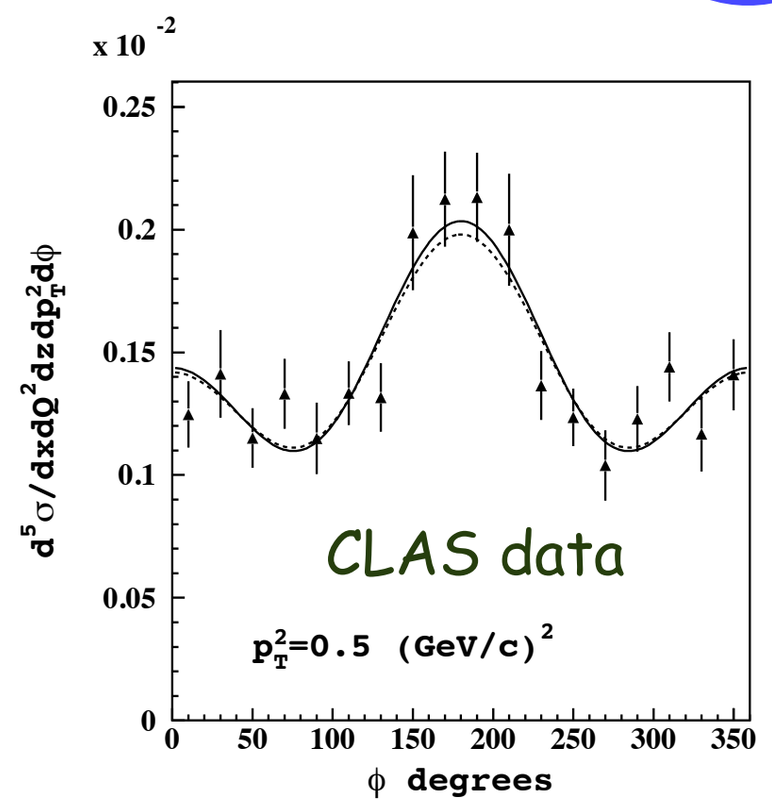
$$f \otimes D \sim \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{p}_\perp \, \delta^{(2)}(\boldsymbol{P}_T - z_h \boldsymbol{k}_\perp - \boldsymbol{p}_\perp) \, w(\boldsymbol{k}_\perp, \boldsymbol{P}_T) \, f(x_B, k_\perp) \, D(z_h, p_\perp)$$

TMDs in unpolarized SIDIS: "Cahn effect" at $\mathcal{O}(k_\perp/Q)$

$$\frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} \sim \underbrace{f_1^q \otimes D_1^q \otimes \mathrm{d}\hat{\sigma}} + \left(h_1^{q\perp} \otimes H_1^{q\perp} \otimes \mathrm{d}\Delta\hat{\sigma}\right)$$

$$d\hat{\sigma}^{\ell q\to\ell q} \propto \hat{s}^2 + \hat{u}^2 = \frac{Q^4}{y^2}\left[1 + (1-y)^2 - 4\,\frac{k_\perp}{Q}\,(2-y)\,\sqrt{1-y}\,\cos\varphi\right]$$





assuming gaussian $k_\perp$ and $p_\perp$ dependences:

$$\langle k_\perp^2\rangle = 0.28\,(\mathrm{GeV})^2 \qquad \langle p_\perp^2\rangle = 0.25\,(\mathrm{GeV})^2$$

M.A., M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin

Siver function phenomenology in SIDIS

M.Anselmino, M.Boglione, J.C.Collins, U.D'Alesio, A.V.Efremov, K.Goeke, A.Kotzinian, S.Menzel, A.Metz, F.Murgia, A.Prokudin, P.Schweitzer, W.Vogelsang, F.Yuan

$$2\,\langle \sin(\phi-\phi_S)\rangle = A_{UT}^{\sin(\phi-\phi_S)} \equiv 2\,\frac{\overbrace{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow})\,\sin(\phi-\phi_S)}^{F_{UT}^{\sin(\phi-\phi_S)}}}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}+\mathrm{d}\sigma^{\downarrow})}$$

extraction of Sivers function based on very simple parameterization, with x and $\mathbf{k}_\perp$ factorization. Typically:

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp) = N x^\alpha (1-x)^\beta\, h(k_\perp) f_{q/p}(x,k_\perp)$$

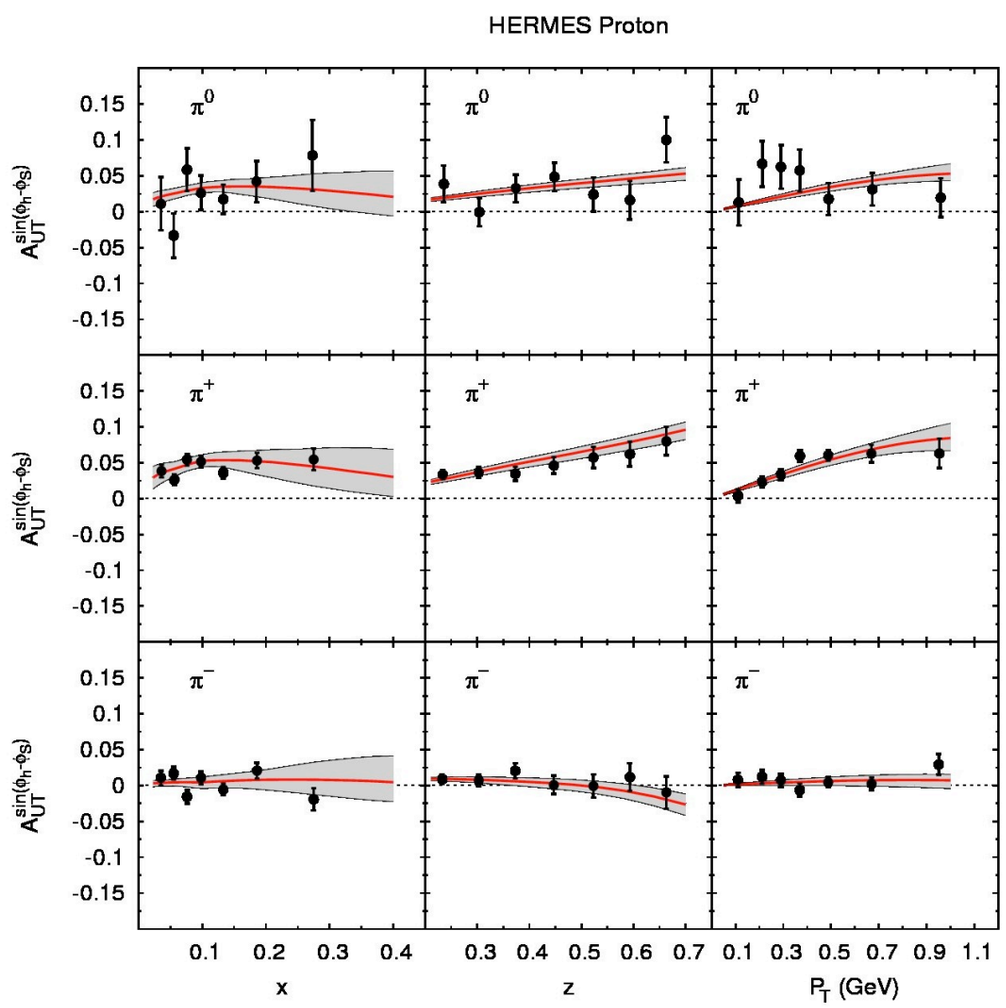
with

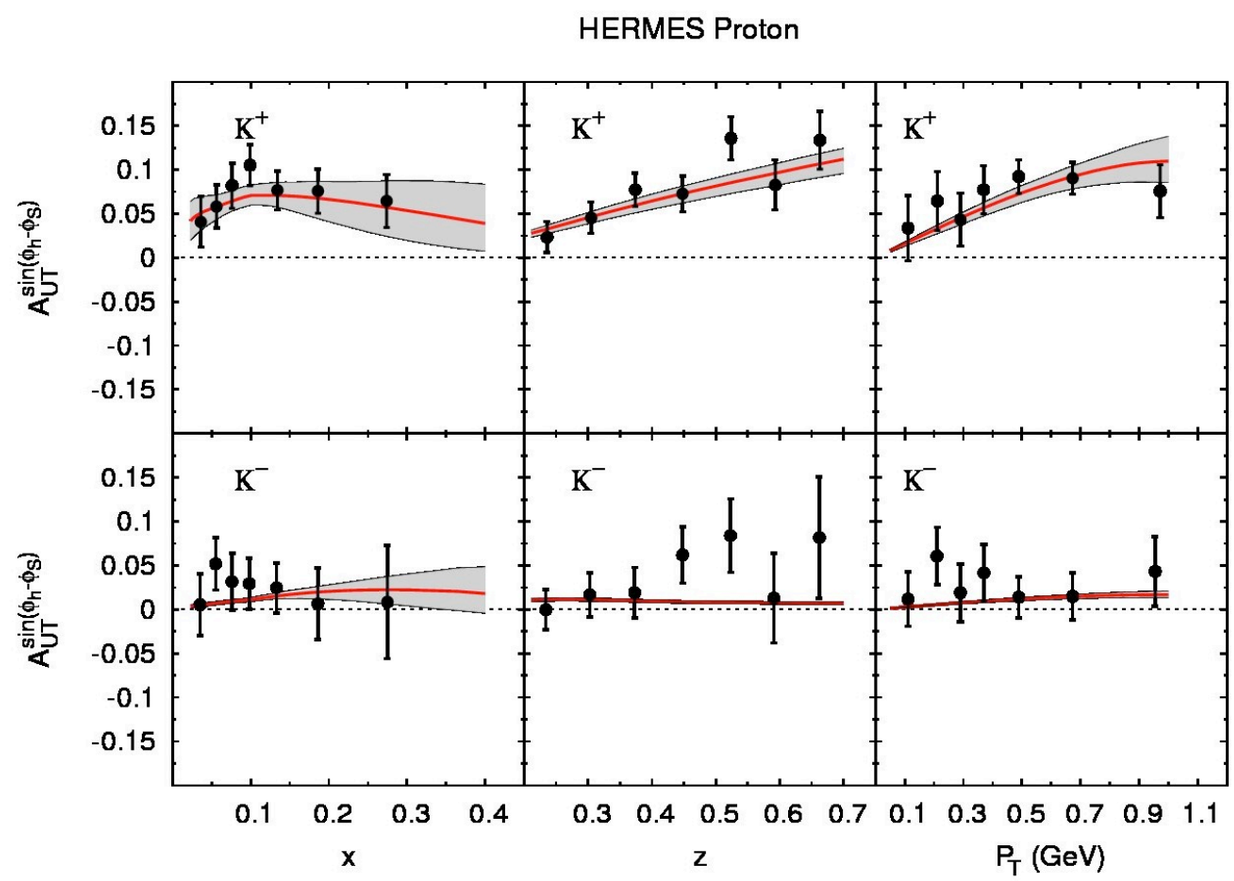
$$f_{q/p}(x,k_\perp) = f_q(x)\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$\langle k_\perp^2\rangle$ constant and flavour independent

