Hadron Structure, Baryon and Meson Form Factors and g-2 from the lattice

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Topics to Cover

- Baryon/Meson Form Factors
  - Provide information of size, shape and internal (charge) densities
  - e.g. Neutron has charge zero, but charge density +/-?
  - Good place to search for chiral non-analytic behaviour
- Nucleon Axial Charge
  - Neutron beta decay, chiral symmetry breaking
  - Study finite size effects
- Quark Momentum Fraction $\langle x \rangle$
  - Will it ever bend down?
- $(g\text{-}2)_{mu}$
  - Hadronic (vacuum polarisation) contribution

Motivation: Electromagnetic Form Factors

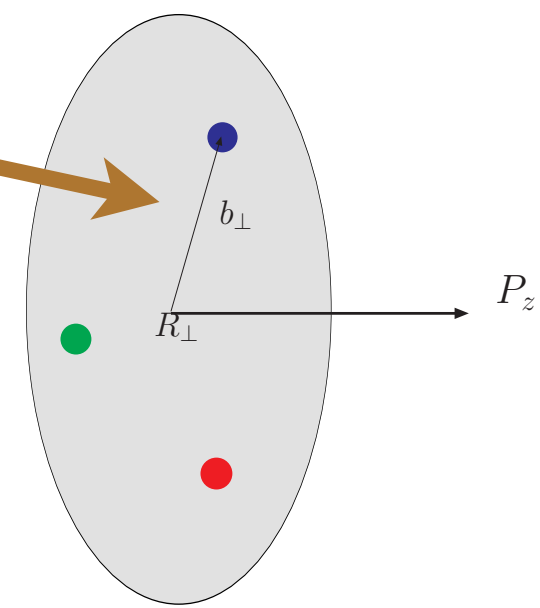
$$\langle p', s'|J^\mu(\vec{q})|p, s\rangle = \bar{u}(p', s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p, s)$$

Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2q_\perp \, \mathrm{e}^{-i\vec{b}_\perp\cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon

Provide information on the size and internal charge densities



Scaling of Form Factors

From dimensional counting *[Brodsky & Farrar, 1973]*

$$\frac{F(0)}{(1+Q^2/M^2)^p}$$

$$F_1 \propto \frac{1}{Q^4} \qquad \text{(dipole?)}$$

$$F_2 \propto \frac{1}{Q^6} \qquad \text{(tripole?)}$$

for $Q^2 > \zeta_{\mathrm{pQCD}}$

$$Q^2 \frac{F_2}{F_1} \propto \text{const}$$

$$\frac{G_E}{G_M} \propto \text{const}$$

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$$Q \frac{F_2}{F_1} \propto$$

JLab

$$\frac{G_E}{G_M} \propto$$

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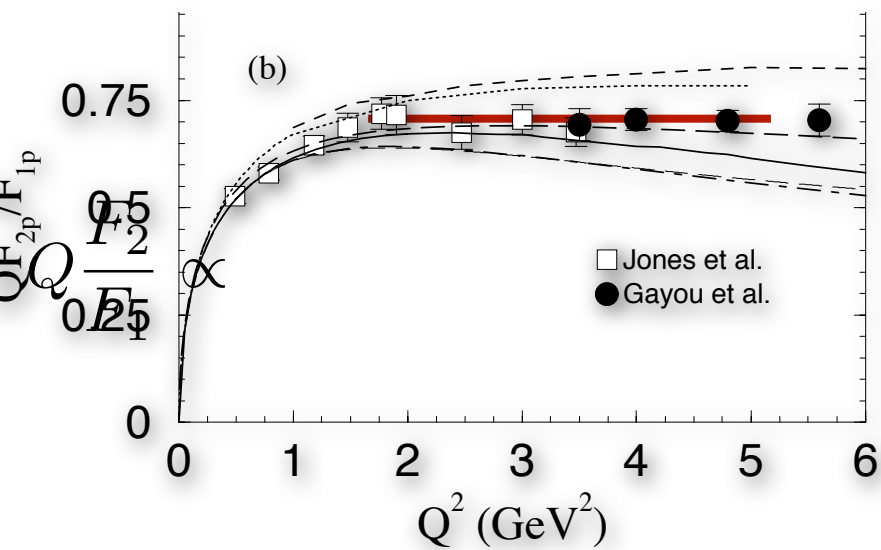

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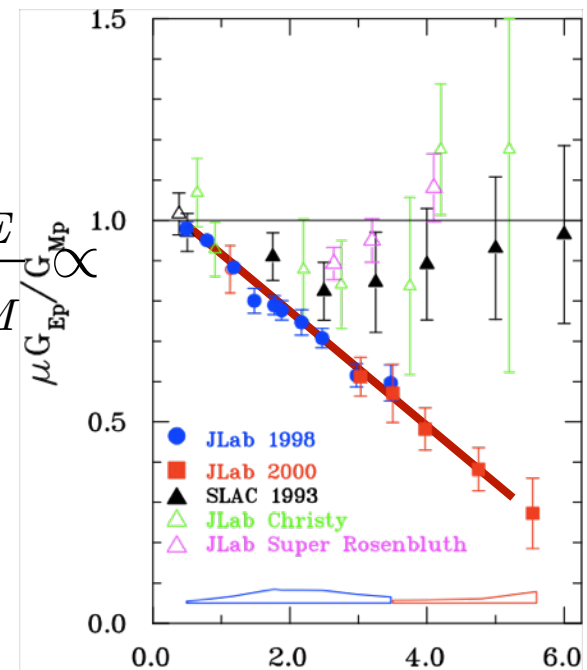


Size of the Proton

- $> 5\sigma$ discrepancy between muonic hydrogen and *e-p* scattering
  - r_p=0.84184(67) fm **[Nature 466, 213 (2010]**
  - r_p=0.875(8)(6) fm **[arXiv:1102.0318]**

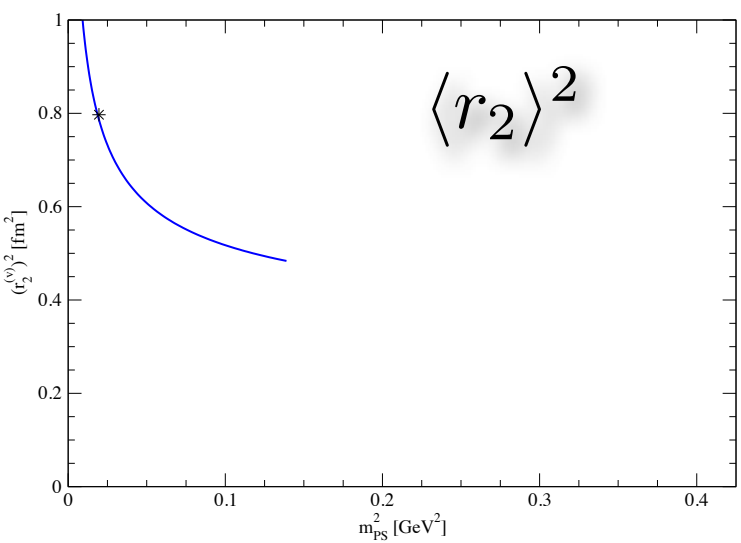
Form Factor Radii & Magnetic Moments

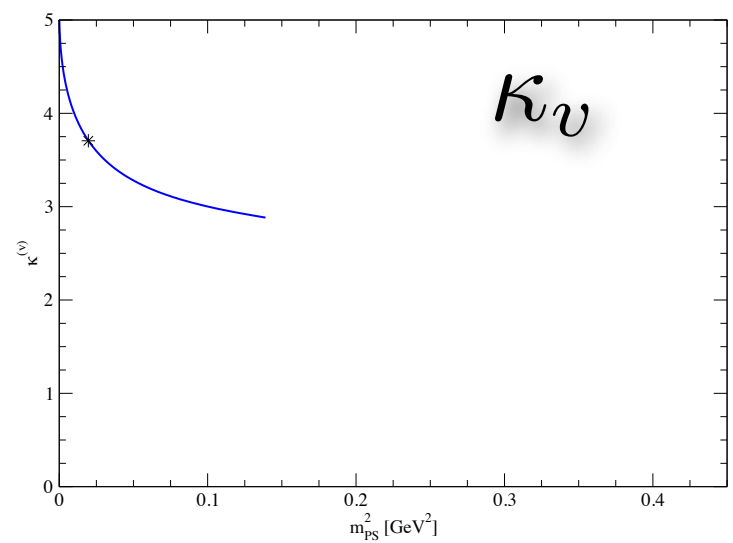
Search for non-analytic behaviour predicted by Chiral Perturbation Theory

- Form factor radii: $$r_i^2 = -6 \frac{dF_i(q^2)}{dq^2}\Big|_{q^2=0}$$

- Magnetic moment μ / anomalous magnetic moment κ

$$\mu = 1 + \kappa = G_m(0)$$





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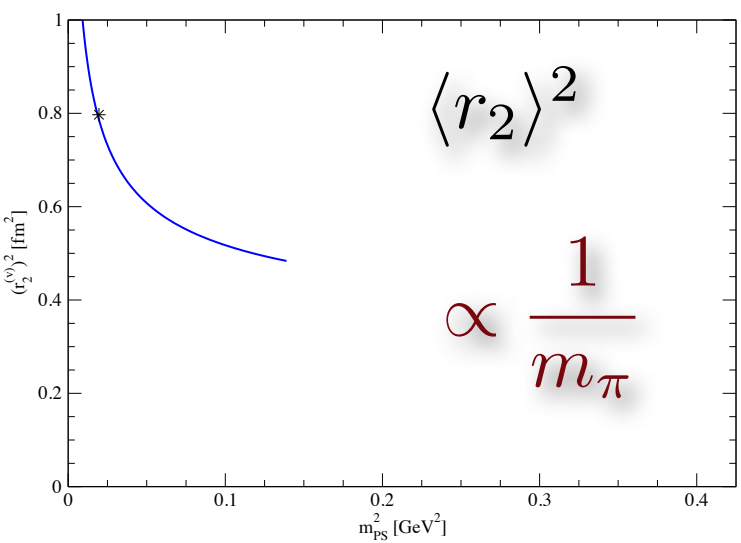
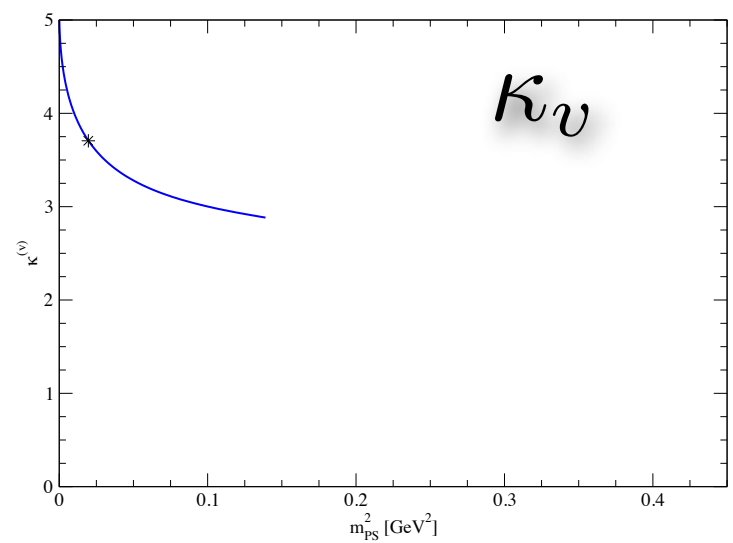
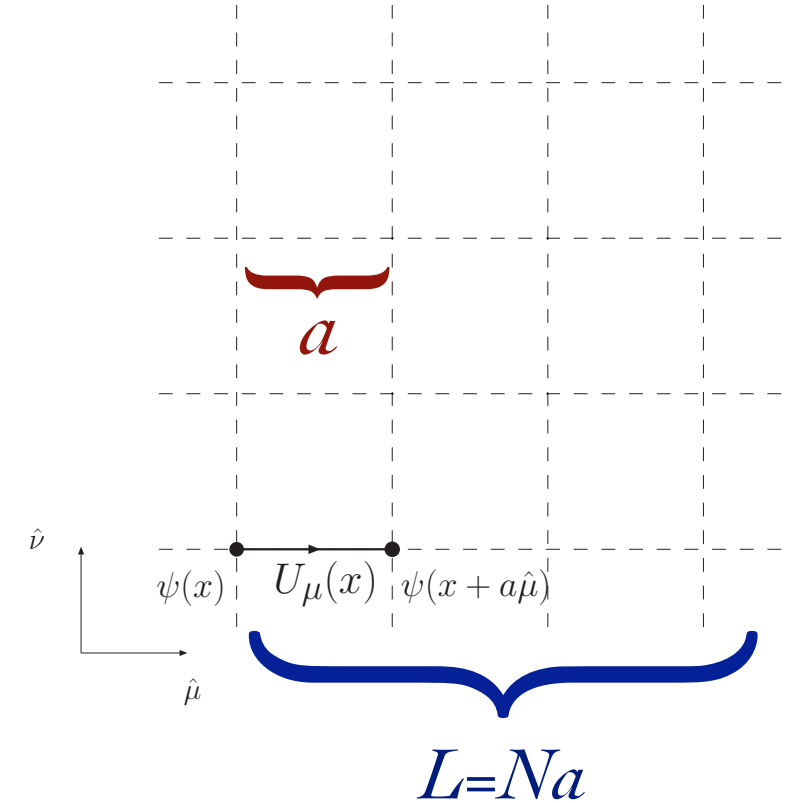




The Lattice

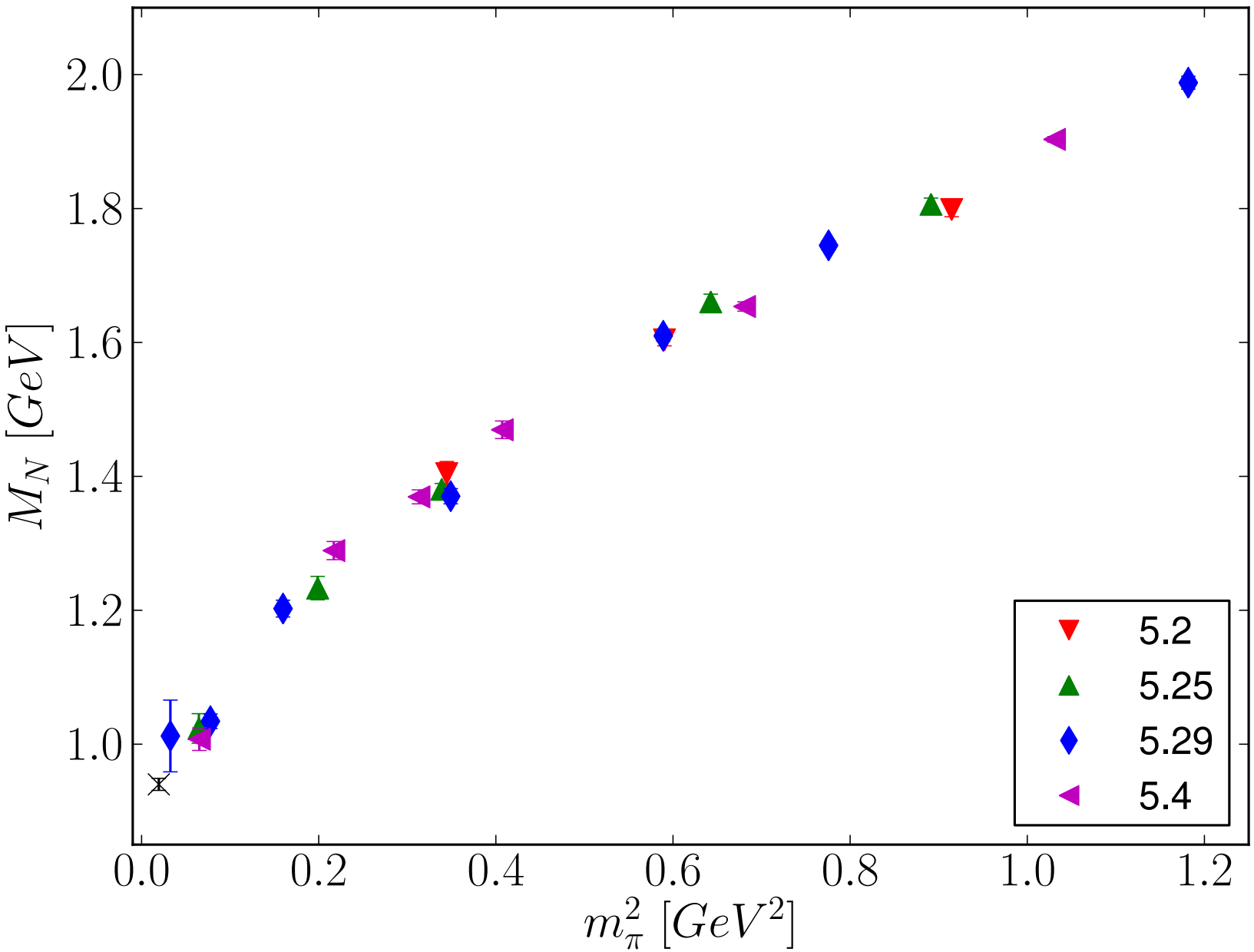
The Lattice



- Discretise space-time with lattice spacing a volume L^3xT
- Quark fields reside on sites
- Gauge fields on the links
- Lattice simulations for QCD give first principle results
- but need to have control of: *'Goal'*
  - Statistical errors, $N_{conf} \sim O(1000)$ $N_{conf} \to \infty$
  - Volume: $L \sim 2\,\mathrm{fm} \to 4\,\mathrm{fm}$ $L \to \infty$
  - Continuum limit: $a \sim 0.1\,\mathrm{fm} \to 0.04\,\mathrm{fm}$ $a \to 0$
  - Chiral extrapolation: $m_\pi \sim 500\,\mathrm{MeV} \to 200\,\mathrm{MeV}$ $m_\pi \to 140\,\mathrm{MeV}$
- difficult, need Tflop++ machines to approach the theoretical goal

