Meson Spectroscopy and Resonances

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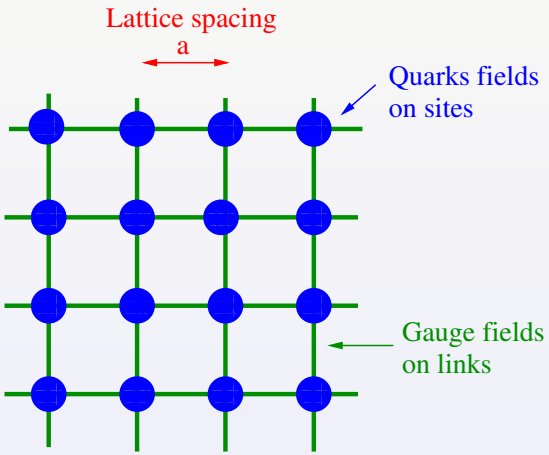
Lattice - a nonperturbative, gauge-invariant regulator for QCD

- Nielson-Ninomiya theorem $\Rightarrow$ chirally symmetric quarks missing, but can discretise quarks by trading-off some symmetries.
- In finite volume, $V = L^4$, finite d.o.f and path-integral is large but finite integral.



- Wick rotation, analytic continuation $t \to i\tau$, $\frac{-i}{\hbar}S \to \frac{i}{\hbar}S$
- Enables importance sampling ie Monte Carlo
- Lose direct access to dynamical properties of the theory like decay widths.

The current landscape

From 2001 [Ukawa, 2001: the “Berlin Wall”]

$$C_{flops} \propto \left(\frac{m_\pi}{m_\rho}\right)^{-6} \times L^5 \times a^{-7}$$

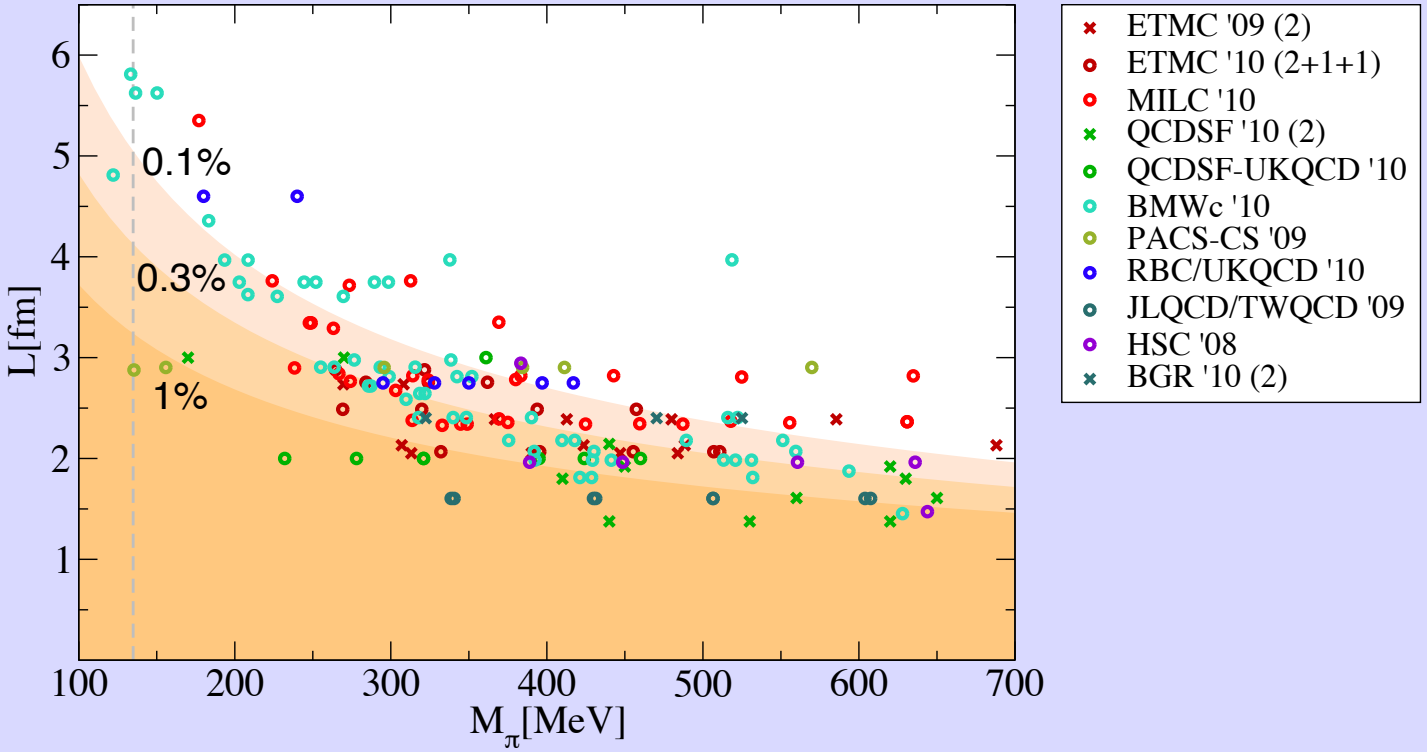
to 2009 [Giusti, 2006]

$$C_{flops} \propto \left(\frac{m_\pi}{m_\rho}\right)^{-2} \times L^5 \times a^{-7}$$

- dramatic improvements in scaling with quark mass [Hasenbusch ’01, Lüscher ’03,04].
- With fall in cost of CPU cores, simulations at physical quark masses possible.

The current landscape

C. Hoelbling, Lattice 2010 arXiv:1102.0410



- Dynamical simulations with $N_f = 2$ or $2 + 1$
- Large volumes, $L \geq$ 3fm $\Rightarrow \mathcal{O}(1\%)$ on m_π.
- Light quark masses, now close to or at m_π.

Disclaimer

Not a review talk. I will discuss challenges and recent progress, showing results from the Hadron Spectrum Collaboration and others.