S. Melis, talk at DIS 2011

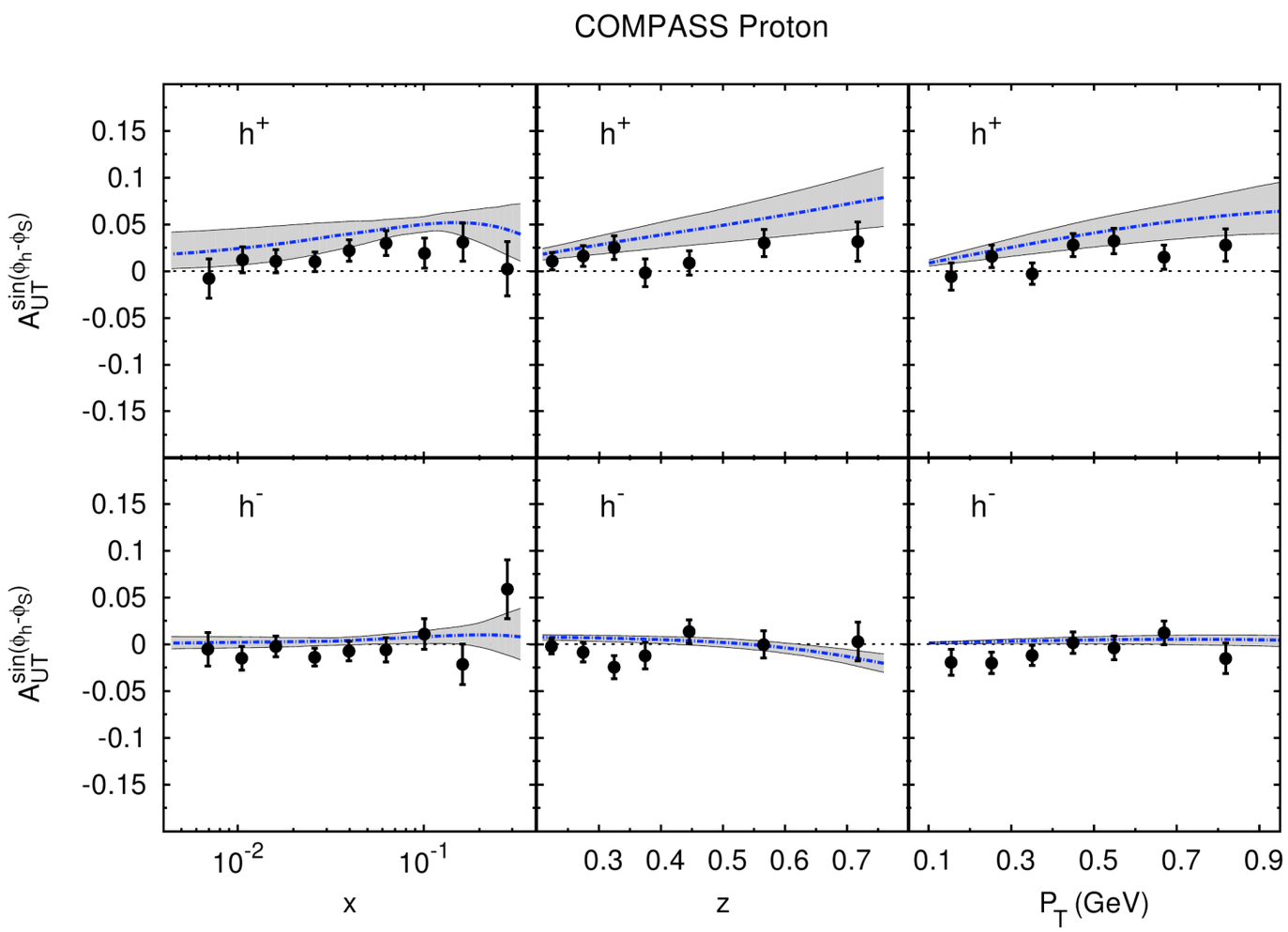
FIT u & d only





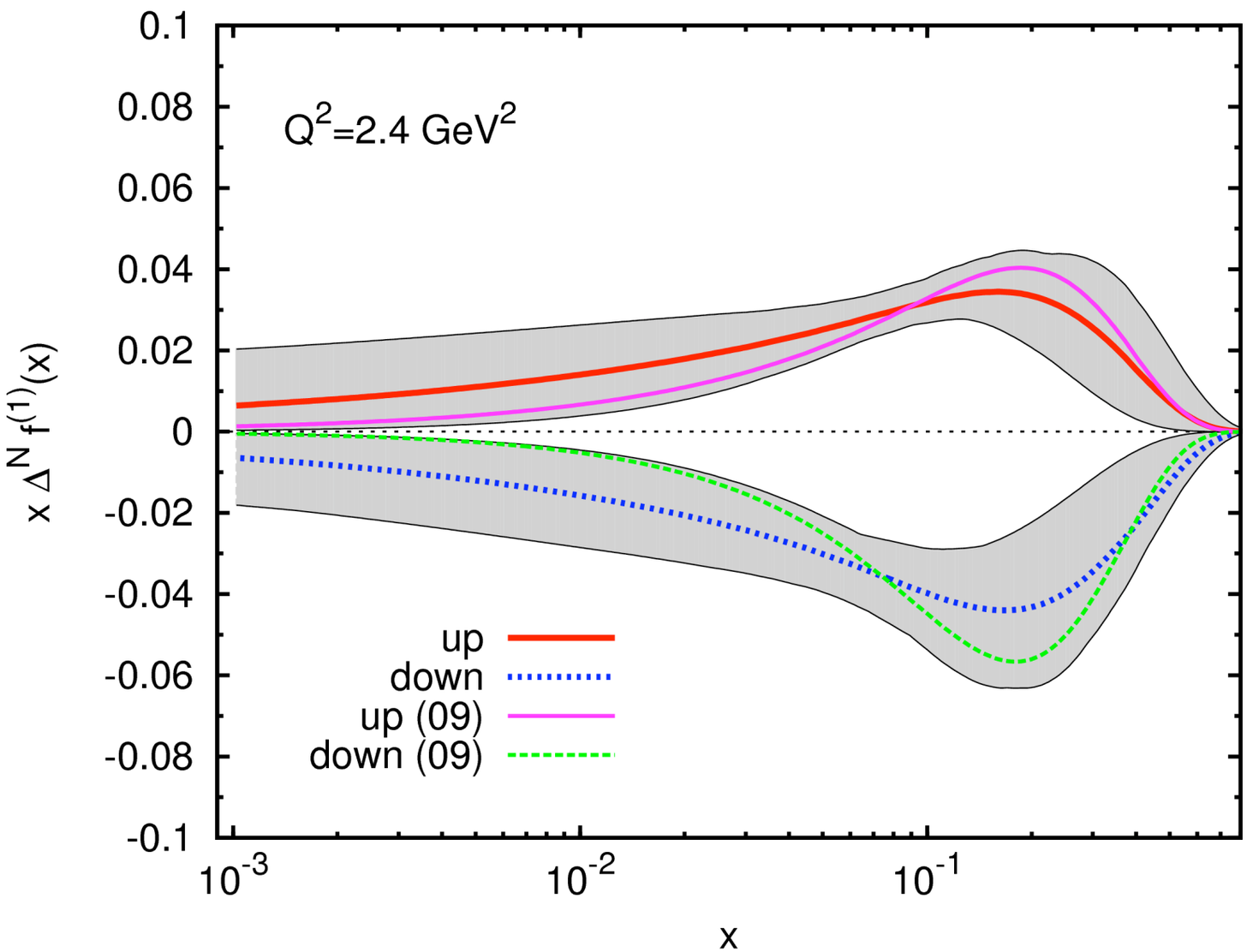
S. Melis, talk at DIS 2011

FIT u & d only



simple Sivers functions for u and d quarks are sufficient to fit the available SIDIS data

large and very small x dependence not constrained by data



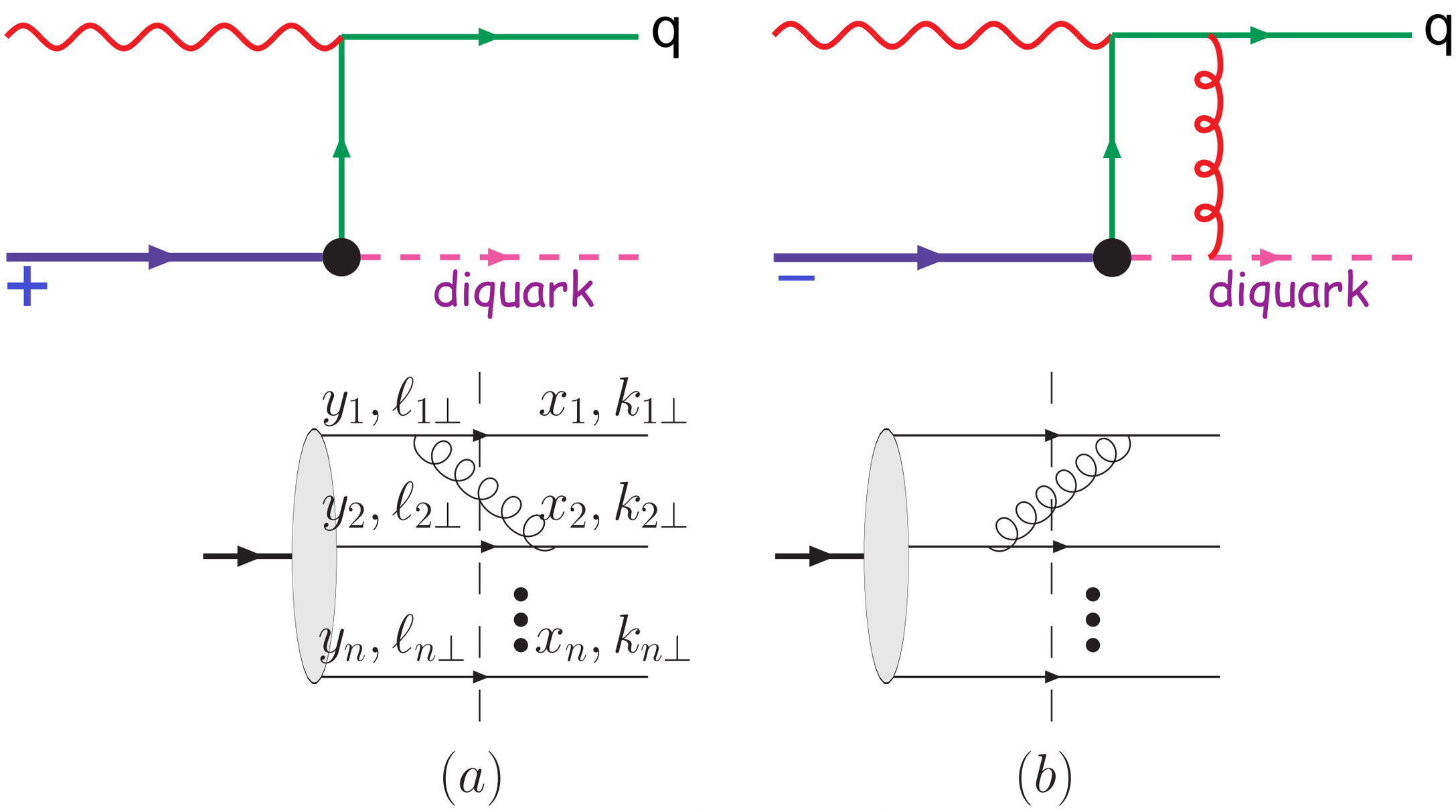
new and previous extraction of u and d Sivers functions

S. Melis and A. Prokudin, preliminary results

Anselmino et al. Eur. Phys. J. A39,89 (2009)

Quark models for Sivers function

Brodsky, Hwang, Schmidt; Bacchetta, Conti, Radici; Bacchetta, Radici, Conti, Guagnelli; Brodsky, Pasquini, Xiao, Yuan; Pasquini, Yuan; Hwang; ....



in all models one has:

$$[f_{1T}^{q\perp}]_{\rm SIDIS} = -[f_{1T}^{q\perp}]_{\rm DY}$$

the azimuthal dependence induced by intrinsic motion in unpolarized SIDIS (Cahn effect) has been confirmed (EMC, HERMES, COMPASS, CLAS)

Gaussian $k_\perp$ distribution of TMDs?

$$\langle k_\perp^2 \rangle (x, Q^2) \quad \langle p_\perp^2 \rangle (z, Q^2)$$

x, z dependence? flavour dependence?
energy dependence?