Nucleon Mass

QCDSF, N_f=2

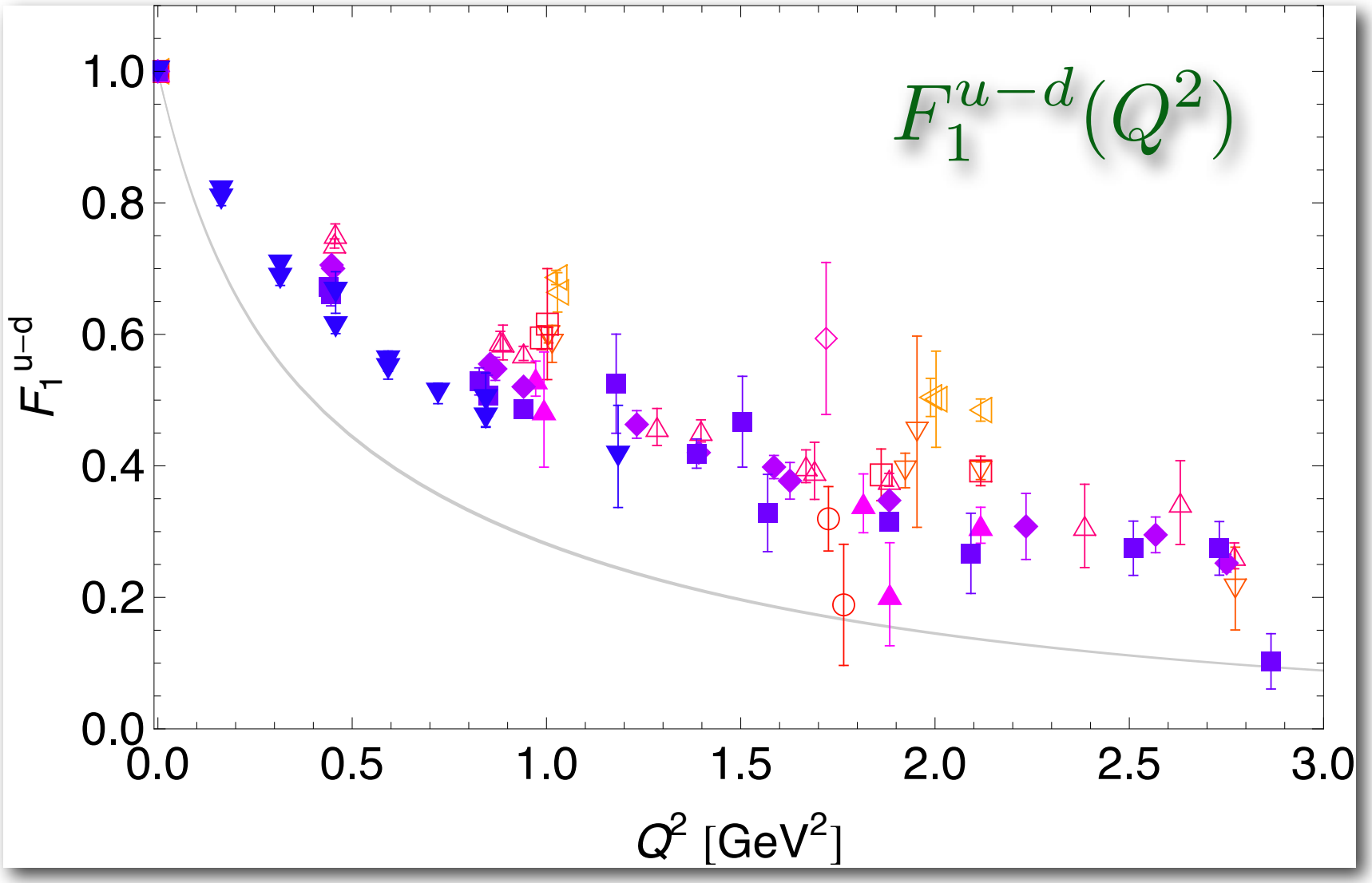


Lattice Form Factors

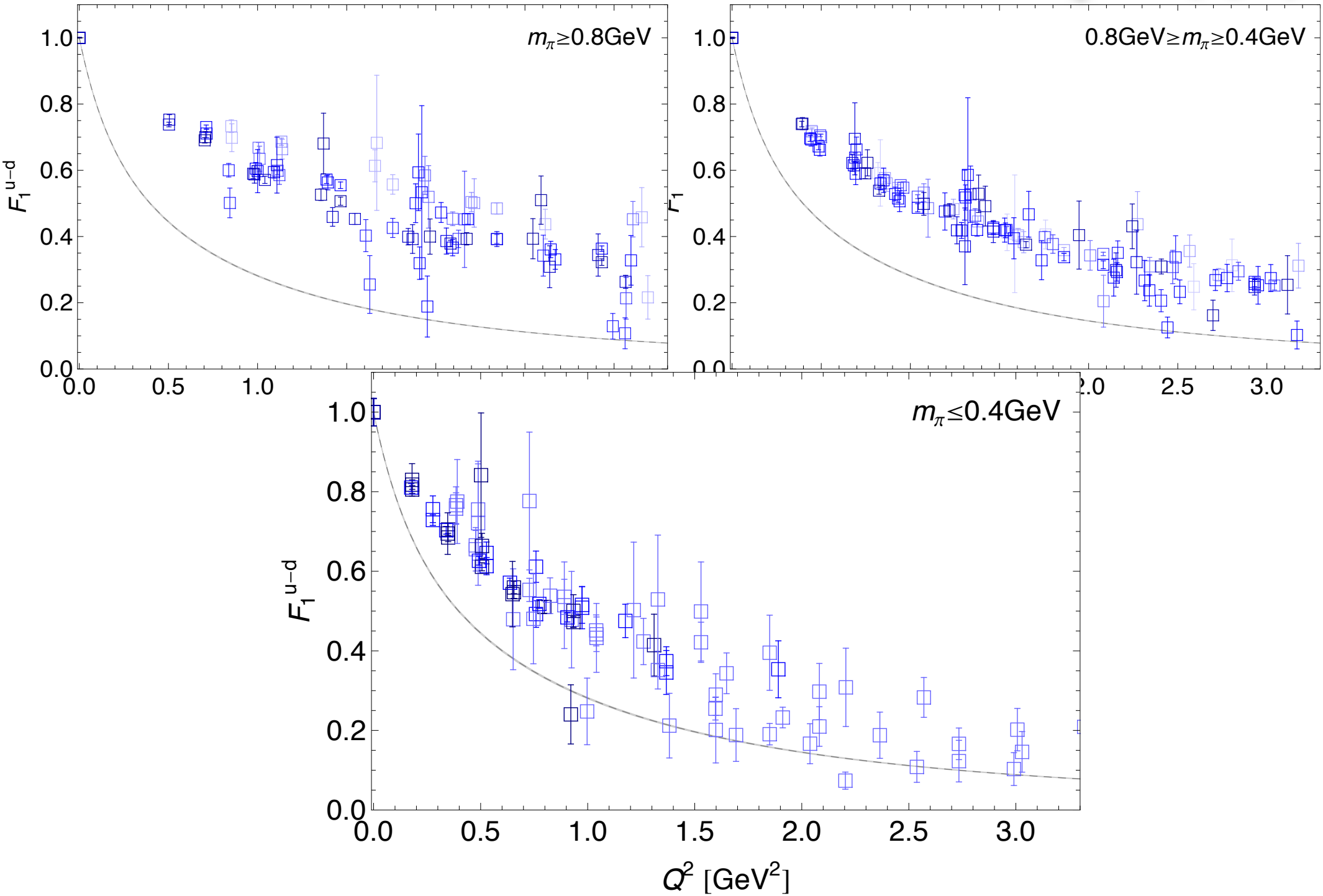
[QCDSF arXiv:1106.XXXX]

Comparison With Experiment

Fixed lattice spacing, wide range of quark masses

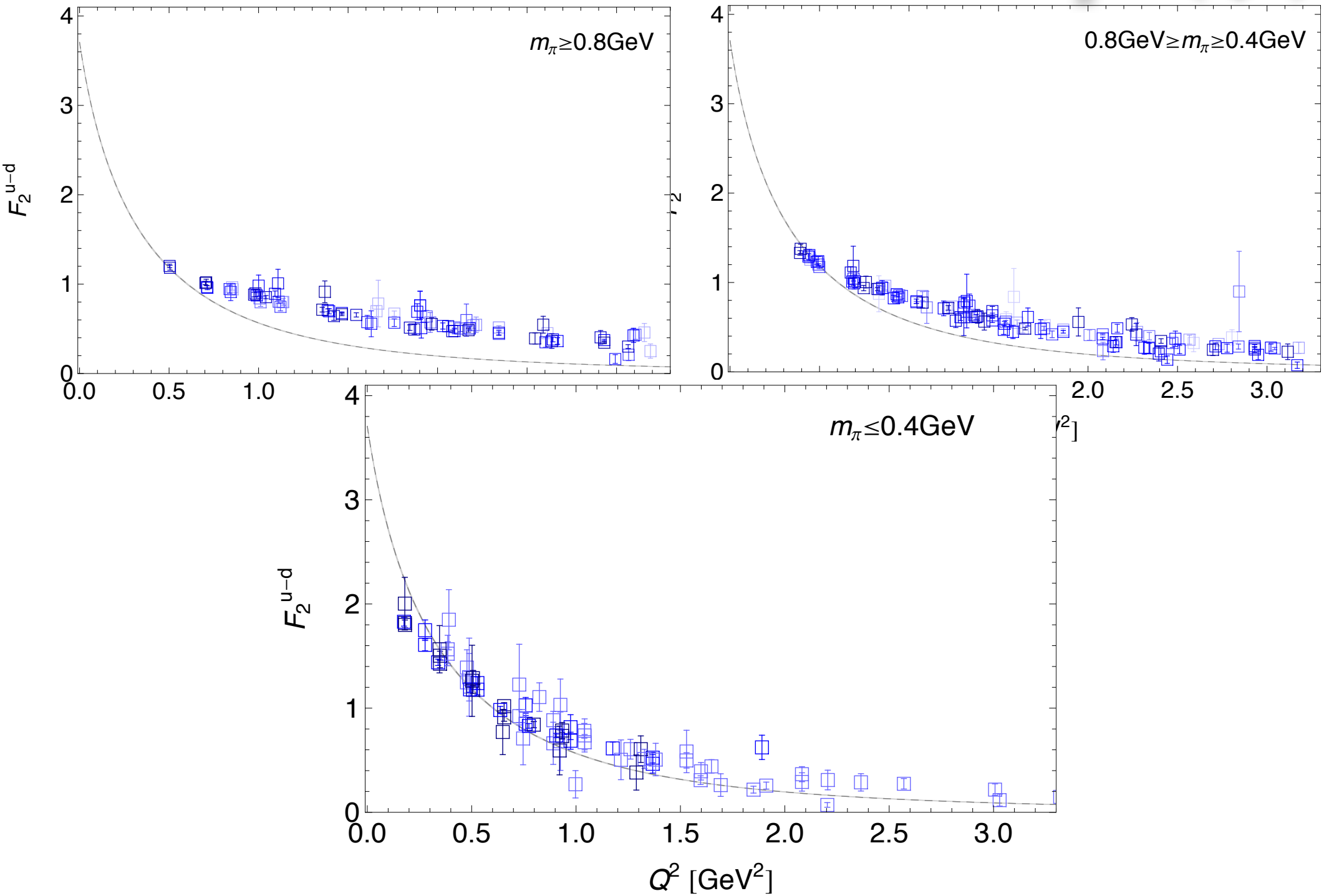


Comparison With Experiment $F_1^{u-d}(Q^2)$



Comparison With Experiment

$F_2^{u-d}(Q^2)$



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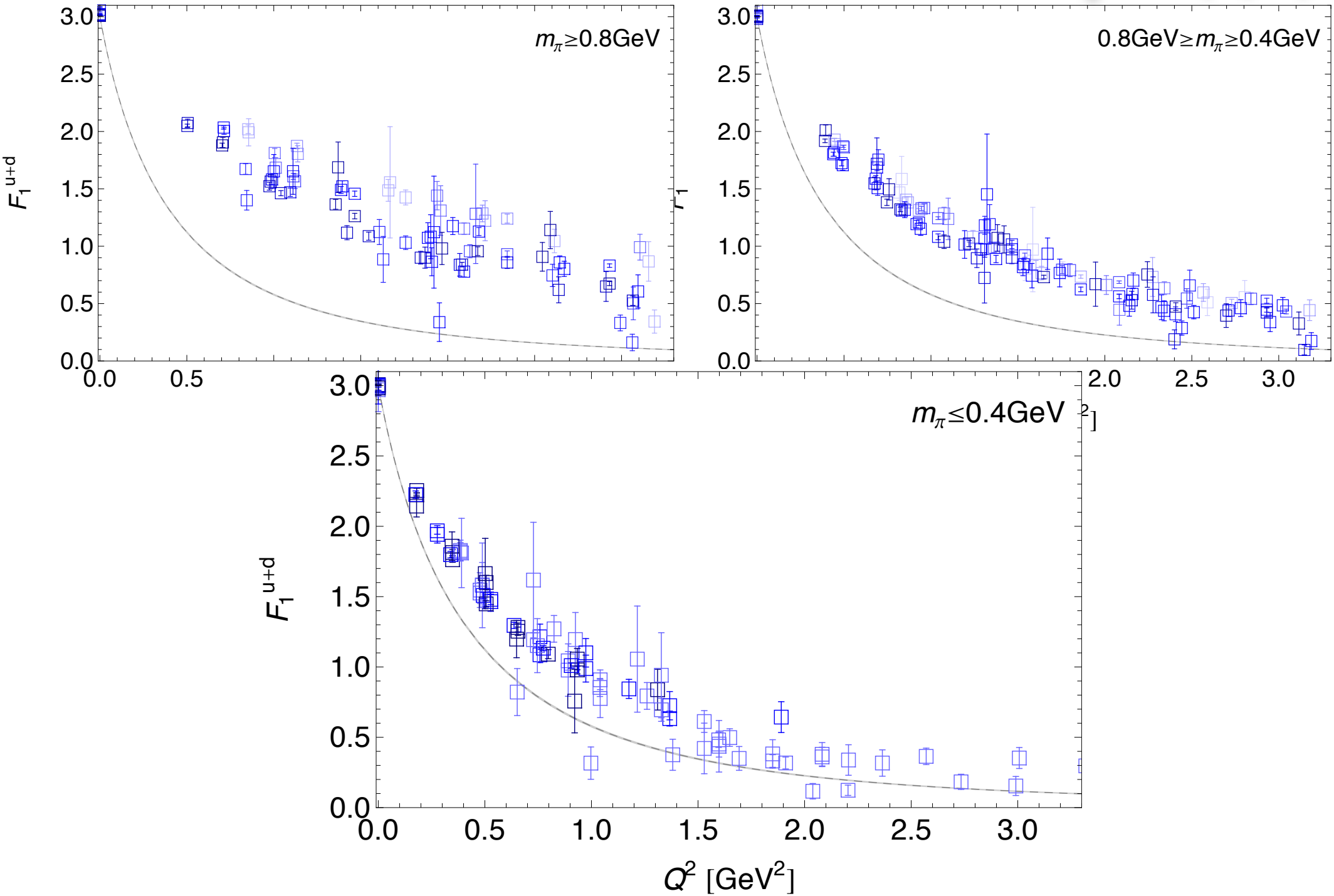