Other lattice talks at this meeting

- Daniel Mohler 13/6 (17:30)
- James Zanotti 14/6 (10.30)
- Bernhard Musch 14/6 (14:30)
- Robert Edwards 15/6 (12.30) [Baryon Spectroscopy and Resonances]

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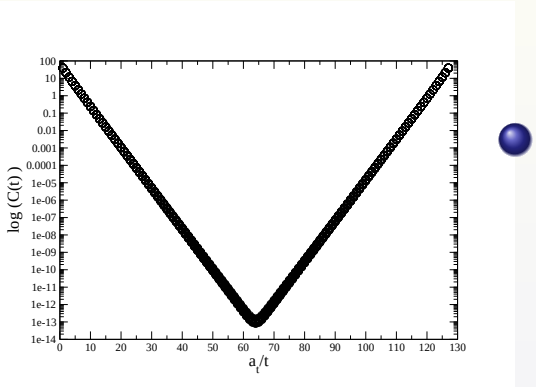
Plan

- spectroscopy: methods, challenges and solutions
- results: light and charm meson spectroscopy
- resonances: challenges and possible solutions
- recent results for light meson resonances

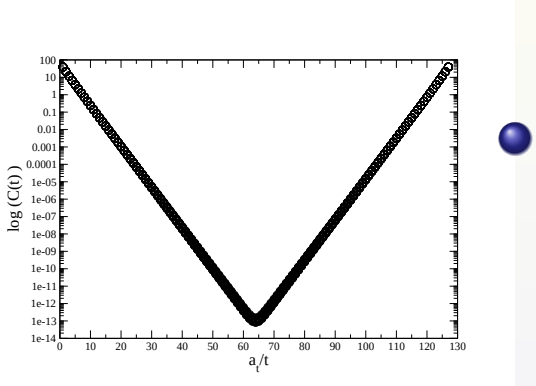
Spectroscopy

Spectroscopy - making measurements



- Energy of a (colorless) QCD state extracted from a two-point function in Euclidean time, $C(t) = \langle \phi(t) | \phi^\dagger(0) \rangle$.
  - Inserting a complete set of states, $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$.
  - Observing the exponential fall of $C(t)$ at large t, the energy can be measured.

Spectroscopy - making measurements



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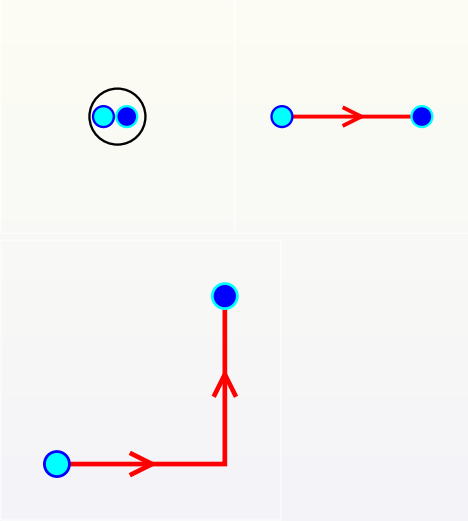
- Observing the exponential fall of $C(t)$ at large t, the energy can be measured.
- Excited state energies from a matrix of correlators: $C_{ij}(t) = \langle \phi_i(t) | \phi_j^\dagger(0) \rangle$.
- Solving a generalised eigenvalue problem $C(t_1)\mathbf{v} = \lambda C(t_0)\mathbf{v}$ gives

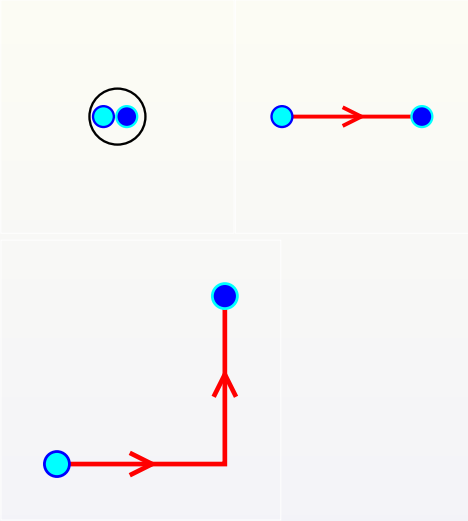
$$\lim_{(t_1-t_0)\to\infty} \lambda_n = e^{-E_n(t_1-t_0)} \ .$$

Spectroscopy - making measurements



- Lattice operators are bilinears with path-ordered products between quark and anti-quark fields; different offsets, connecting paths and spin contractions give different projections into lattice symmetry channels.
- Need ops with good overlap onto low-lying spectrum
- Good idea to smooth fields spatially before measuring: smearing

Spectroscopy - making measurements



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- Need ops with good overlap onto low-lying spectrum
- Good idea to smooth fields spatially before measuring: smearing
  - Distillation [Hadron Spectrum Collab.]
  - Reduce the size of space of fields (on a time-slice) preserving important features.
  - all elements of the (reduced) quark propagator can be computed: allows for many operators, disconnected diagrams and multi-hadron operators.
  - combined with stochastic methods to improve volume scaling.

Spectroscopy

The "naive" spectroscopy of heavy and light mesons
As well as control of usual lattice systematics
($a \to 0$,$L \to \infty$, m_π realistic) need

- statistical precision at % percent level
- reliable spin identification
- heavy quark methods

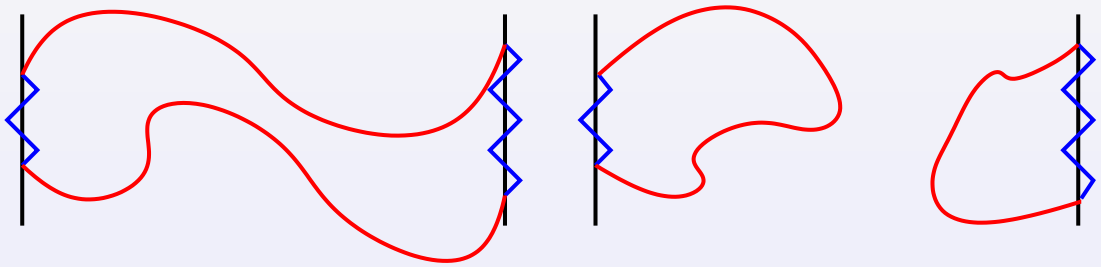
Spectroscopy

The "naive" spectroscopy of heavy and light mesons
As well as control of usual lattice systematics
($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) need

- statistical precision at percent level
  - to include multi-hadrons and study resonances
- reliable spin identification
- heavy quark methods

statistical precision at percent level

- “distillation” - a new approach to simulating correlators. Particularly good for spectroscopy.
- enables precision determination of disconnected diagrams, crucial for isoscalar spectroscopy



- large bases of interpolating operators now feasible, for better determination of excited states via variational method.