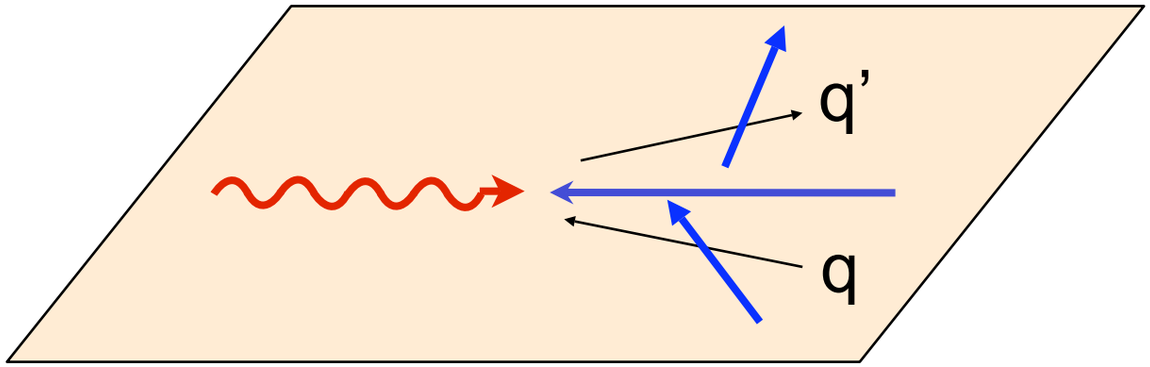
Sivers effect now observed by two experiments (+ HALL-A A_{UT} on neutrons), but needs further measurements (wider kinematical coverage)

Q^2 of data not so high, role of higher twists?
more sophisticated parameterization...
(non) universality of Sivers function?

recent improvement in study of QCD evolution (Aybat, Rogers, arXiv:1101.5057)

Collins effect in SIDIS - $F_{UT}^{\sin(\phi+\phi_S)}$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$



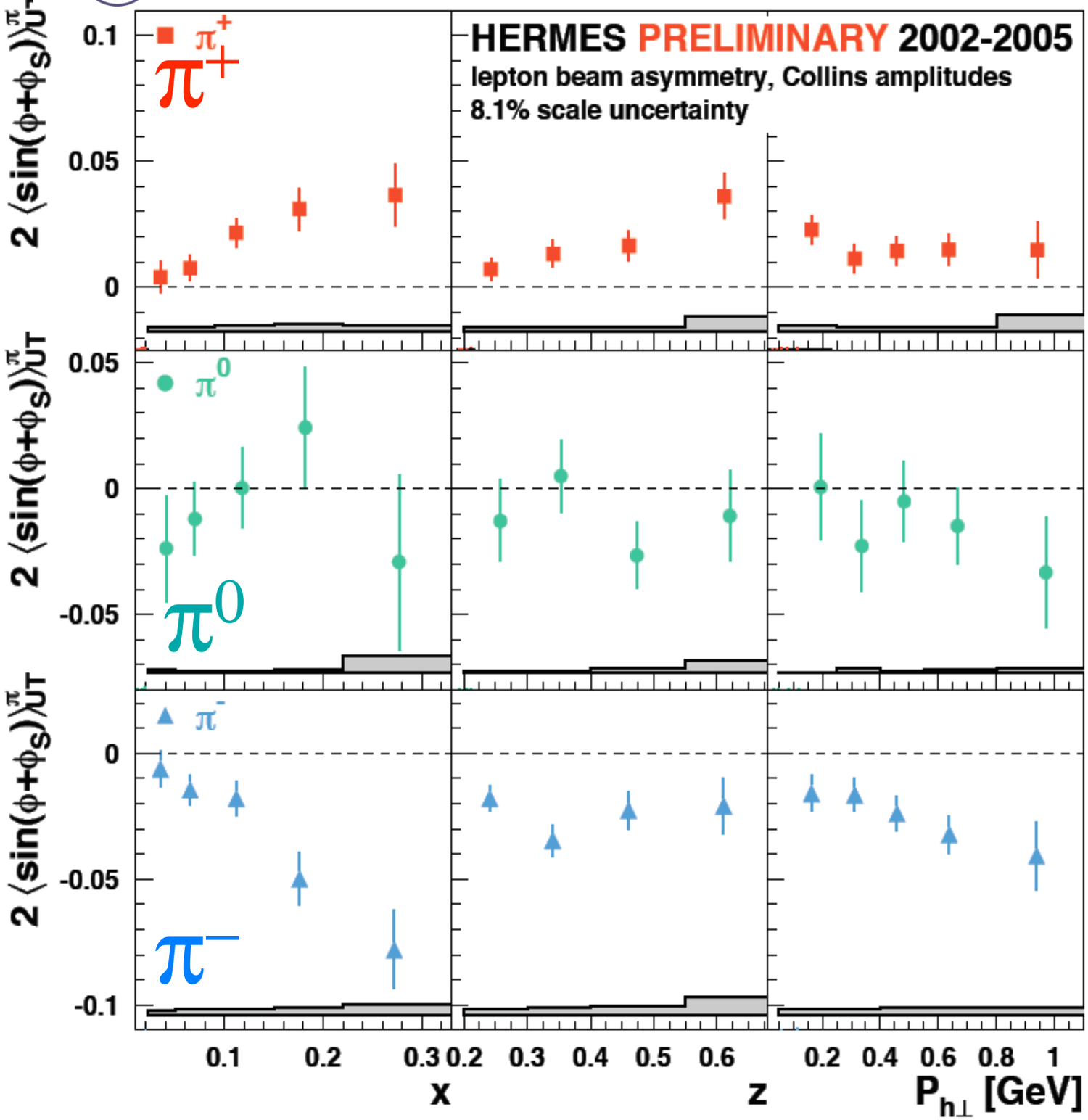
$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q h_{1q}(x,k_\perp)\otimes \mathrm{d}\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp)\otimes\Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

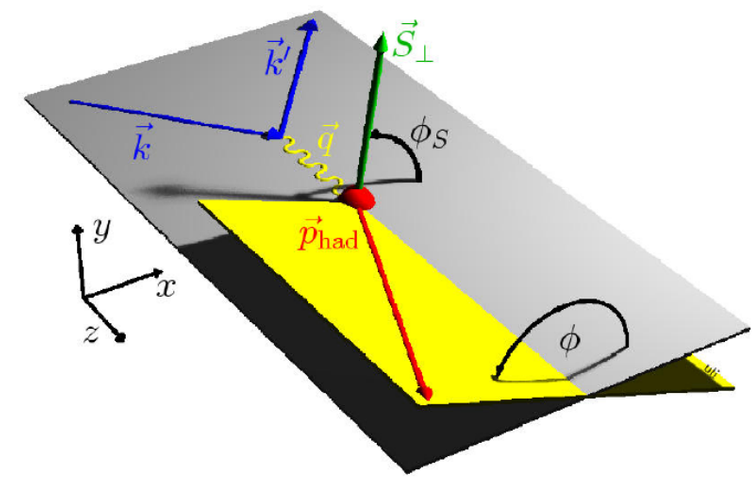
$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$

$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\uparrow} - \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\downarrow}$$

