

# Quark mass dependence of light resonances and phase shifts in elastic $\pi\pi$ and $\pi K$ scattering

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- Motivation
- Phase shifts  $M_\pi$  dependence in **Standard ChPT**
- Phase shifts  $M_\pi$  dependence in **Unitarized ChPT**
- Comparison of **ChPT and lattice results**
- **Light resonances** dependence on  $\hat{m}$
- Summary

# Motivation

**Lattice:** rigorous QCD results with quarks and gluons.  
 Growing interest in scattering and scalar sector.  
 Caveat: small, realistic quark masses are difficult to implement.

**ChPT:** QCD dependence on quark masses as an expansion.

We can compare:

Lattice multi-hadron states calculations $\rightarrow$ phase shifts and scattering lengths	vs.	standard ChPT (model independent) or UChPT (to go higher in $\sqrt{s}$ )
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Lattice spectrum calculations $\rightarrow$ masses	vs.	UChPT
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# Standard Chiral Perturbation Theory

# Chiral Perturbation Theory Weinberg, Gasser & Leutwyler

Low energy effective theory of QCD with:

- DOF: Pseudo-Goldstone Bosons of the spontaneous chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

$$N_f=2 \rightarrow \pi\text{'s}$$

$$N_f=3 \rightarrow \pi\text{'s, } K\text{'s and } \eta$$

- expansion in masses and momenta

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

- parameters: Low Energy Constants (LECs)
  - $N_f=2 \rightarrow 4$   $l$ 's (one loop) and 7  $r$ 's (two loops)
  - $N_f=3 \rightarrow 8$   $L$ 's (one loop)

## $\pi\pi$ scattering in SU(2) standard ChPT:

- Already calculated to 1 and 2 loops\*, we study the phases dependence on  $\hat{m} = \frac{m_u+m_d}{2}$ .
- Advantages:
  - **SISTEMATIC EXPANSION, MODEL INDEPENDENT**
  - some lattice groups already giving results for l=2 phases and scattering lengths\*\*
- Limitations:
  - only low energy region
  - no resonances.

\*J. Bijnens, G. Colangelo, G. Ecker, J. Gasser and M. E. Sainio, Phys. Lett. B **374**, 210 (1996)

\*\* K. Sasaki and N. Ishizuka, Phys. Rev. D **78**, 014511 (2008)

## Standard SU(2) ChPT amplitudes with LECs from

G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603**, 125 (2001)

$O(p^4)$ LECs ( $\times 10^{-3}$ )		$O(p^6)$ LECs ( $\times 10^{-4}$ )	
$l_1^r$	$-3.98 \pm 0.62$	$r_1^r$	$-0.60$
$l_2^r$	$1.89 \pm 0.23$	$r_2^r$	$1.28$
$l_3^r$	$0.82 \pm -3.80$	$r_3^r$	$-1.68$
$l_4^r$	$6.17 \pm 1.39$	$r_4^r$	$-1.00$
		$r_5^r$	$1.52 \pm 0.42$
		$r_6^r$	$0.40 \pm 0.04$

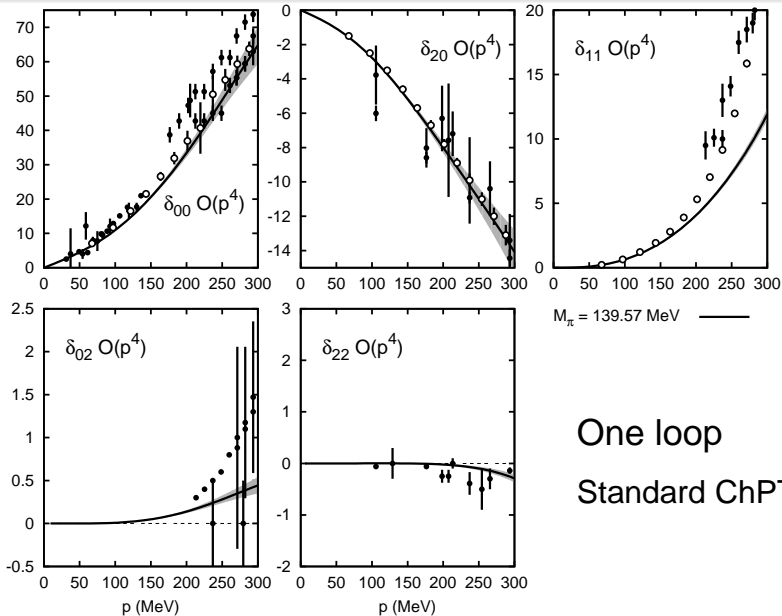
Statistical error, not systematic

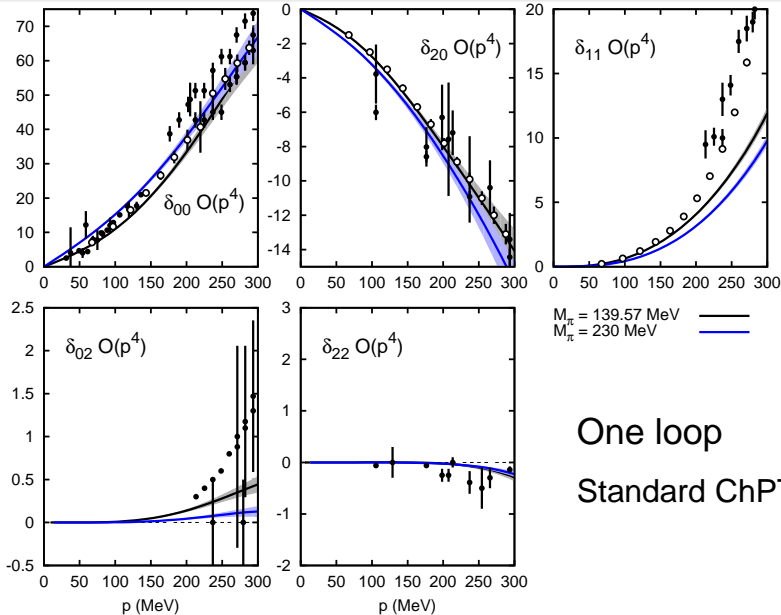
Change  $\hat{m} \Rightarrow$  change on  $M_\pi^2 = 2\hat{m}B_0 \Rightarrow$  change on  $f_\pi$ (one more  $O(p^6)$  parameter:  $r_4^r \approx 0 \pm 1.2 \times 10^{-4}$ )Uncertainties in phase shifts: **Montecarlo Gaussian Sampling.**

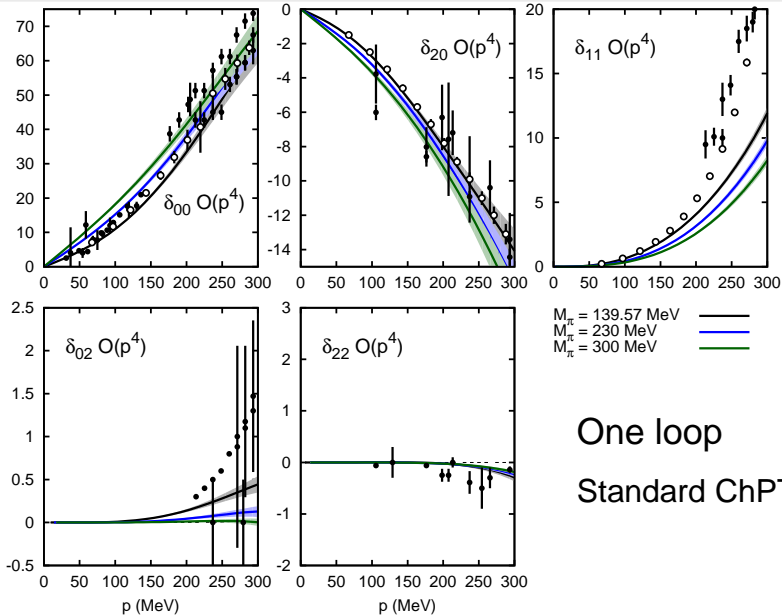
## Phase shifts vs. Momentum, increasing $M_\pi$

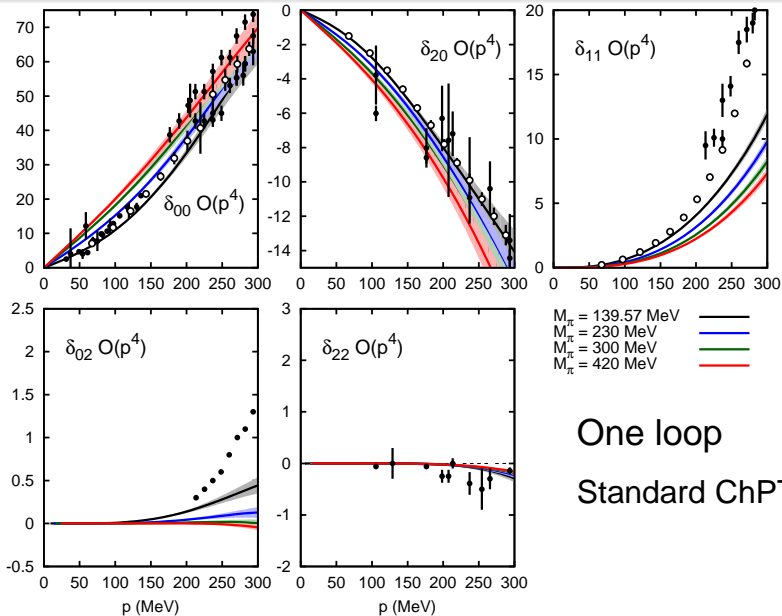
Phases vs. energy  $\rightarrow \hat{m}$  dependence from the threshold's shift.  
Better to plot phases vs. momentum.