Spectroscopy

The "naive" spectroscopy of heavy and light mesons
As well as control of usual lattice systematics
($a \to 0$,$L \to \infty$, m_π realistic) need

- statistical precision at % percent level
- reliable spin identification
  - understanding symmetries and connection between lattice and continuum
  - designing operators with overlap onto J^{PC} of interest.
- heavy quark methods

Reliable spin identification

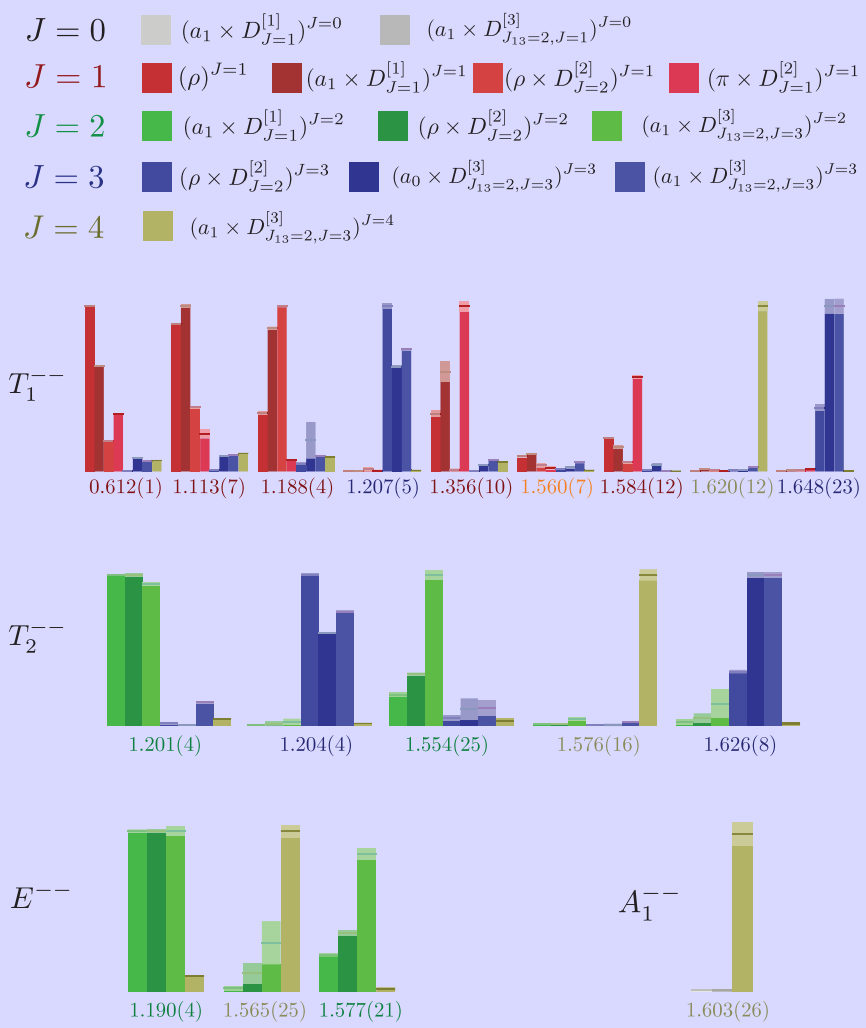
- Continuum: states classified by irreps (J^P) of $O(3)$. The lattice breaks $O(3) \to O_h$.
- O_h has 10 irreps: $\{A_1^{(g,u)}, A_2^{(g,u)}, E^{(g,u)}, T_1^{(g,u)}, T_2^{(g,u)}\}$
- Continuum spin assignment then by subduction

J	0	1	2	3	4	...
A_1	1				1	...
A_2				1		...
E			1		1	...
T_1		1		1	1	...
T_2			1	1	1	...

- Design good operators: start from continuum, "latticize" (D_{latt} for D) continuum operators.
- These lattice operators subduced from J should have good overlap with states of continuum spin J. Study overlaps (Z).

Reliable spin identification - overlaps

Hadron Spectrum Collaboration, 2010



- overlaps for J^{--}
- 16^3 lattice
- $m_\pi \approx 700$ MeV.

Spectroscopy

The "naive" spectroscopy of heavy and light mesons
As well as control of usual lattice systematics
($a \to 0$,$L \to \infty$, m_π realistic) need

- statistical precision at % percent level
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Heavy quarks in lattice QCD

$\mathcal{O}(am_Q)$ errors are significant for charm and large for bottom. These sectors require particular methods.

Relativistic actions

- Isotropic ($a_s = a_t$): needs very fine lattices. Working well for charm, extended to (nearly) bottom [arXiv:1010.3848].
- Anisotropic ($a_s \neq a_t$): reduce relevant temporal $a_t m_Q$ errors. Works well for charm (see later).

Effective Theories

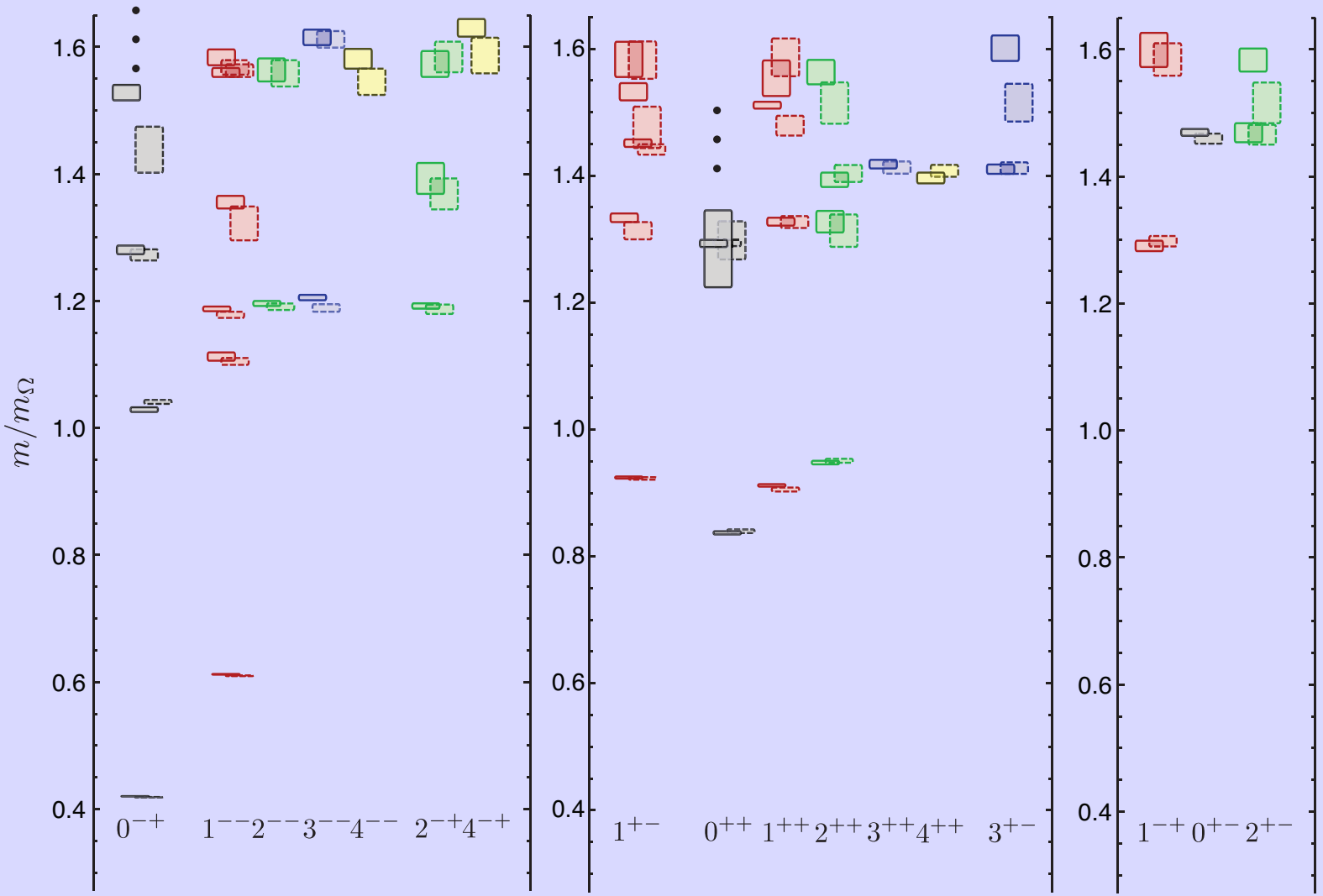
- NRQCD: m_c not heavy enough? Good for bottomonium.
- Fermilab: works well but difficult to improve. Also works for bottomonium. [See talk by Mohler]