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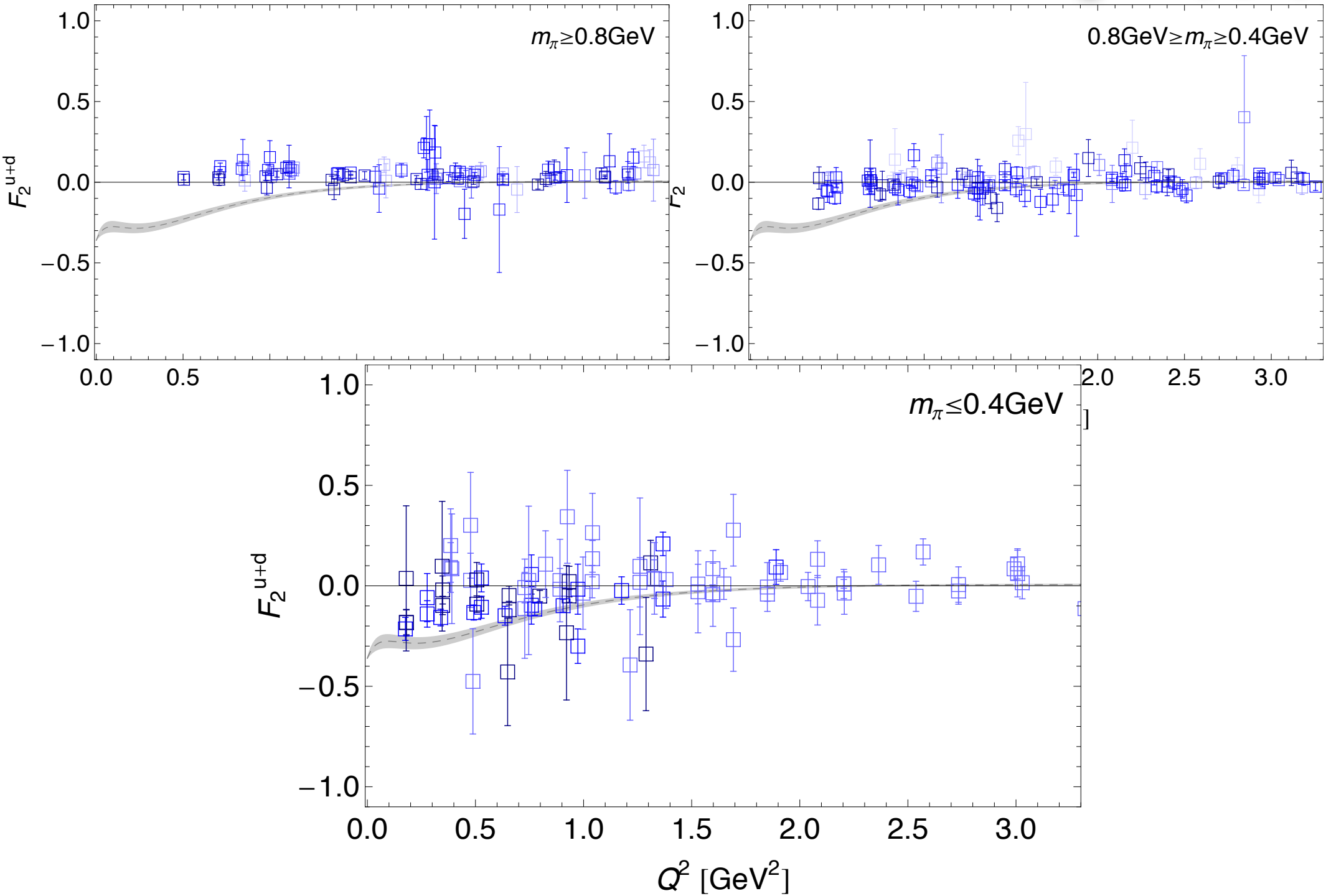

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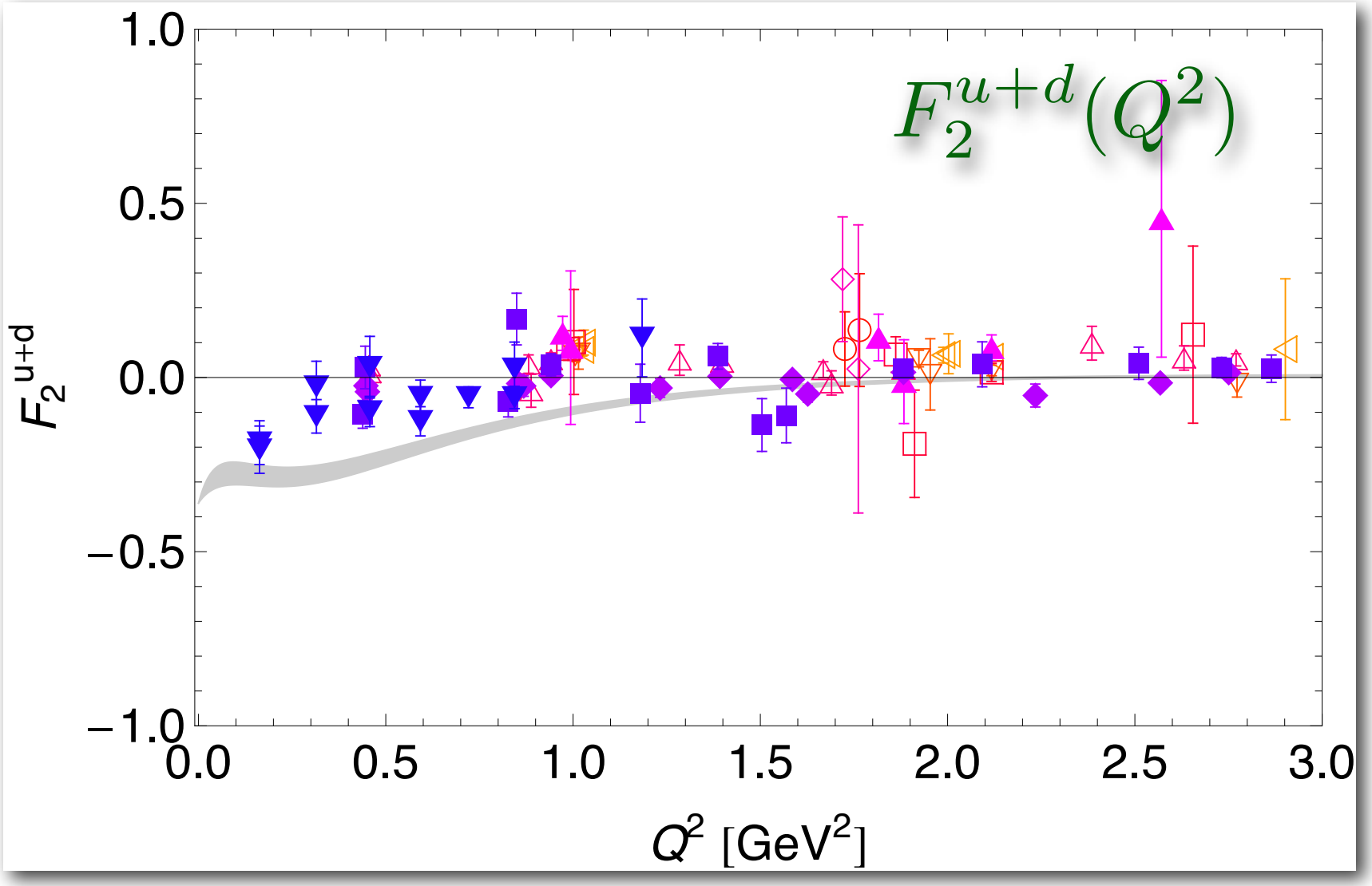
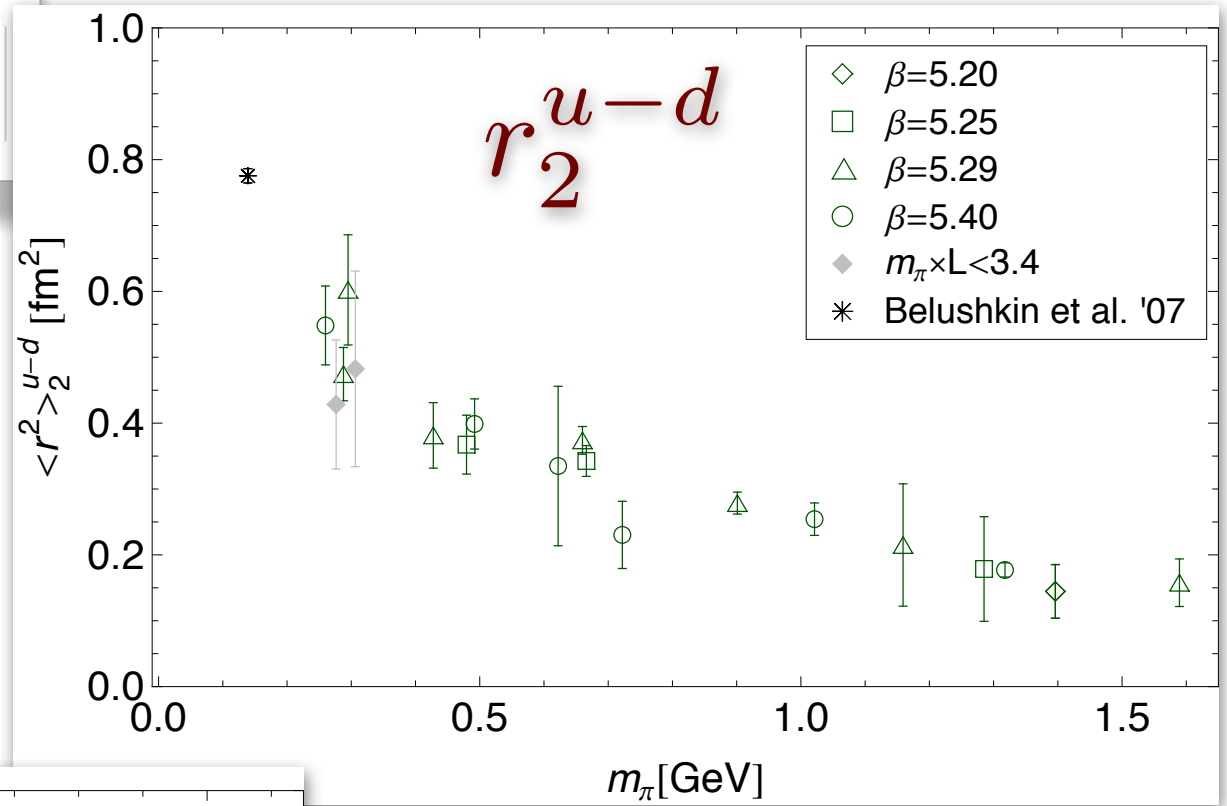
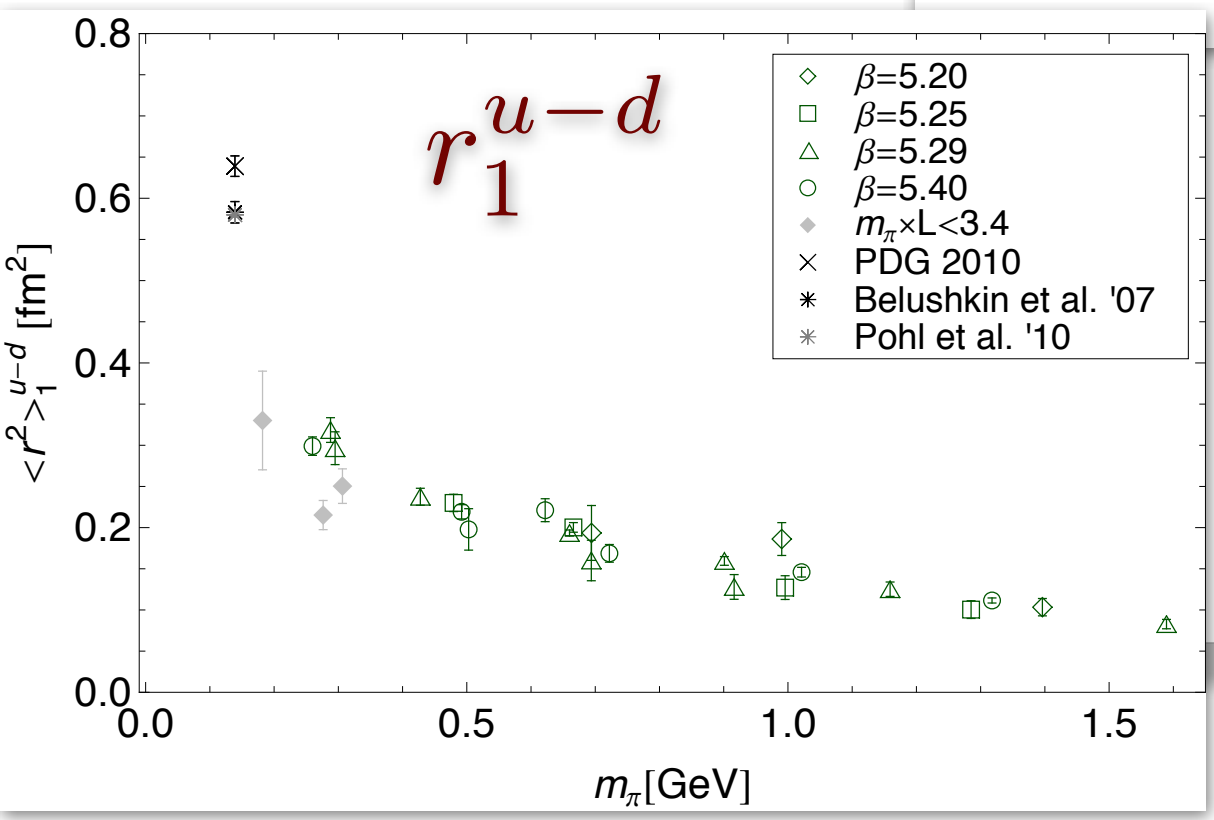


Form Factor Radii

Isovector

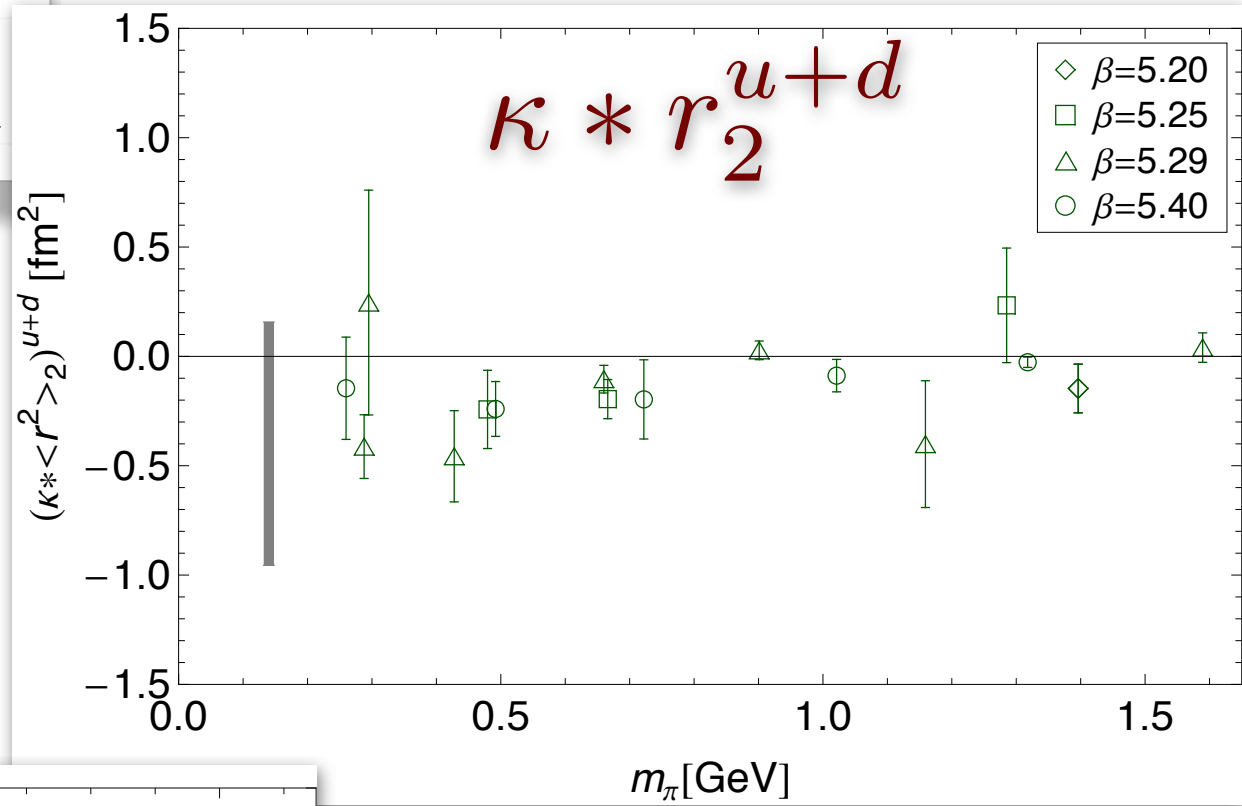


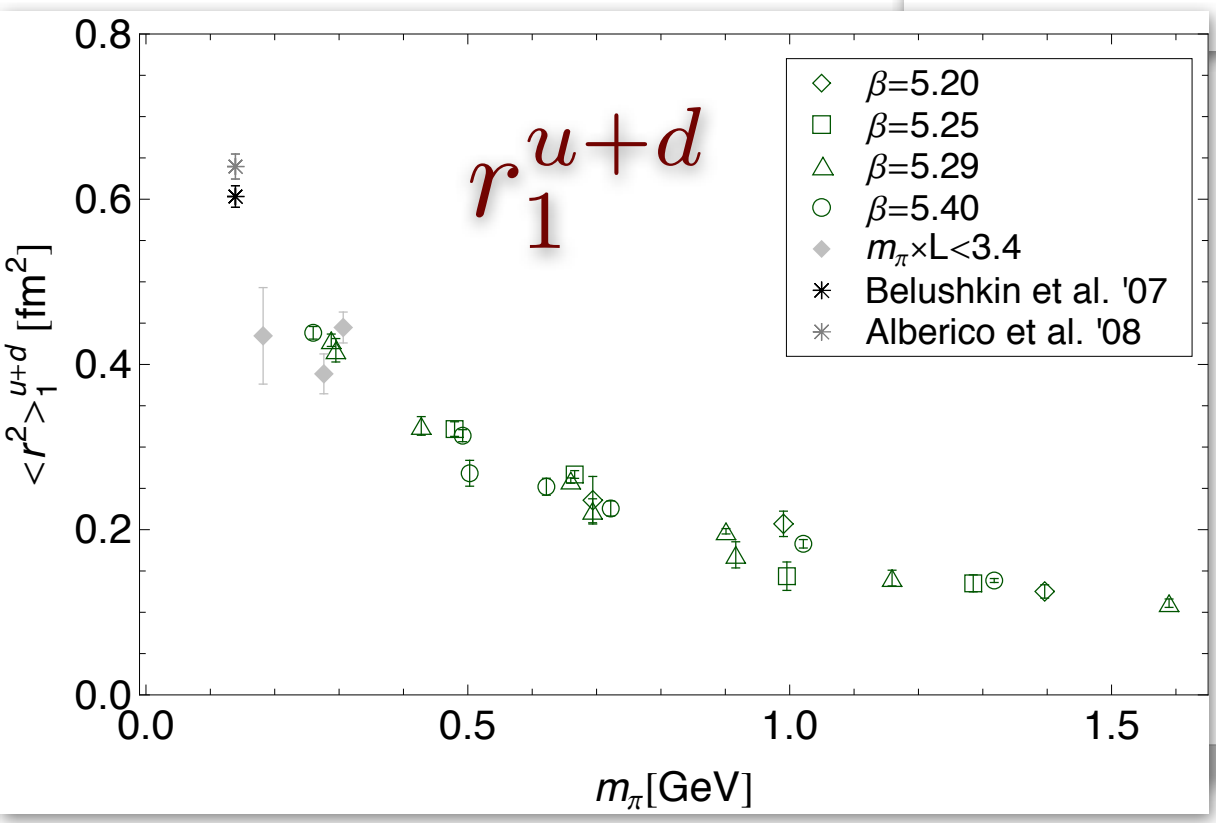


- Radii suppressed at large masses and small volumes
- Hint of sharp rise at small masses
- r_2 approaching experimental result