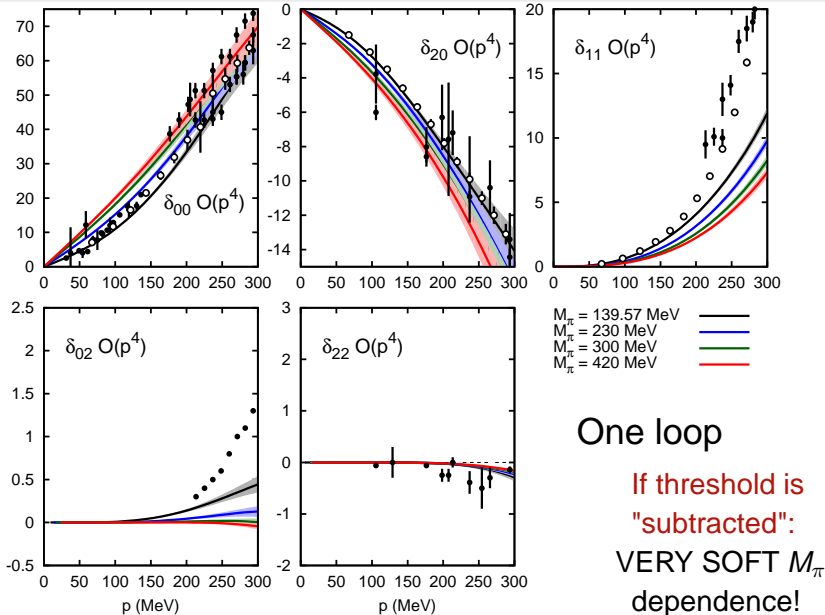


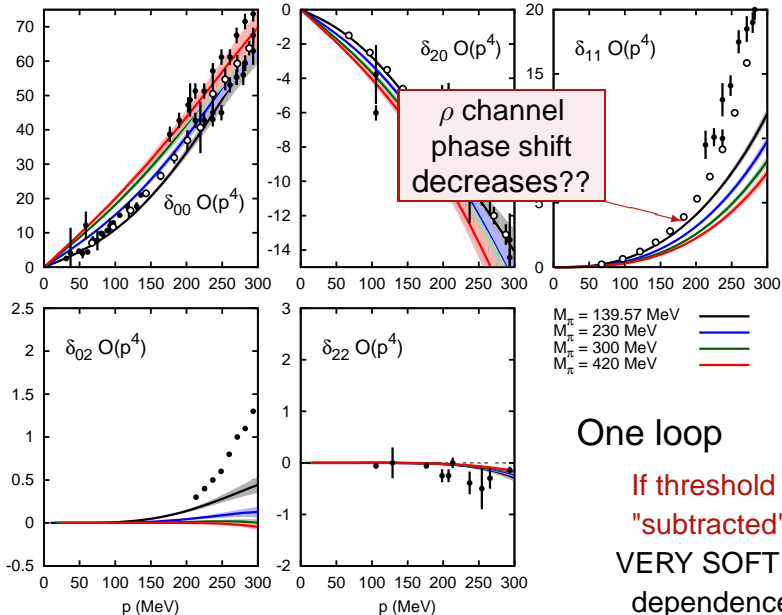


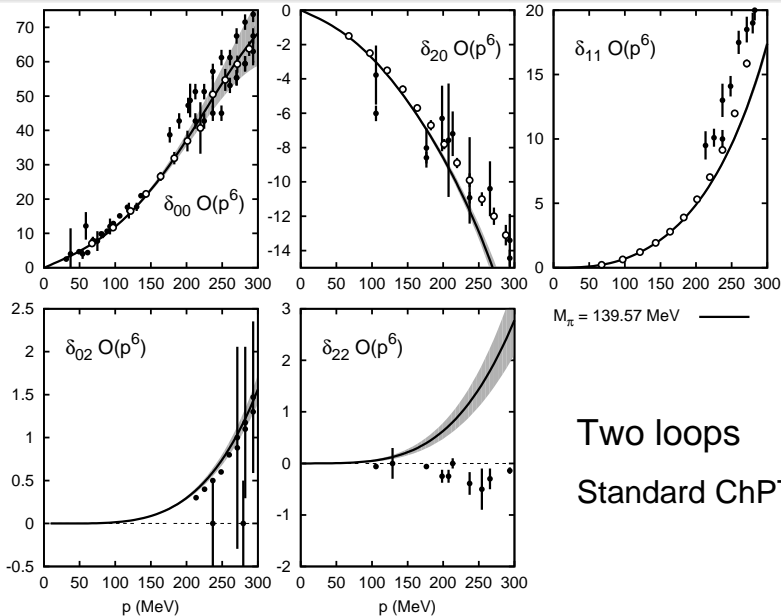


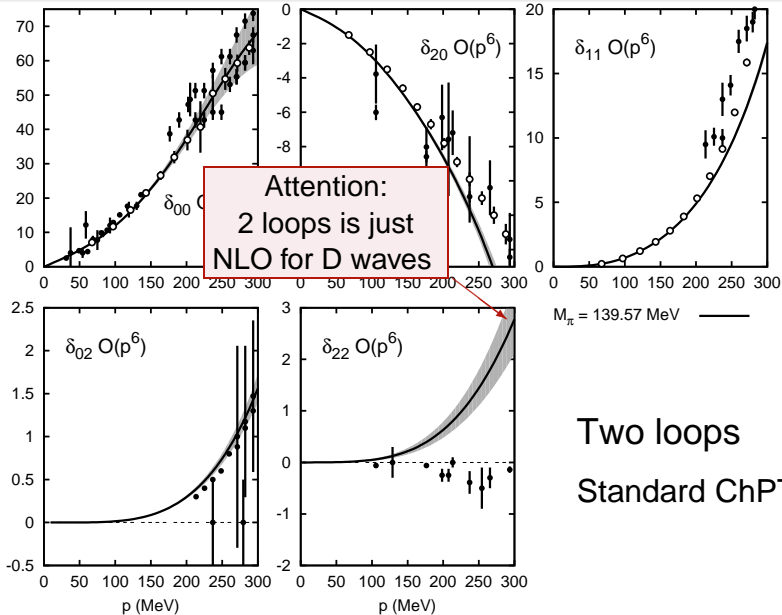




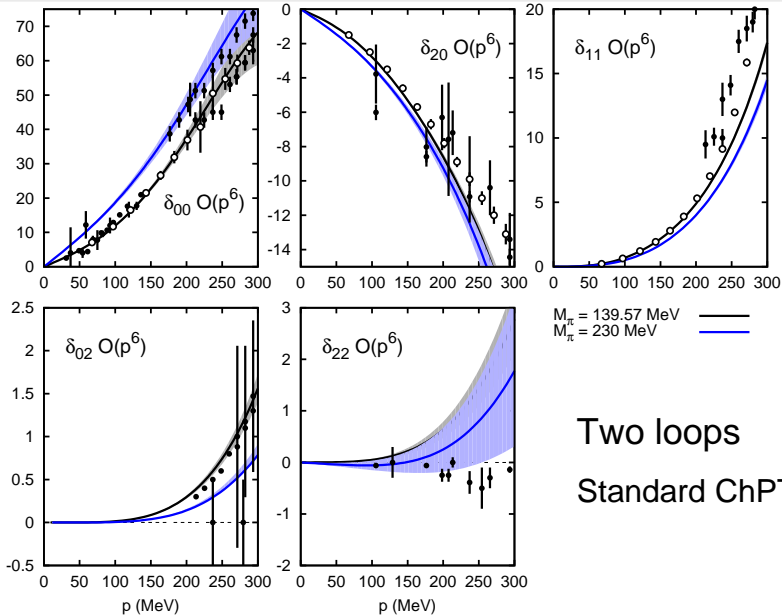


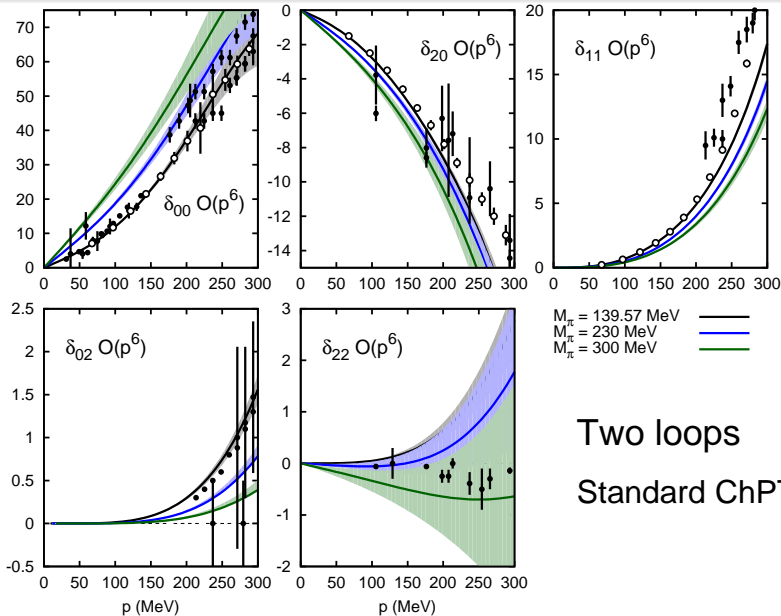


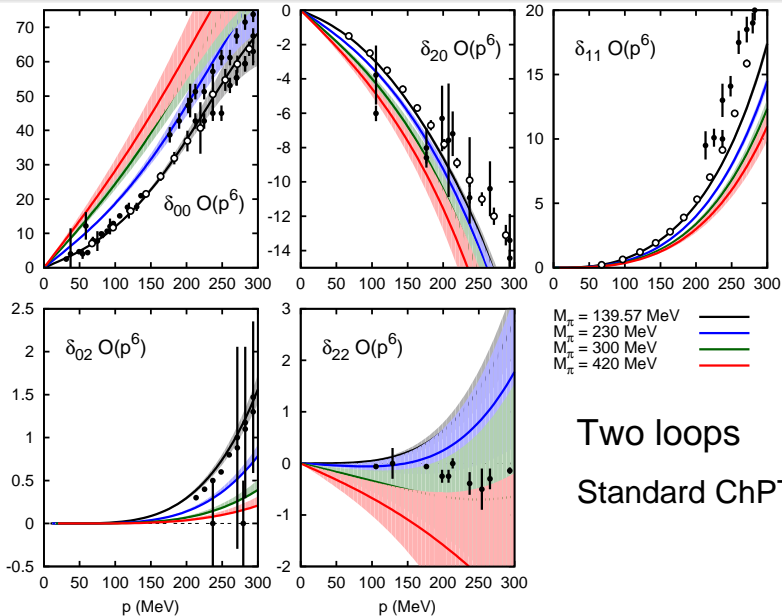


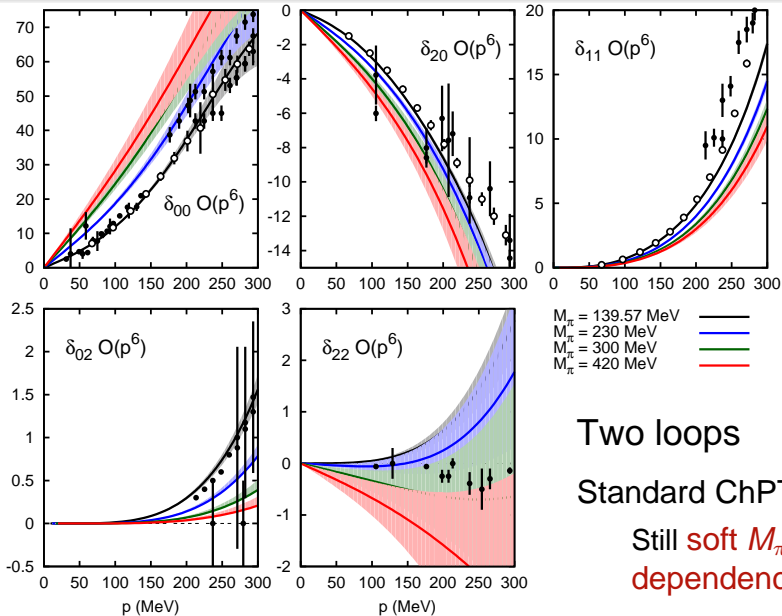


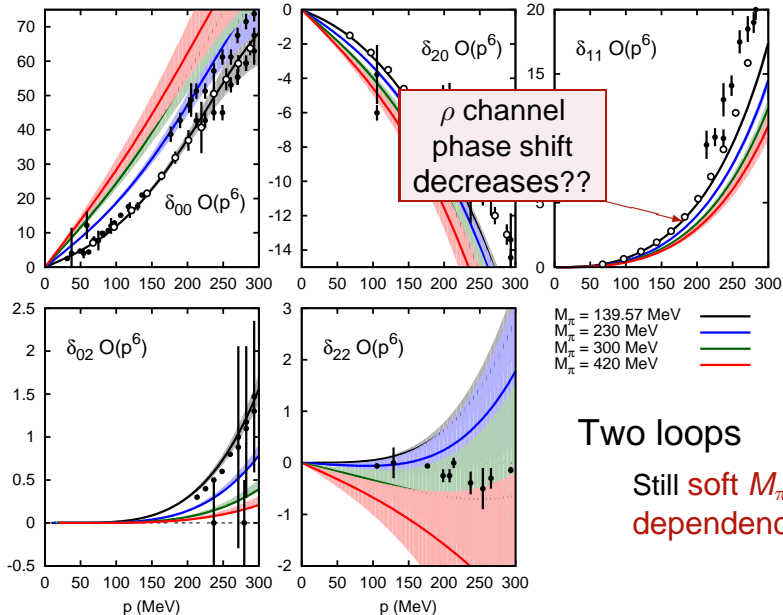












## Unitarized ChPT

# Inverse Amplitude Method

 Truong, Dobado, Herrero, Peláez

Elastic IAM partial waves satisfy exact unitarity

$$\mathbf{S}\mathbf{S}^\dagger = 1 \Rightarrow \text{Im } t^{-1} = -\sigma$$

$O(p^4)$  IAM partial waves:

$$t(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

It is derived from a **dispersion relation**:

- exact on the elastic right cut,
- left cut and subtraction constants approximated within NLO ChPT,
- fully renormalized,
- no spurious parameters.

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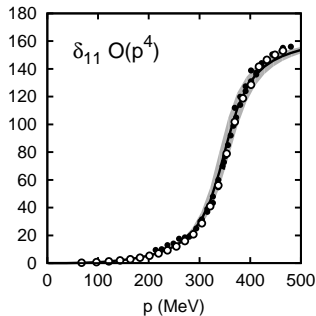
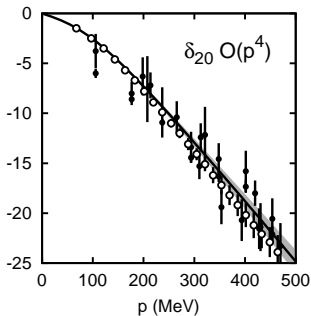
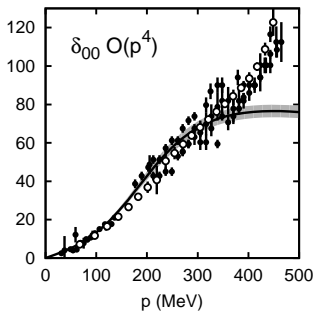


## SU(2) Unitarized ChPT phase shifts vs. Momentum

## Unitarized SU(2) ChPT amplitudes with LECs:

		Two loops		
		Set A	Set D	
<p style="text-align: center;">One loop</p> <hr/> $O(p^4)$ LECs ( $\times 10^{-3}$ ) <hr/>		$O(p^4)(\times 10^{-3})$		
		$l_1^r(\mu)$	-5.0	-4.0
		$l_2^r(\mu)$	1.7	1.2
		$l_3^r(\mu)$	0.8	0.8
		$l_4^r(\mu)$	6.5	6.5
<hr/>		$O(p^6)(\times 10^{-4})$		
		$r_1^r(\mu)$	-0.6	-0.6
		$r_2^r(\mu)$	1.3	1.5
		$r_3^r(\mu)$	-1.7	-3.3
		$r_4^r(\mu)$	2.0	0.9
		$r_5^r(\mu)$	2.0	1.7
		$r_6^r(\mu)$	-0.6	-0.7
		$r_f^r(\mu)$	-1.4	-1.8

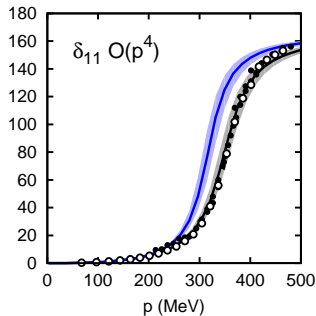
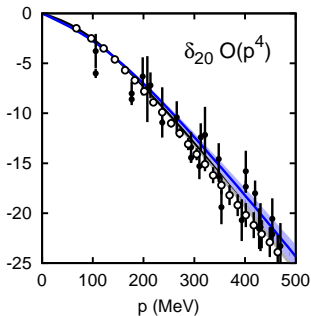
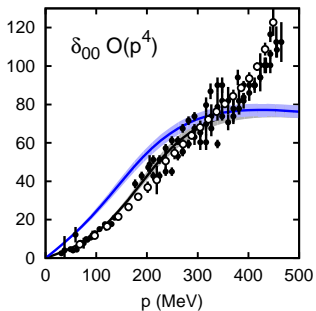
## Unitarized ChPT



$M_\pi = 139.57$  MeV ———

One loop

## Unitarized ChPT

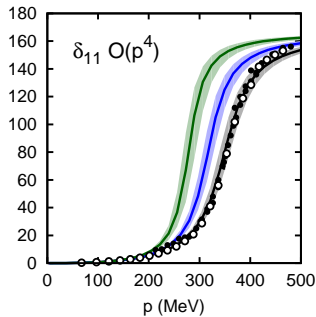
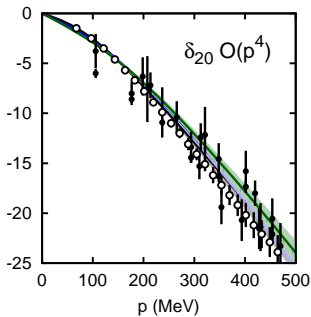
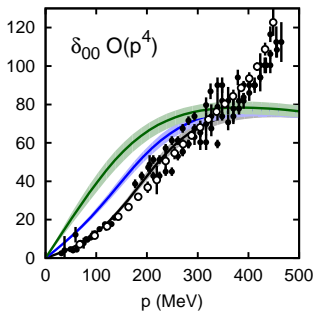


$M_\pi = 139.57$  MeV  
 $M_\pi = 230$  MeV



One loop

## Unitarized ChPT

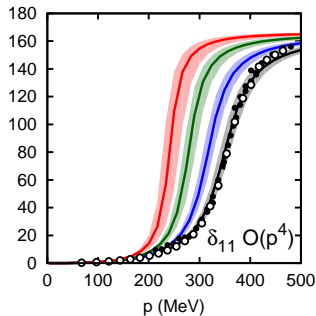
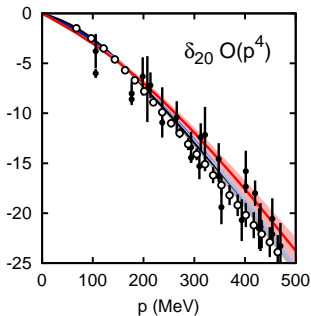
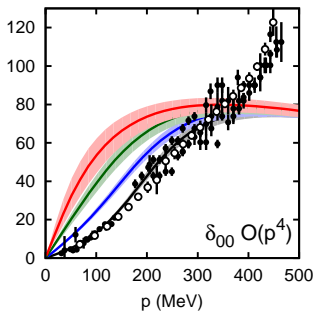


$M_\pi = 139.57$  MeV  
 $M_\pi = 230$  MeV  
 $M_\pi = 300$  MeV



One loop

## Unitarized ChPT

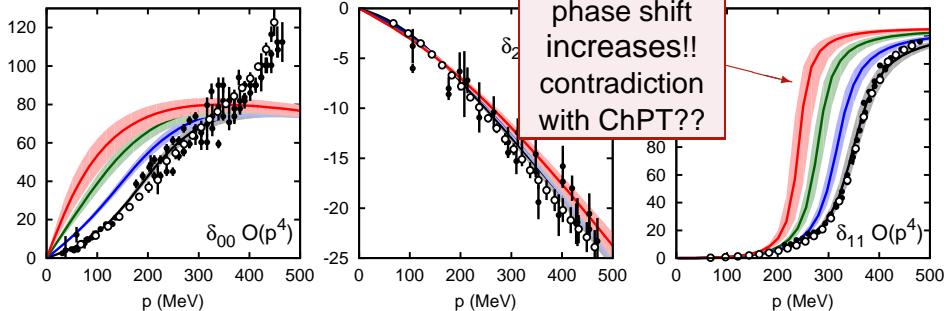


$M_\pi = 139.57$  MeV  
 $M_\pi = 230$  MeV  
 $M_\pi = 300$  MeV  
 $M_\pi = 420$  MeV



One loop

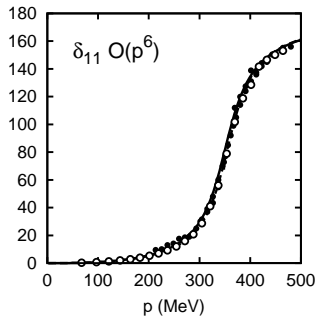
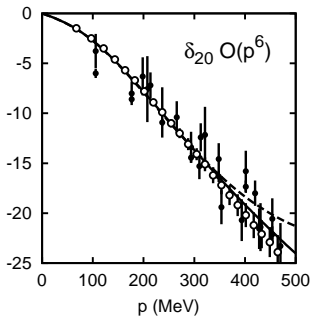
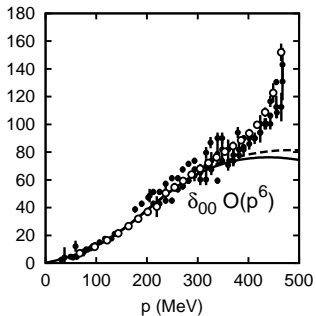
## Unitarized ChPT



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One loop

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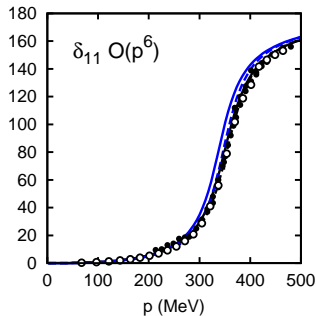
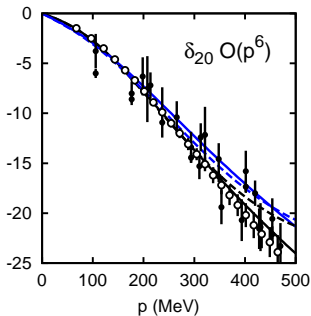
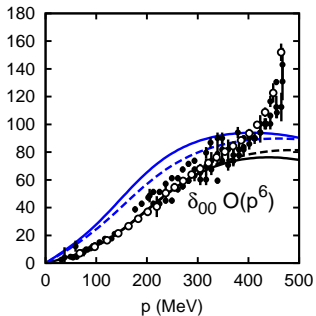