In general, $\mathcal{O}(am_Q)$ can be controlled and methods have been shown to agree.

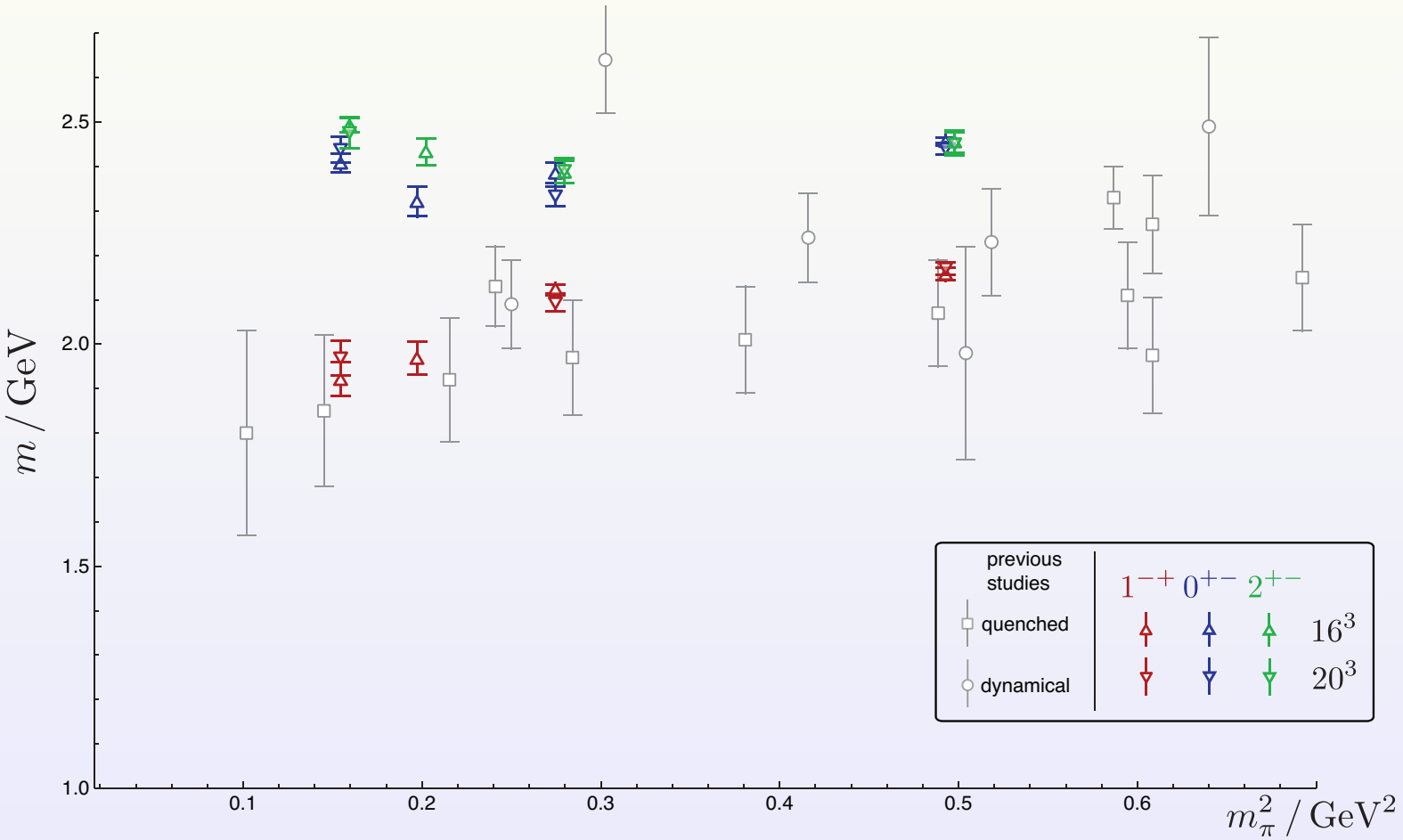
Results

Results: Light Isovector Spectrum

Hadron Spectrum Collaboration, 2010



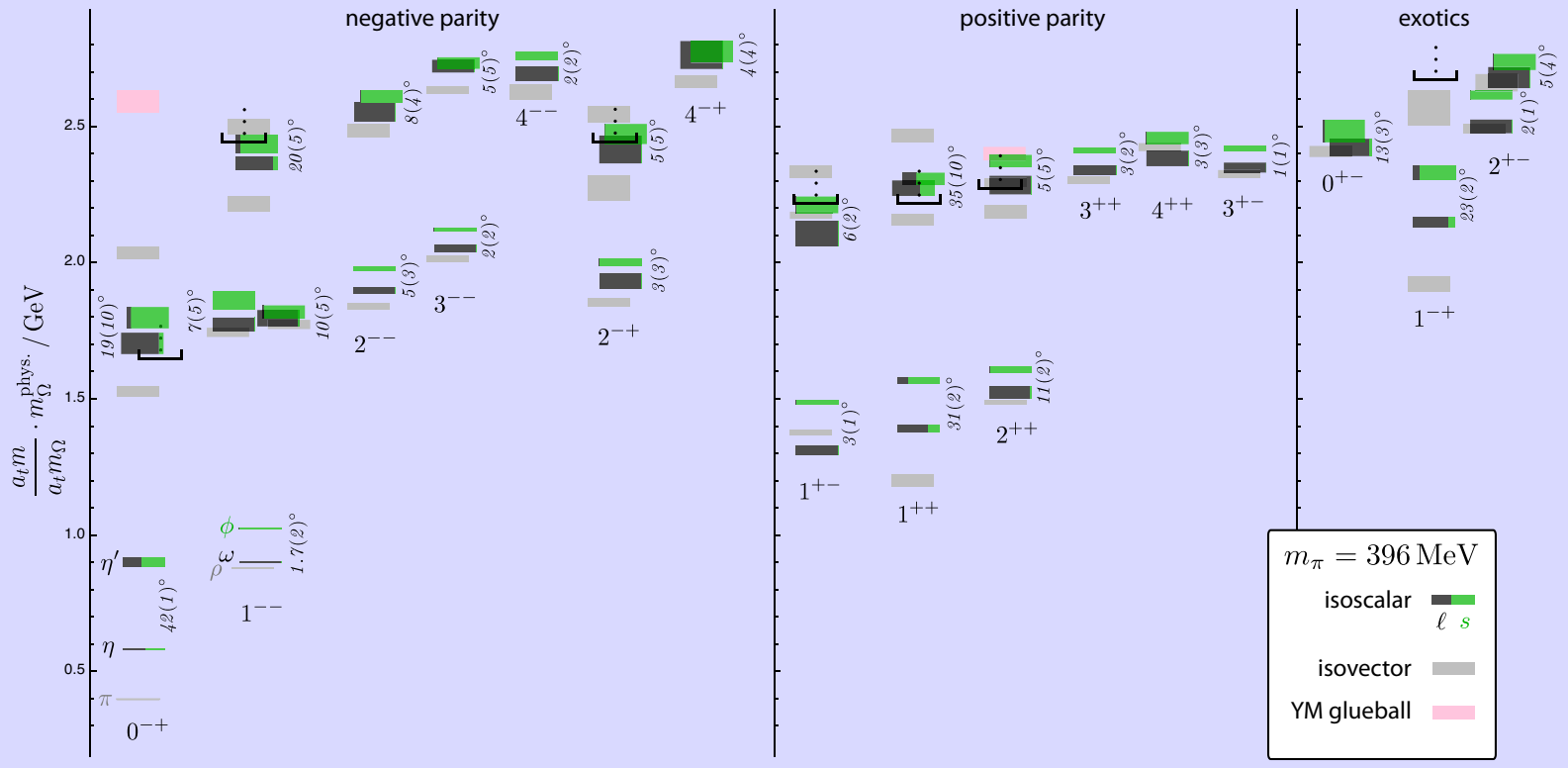
Results: isovector exotic summary



Recent isovector exotics compared with older results.
Note the improvement in precision.

Results: Light Isoscalars