Form Factor Radii

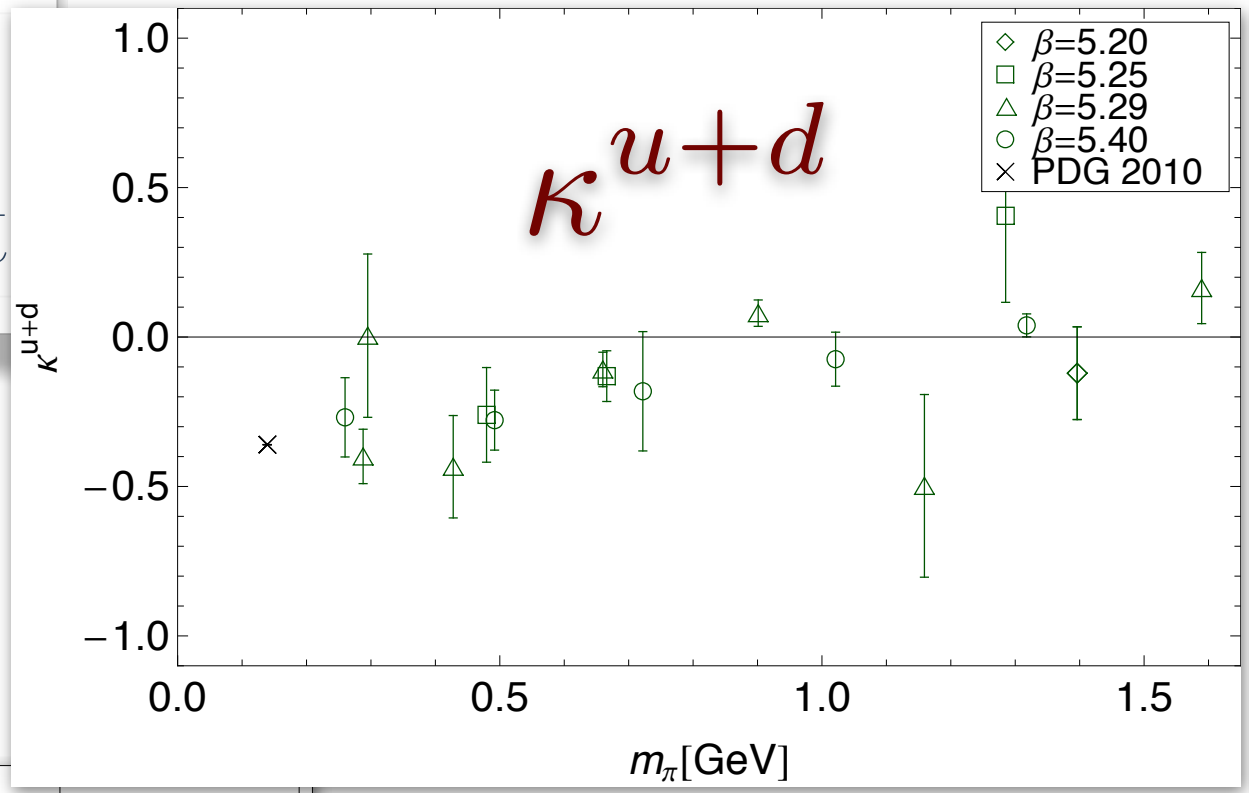
Isoscalar

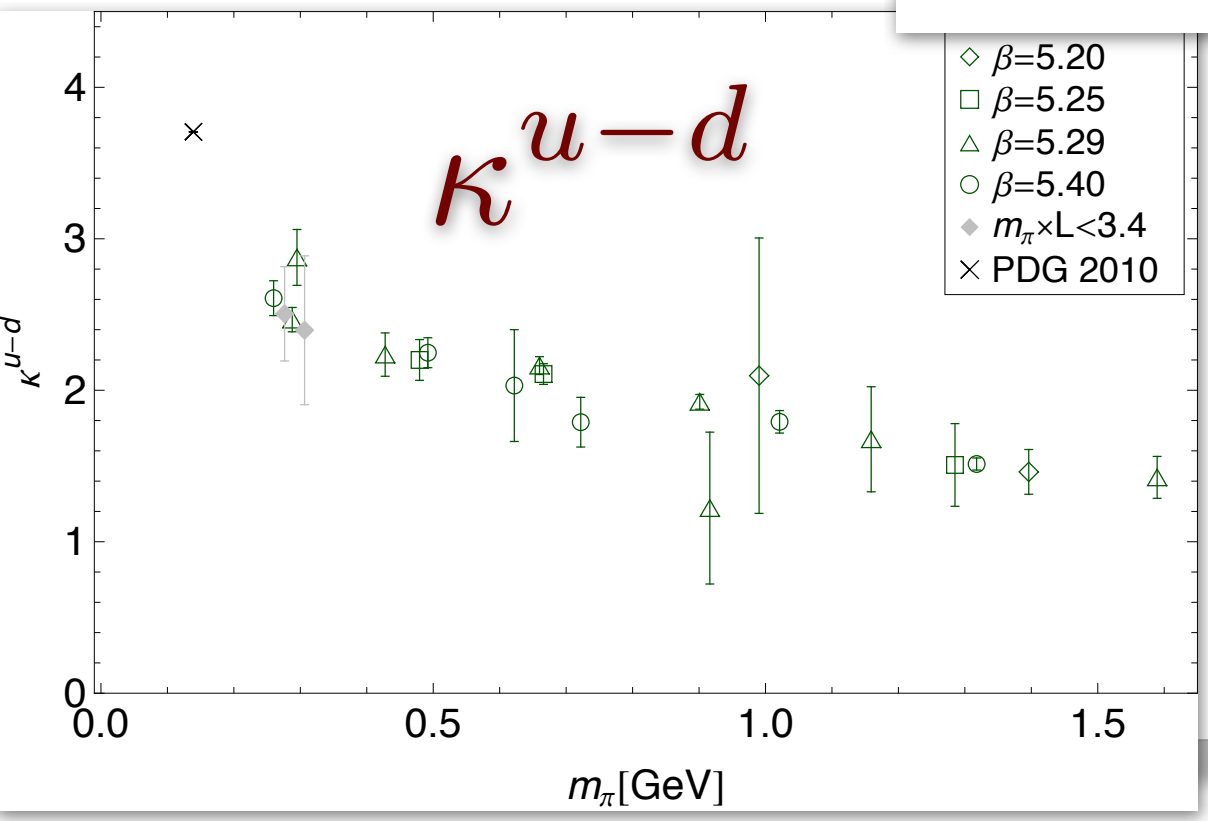




- r_1 shows clear curvature at small masses
- favour a small but negative value for $\kappa * r_2^{u+d}$

Anomalous Magnetic Moment



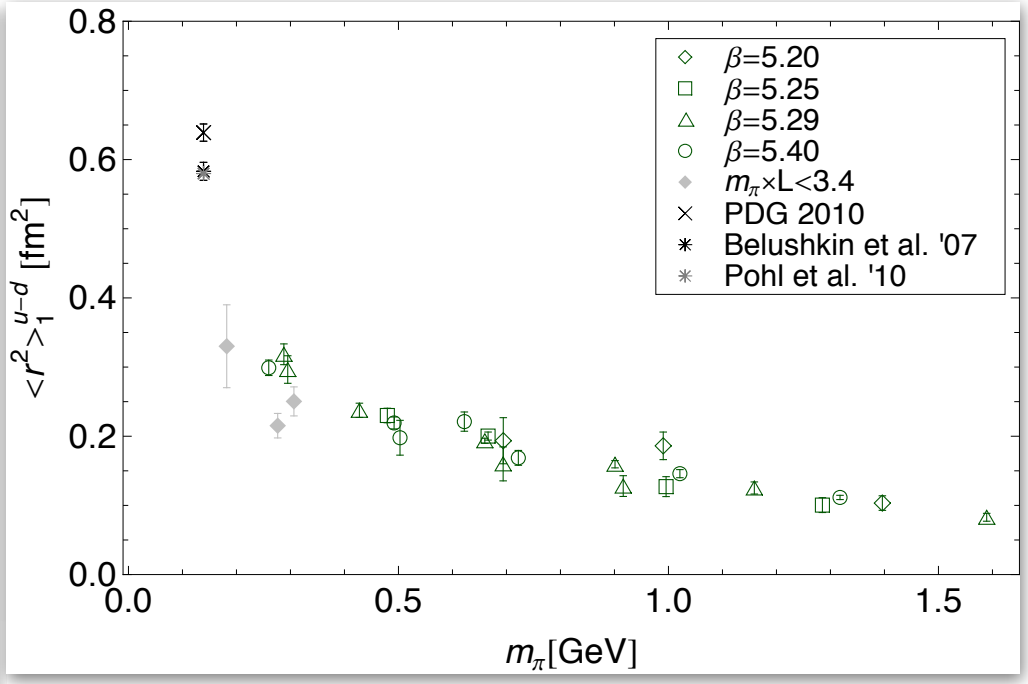


- κ^{u-d} shows clear curvature at small masses
- favour a small but negative value for κ^{u+d} in agreement with exp

Quark Mass Dependence

Contact with ChPT?

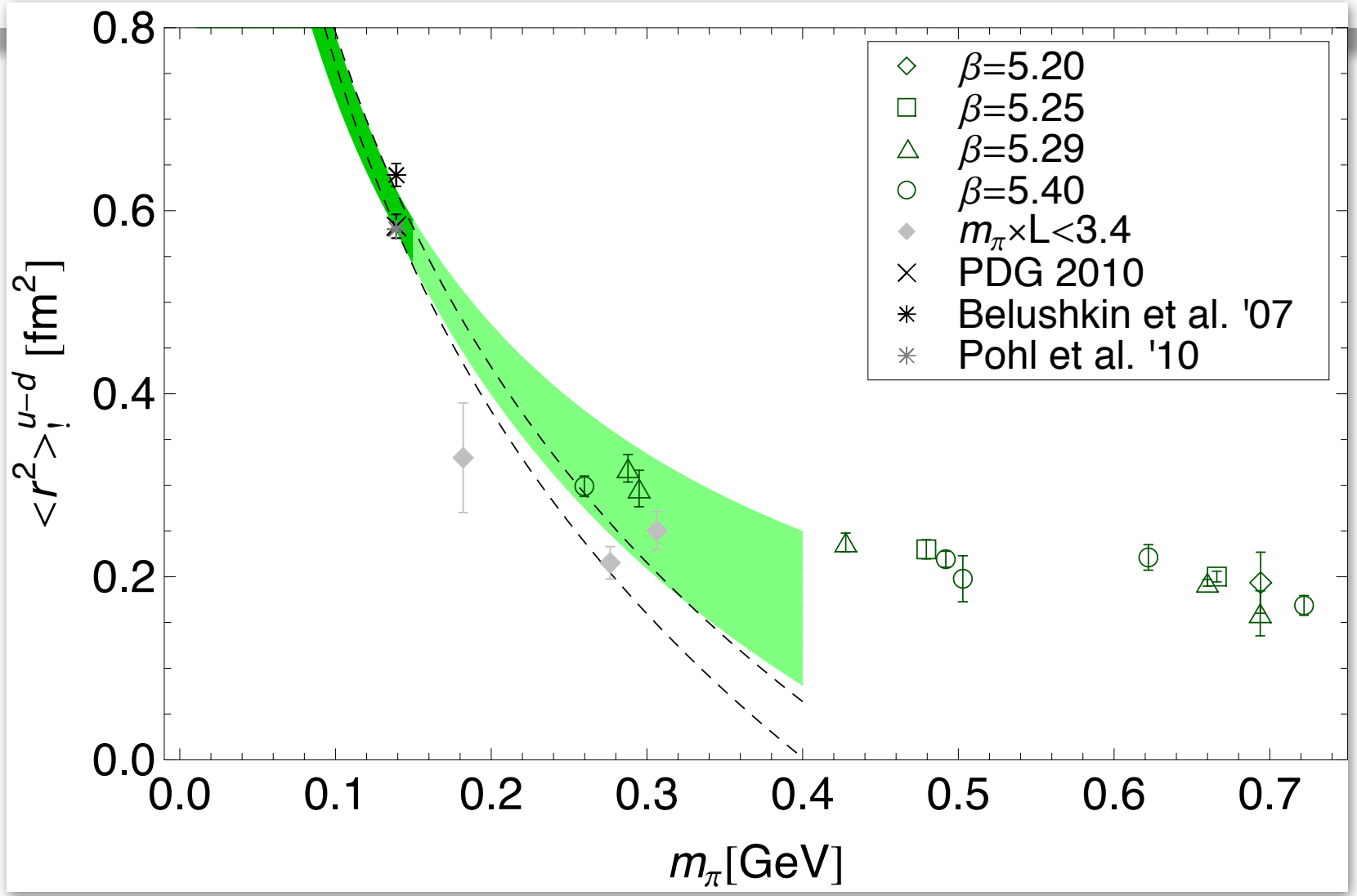
- Systematics of our simulations appear to be under control
- Remaining discrepancy must come from unphysical quark masses
- Contact with ChPT?