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Two loops



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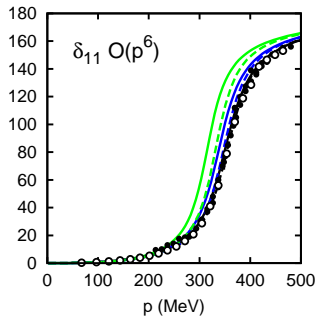
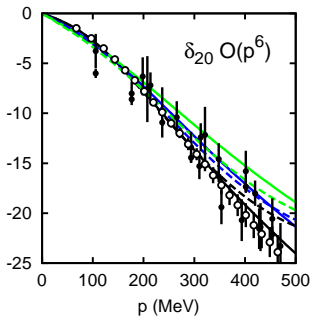
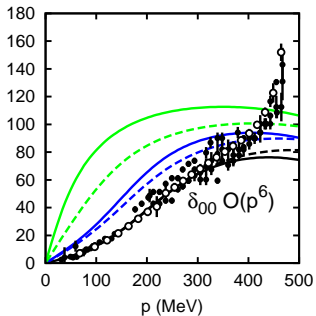


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Two loops

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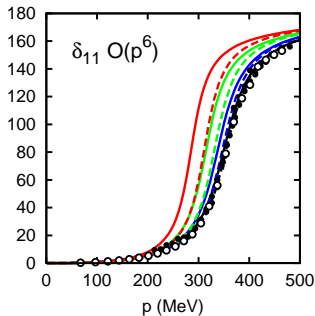
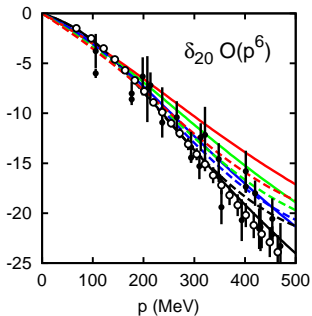
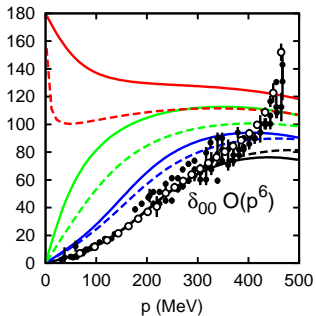


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Two loops

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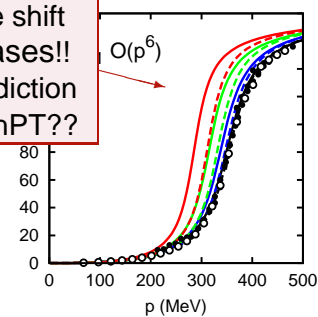
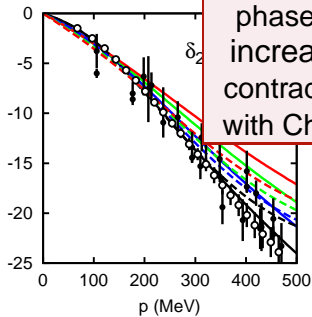
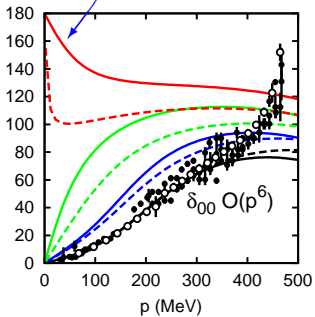
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Two loops

Bound state: phase jumps  $2\pi$   
(Levinson's theorem)

$\rho$  channel  
phase shift  
increases!!  
contradiction  
with ChPT??



$M_\pi = 139.57$  MeV  
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Two loops

Crude, intuitive model of  $I=1$   $J=1$  channel behavior

For a simple Breit-Wigner parametrization:

$$t(s) = \frac{-\sqrt{s}M\Gamma(\rho)/2\rho}{s - M^2 + iM\Gamma(\rho)} \quad \text{with} \quad \Gamma(\rho) = \Gamma_R \left( \frac{\rho}{\rho_R} \right)^3$$

we get a **positive** phase shift derivative:

$$\frac{\partial\delta(\rho)}{\partial(M_\pi^2)} = -\frac{\partial\delta(\rho)}{\partial(\rho_R^2)} = \frac{4M\Gamma(\rho)}{(4\rho^2 - 4\rho_R^2)^2 + M^2\Gamma(\rho)^2} > 0.$$

The **phase shift grows** as the  $\rho$  approaches threshold.  
 Intuitive behavior but **opposed** to ChPT at low momentum.

For a simple Breit-Wigner parametrization:

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Introducing Blatt-Weisskopf modification:

$$\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_I(p_R r)}{D_I(p r)} \equiv \tilde{\Gamma}(p) \frac{D_I(p_R r)}{D_I(p r)}$$

the phase shift derivative is given by:

$$\frac{\partial \delta(p)}{\partial (M_\pi^2)} \simeq \frac{1 + p_R^4 (r^2)'}{4p_R^4} M \tilde{\Gamma}(p)$$

Estimation of  $r^2$  matching LO ChPT at low  $p$ :

$$r^2 = \frac{1}{g^2 f_\pi^2} \frac{M}{M_\pi} + O(M_\pi^0) \Rightarrow 1 + p_R^4 (r^2)' = 1 - \frac{M p_R^4}{2g^2 f_\pi^2 M_\pi^3} < 0$$

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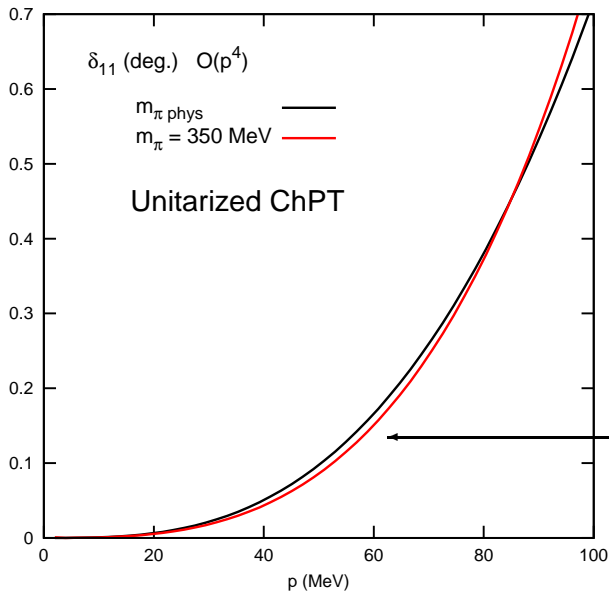
$$\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_I(p_{Rr})}{D_I(pr)} \equiv \tilde{\Gamma}(p) \frac{D_I(p_{Rr})}{D_I(pr)}.$$

the phase shift derivative is given by:

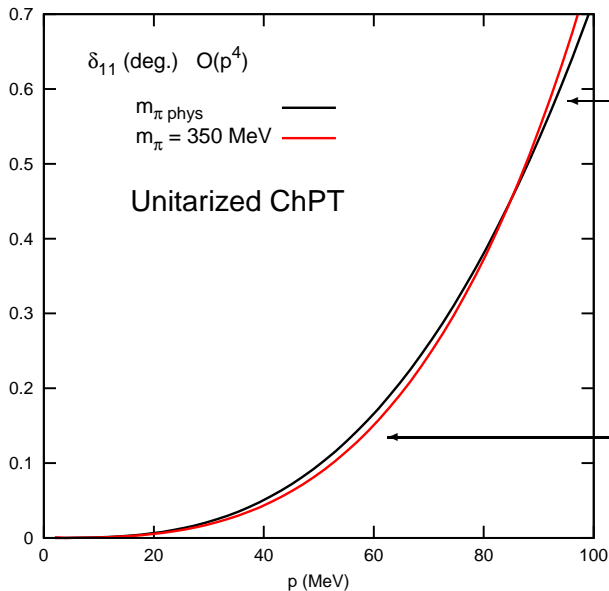
$$\frac{\partial \delta(p)}{\partial (M_\pi^2)} \simeq \frac{1 + p_R^4 (r^2)'}{4p_R^4} M \tilde{\Gamma}(p) < 0$$

The **phase shift goes down** for low  $p$  and near  $M_\pi = M_\pi^{phys}$

**Agreement with standard and unitarized ChPT.**



At low  $p$  the phase shift **decreases** as in standard ChPT



However at higher  $p$   
the phase shift **grows**

At low  $p$  the phase shift  
**decreases**  
as in standard ChPT



## Standard and unitarized ChPT phase shifts vs. lattice results

### ChPT

J. Nebreda, J.R. Peláez and G. Ríos, Phys. Rev. D 83: 094011(2011)

### Lattice

J. Dudek et al., Phys.Rev. D 83: 071504 (2011)

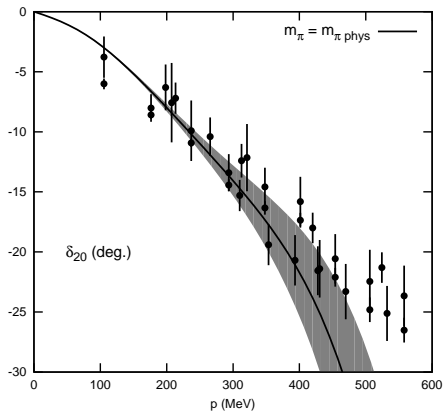
K. Sasaki and N. Ishizuka, Phys. Rev. D 78, 014511 (2008)