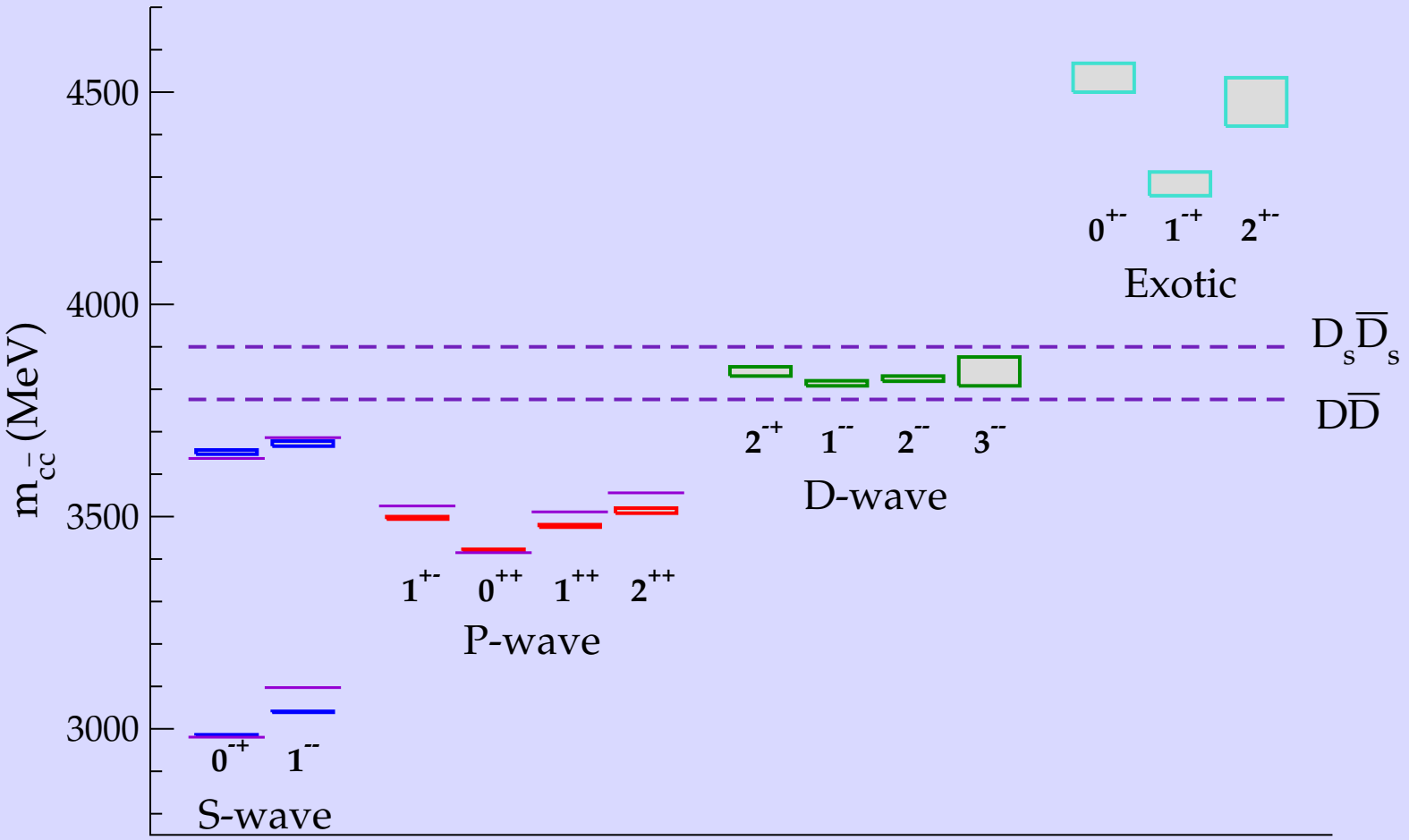
Hadron Spectrum Collaboration, 2011



- 16^3 lattice ($\sim 2fm$), $m_\pi \approx 400$MeV
- Green/black bars - flavour mixing: note PS and axial. $\omega - \rho = 21(5)$ MeV .
- Made possible by distillation.

Charmonium spectroscopy

Liu et al, Hadron Spectrum Collab.(Preliminary)



- Preliminary result; precision is $< 1\%$ on S-waves.
- Ordering of states correct.
- Exotic (hybrids) determined.