- Popular expressions from *Phys. Rev. D71, 034508 (2005)* (SSE)
- But are they valid up to $m_\pi < 300\ \mathrm{MeV}$?
- Our method: Vary unknown parameters over a “reasonable” range and extrapolate up from the chiral limit with the only constraint provided by the experimental point

Dirac Radius

- Rapidly decreasing isovector Dirac ms radius as pion mass increases
- Overlap with the lattice data points at $m_\pi \approx 250 \ldots 300$ MeV
- Similar observations for Pauli radius and anomalous magnetic moment



g_A

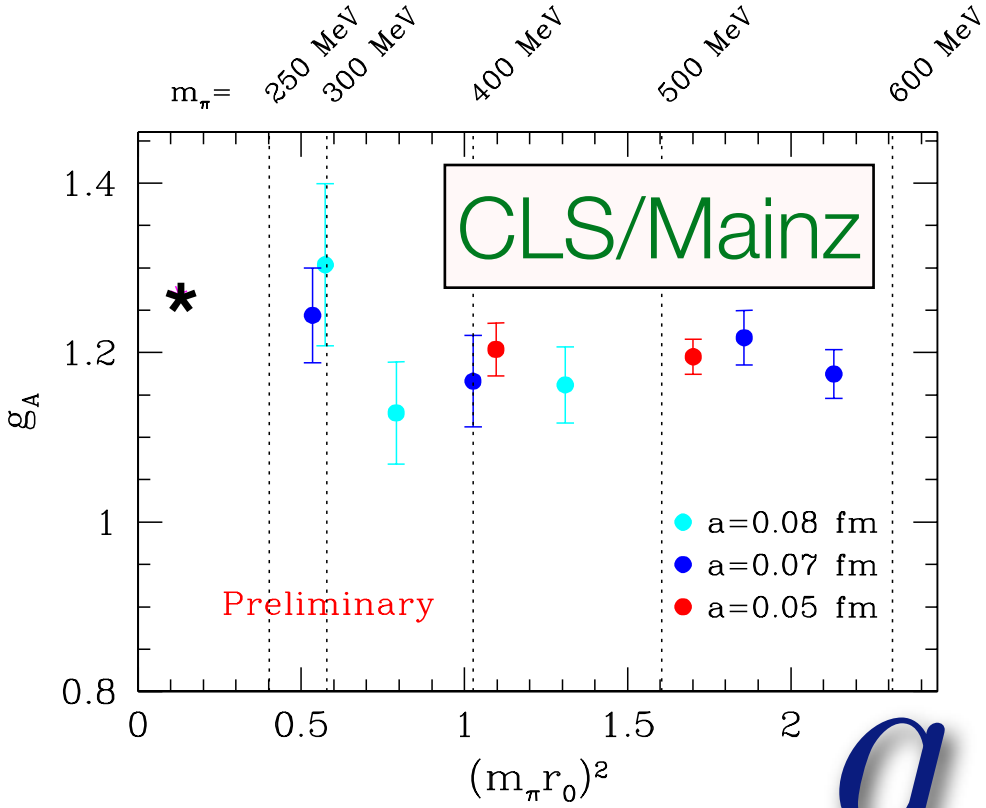
Axial Charge, g_A

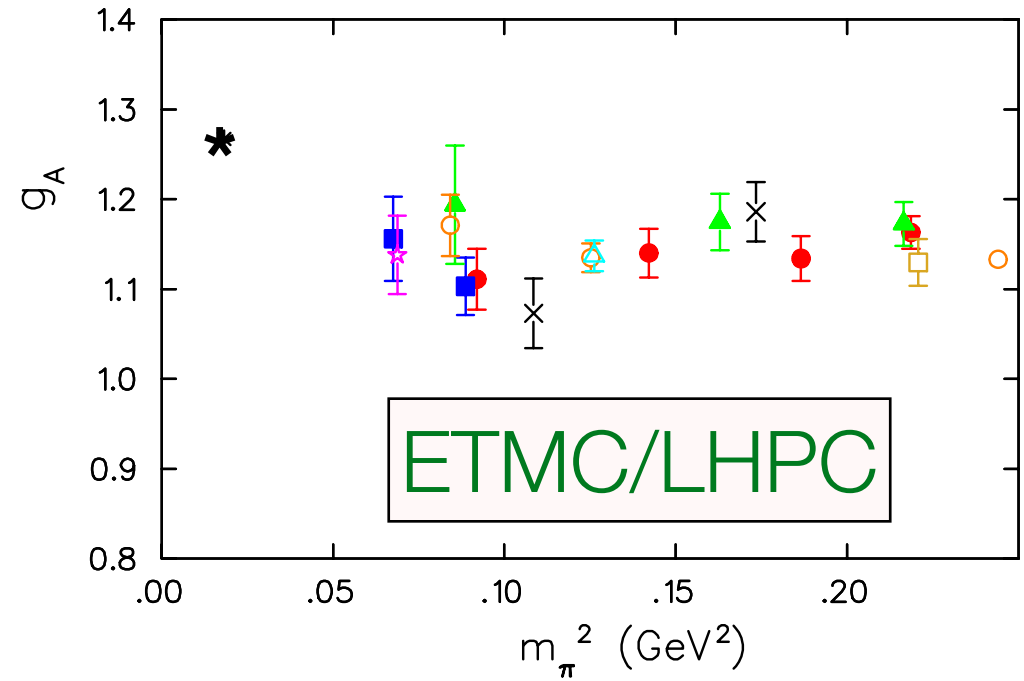
- Governs neutron ß decay
- Measure of spontaneous chiral symmetry breaking
- Related to 1st moment of helicity-dependent quark distribution functions

$$g_A = \Delta u - \Delta d$$

- Lattice calculations:
  - benchmark calculation
    - zero momentum, isovector
  - Known to suffer from large finite size effects
  - What size lattice is needed at low m_π?

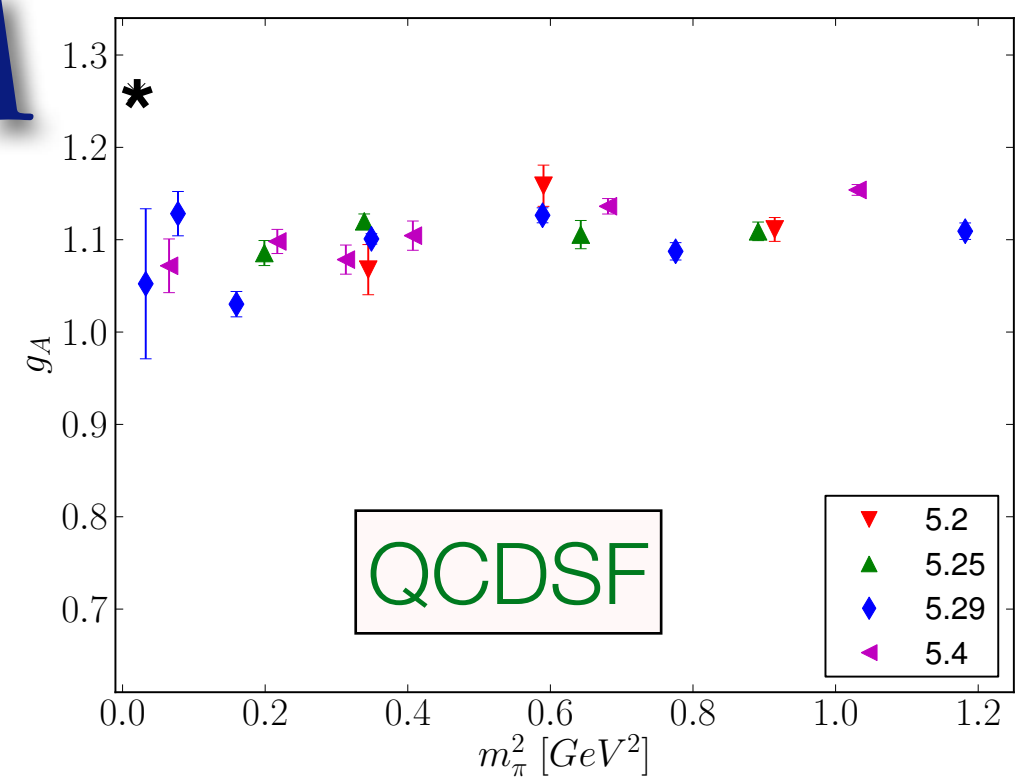
- Increased interest from several lattice collaborations
  - QCDSF
    PRD 74, 094508 (2006)
  - LHPC
    PRL 96, 052001 (2006)
  - RBC/UKQCD
    PRL 100, 171602 (2008)
  - ETMC
    PRD 83, 045010 (2011)
  - CLS/Mainz
    arXiv:1106.1554 [hep-lat]





g_A

- Lattice results for g_A are systematically low, even at small quark masses
- Systematic errors?



Excited State Contamination?

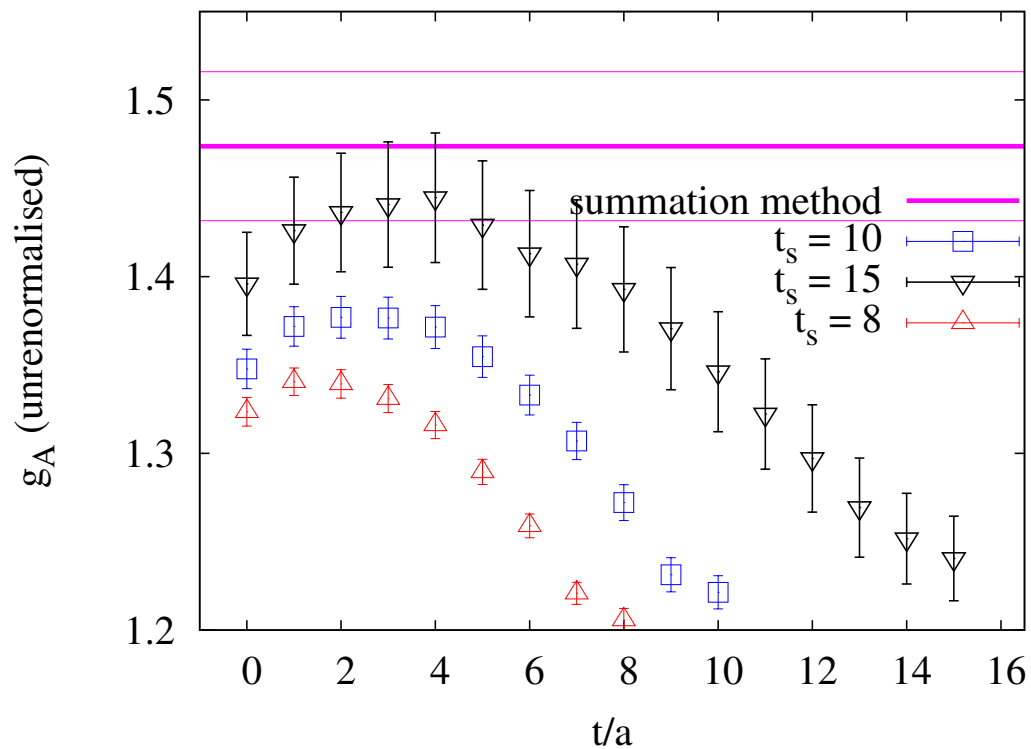
g_A

CLS/Mainz

arXiv:1106.1554 [hep-lat]

$$G(t,\tau,p,p') = \sum_{\vec{x},\vec{y}} \sum_{s,s'} e^{-E_{p'}(t-\tau)} e^{-E_p \tau} e^{-i\vec{p}'\cdot(\vec{y}-\vec{x})} e^{-i\vec{p}\cdot\vec{x}} \langle \Omega | \chi | p', s' \rangle \langle p', s' | \mathcal{O} | p, s \rangle \langle p, s | \overline{\chi} | \Omega \rangle$$

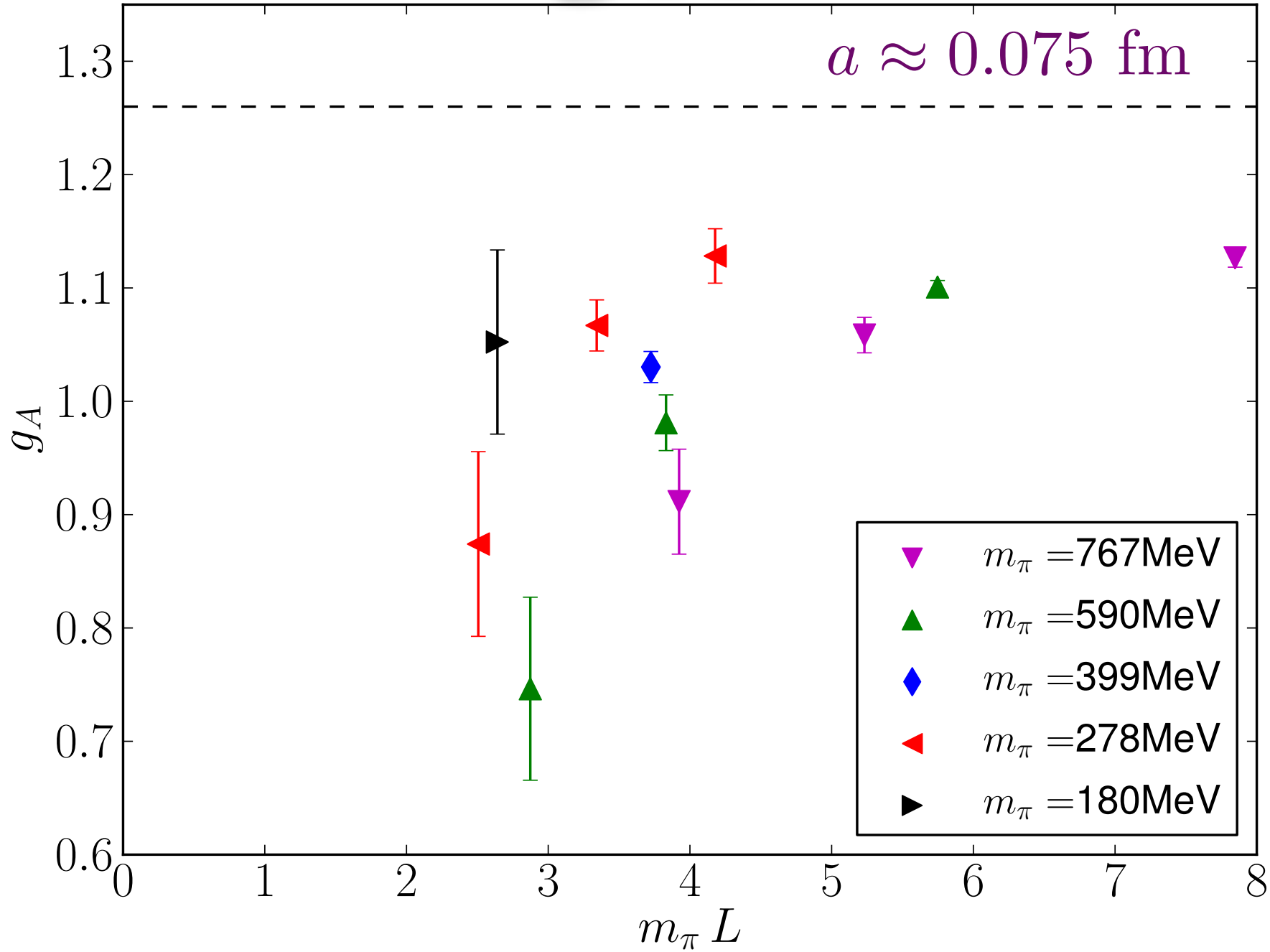
- Lattice correlation functions overlap with ground + excited states of, eg. proton
- Ground state isolated at large Euclidean times
- Lattice 3-point functions used a fixed sink
  - Residual excited state contamination?



Finite Volume Effects?

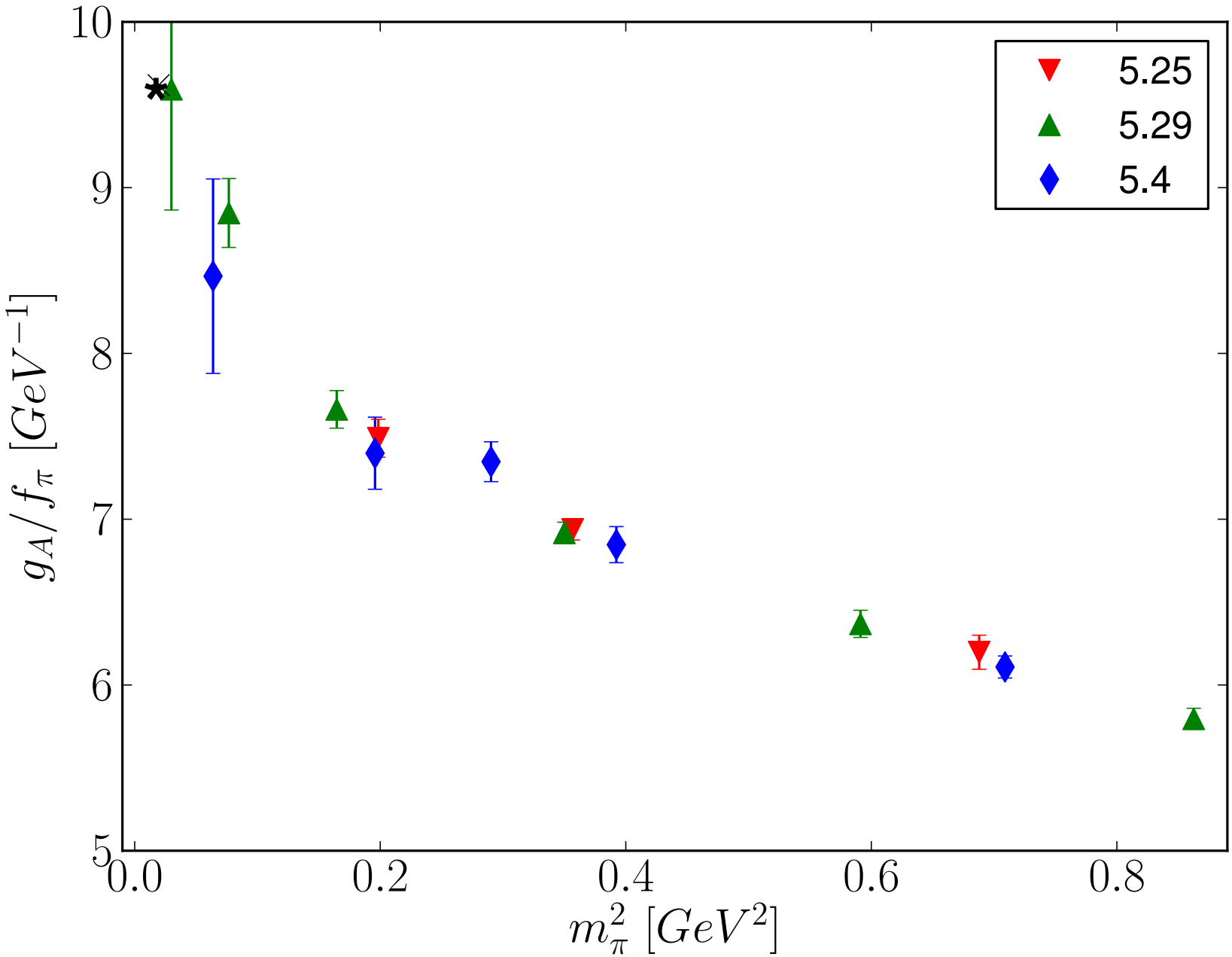
g_A

QCDSF



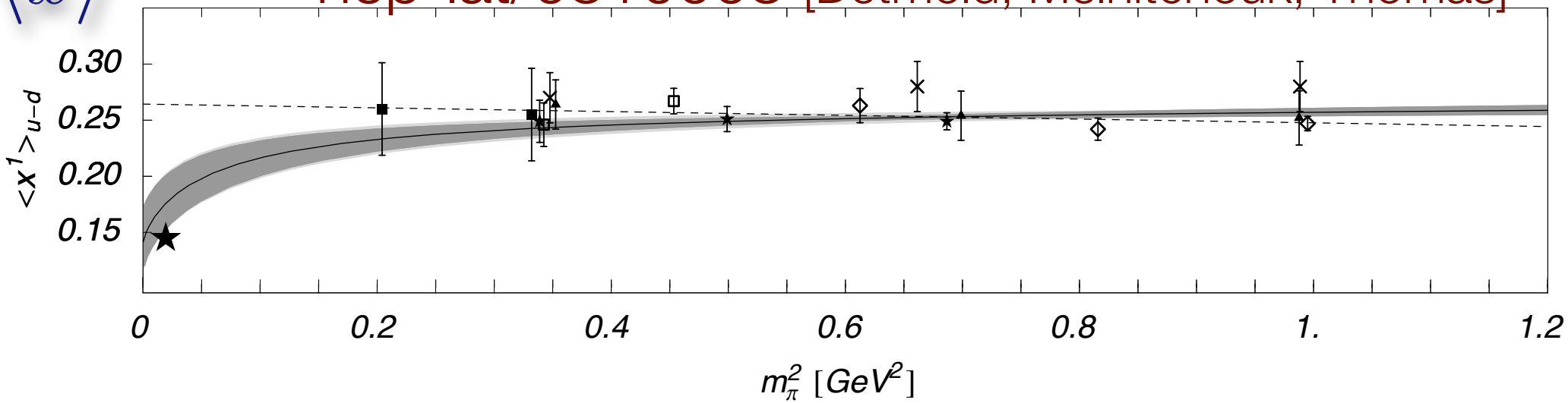
g_A/f_π [GeV^{-1}]