Collins effect in SIDIS couples to transversity

$ep^{\uparrow} \rightarrow \pi X$ E_b=27GeV, $\sqrt{s}$~7 GeV

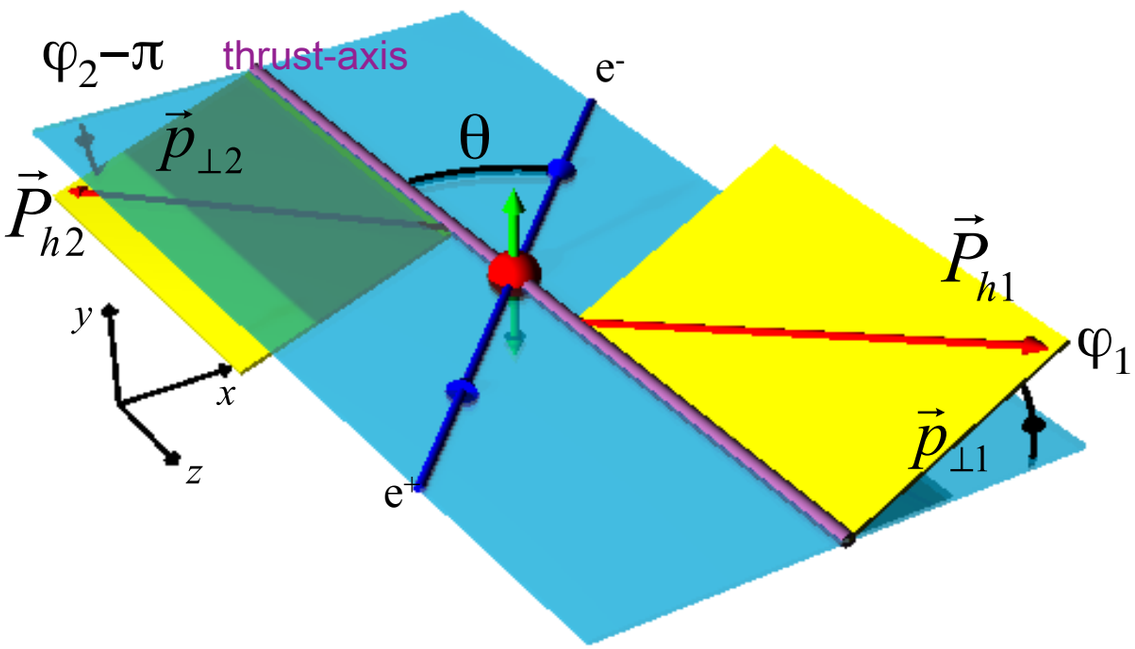


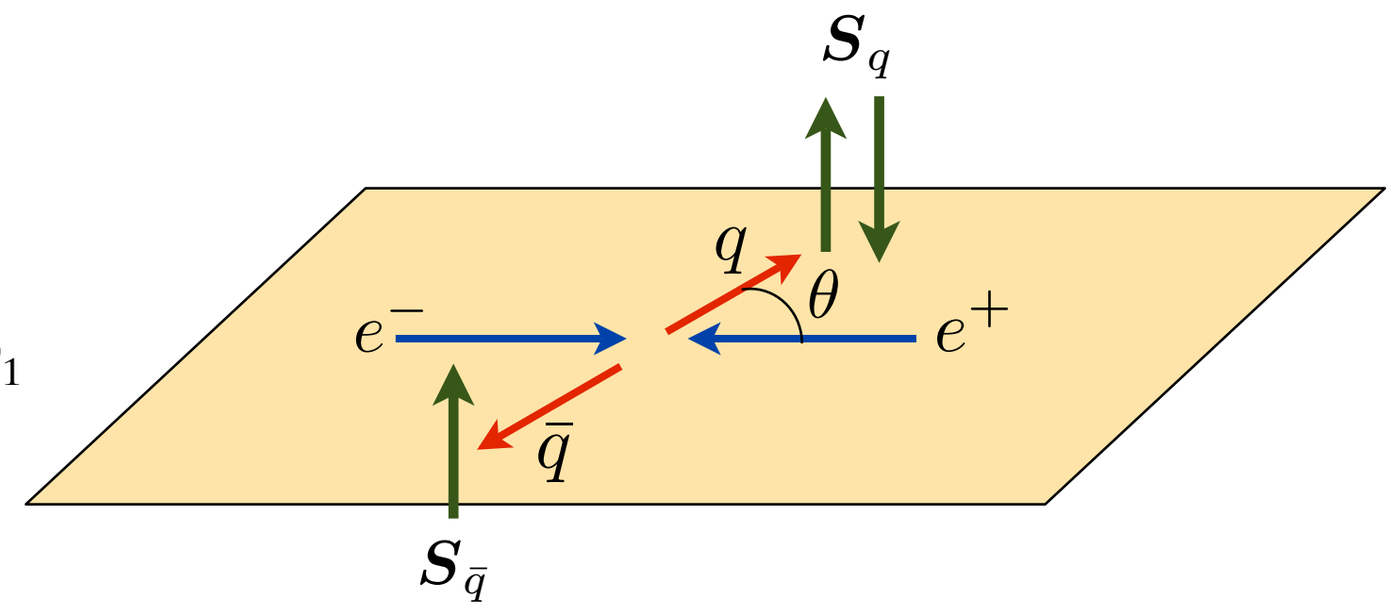


HERMES Collins asymmetry

independent information on Collins function from e^+e^- processes

BELLE @ KEK





$$A_{12}(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$

$$= 1 + \frac{1}{4} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

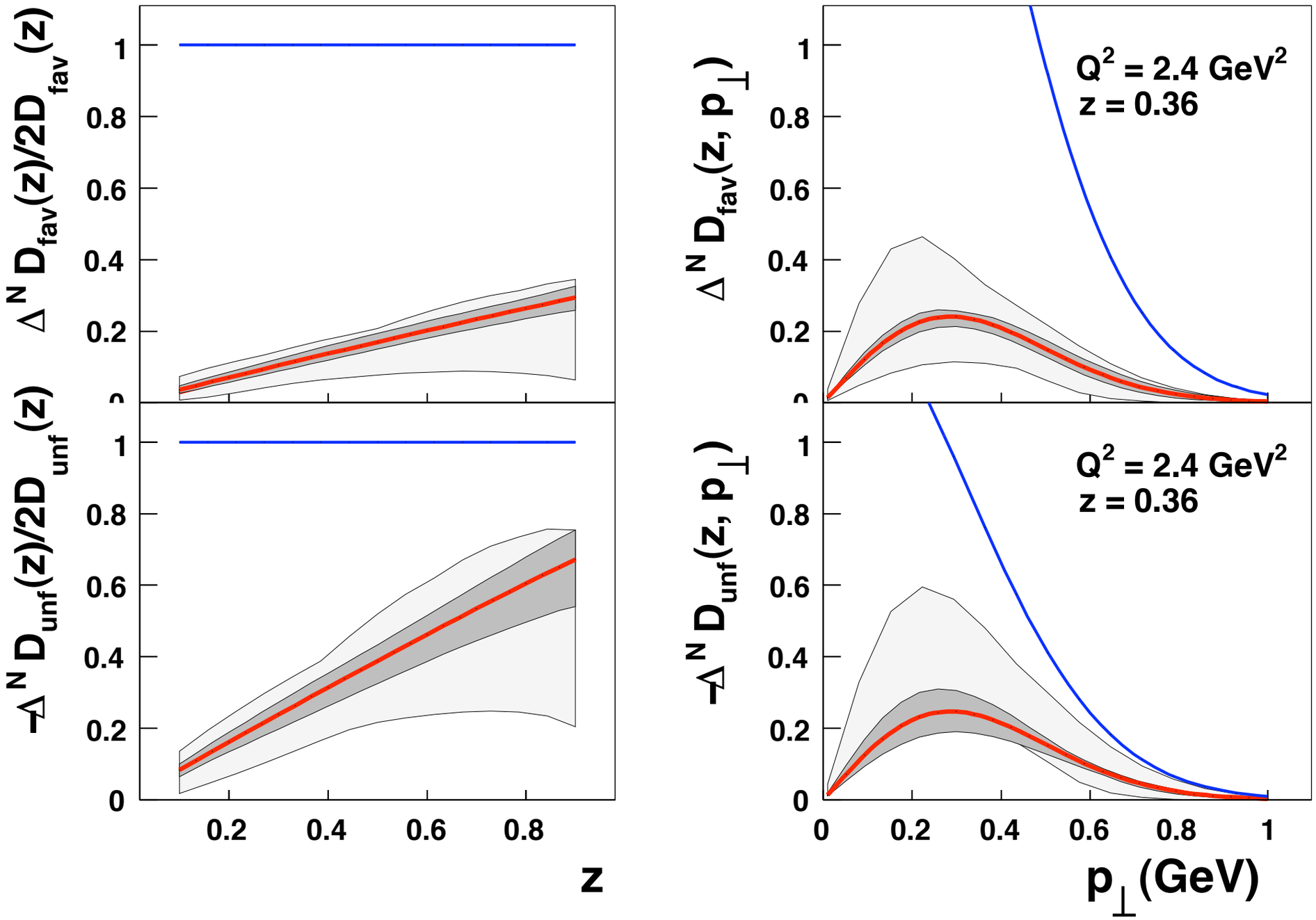
Transversity & Collins function phenomenology in SIDIS and e+e-

Same simple parametrization as for Sivers, but Collins effect has been clearly observed by three independent experiments: HERMES, COMPASS and BELLE

Collins function expected to be universal

QCD evolution important, as BELLE data are at a much higher energy than SIDIS data

extracted Collins functions



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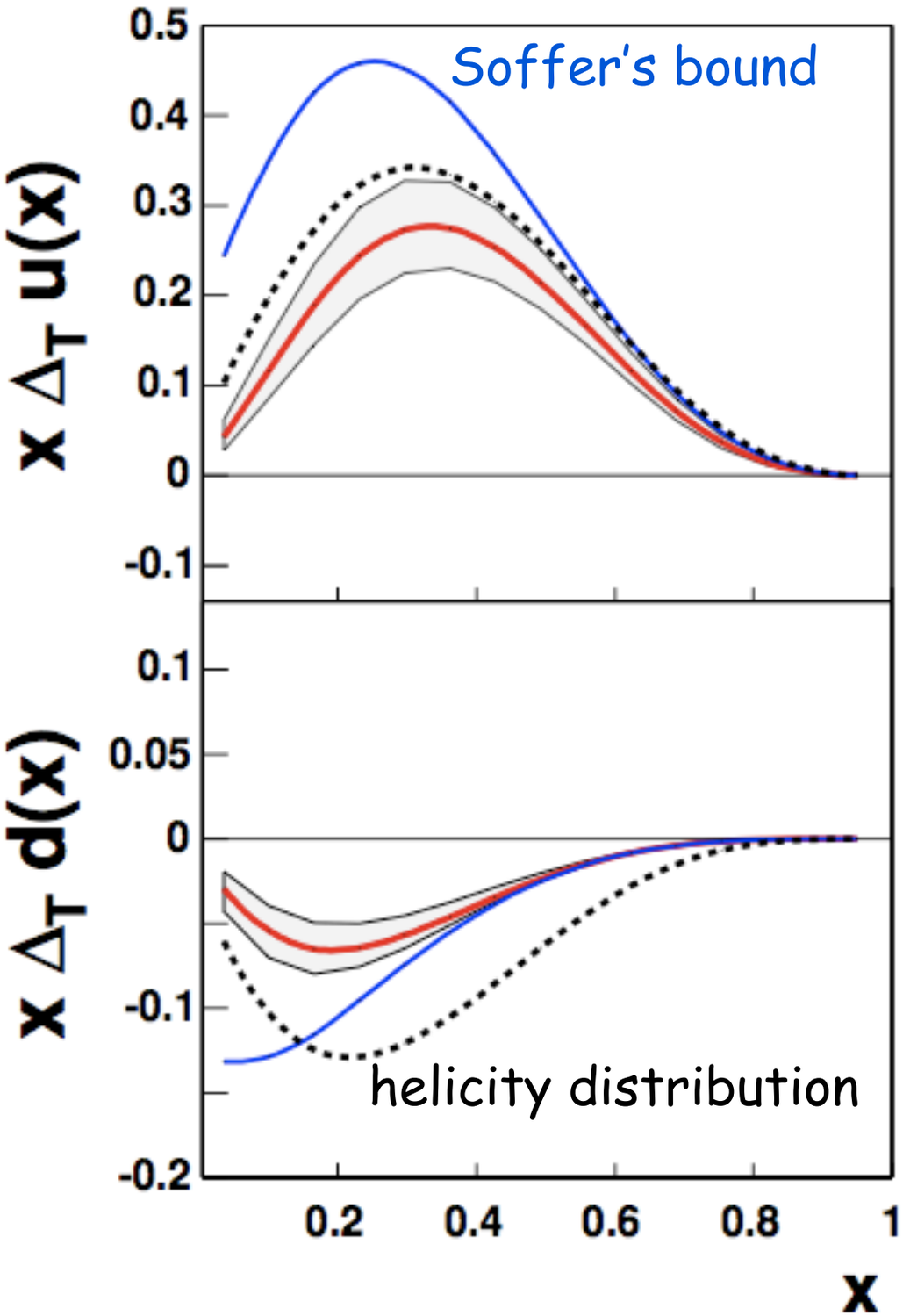
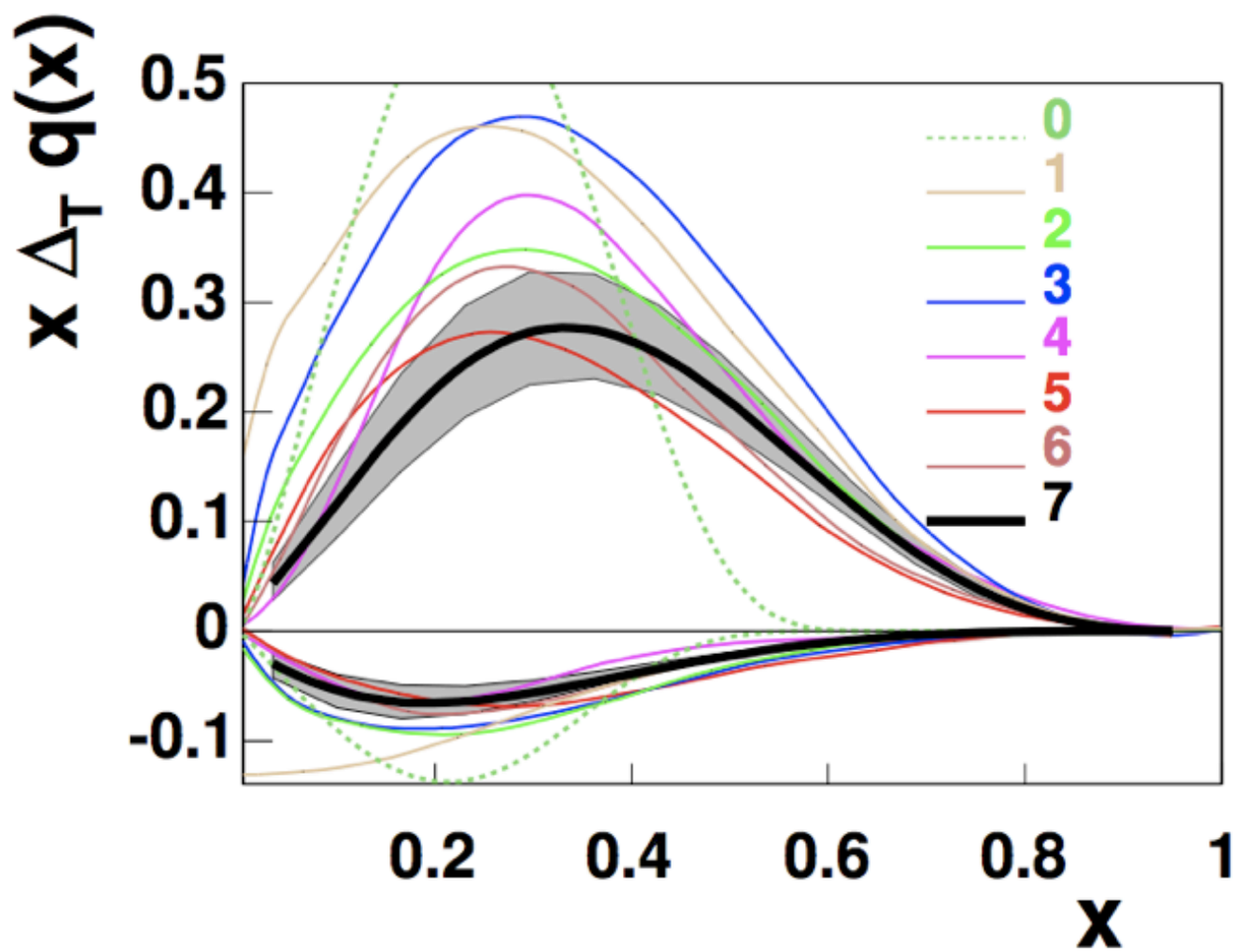
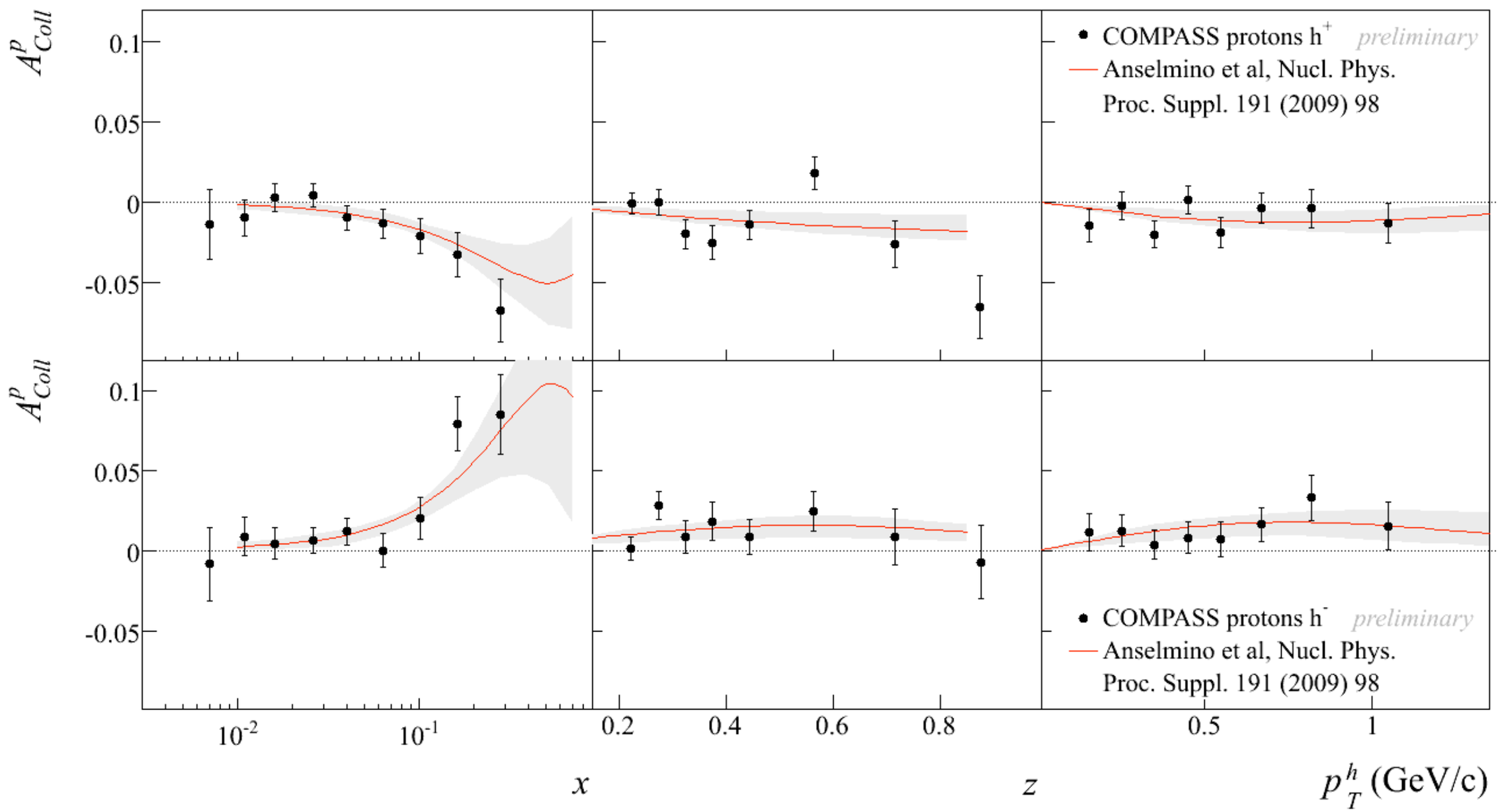