## Scalar $I=2$ wave - one loop

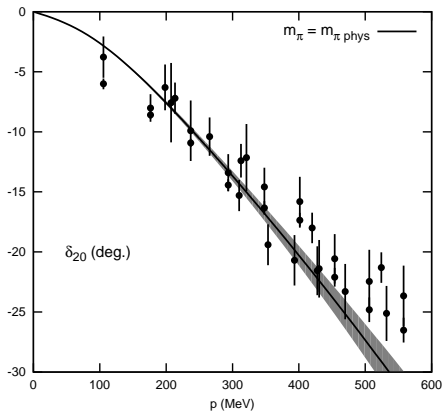
$I=2$   $J=0$  phase shift at one loop

$$M_{\pi} = 139.57 \text{ MeV}$$

Standard ChPT

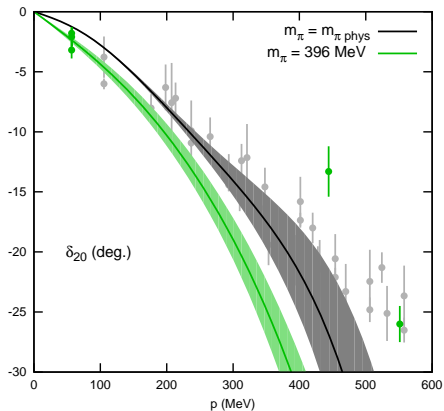


Unitarized ChPT

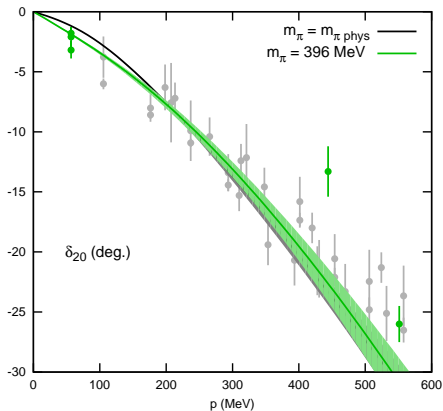


$I=2$   $J=0$  phase shift at one loop $M_{\pi}=396$  MeV

Standard ChPT

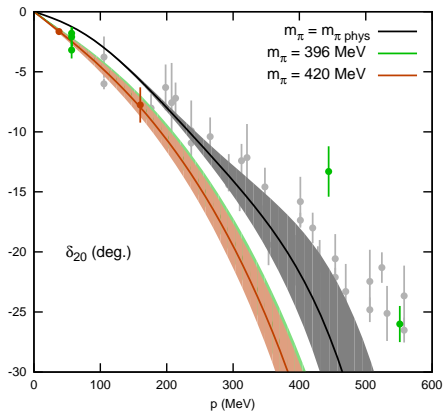


Unitarized ChPT

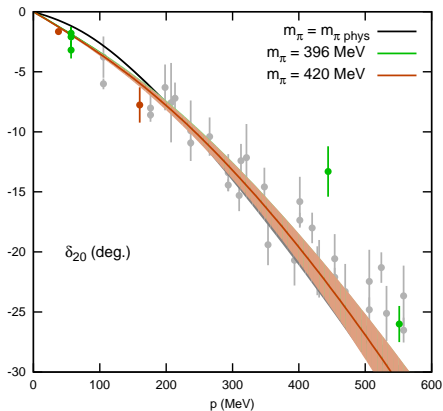


$I=2$   $J=0$  phase shift at one loop $M_{\pi}=420$  MeV

Standard ChPT

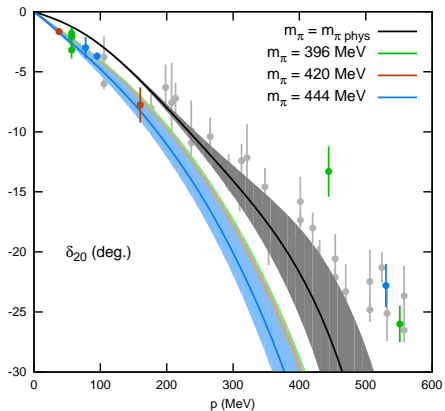


Unitarized ChPT

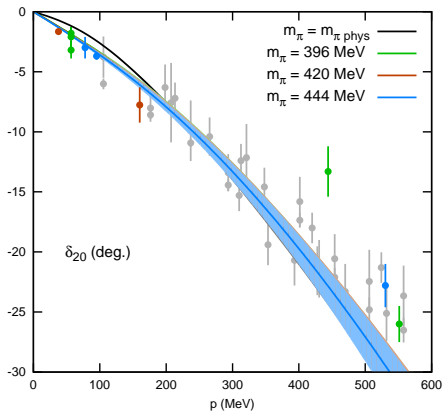


$I=2$   $J=0$  phase shift at one loop $M_{\pi}=444$  MeV

Standard ChPT

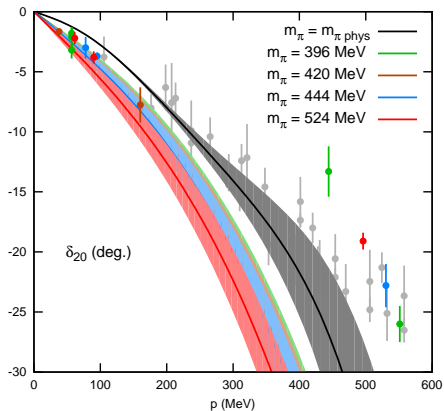


Unitarized ChPT

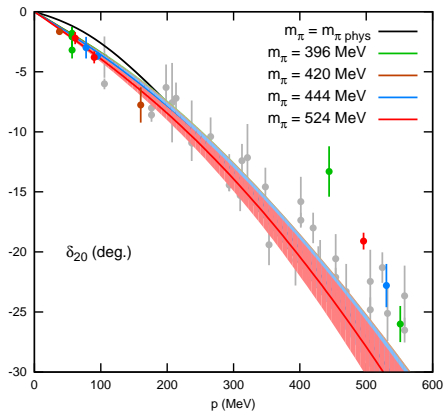


$I=2$   $J=0$  phase shift at one loop $M_\pi=524$  MeV

Standard ChPT

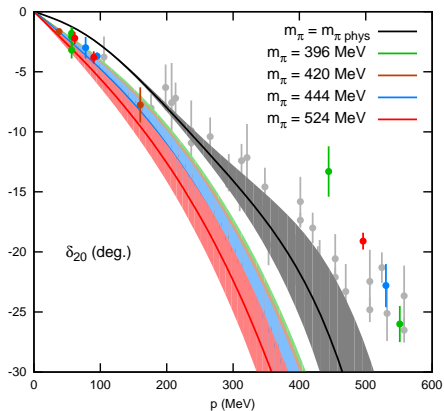


Unitarized ChPT



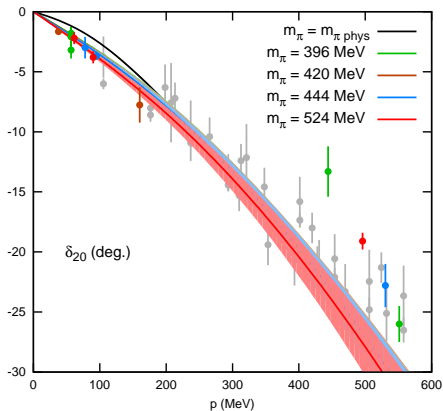
$I=2$   $J=0$  phase shift at one loop $M_\pi=524$  MeV