QCDSF



$$\langle x \rangle$$

$\langle x \rangle$ hep-lat/0310003 [Detmold, Melnitchouk, Thomas]



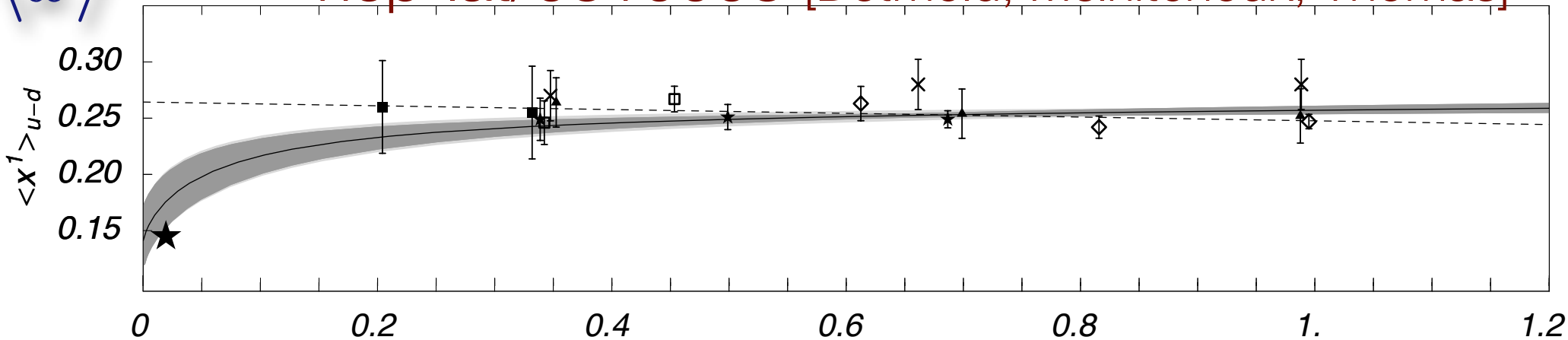
- First moment of the (isovector) nucleon parton distribution function

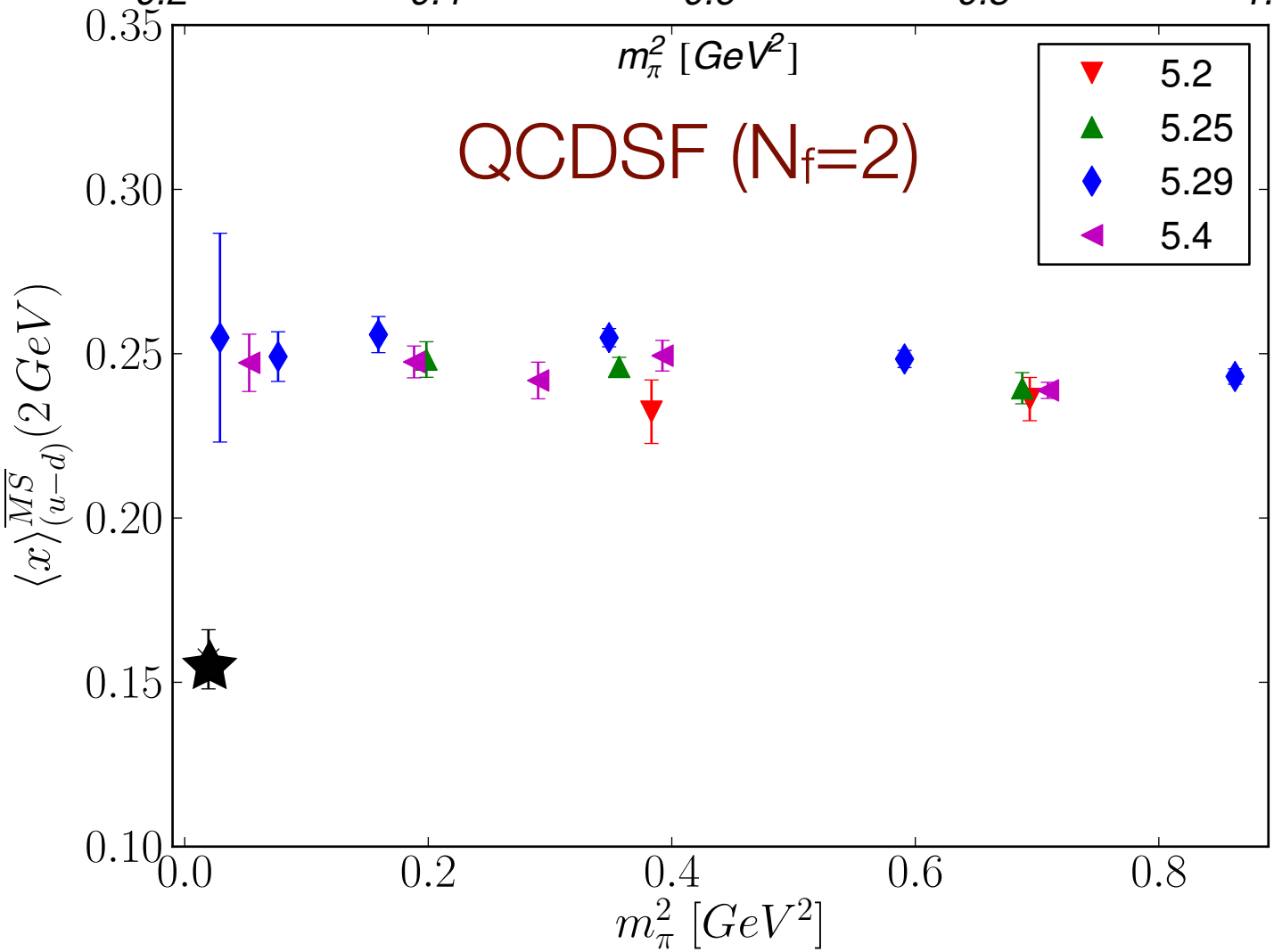
$$\langle x \rangle_\mu^{u-d} = \int_0^1 dx\, x(u(x,\mu) - d(x,\mu)) + \int_0^1 dx\, x(\bar{u}(x,\mu) - \bar{d}(x,\mu))$$

- Notorious for producing lattice results ≈ 2x too large for isovector nucleon
  - What are the possible systematic errors that could account for this
    - Quenching? Chiral physics? Finite volume effects?

$\langle x \rangle$

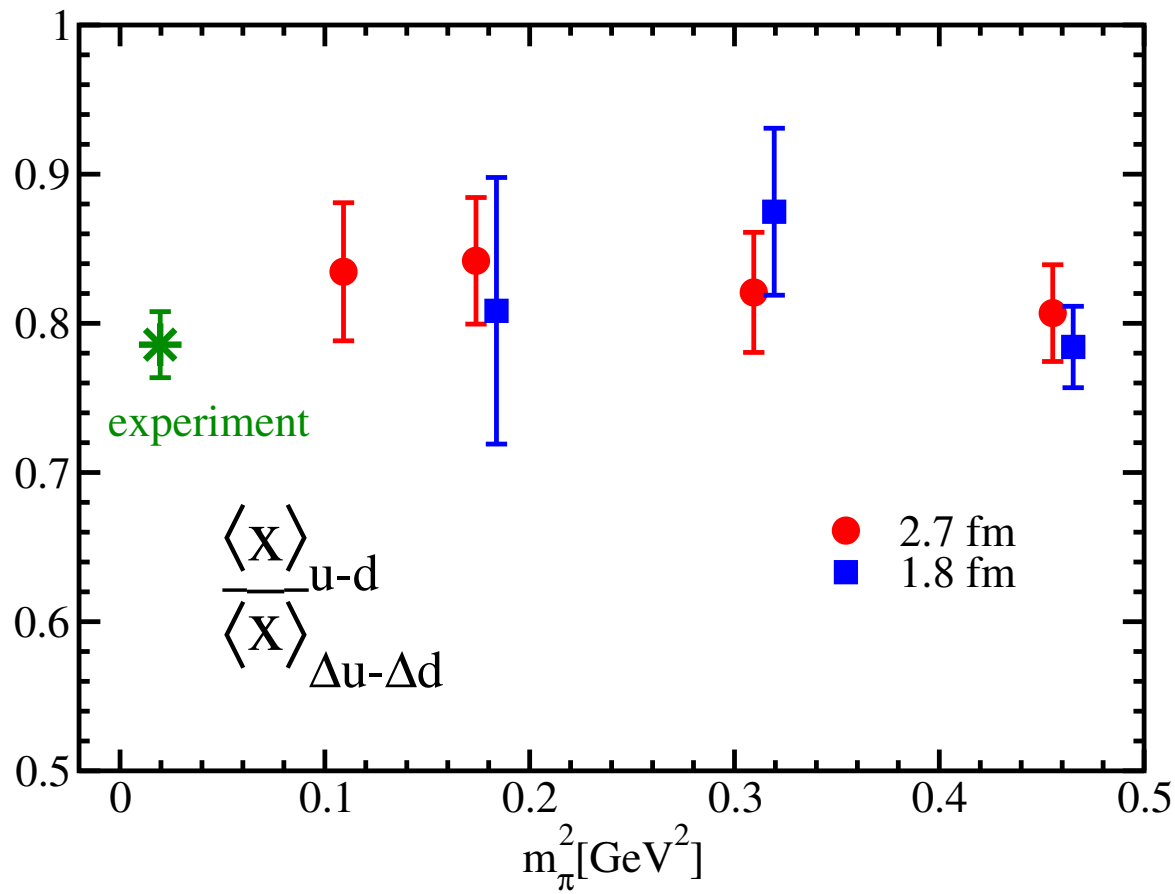
hep-lat/0310003 [Detmold, Melnitchouk, Thomas]





- However ratios of lattice results again look good, e.g.

RBC/UKQCD PRD 82, 014501 (2010)



Pion Form Factor

Pion Form Factor

- Asymptotic normalisation known from $\pi \to \mu + \nu$ decay

$$F_\pi(Q^2 \to \infty) = \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

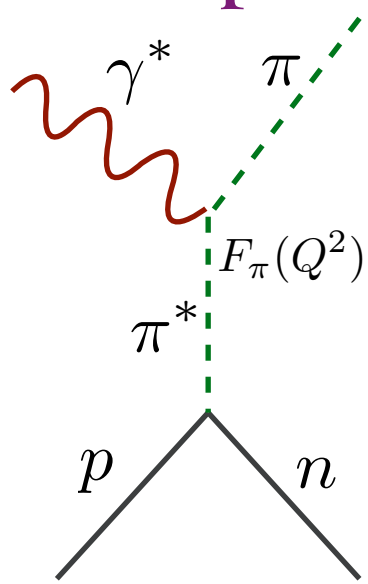
- Allows to study the transition from the soft to hard regimes

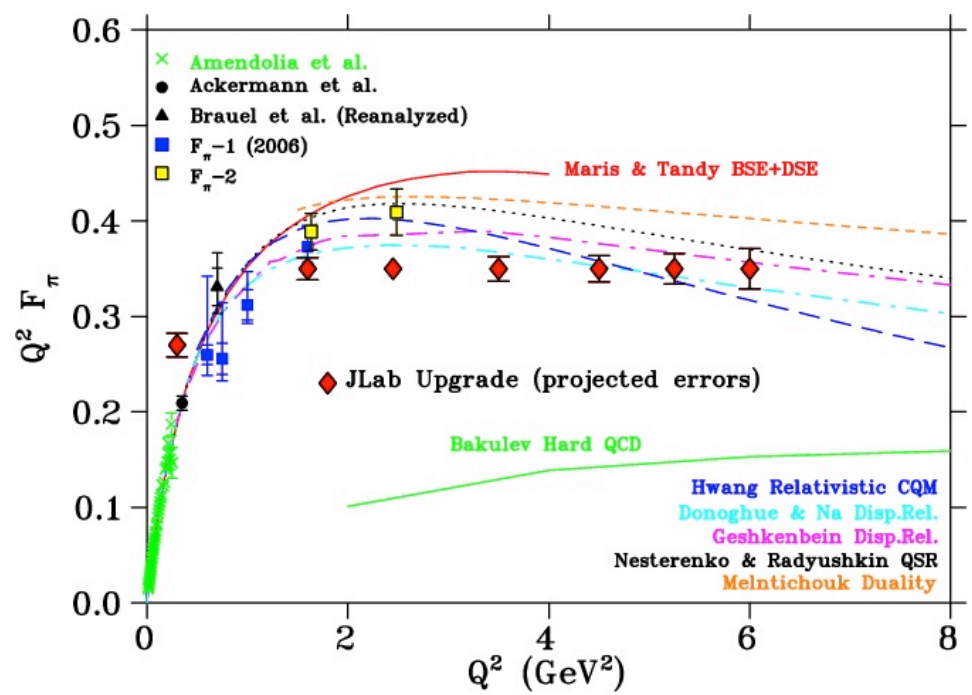
- Low Q^2: measured directly by scattering high energy pions from atomic electrons

$$r_\pi = 0.657 \pm 0.006 \text{ fm}$$

- High Q^2: quasi-elastic scattering off virtual pions

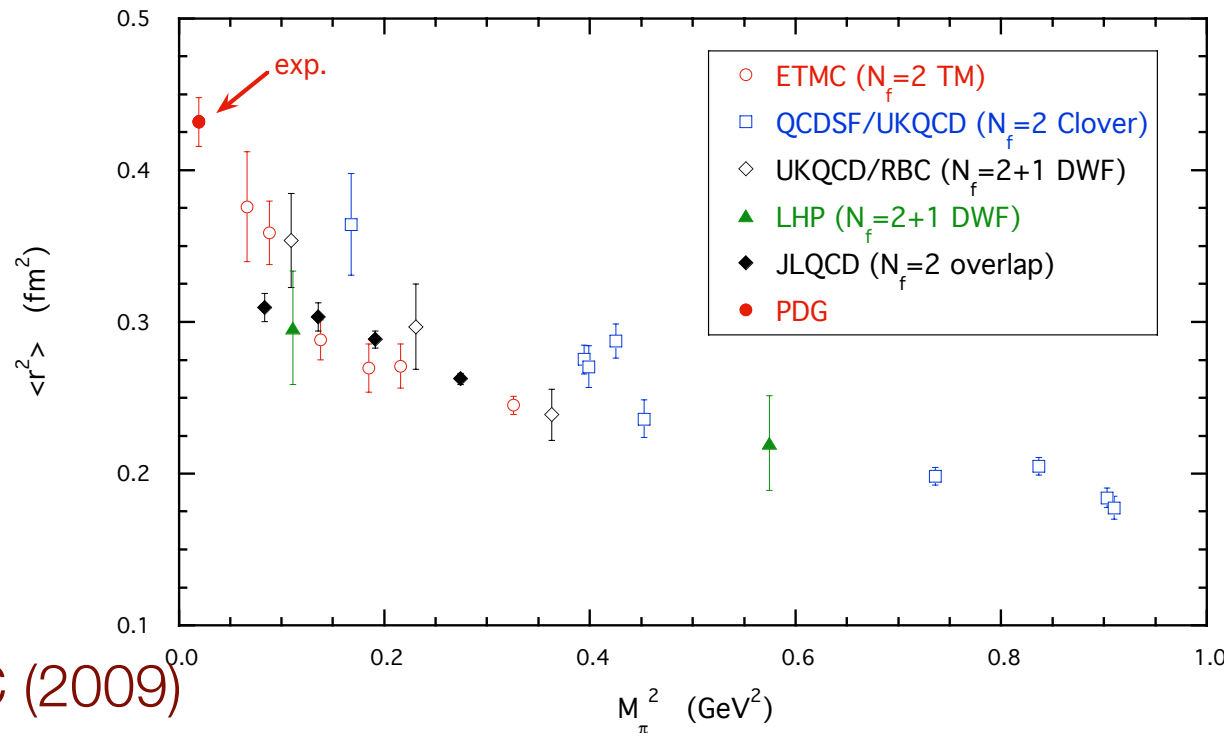
Model dependence





Pion Form Factor

- Under investigation by several lattice collaborations
  - CLS/Mainz arXiv:1106.1554 [hep-lat]
  - PACS/CS arXiv:1003.3321 [hep-lat]
  - ETMC PRD 79, 074506 (2009)
  - QCDSF EPJ C51,335 (2007)
  - JLQCD arXiv:1012.0137 [hep-lat]
  - RBC/UKQCD JHEP 0807:112 (2008)