extracted transversity and comparison with models

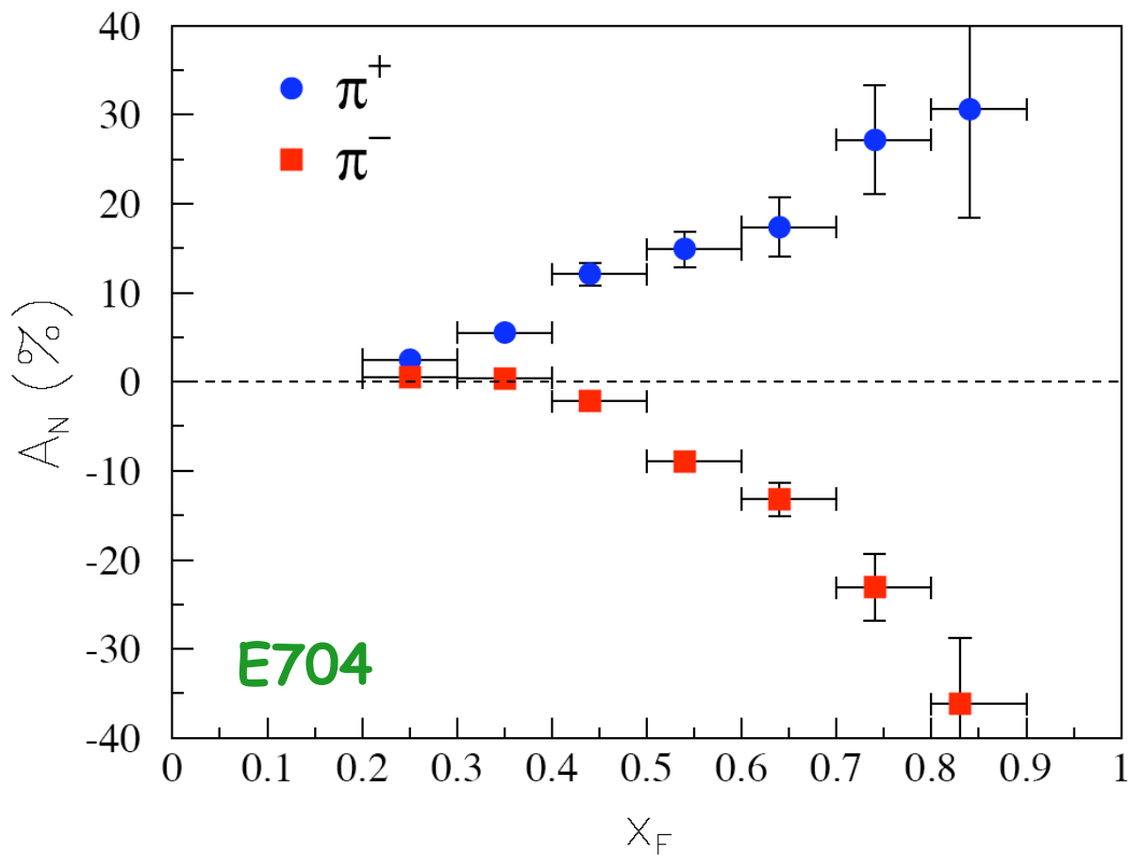


Predictions for COMPASS, with a proton target, and comparison with data



A. Martin, DIS2010

A_N in p p $\rightarrow \pi$ X, the big challenge

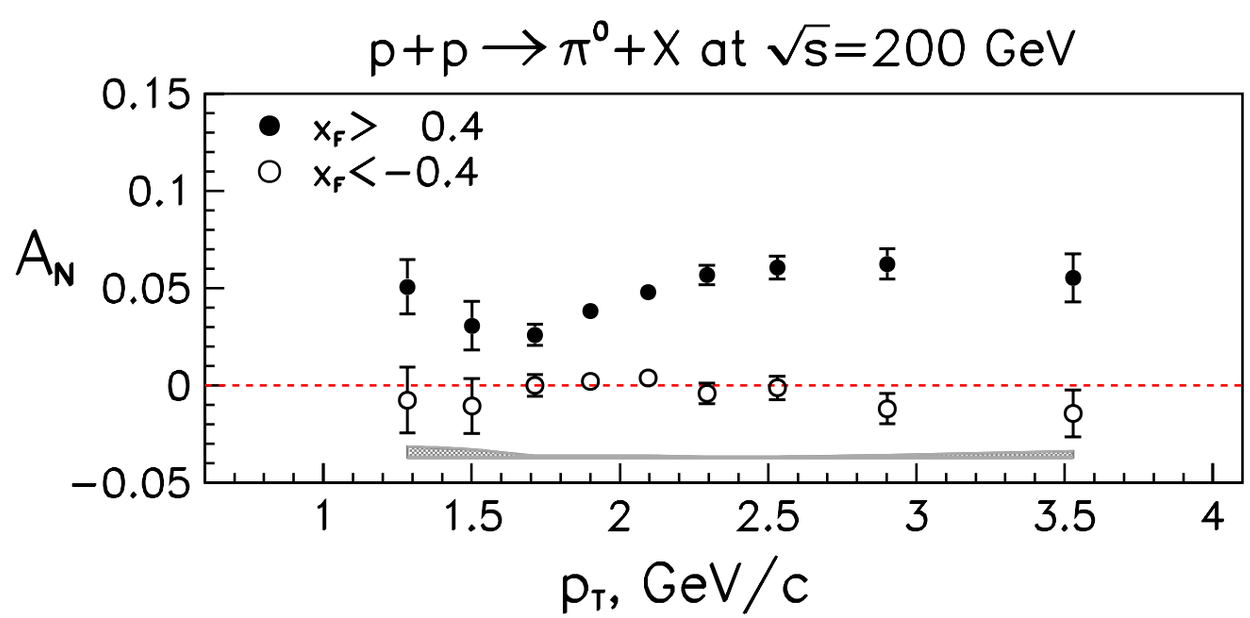


$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\uparrow}}{d\sigma^{\uparrow} + d\sigma^{\uparrow}}$$