Resonances

In quenched QCD all states are stable, not true in dynamical QCD - measure resonances.

First Problem: A No-Go Theorem

Maini-Testa $\Rightarrow$ matrix elements measured in Euclidean field theory do not contain information on strong decay widths

Solutions:

- Lüscher ['86], changes to energy spectrum in a finite box as size of box changes $\rightarrow$ information about widths (elastic region).
- Extension by Rummukainen & Gottlieb ['95].
- alternative: Bernard et al['08], use binning algorithm to measure widths.

Resonances on a lattice

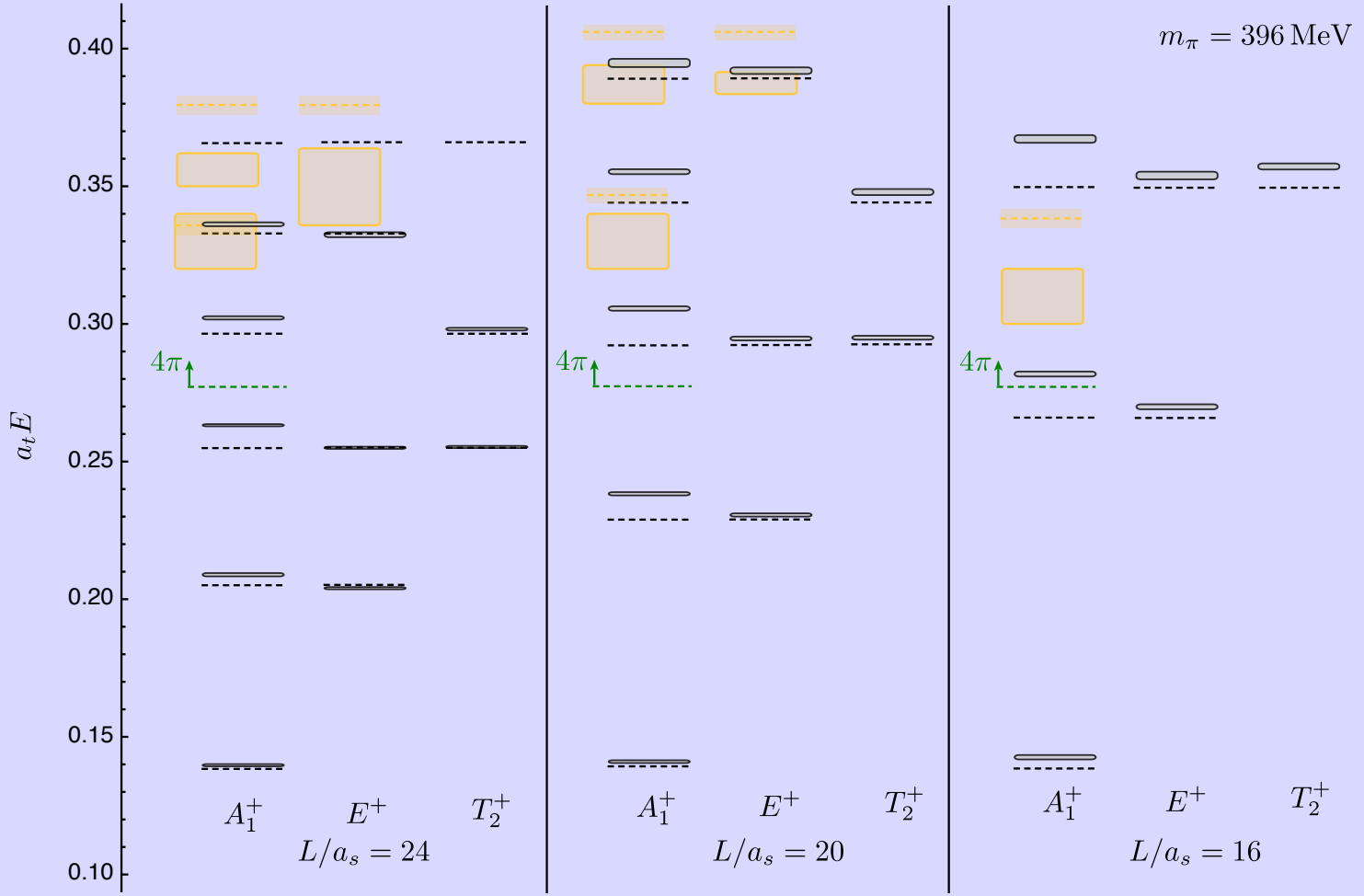
- On a lattice, extent L, with periodic b.c., momenta are: $p = \frac{2\pi}{L}(n_x, n_y, n_z)$, $n_i \in \{0, 1, \ldots, L-1\}$.
- Energy spectrum a set of discrete levels, classified by p

$$E = \sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 N^2}, \;\; N^2 = n_x^2 + n_y^2 + n_z^2.$$

- Interactions modify the finite-volume energy spectrum.
- E as a function of L - avoided level crossings.
- Lüscher's method relates spectrum in a finite box to scattering phase shift and so to resonance properties (in the elastic region).

Resonances on a lattice: I=2 $\pi\pi$ scattering

Dudek et al



The problem: resolve shifts in masses away from non-interacting values.

Resonances on a lattice

Follow-on problems:

- Naively, expect to see effects of multi-hadron states (with just $q\bar{q}$ operators).
- No effect observed so multi-hadron operators must be included.
- Notoriously difficult, due to statistical noise.
- Evidence that a large basis of these operators needed.
- In addition multiple volumes and/or momenta needed.
- Statistical precision to see energy shifts.

This is a difficult problem but it is being tackled!