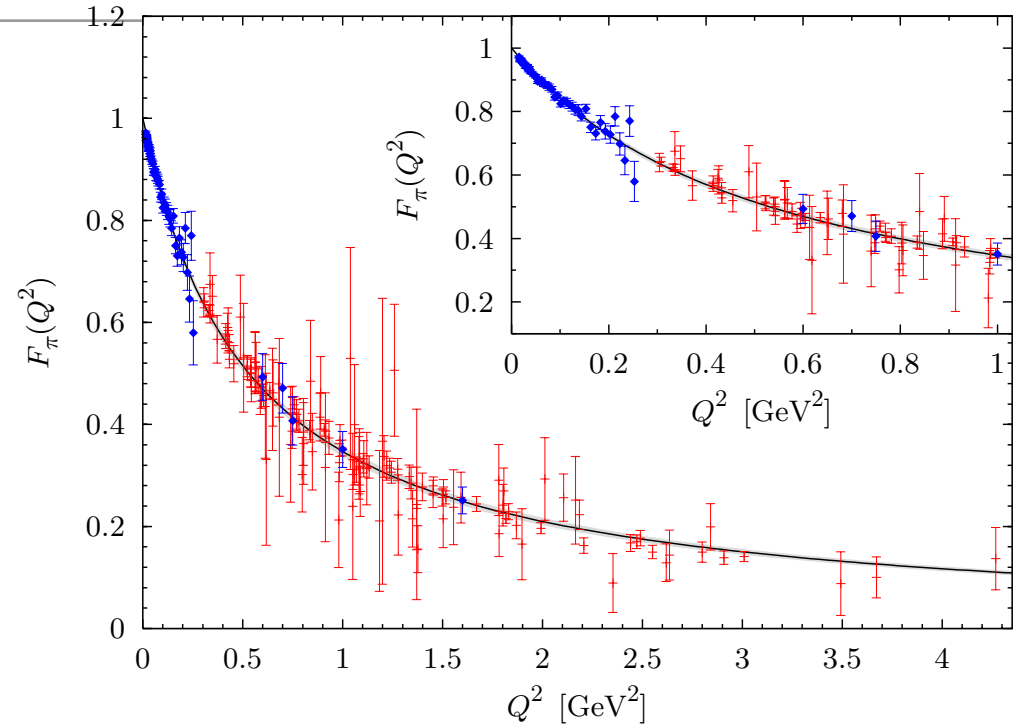


Plot from ETMC (2009)

Pion Form Factor

Accessing small Q^2

- On a periodic lattice with spatial volume L^3, momenta are discretised in units of $2\pi/L$



[QCDSF, hep-lat/0608021]

Pion Form Factor

hep-lat/0411033, hep-lat/0703005

Accessing small Q^2

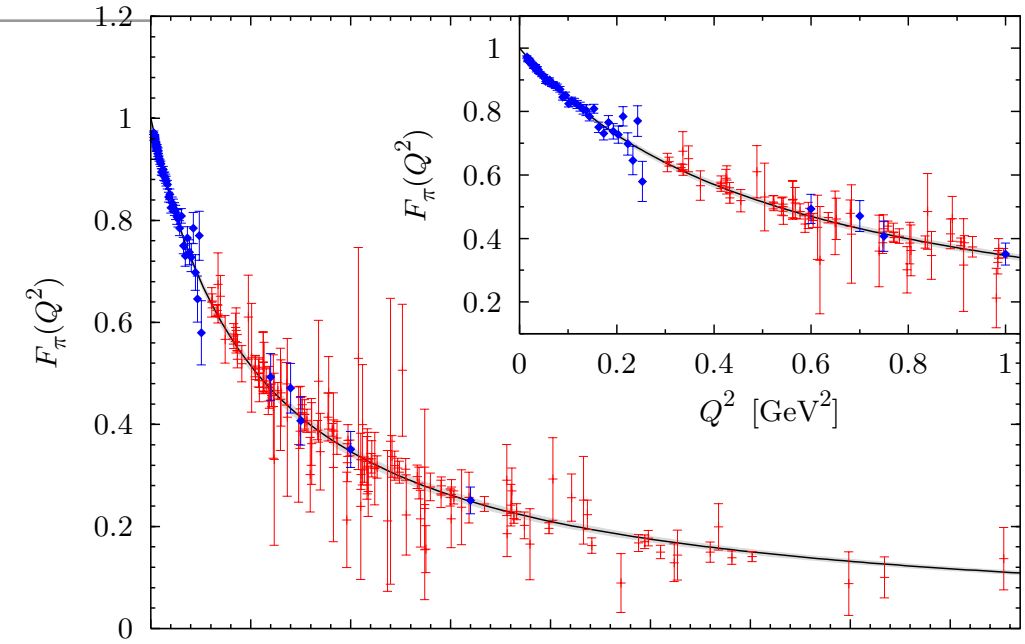
- On a periodic lattice with spatial volume L^3, momenta are discretised in units of $2\pi/L$

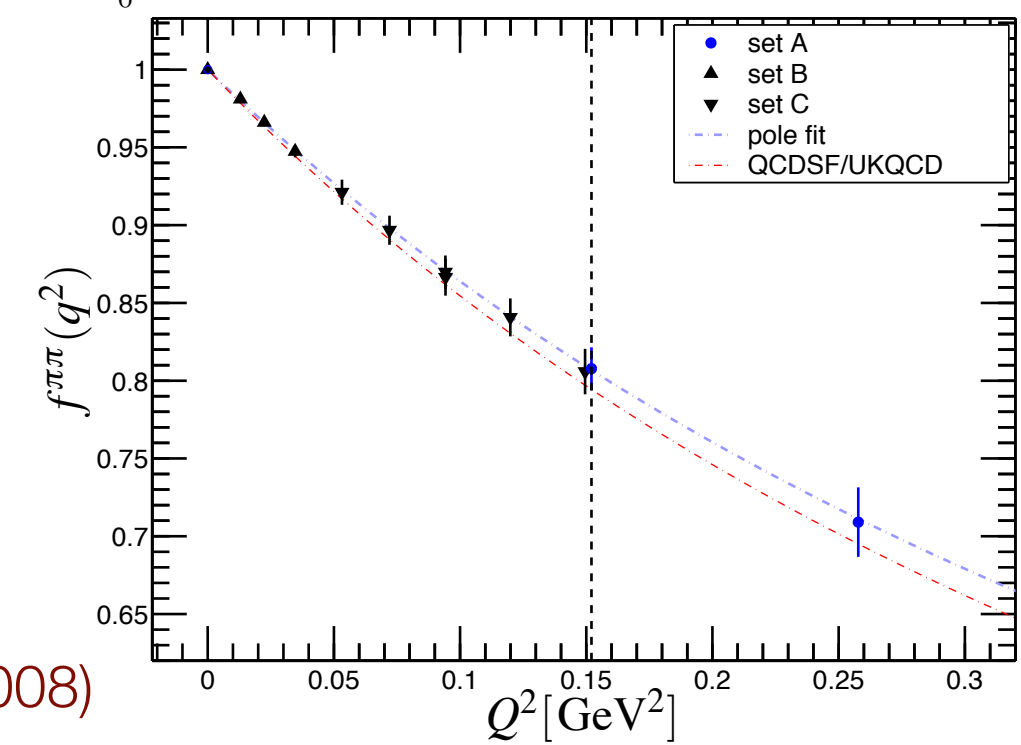
- Modify boundary conditions on the valence quarks

$$\psi(x_k + L) = e^{i\theta_k}\psi(x_k), \qquad (k = 1, 2, 3)$$

- allows to tune momenta continuously $\vec{p}_{\mathrm{FT}} + \vec{\theta}/L$

- Introduces additional (small) finite volume effect $\sim e^{-m_\pi L}$



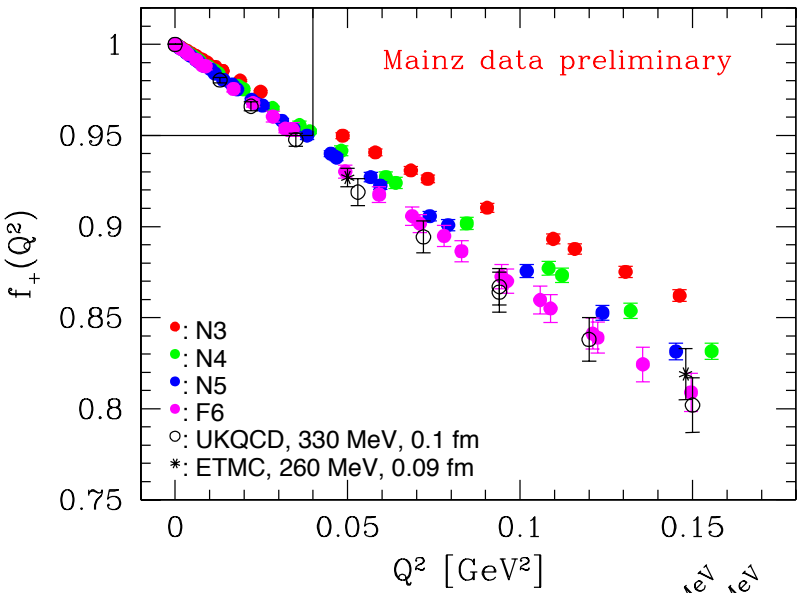


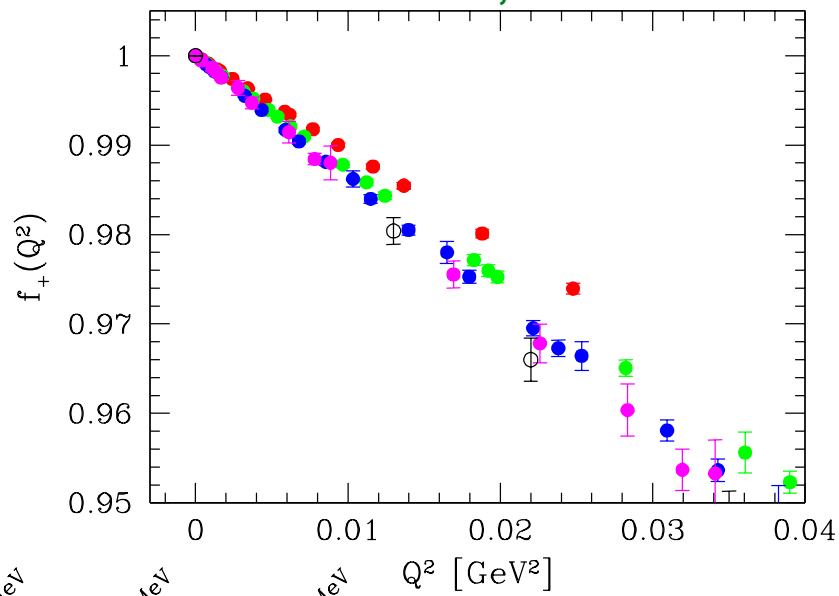
RBC/UKQCD JHEP 0807:112 (2008)

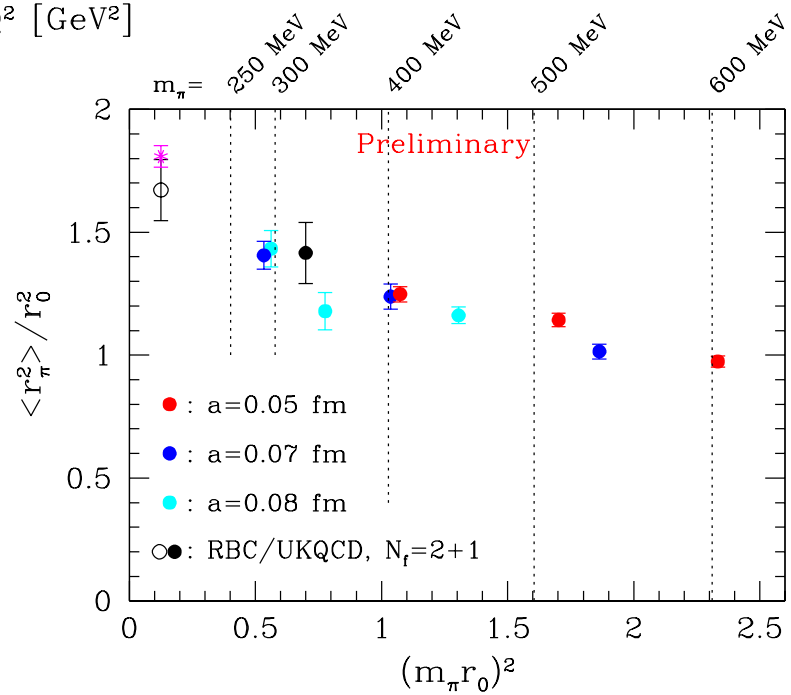
Pion Form Factor

B. Brandt 16:50

Brandt et al., arXiv:1106.1554 [hep-lat]







$$a_\mu = \frac{(g-2)_\mu}{2}$$

Muon Anomalous Magnetic Moment

- Describes contribution from quantum effects to the magnetic moment
- Provides a precision test of the Standard Model
- Studied experimentally > 50 years and now achieving 0.5ppm precision
- Standard Model contributions
  - QED
  - Weak
  - QCD

  } perturbation theory

  non-perturbative

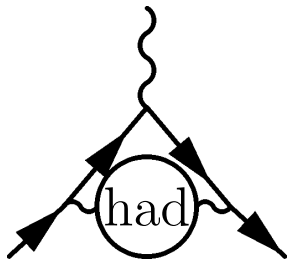
  [exp: e^+e^- annihilation, hadronic tau decays]
- Sensitive to physics Beyond the Standard Model
- Theory - Experiment differ by 3.2σ
- Dominant source of error from the hadronic (QCD) contribution

Muon Anomalous Magnetic Moment

- Hadronic (QCD) contribution are of two types
  - Vacuum polarisation
    - Early lattice work: Blum, PRL 91, 052001 (2003), Göckeler et al., NPB 688, 135 (2004)
    - A lot of recent attention ETMC, Mainz, RBC/UKQCD
  - Light-by-light scattering
    - A challenge on the lattice (4 point function)
    - Two groups currently investigating (T. Blum et al., P. Rakow et al.)
    - No definite results (Hadron 2013?)