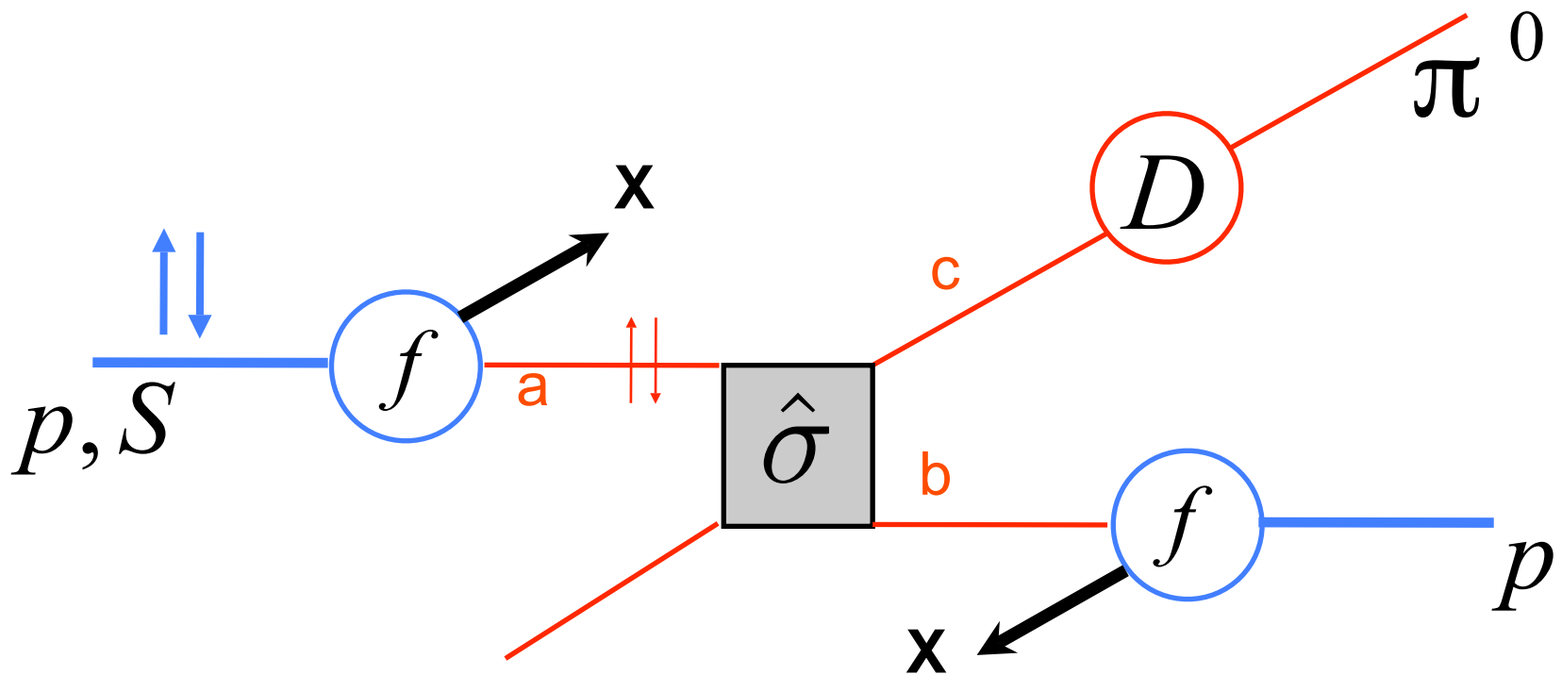
E704 $\sqrt{s}$ = 20 GeV

$0.7 < p_T < 2.0$

and all beautiful RHIC data, persisting at high energy...



no SSA in collinear factorization



$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[\mathrm{d}\hat{\sigma}^{\uparrow} - \mathrm{d}\hat{\sigma}^{\downarrow}]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

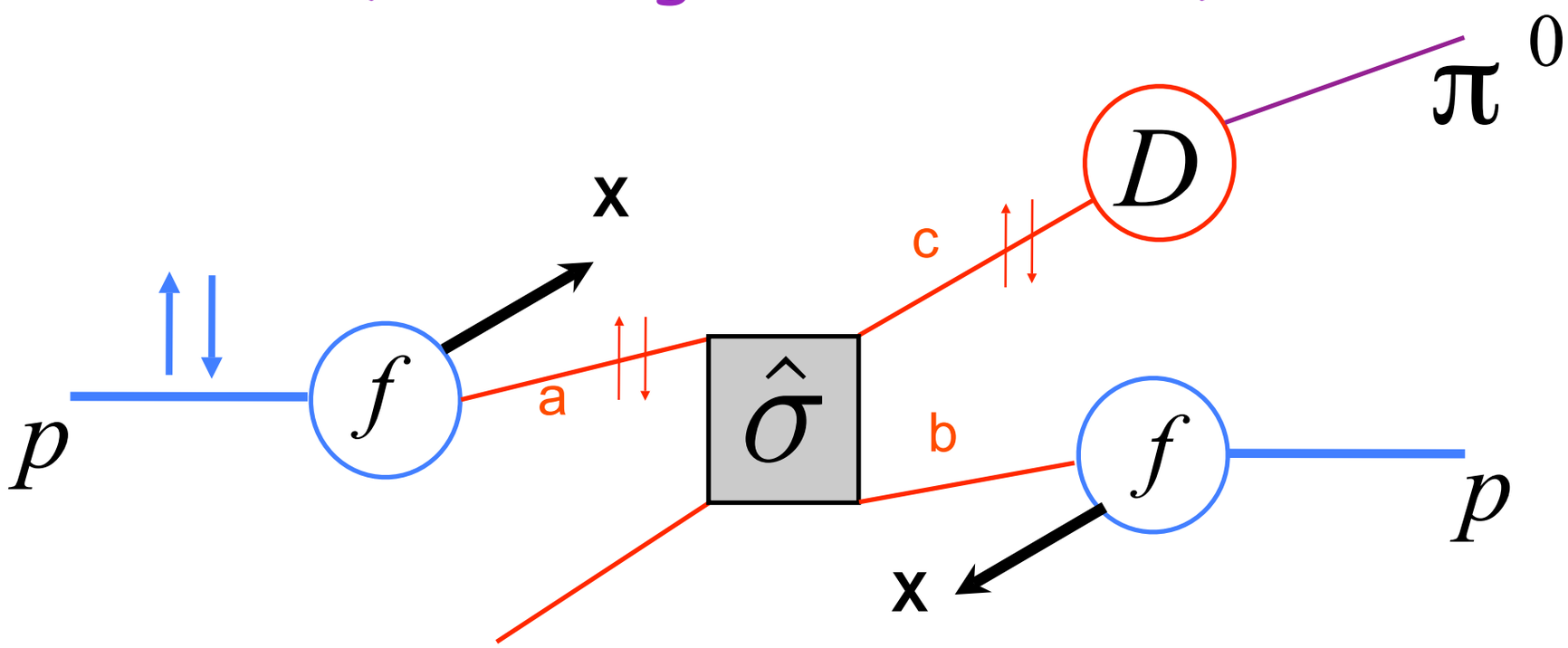
$$A_N = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \hat{a}_N \propto \frac{m_q}{E_q}\,\alpha_s$$

was considered almost a theorem

Only one large scale, P_T. Any role for TMDs?

TMD factorization not proven

1. Generalization of collinear scheme (assuming factorization)



$$\mathrm{d}\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

single spin effects in TMDs

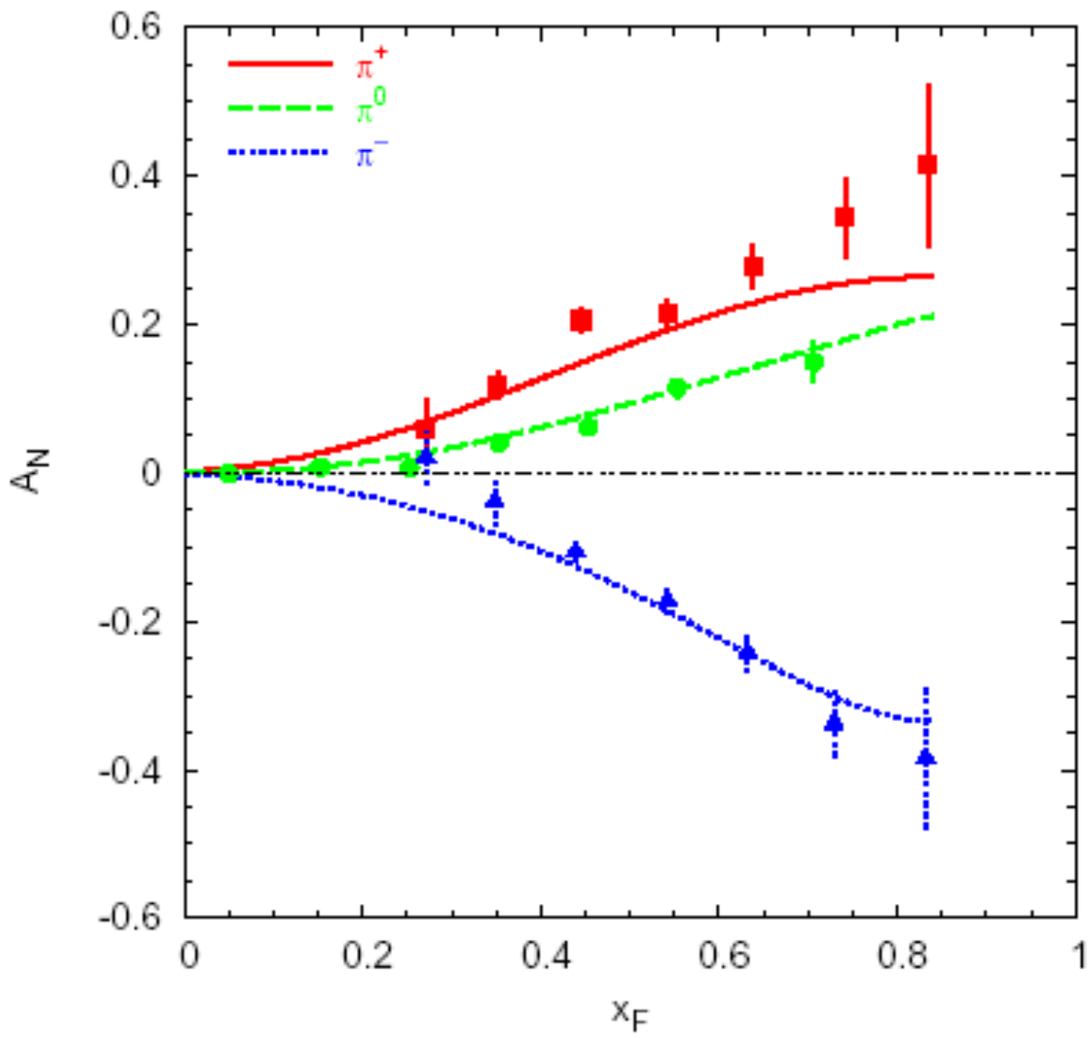
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...

(Field-Feynman in unpolarized case)

TMD factorization at work ....