Standard ChPT



■ Limited to very low momenta

Unitarized ChPT



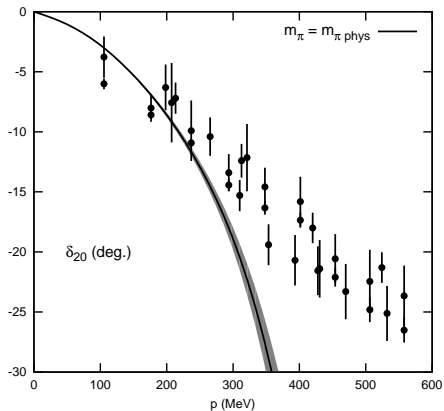
■ Improves behavior at higher momenta



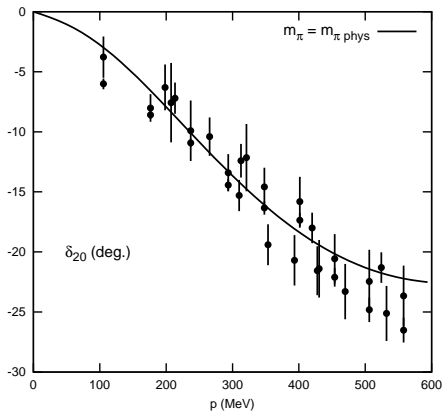
## Scalar $I=2$ wave - two loops

$I=2$   $J=0$  phase shift at two loops $M_\pi = 139.57$  MeV

Standard ChPT

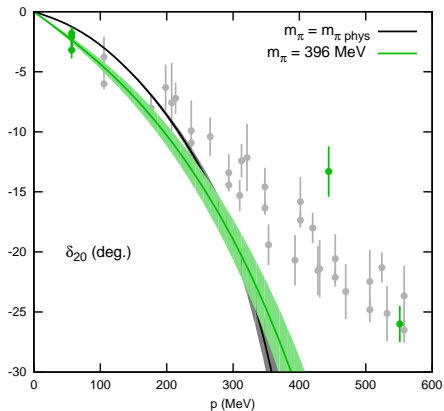


Unitarized ChPT

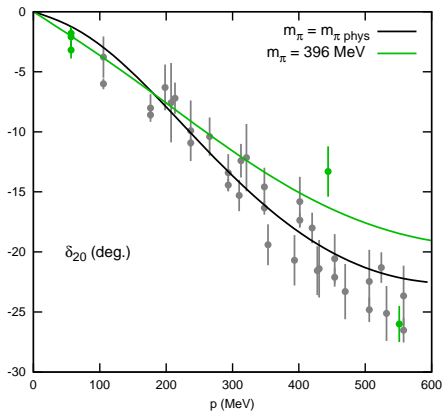


$I=2$   $J=0$  phase shift at two loops $M_\pi=396$  MeV

Standard ChPT



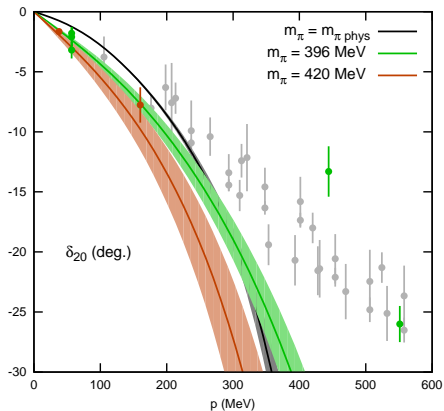
Unitarized ChPT



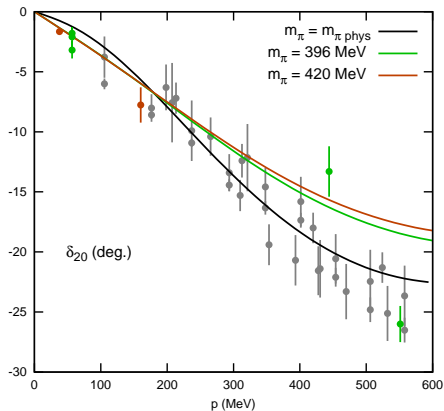
$I=2$   $J=0$  phase shift at two loops

$M_\pi=420$  MeV

Standard ChPT

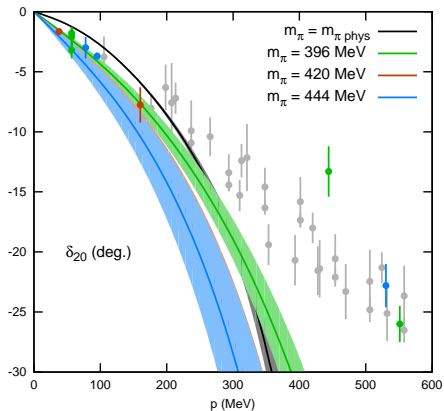


Unitarized ChPT

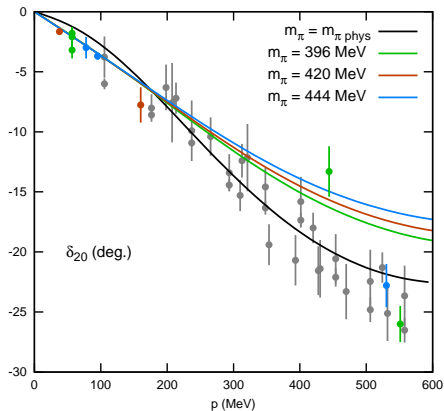


$I=2$   $J=0$  phase shift at two loops $M_\pi=444$  MeV

Standard ChPT

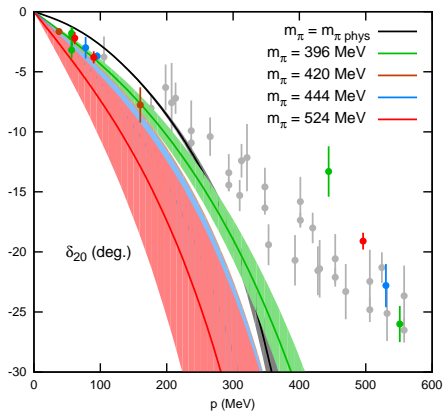


Unitarized ChPT

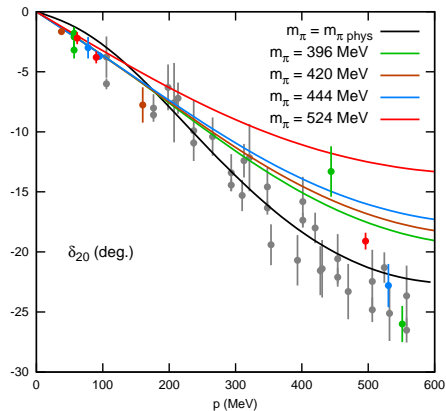


$I=2$   $J=0$  phase shift at two loops $M_\pi=524$  MeV

Standard ChPT



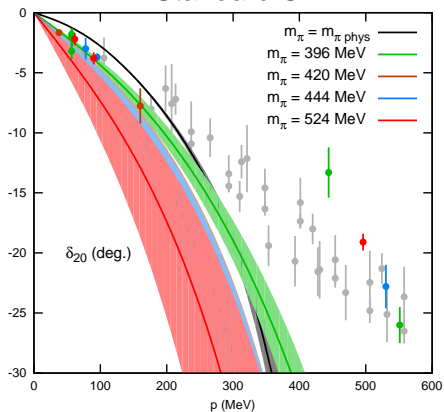
Unitarized ChPT



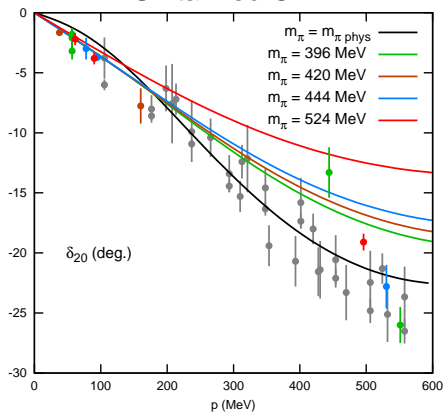
$I=2$   $J=0$  phase shift at two loops

$$M_\pi = 524 \text{ MeV}$$

Standard ChPT



Unitarized ChPT



- Bends down faster than 1 loop
- No improvement

- Works better than Standard ChPT at high  $p$
- No clear improvement either

## Tensor $I=2$ wave

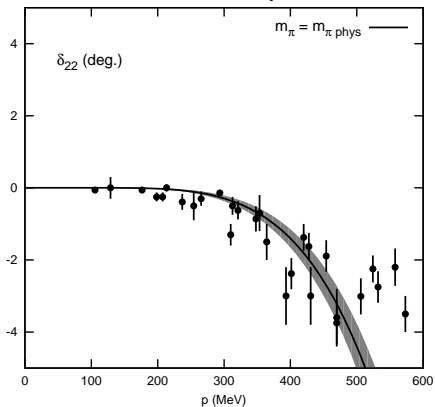
D waves are zero at tree level:

- IAM cannot be applied at one or two loops
- one and two-loops amplitudes are only LO and NLO

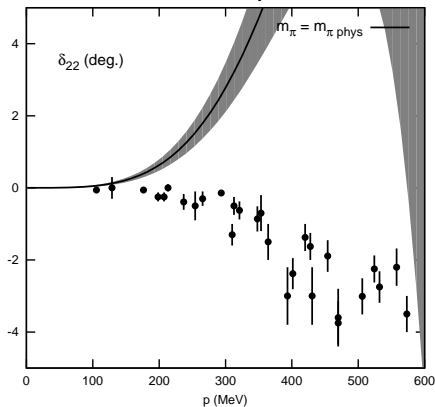


$I=2$   $J=2$  phase shift in standard ChPT  $M_\pi=139.57$  MeV

One loop

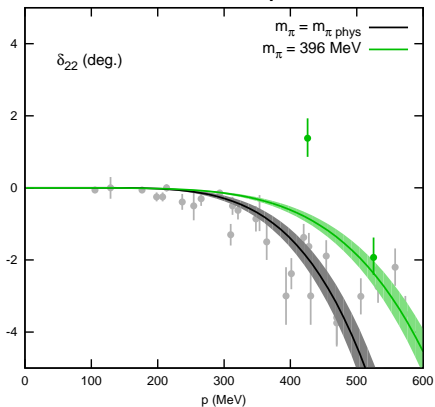


Two loops

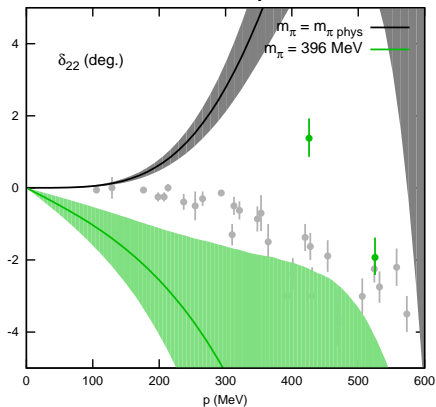


$I=2$   $J=2$  phase shift in standard ChPT $M_\pi=396$  MeV

One loop

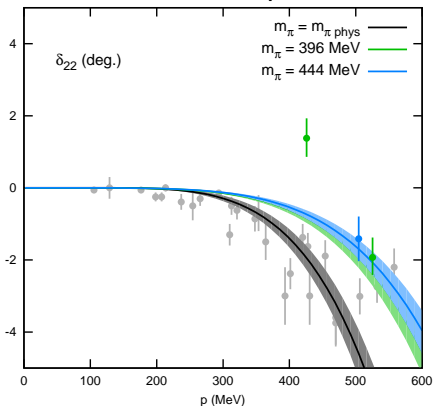


Two loops

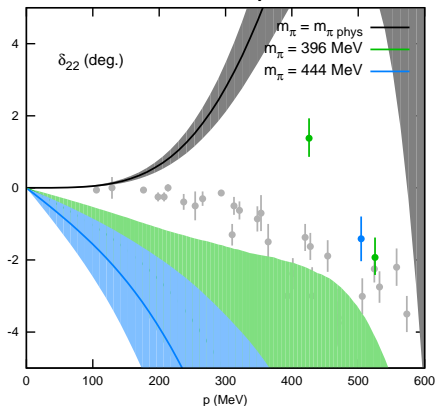


$I=2$   $J=2$  phase shift in standard ChPT $M_\pi=444$  MeV

One loop

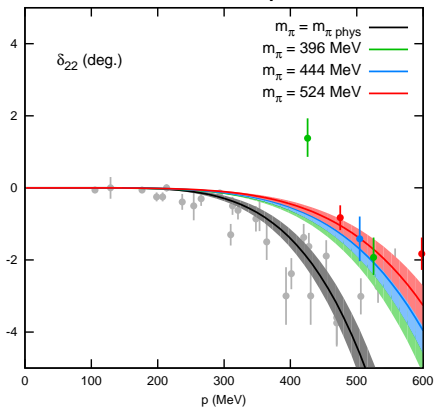


Two loops

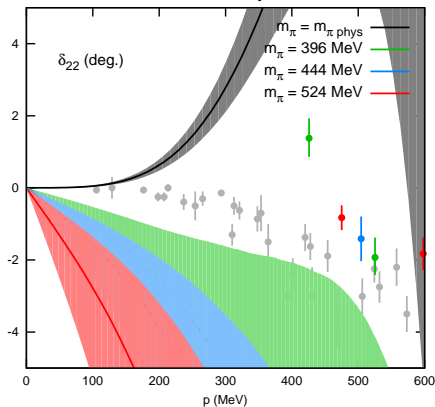


$I=2$   $J=2$  phase shift in standard ChPT $M_\pi=524$  MeV

One loop

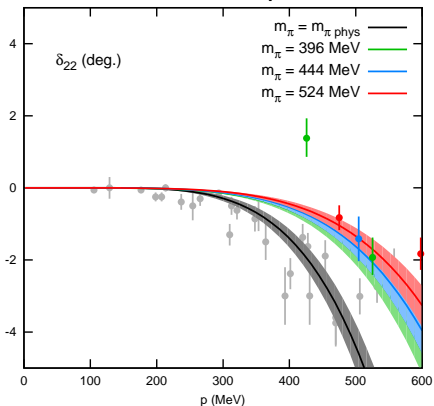


Two loops



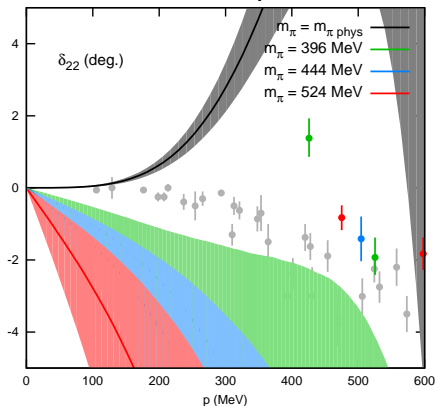
$I=2$   $J=2$  phase shift in standard ChPT $M_\pi=524$  MeV

## One loop



■ Works up to higher  $p$

## Two loops



■ No improvement

## Scalar and vector mesons dependence on $M_\pi$

# Quark mass dependence

Generalization to  $SU(3)$  of previous work on  $SU(2)^*$ .

Elastic channels:

- $\pi\pi \rightarrow \pi\pi$ : resonances  $\rho$  and  $\sigma$  (comparison to  $SU(2)$  results)
- $\pi K \rightarrow \pi K$ : resonances  $K^*(892)$  and  $\kappa$ .

Change of  $\hat{m} = \frac{m_u+m_d}{2}$  and  $m_s \Rightarrow$

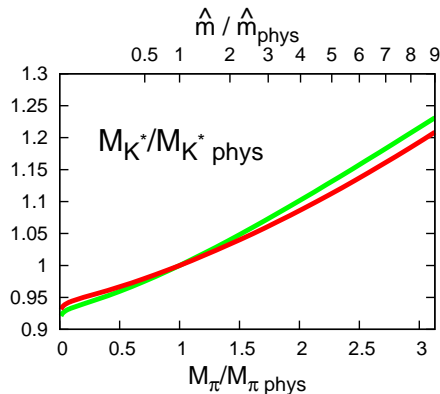
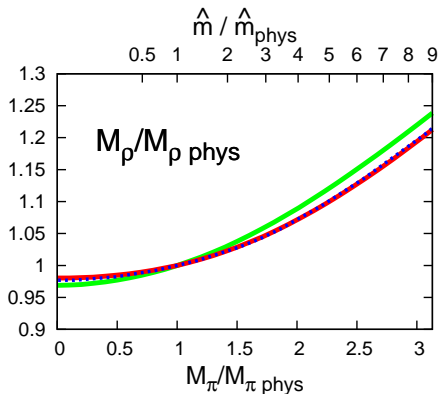
change of  $M_\pi^2, M_K^2, M_\eta^2, f_\pi, f_K, f_\eta$ .