A good test-ground: I=2 $\pi\pi$ scattering

Some results from 2010,11 only

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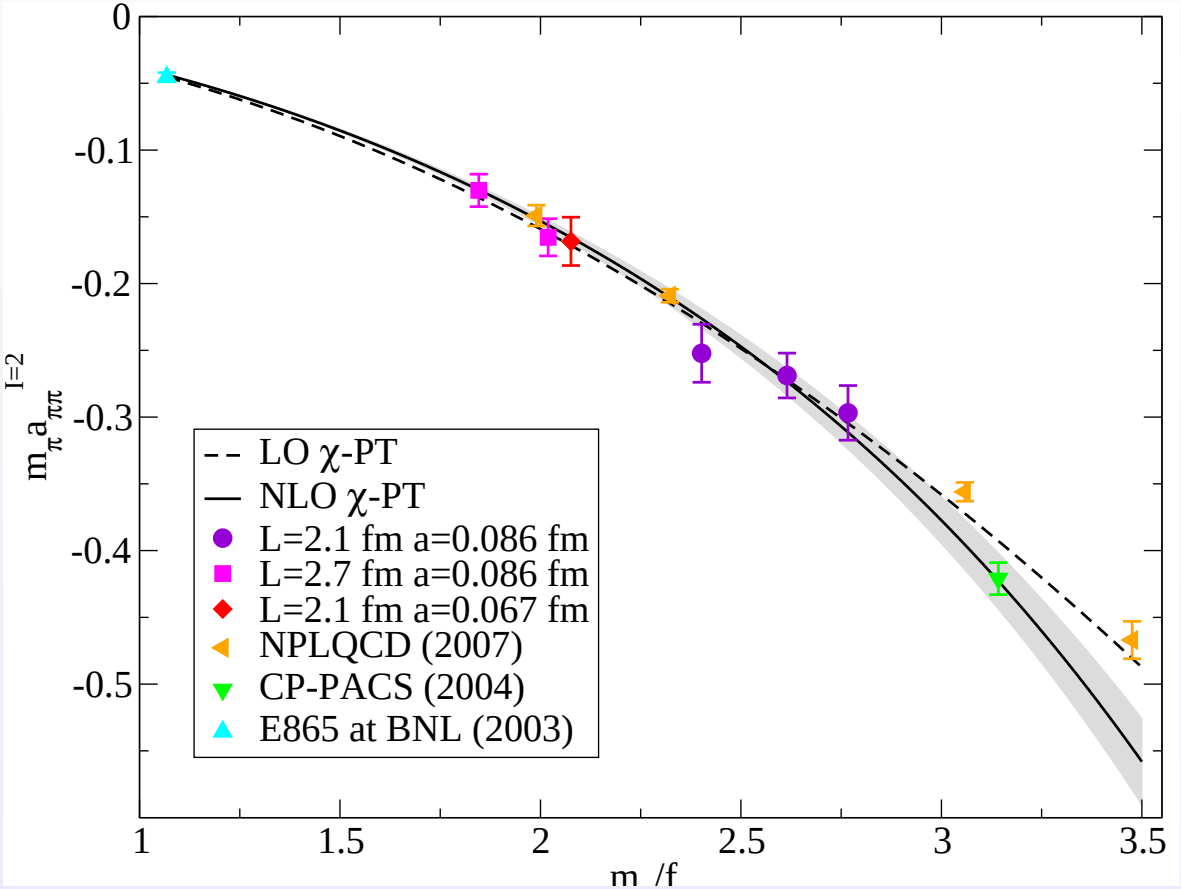
Some results from 2010,11 only

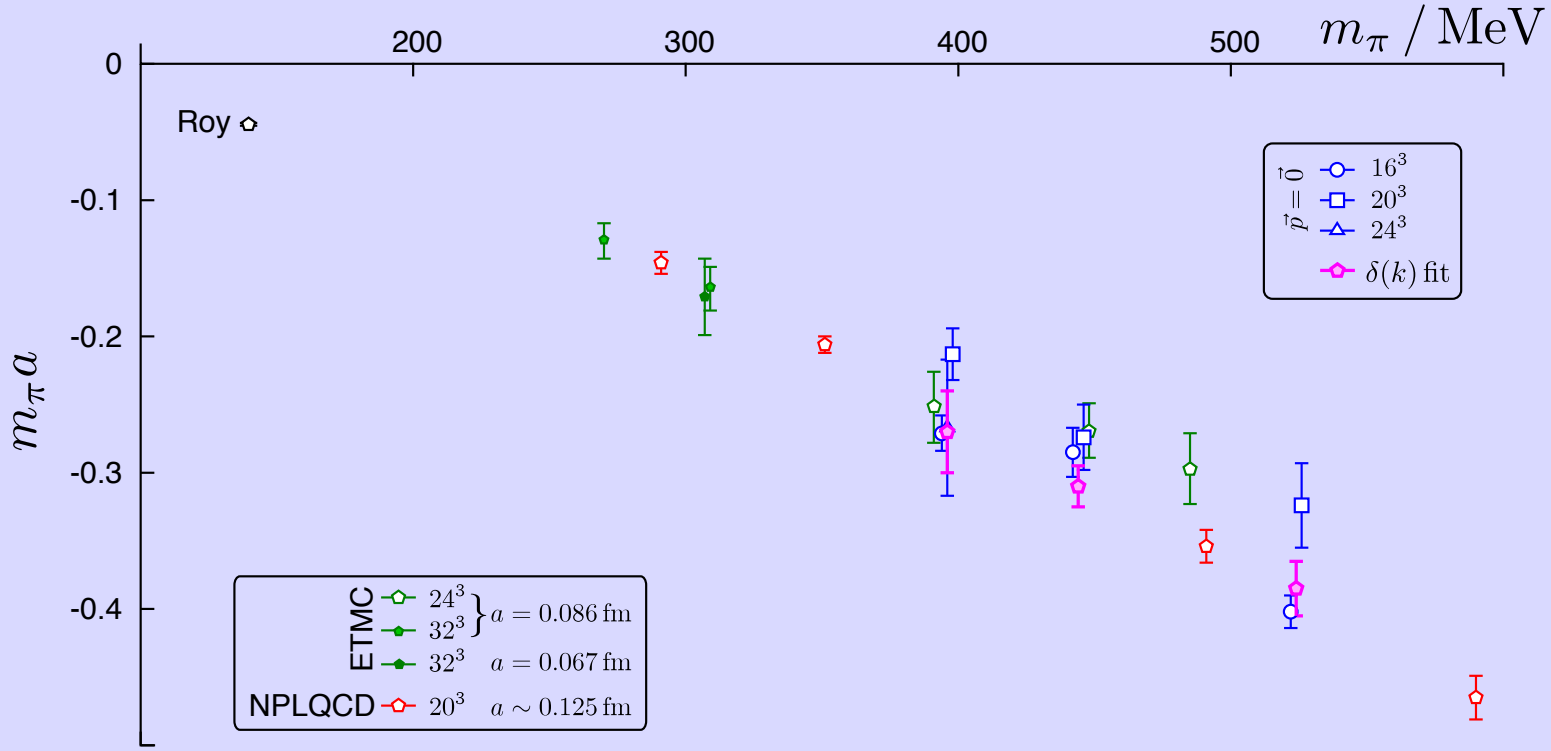
ETMC'10, χ-extrap $\pi\pi$ scattering



- 270 MeV$\leq m_\pi \leq$ 485 MeV.
- $m_\pi a_{\pi\pi}^{I=2} = -0.04385(28)(28)$