Muon Anomalous Magnetic Moment

- Hadronic vacuum polarisation



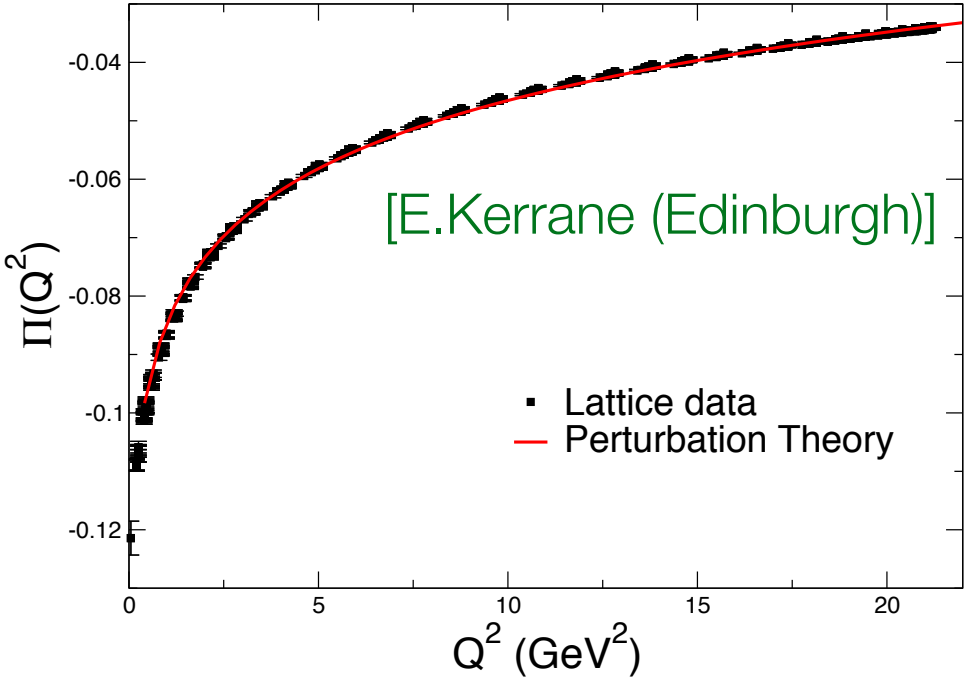
$$\longrightarrow a_\mu^{(2)had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty d(Q^2) f(Q^2) \times \hat{\Pi}(Q^2)$$

where

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) \qquad (\Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2))$$

is the (infrared-subtracted) vacuum polarisation amplitude

$$\Pi_{\mu\nu}(q) = \int d^4x\, e^{iq\cdot(x-y)} \langle J_\mu^{\rm EM}(x) J_\nu^{\rm EM}(y)\rangle$$



Muon Anomalous Magnetic Moment

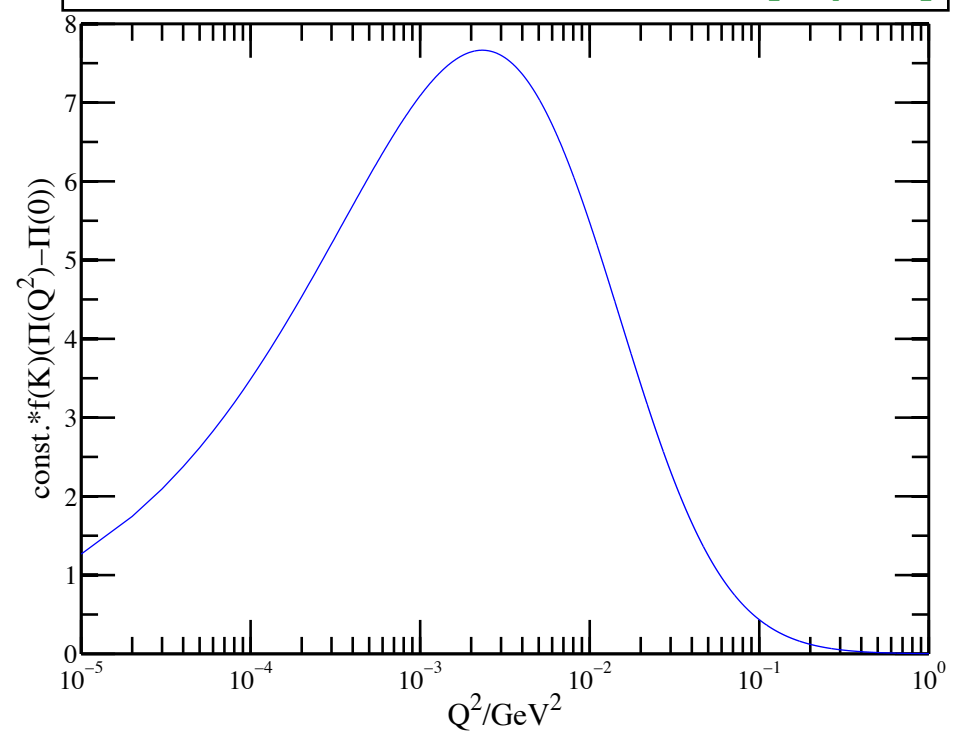
The Integral & The Kernel

- Usual to break up Integral into low and high Q^2 regions

$$a_\mu^{(2)had} = 4\alpha^2 \left[\int_0^{Q_C^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) + \int_{Q_C^2}^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \right]$$

- integrand peaked around muon mass (106 MeV)

Della Morte et al., arXiv:1011.5793 [hep-lat]



Muon Anomalous Magnetic Moment

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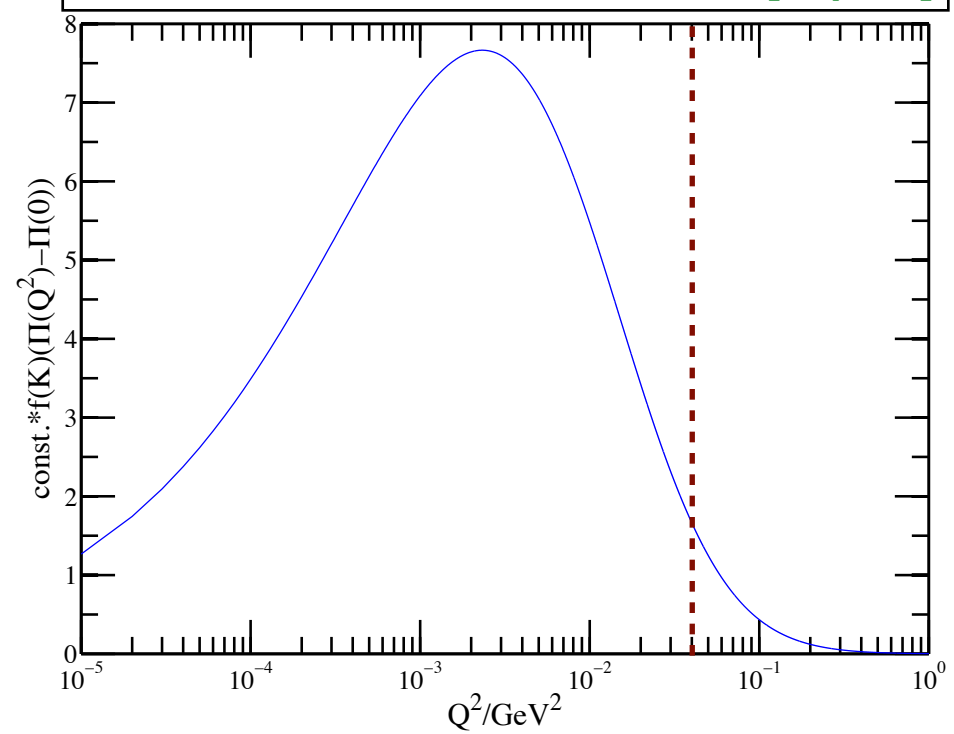


- Smallest momentum on the lattice

$$2\pi/T \approx 200\,\mathrm{MeV}$$

→ **Model dependence**

- Twisted boundary conditions

Brandt et al., arXiv:1010.2390 [hep-lat]

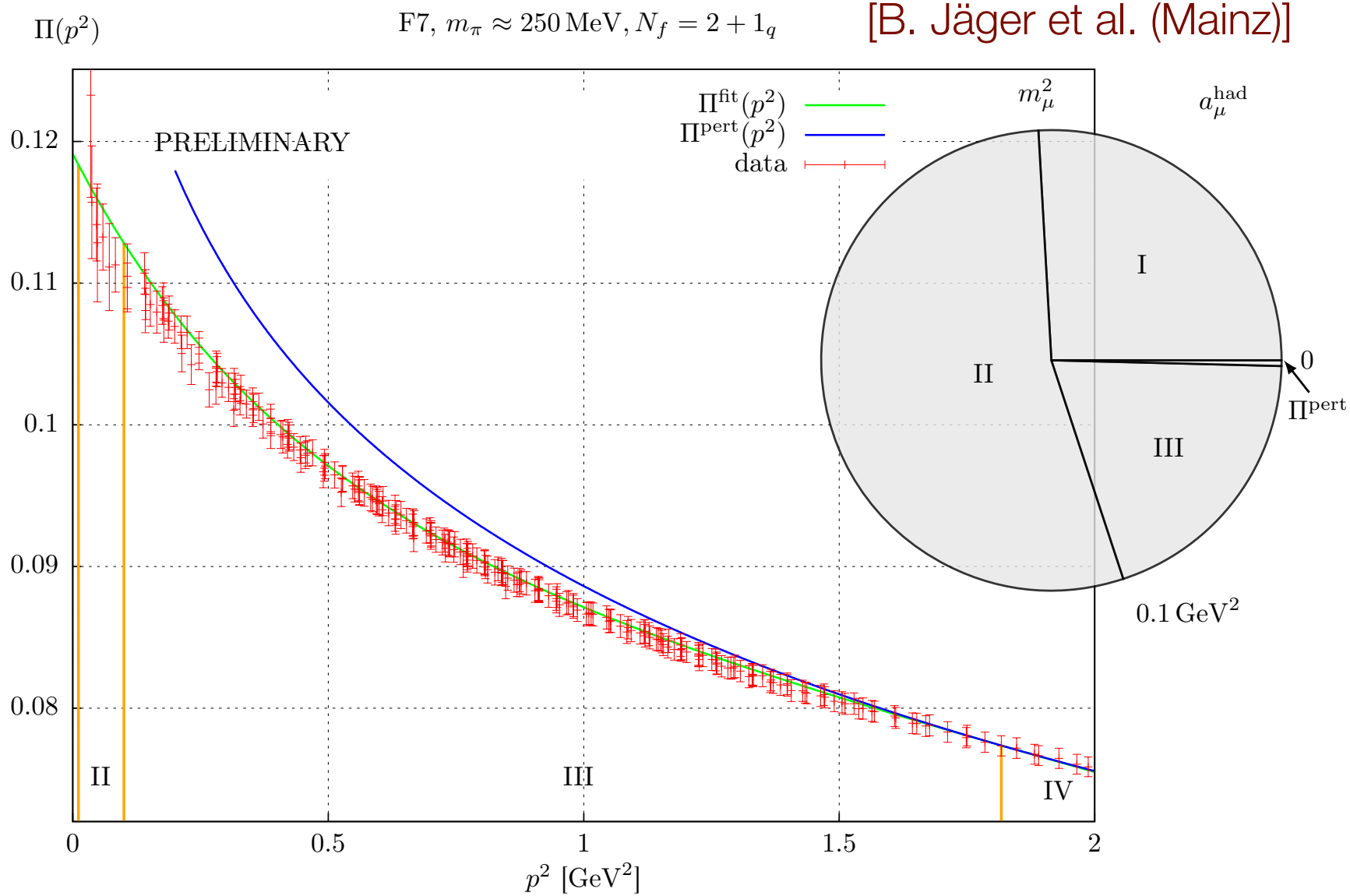
- Disconnected contributions from ChPT

Della Morte and Jüttner, JHEP 1011,154 (2010)

Muon Anomalous Magnetic Moment

The Integral & The Kernel

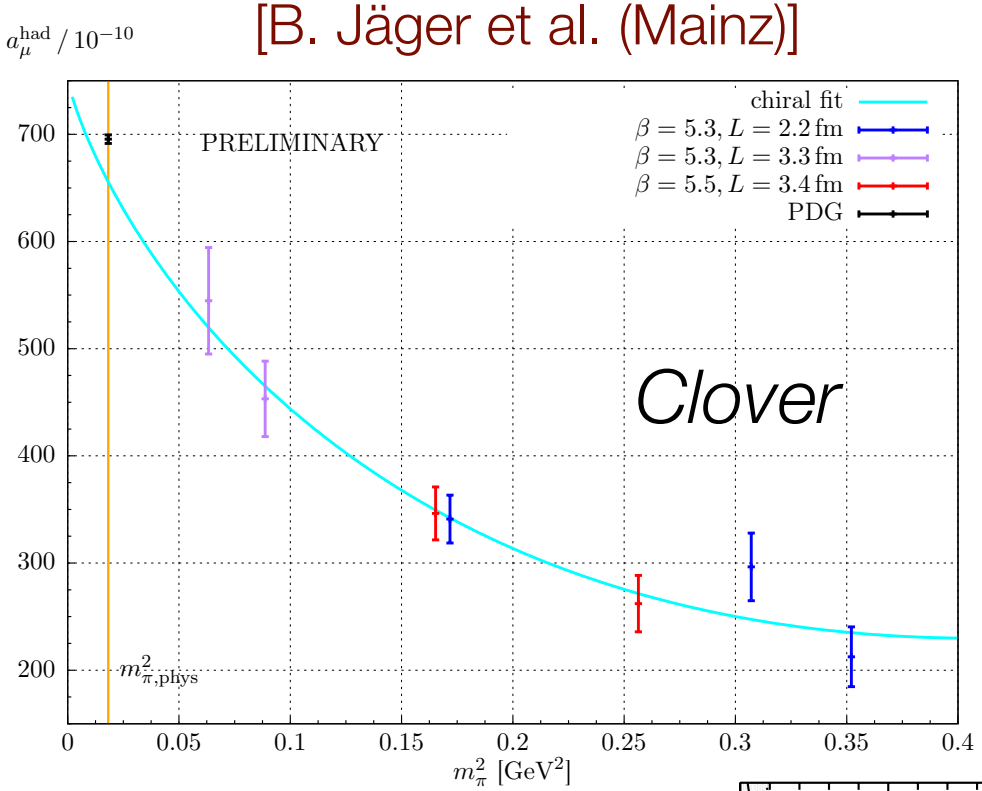
[B. Jäger et al. (Mainz)]



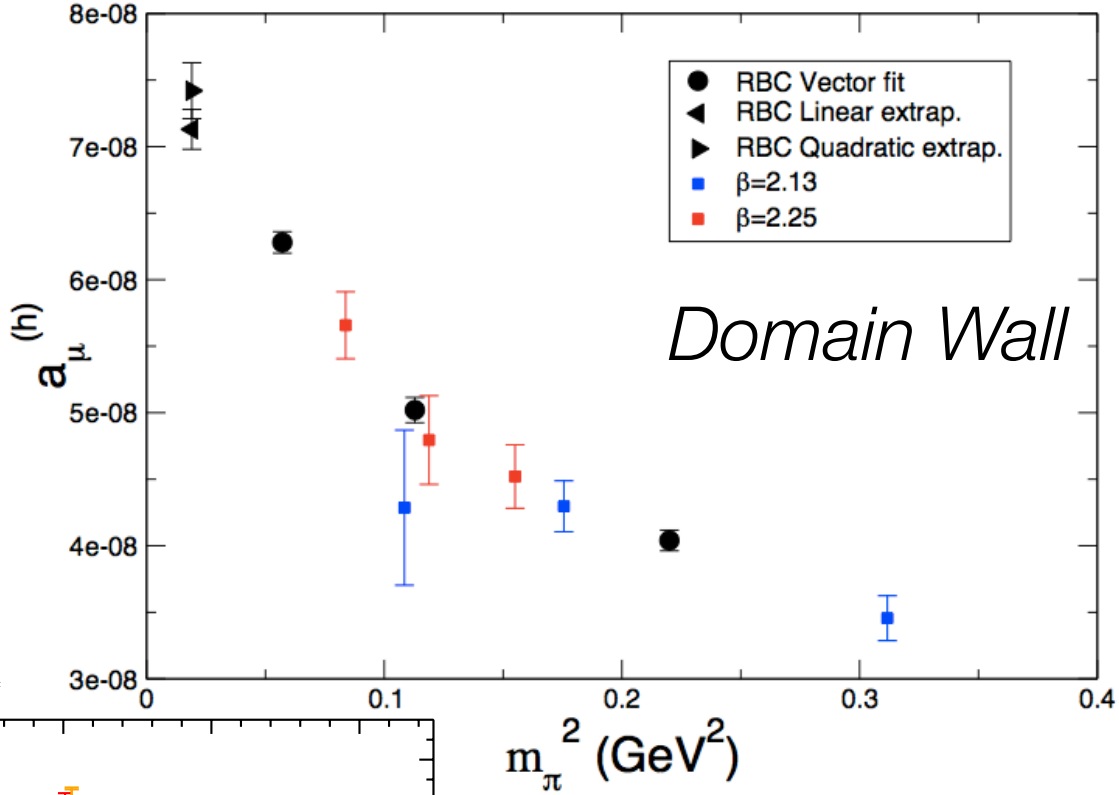
Muon Anomalous Magnetic Moment

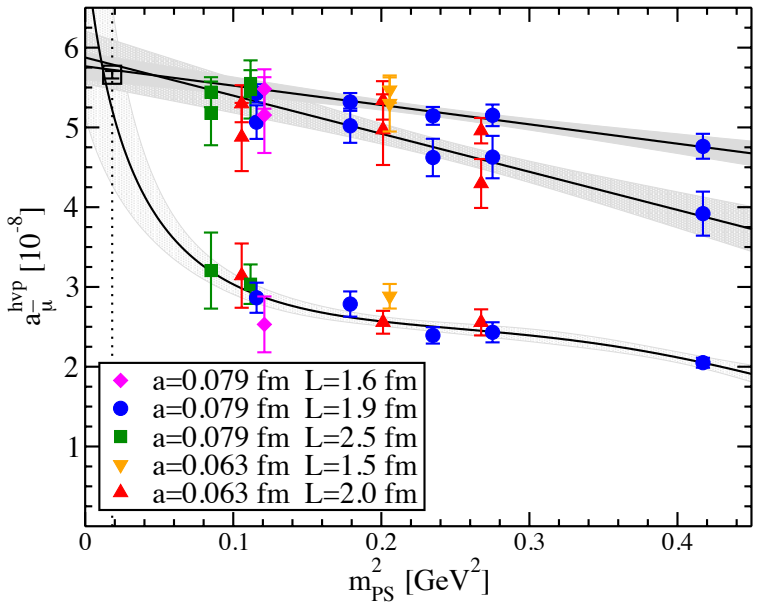
Recent results

[B. Jäger et al. (Mainz)]



[E. Kerrane et al., (Edinburgh)]





Twisted Mass

Feng et al., (ETMC)
arXiv:1103.4818 [hep-lat]

Conclusions & Outlook

- Lattice hadronic matrix elements becoming available with near physical masses
- Nucleon Form Factors: Evidence for chiral curvature in the lqm regime for r_1, r_2, κ
- Pion Form Factor: twisted boundary conditions allow results to be obtained at small Q^2 - reliable determination of charge radius
- g_A still a challenge:
  - Finite volume effects go in the right direction, but are they enough
  - Renormalisation? Excited state contamination?
- $\langle x \rangle$results still show no sign of “bending down” towards phenomenological point. Finite effects? Renormalisation?
- Muon g-2: results with several fermion actions becoming available at light masses
  - Sensitivity to Q^2 region helped with tbcs