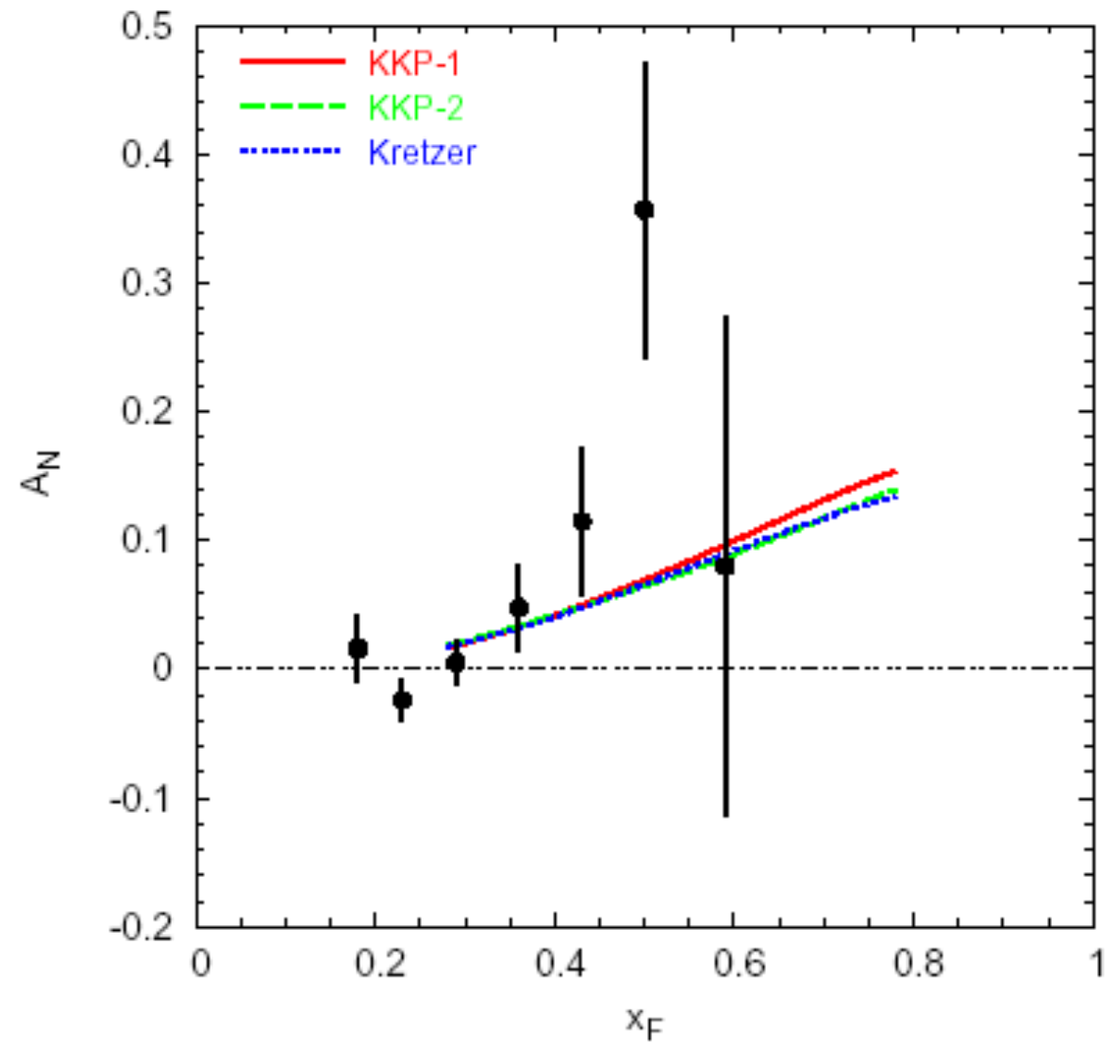
U. D'Alesio, F. Murgia

E704 data



fit

STAR data



prediction

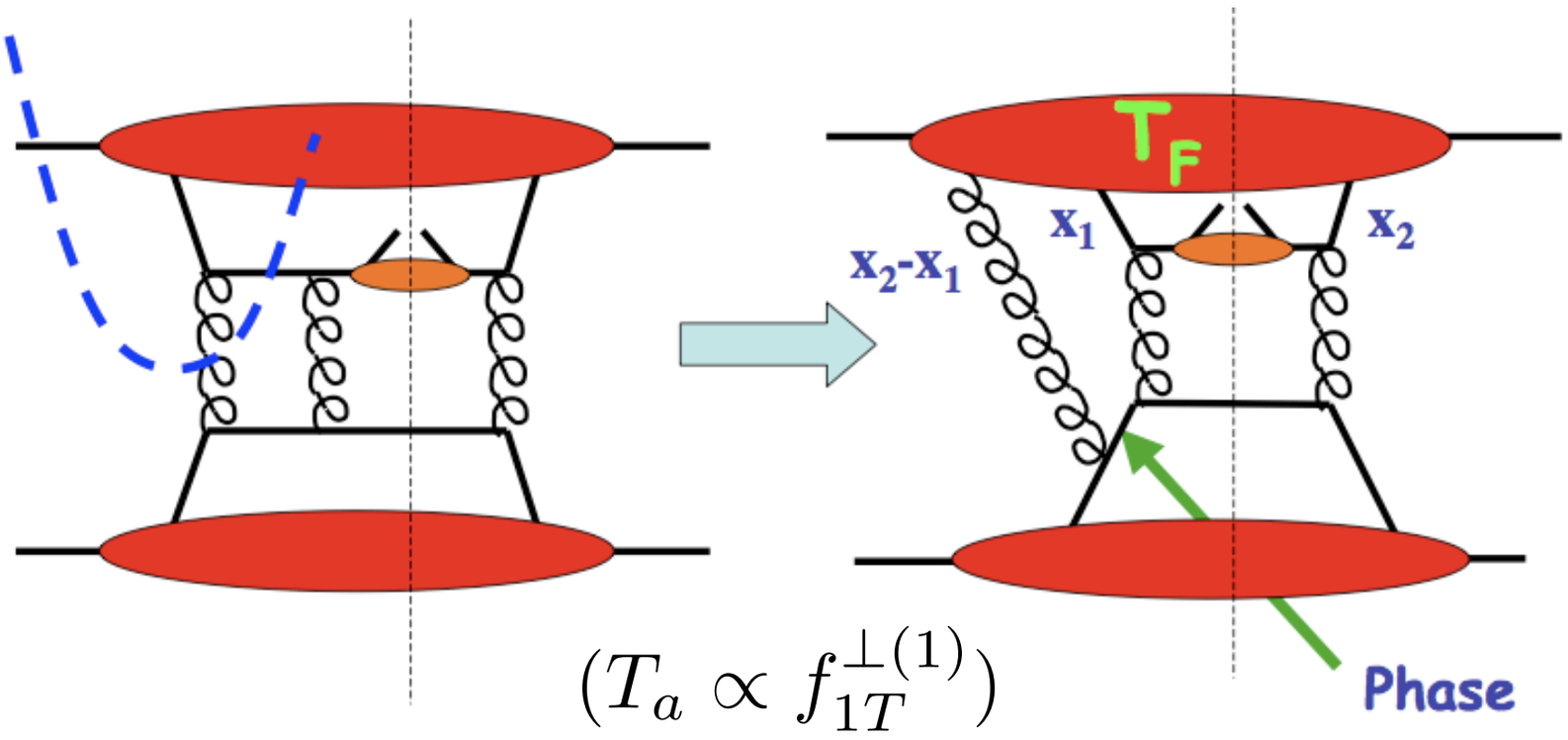
Sivers effect $p\,p \to \pi\, X$

2. Higher-twist partonic correlations

(Efremov, Teryaev; Qiu, Sterman; Kouvaris, Vogelsang, Yuan; Bacchetta, Bomhof, Mulders, Pijlman; Koike ...)

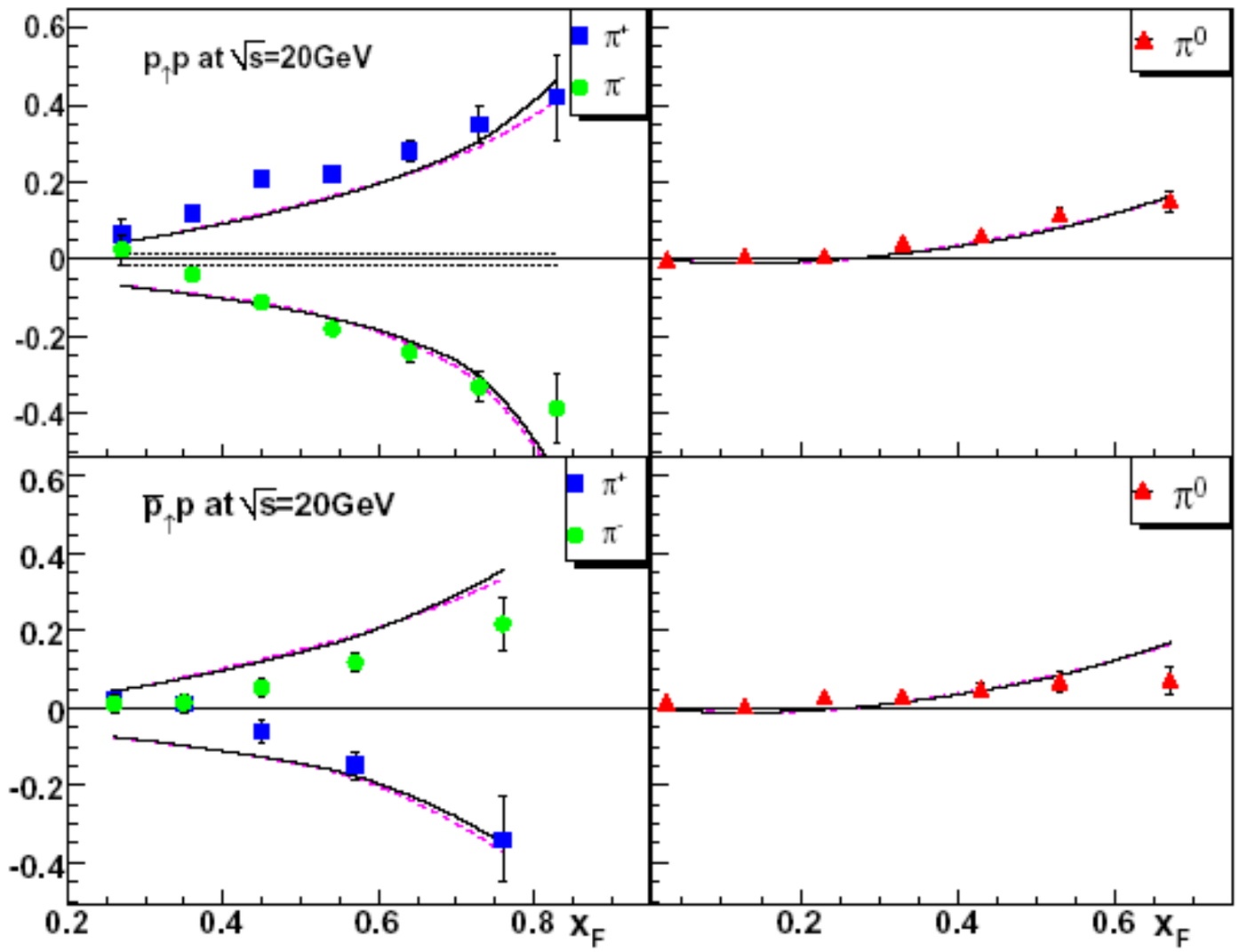
higher-twist partonic correlations - factorization OK

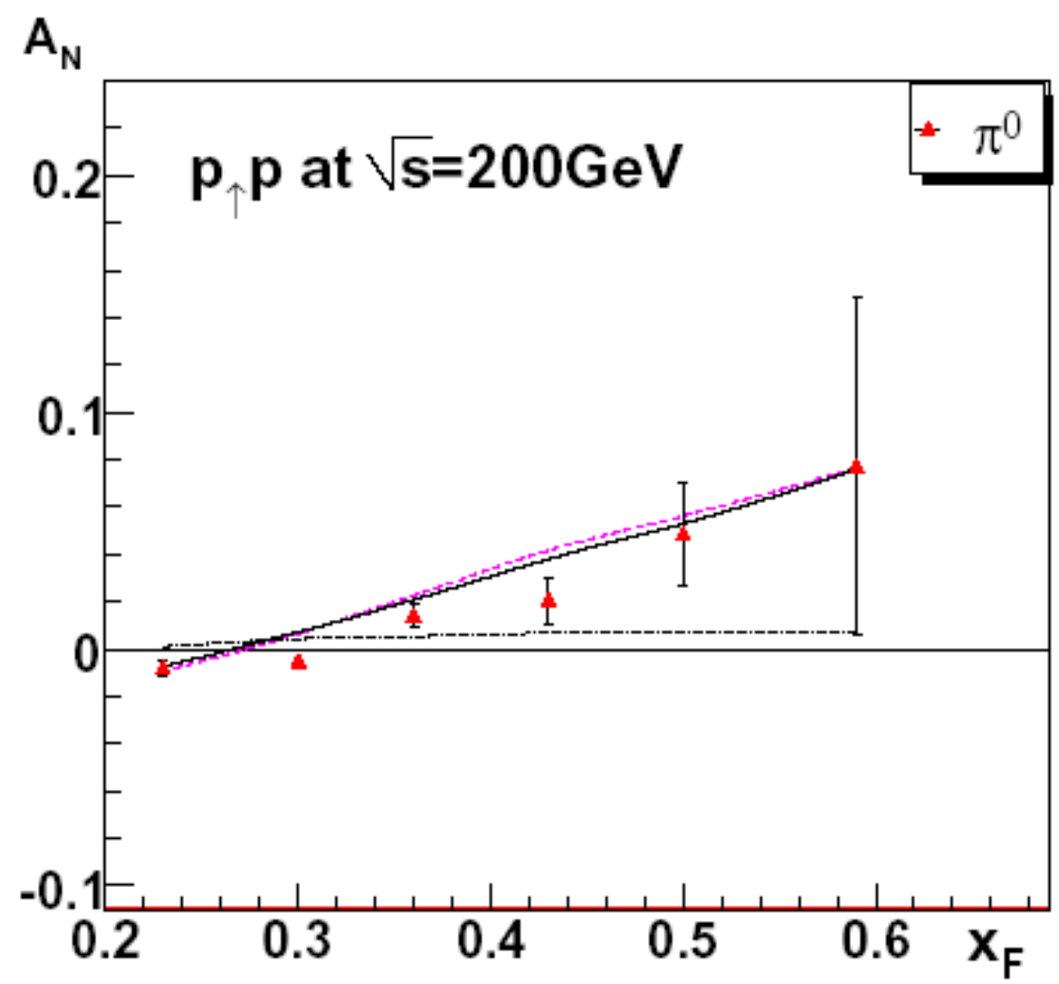
$$\mathrm{d}\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, \boldsymbol{S}_\perp)}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\to c}(k_1, k_2)}_{\text{hard interaction, not a cross section}} \otimes D_{h/c}(z)$$