Applicability in  $SU(3)$ :  $0 < M_\pi \lesssim 400 \text{ MeV} \Rightarrow M_K \lesssim 600 \text{ MeV}$   
(Being optimistic!)

\* C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)

Light vector mesons:  $\rho$  and  $K^*(892)$

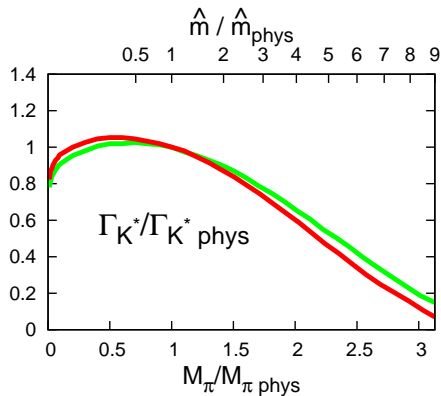
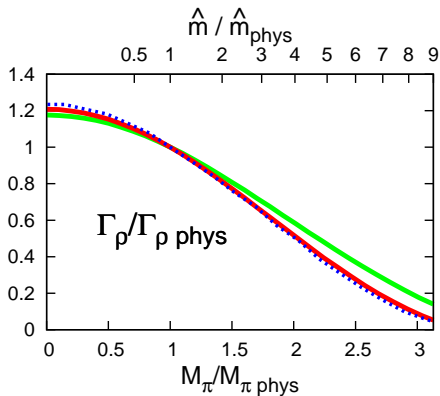


$\hat{m}$  dependence - Light vector mesons - Mass

- Both masses increase slower than  $M_\pi$
- Agreement with SU(2) analysis (blue line)\*

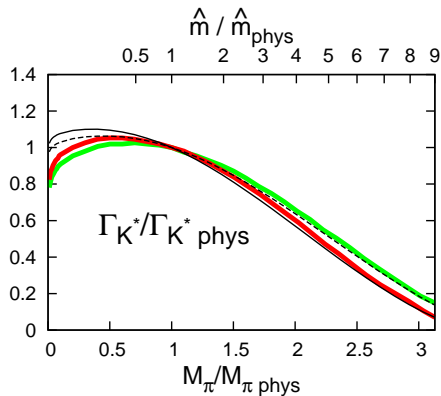
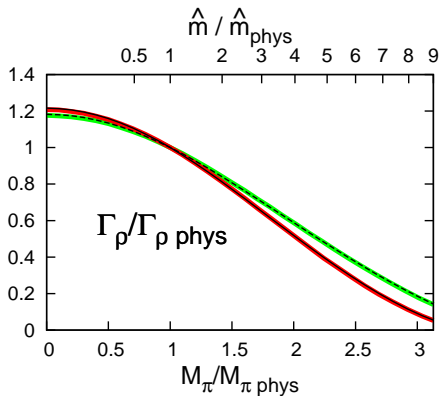
\* C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)

# $\hat{m}$ dependence - Light vector mesons - Width



C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)

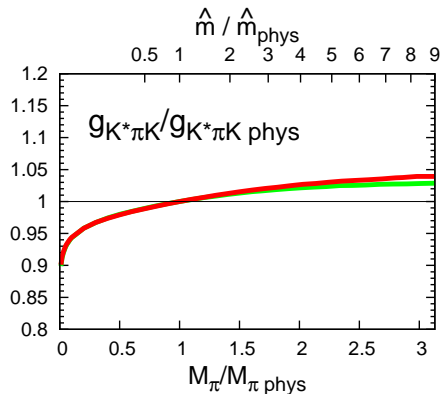
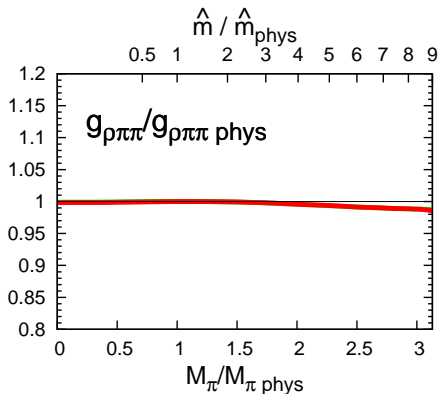
# $\hat{m}$ dependence - Light vector mesons - Width



- Width decrease in accordance with phase space reduction:

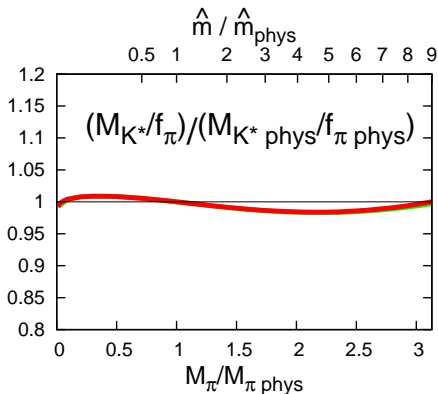
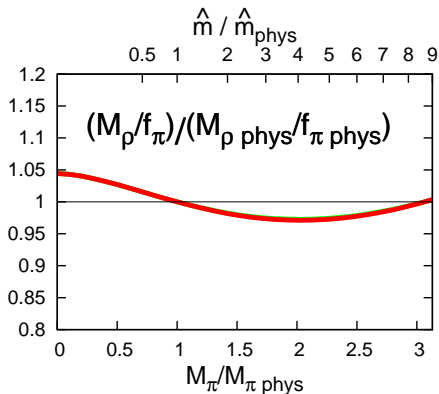
$$\Gamma_V = g^2 \frac{1}{8\pi} \frac{|\mathbf{p}|^3}{M_V^2} \quad (\text{black lines})$$

# $\hat{m}$ dependence - Light vector mesons - Coupling



- Coupling to two mesons independent of  $\hat{m}$  (assumption in some lattice works)

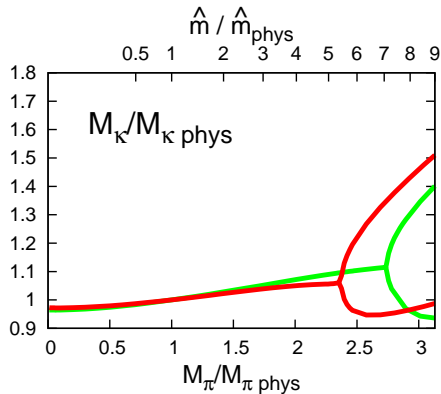
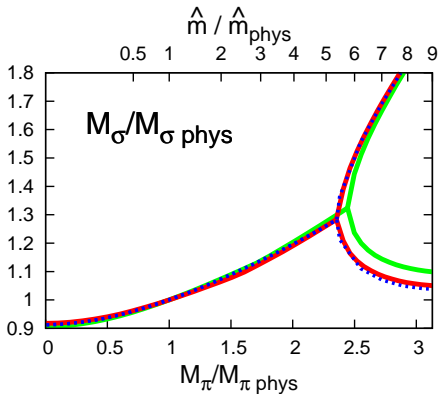
# $\hat{m}$ dependence - Light vector mesons - KSFR



- Fulfill the KSFR relation for different  $\hat{m}$ :

$$g \simeq M_V/2\sqrt{2}f_\pi$$

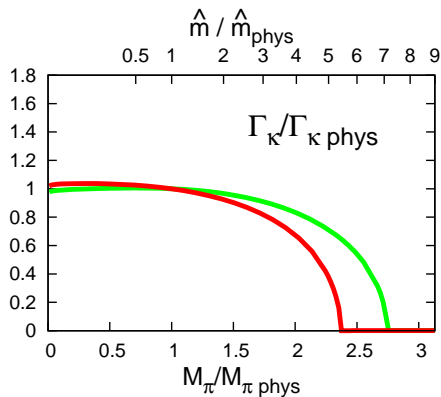
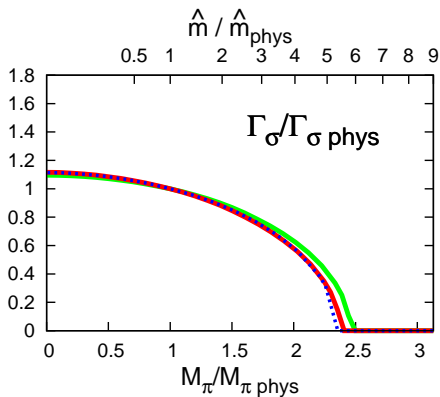
## Light scalar mesons: $\sigma$ and $\kappa$

$\hat{m}$  dependence - Light scalar mesons - Mass

- Mass split into two branches
- Agreement with SU(2) analysis

\* C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)

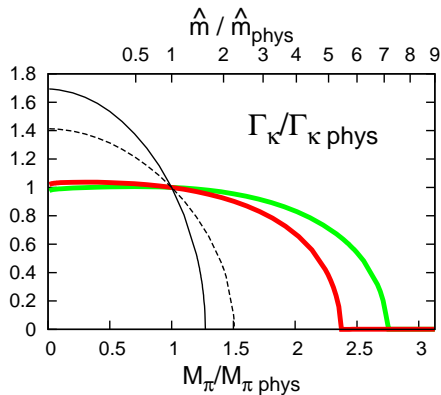
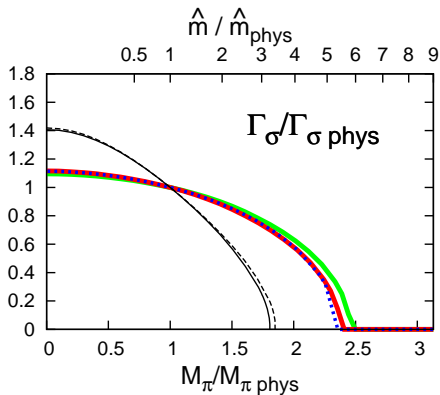
# $\hat{m}$ dependence - Light scalar mesons - Width



C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)



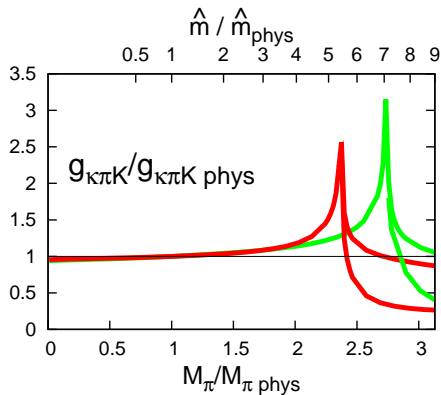
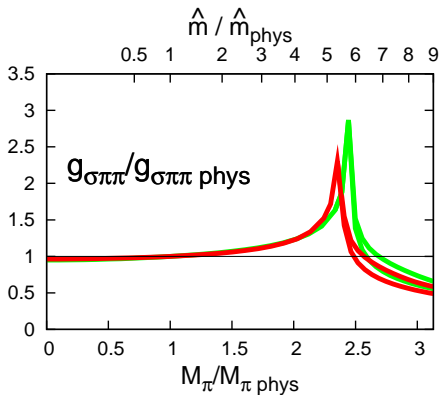
# $\hat{m}$ dependence - Light scalar mesons - Width



- Width decrease not explained by phase space reduction:

$$\Gamma_S = g^2 \frac{1}{8\pi} \frac{|\mathbf{p}|}{M_S^2}$$

# $\hat{m}$ dependence - Light scalar mesons - Coupling



■ Strong  $\hat{m}$  dependence of coupling to two mesons

## Summary

# Summary

Chiral extrapolation of the parameters of the  $\sigma$  ( $f_0(600)$ ),  $\kappa(800)$ ,  $\rho(770)$  and  $K^*(892)$  resonances increasing  $\hat{m}$ .