I=2 $\pi\pi$ scattering

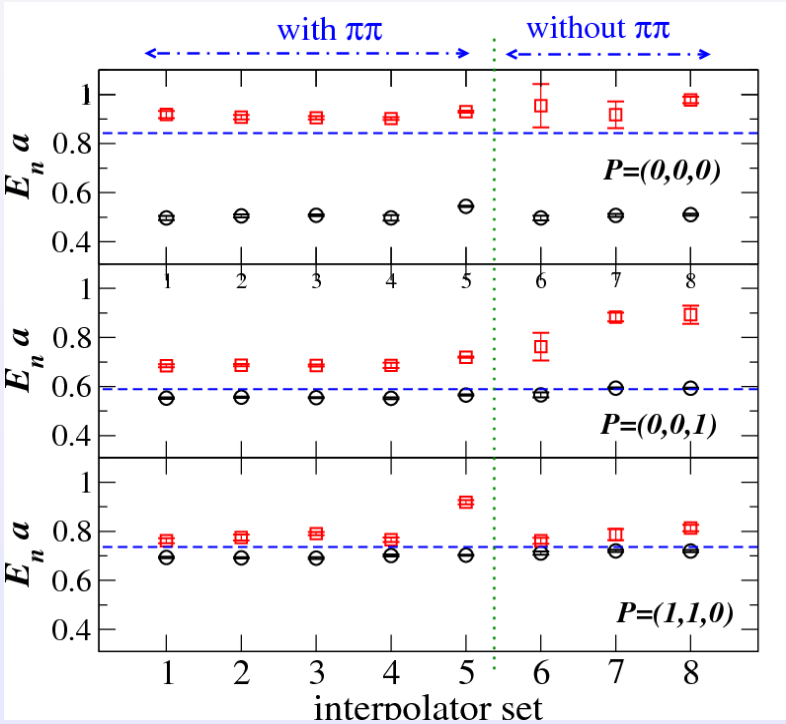
HadSpec'10, $\pi\pi$ scattering

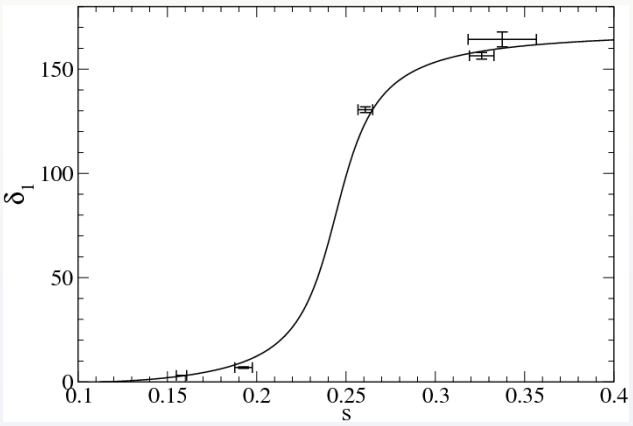


- $400 \leq m_\pi \leq 500$MeV, multiple volumes, large operator basis.
- Little quark mass dependence (agreeing with other studies).

I=1 $\rho \to \pi\pi$ Resonance

Lang et al,
arXiv.1105.5636



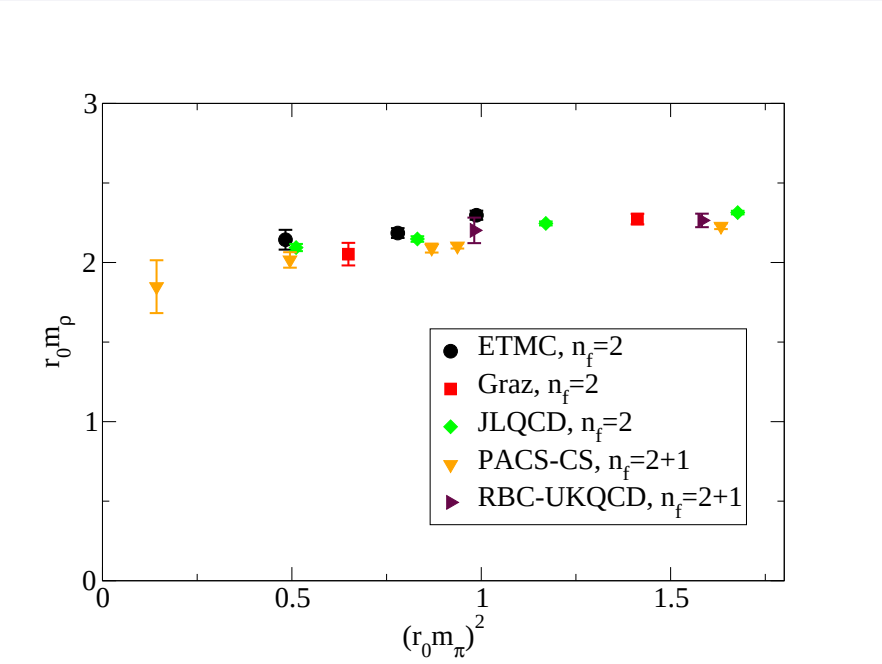


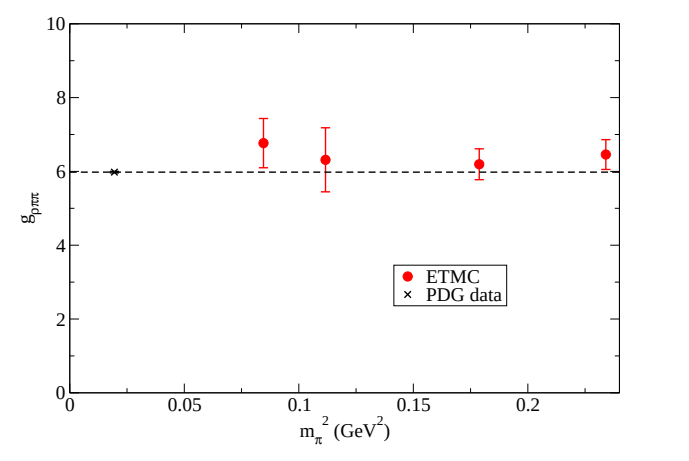
Observations:

- large basis of interpolating operators required.
- “distillation” improves all signals - particularly meson-meson signals.

I=1 $\rho \to \pi\pi$ summary

- ETMC: from fit to effective range formula: $m_\rho = 0.850(35) GeV, \Gamma_\rho = 0.166(49) GeV$.
- Lang et al: $g_{\rho\pi\pi} = 5.13(20), m_\rho = 792(7)(8)$.
- little pion mass dependence in $g_{\rho\pi\pi}$.





[ETMC'10, Feng et al]

Encouraging recent results

Summary and Prospects

Spectroscopy

- Dynamical simulations are here: large volumes, fine lattices, light quarks.
- Precision analysis of isoscalar and isovector spectra, including exotic and crypto-exotic states possible.

Expect to see more plots like those I have shown!

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- Until recently, practically impossible.
- New tools in place to extract resonance information and early studies are promising.
- Expect to see many more results in the next 5 years.

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Entering a Golden Age of lattice spectroscopy??