$$(T_a \propto f_{1T}^{\perp(1)})$$

possible project: compute T_a using SIDIS extracted Sivers functions





fits of E704 and STAR data

Kouvaris, Qiu, Vogelsang, Yuan

sign mismatch

(Kang, Qiu, Vogelsang, Yuan)

compare

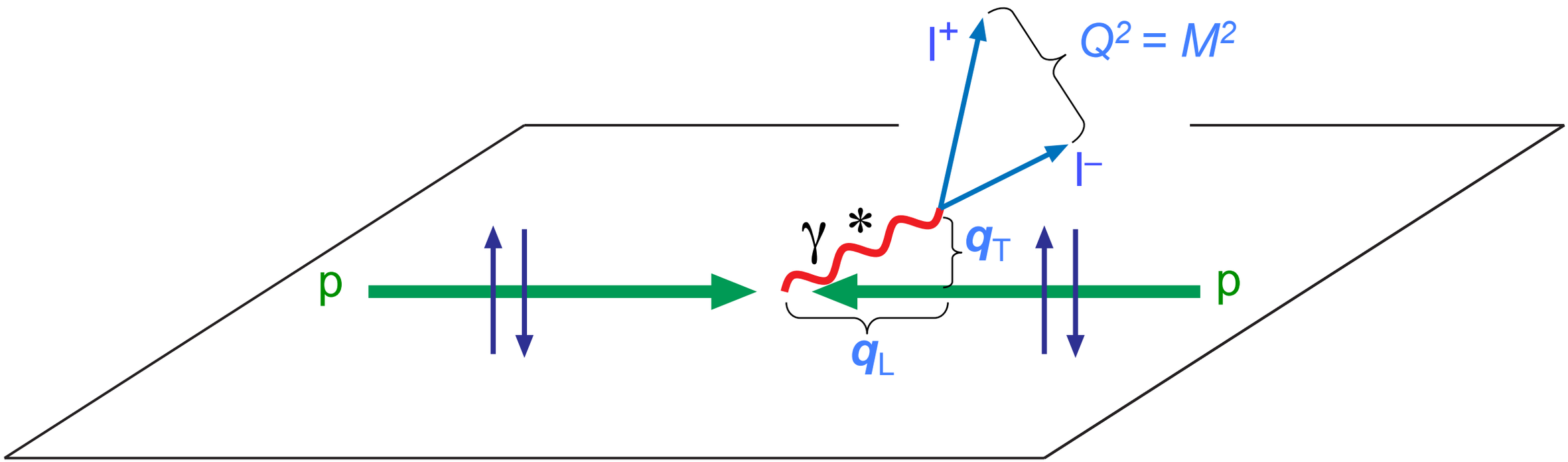
$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\rm SIDIS}$$

as extracted from fitting A_N data, with that obtained by inserting in the the above relation the SIDIS extracted Sivers functions

similar magnitude, but opposite sign!

the same mismatch does not occurr adopting TMD factorization; the reason is that the hard scattering part in higher-twist factorization is negative

TMDs in Drell-Yan processes



factorization holds, two scales, M^2, and $q_T \ll M$

$$\mathrm{d}\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, \mathrm{d}\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

cross-section: most general pp leading-twist expression

S. Arnold, A. Metz and M. Schlegel, arXiv:0809.2262 [hep-ph]

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times\; & \Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big) \\
& + S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big) \\
& + S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big) \\
& + |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big) \\
& \qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big] \\
& + |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big) \\
& \qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big] \\
& + S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big) \\
& + S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big) \\
& \qquad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big] \\
& + |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big) \\
& \qquad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big] \\
& + |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big) \\
& \qquad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big) \\
& \qquad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big) \\
& \qquad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}
\end{aligned}
$$

Drell-Yan

48 structure functions ...

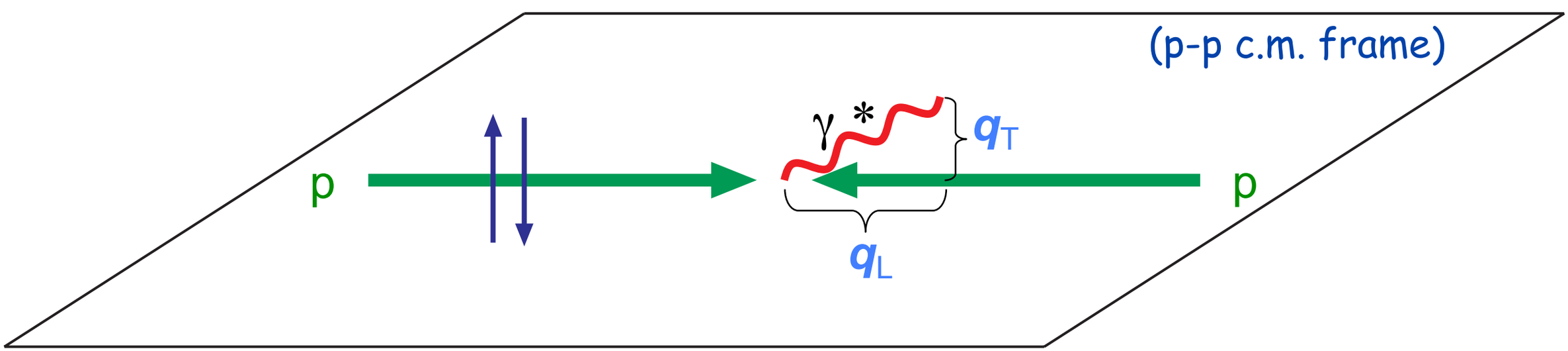
Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \propto \sum_q \Delta^N f_{q/p^{\uparrow}}(x_1, \boldsymbol{k}_{\perp}) \otimes f_{\bar{q}/p}(x_2) \otimes \mathrm{d}\hat{\sigma}$$

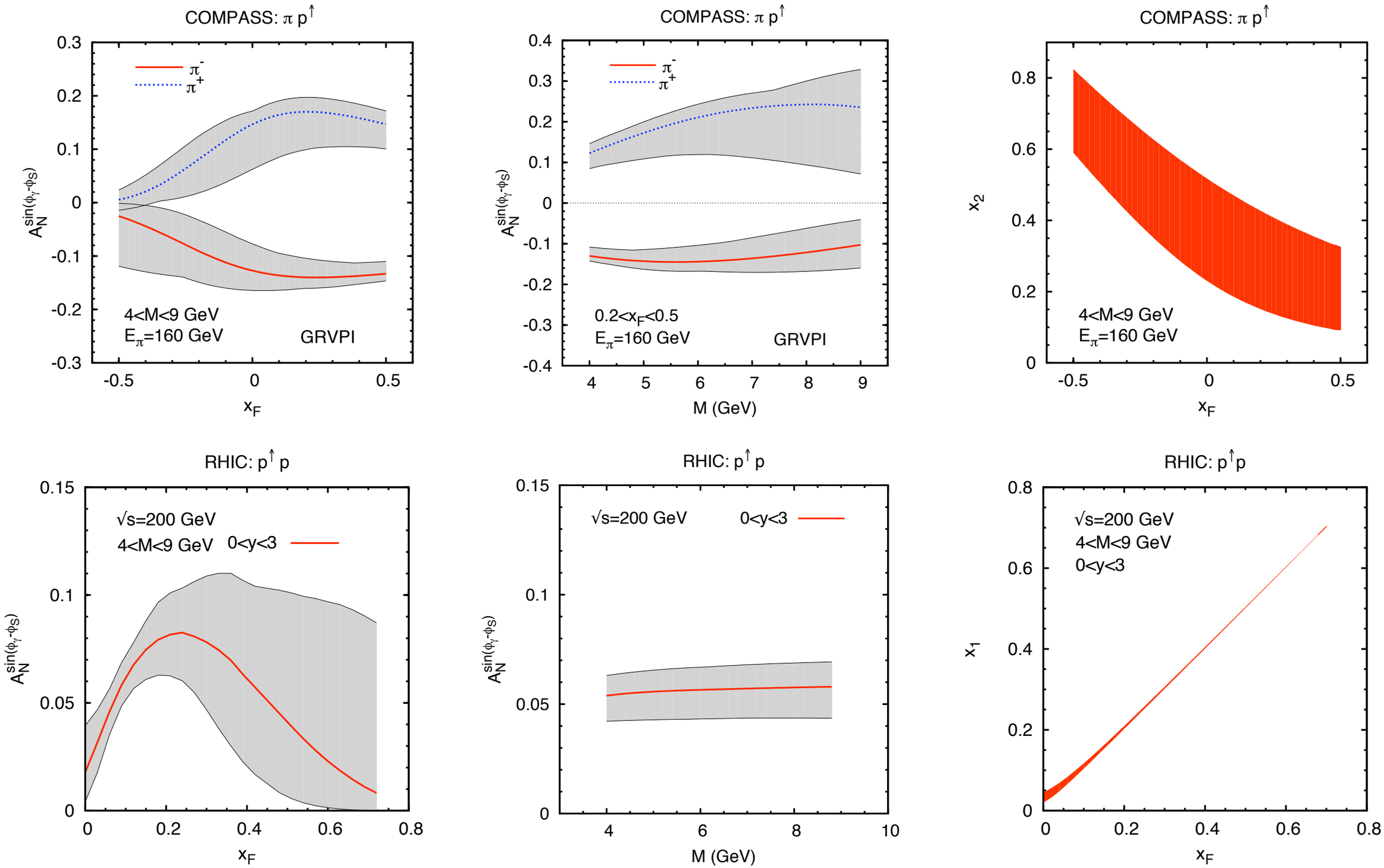
$$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} \mathrm{d}\phi_\gamma \left[\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}\right] \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} \mathrm{d}\phi_\gamma \left[\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}\right]}$$



Predictions for A_N

Sivers functions as extracted from SIDIS data, **with opposite sign**



M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, e-Print: arXiv:0901.3078

Conclusions

The3-dimensional exploration of the nucleon has just started: collect as much data as possible and try to reconstruct the nucleon phase-space structure

TMDs describe the momentum distribution; the actual knowledge covers limited kinematical regions, and assumes (too) simple functional forms

The properties of the Sivers function and its different role in different processes, have to be investigated

and much more to do .....

new experiments would help so much (ENC/EIC, Drell-Yan, ...)