Vector mesons

Scalar mesons

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- vector resonances mass grows slower than  $M_\pi$ ,

## Scalar mesons

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- very different behavior from vector mesons: two branches,



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- $\sigma$  and  $\kappa$  show different quantitative but similar qualitative behavior,
- coupling to two mesons shows stronger  $M_\pi$  dependence.

# Summary

We have presented recent results for the phase shifts  $M_\pi$  dependence :

Standard ChPT

Unitarized ChPT

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We have presented recent results for the phase shifts  $M_\pi$  dependence :

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- very soft  $M_\pi$  dependence once threshold is "subtracted",

## Unitarized ChPT

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- S2 wave: better agreement with lattice at high  $p$ ,



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- S2 wave: better agreement with lattice at high  $p$ ,
- similar results at one and two loops,
- reconciles  $\rho$  ChPT behavior with naive expectation,

# Summary

We have presented recent results for the phase shifts  $M_\pi$  dependence :

## Standard ChPT

- very soft  $M_\pi$  dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low  $p$ ,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

## Unitarized ChPT

- S2 wave: better agreement with lattice at high  $p$ ,
- similar results at one and two loops,
- reconciles  $\rho$  ChPT behavior with naive expectation,
- bound states seen as  $2\pi$  jump in phase shift (Levinson's).

# Summary

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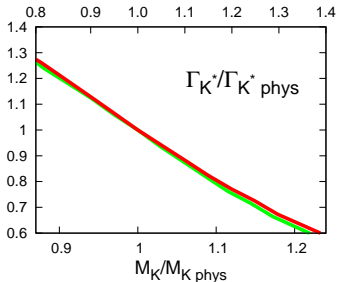
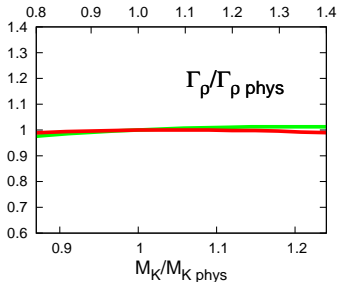
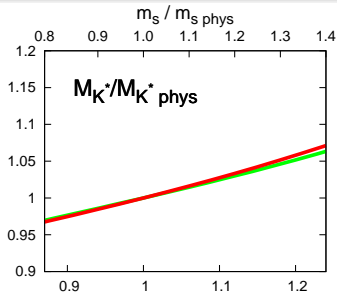
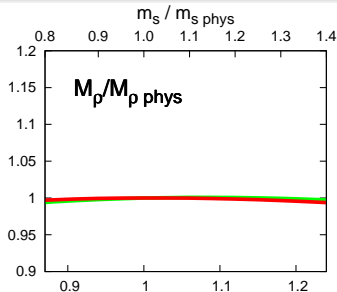
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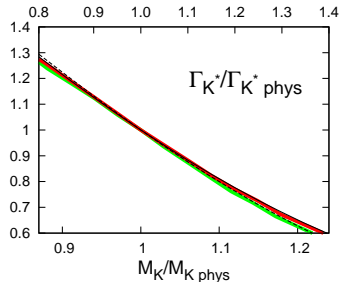
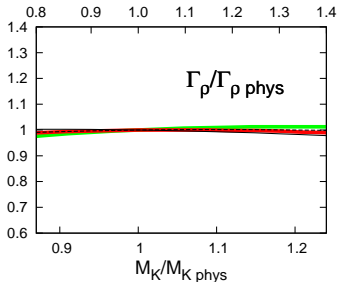
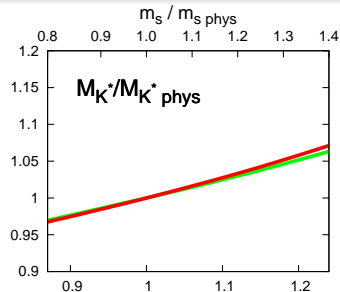
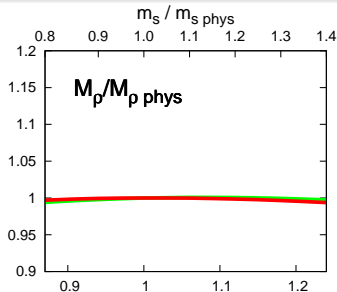
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\*C. Hanhart, J.R. Pelaez and G. Rios, *Phys. Rev. Lett.* **100**, 152001 (2008)

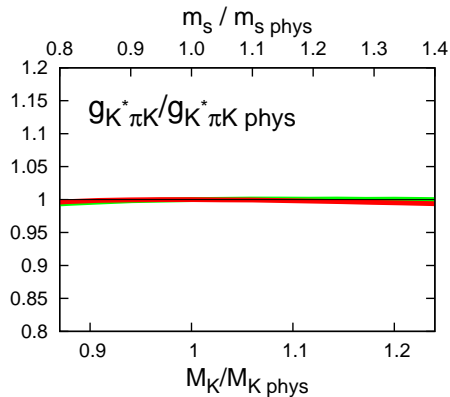
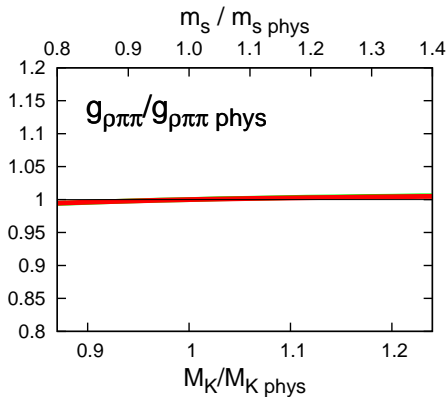
$m_s$  dependence of  $\sigma$ ,  $\rho$ ,  $\kappa$  and  $K^*(892)$

Light vector mesons:  $\rho$  and  $K^*(892)$

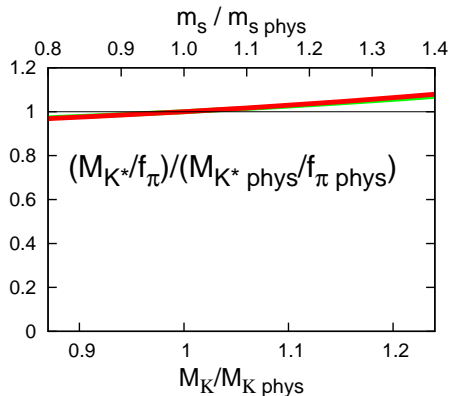
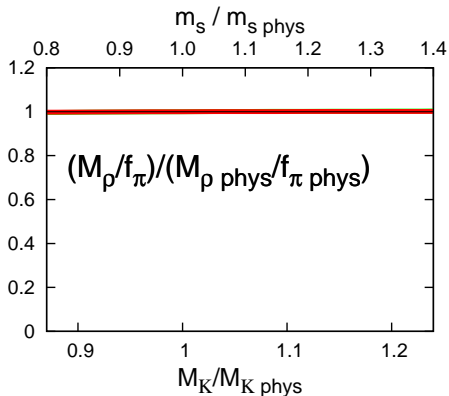
$m_s$  dependence - Light vector mesons - Mass & Width

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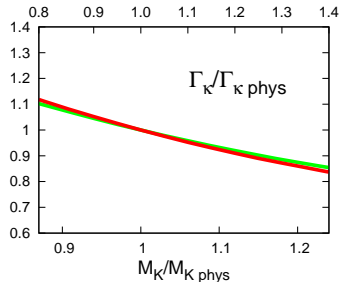
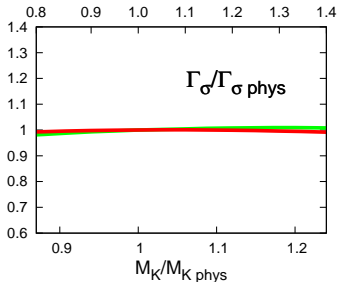
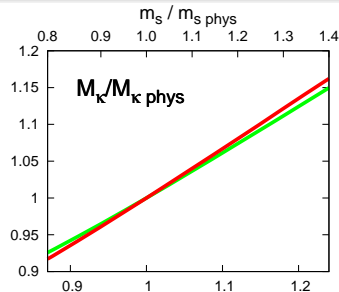
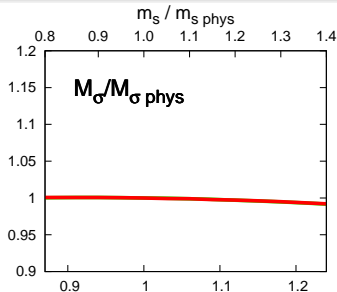
$m_s$  dependence - Light vector mesons - Coupling

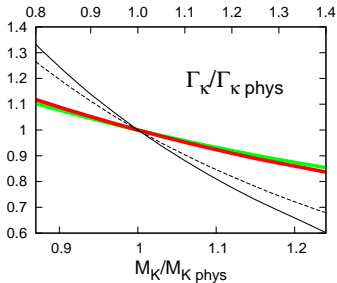
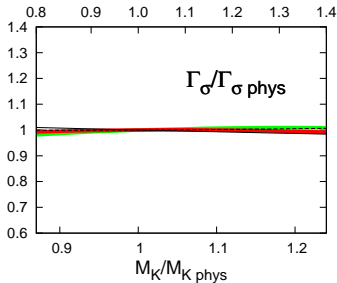
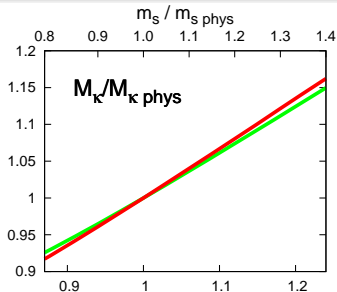
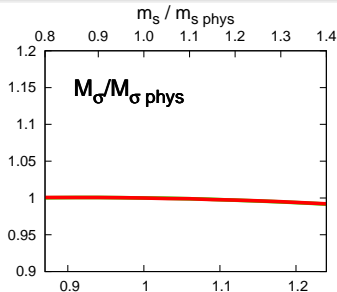
■ Coupling to two mesons constant

$m_s$  dependence - Light vector mesons - KSFR

- KSFR relation well satisfied for different  $m_s$

## Light scalar mesons: $\sigma$ and $\kappa$

$m_s$  dependence - Light scalar mesons - Mass & Width

$m_s$  dependence - Light scalar mesons - Mass & Width

# $m_s$ dependence - Light scalar mesons - Coupling

