$D$ and $D_s$ meson spectroscopy from lattice QCD

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Vancouver, B.C., Canada

München, June 13 2011

Collaborators: Richard Woloshyn
Established states:

- $D_s (J^P = 0^-)$ and $D_s^* (1^-)$
- $D_{s0}^* (2317) (0^+)$, $D_{s1} (2460) (1^+)$, $D_{s1} (2536) (1^+)$, $D_{s2}^* (2573) (2^+)$

More recent discoveries:

- $D_{s1}^* (2710)$ seen by BaBar, Belle ($1^-$)
- $D_{sJ}^* (2860)$ seen by BaBar ($3^-?, 0^+?)$
- $D_{sJ}^* (3040)$ seen by BaBar ($1^+?, 2^-?)$
- $D_{sJ}^* (2632)$ seen by SELEX ($1^-?$)

There is a zoo of phenomenological models and lattice results are getting dated

Some models suggest a tetraquark/molecular interpretations for controversial states
Gauge ensembles used

- We use 2+1 flavor Clover-Wilson ensembles of size $32^3 \times 64$ generated by the PACS-CS collaboration
- (Sea) Pion masses range from $156\text{MeV}$ to $702\text{MeV}$
- We use the lattice spacing as determined by PACS-CS ($a = 0.0907(13)\text{fm}$)

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$c_{sw}^{(h)}$</th>
<th>$\kappa_{u/d}$</th>
<th>$\kappa_s$</th>
<th>#configs $D/D_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.52617</td>
<td>0.13700</td>
<td>0.13640</td>
<td>200/200</td>
</tr>
<tr>
<td>2</td>
<td>1.52493</td>
<td>0.13727</td>
<td>0.13640</td>
<td>-/200</td>
</tr>
<tr>
<td>3</td>
<td>1.52381</td>
<td>0.13754</td>
<td>0.13640</td>
<td>200/200</td>
</tr>
<tr>
<td>4</td>
<td>1.52327</td>
<td>0.13754</td>
<td>0.13660</td>
<td>-/200</td>
</tr>
<tr>
<td>5</td>
<td>1.52326</td>
<td>0.13770</td>
<td>0.13640</td>
<td>200/348</td>
</tr>
<tr>
<td>6</td>
<td>1.52264</td>
<td>0.13781</td>
<td>0.13640</td>
<td>198/198</td>
</tr>
</tbody>
</table>
Charm quark treatment I

- We use the *Fermilab method* for the heavy (charm) quark
  
  El-Khadra et al., PRD 55, 3933

- We tune $\kappa$ for the spin averaged kinetic mass $(M_{D_s} + 3M_{D_s^*})/4$ to assume its physical value

- General form for the dispersion relation
  
  Bernard et al. PRD83:034503, 2011

  $$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \ldots$$

- We compare results from two different fits:
  
  1. Neglect the term with coefficient $W_4$
  2. Fit $E^2(p)$ and neglect $(p^2)^2$ term from mismatch of $M_1$, $M_2$ and $M_4$

  $$E^2(p) \approx M_1^2 + \frac{M_1}{M_2} p^2 - \frac{M_1 a^3 W_4}{3} \sum_i (p_i)^4$$  \hfill (1)
Charm quark treatment II

Method 1:

<table>
<thead>
<tr>
<th>$\kappa_c$</th>
<th>$M_1$ [GeV]</th>
<th>$M_2$ [GeV]</th>
<th>$M_4$ [GeV]</th>
<th>$\frac{M_2}{M_1}$</th>
<th>$M_2$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.128</td>
<td>0.86334(50)</td>
<td>0.9337(73)</td>
<td>0.8638(274)</td>
<td>1.0815(86)</td>
<td>2.0315(158)(291)</td>
</tr>
<tr>
<td>0.127</td>
<td>0.89314(51)</td>
<td>0.9716(76)</td>
<td>0.8855(284)</td>
<td>1.0878(88)</td>
<td>2.1137(166)(303)</td>
</tr>
</tbody>
</table>

Method 2:

<table>
<thead>
<tr>
<th>$\kappa_c$</th>
<th>$M_1$ [GeV]</th>
<th>$M_2$ [GeV]</th>
<th>$M_4$ [GeV]</th>
<th>$\frac{M_2}{M_1}$</th>
<th>$M_2$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.128</td>
<td>0.86342(50)</td>
<td>0.9337(73)</td>
<td>0.8638(274)</td>
<td>1.0889(116)</td>
<td>2.0454(215)(293)</td>
</tr>
<tr>
<td>0.127</td>
<td>0.89322(51)</td>
<td>0.9716(76)</td>
<td>0.8855(284)</td>
<td>1.0955(118)</td>
<td>2.1293(227)(305)</td>
</tr>
</tbody>
</table>

D and $D_s$ meson spectroscopy

Daniel Mohler (TRIUMF)
Variational method (Michael; Lüscher and Wolff; Blossier et al.)

- Matrix of correlators projected to fixed momentum

\[ C(t)_{ij} = \sum_n e^{-tE_n} \langle 0|O_i|n\rangle \langle n|O_j^\dagger|0\rangle \]

- Solve the generalized eigenvalue problem:

\[ C(t)\psi^{(k)}(t) = \lambda^{(k)}(t)C(t_0)\psi^{(k)} \]

\[ \lambda^{(k)}(t) \propto e^{-tE_k} \left( 1 + \mathcal{O}(e^{-t\Delta E_k}) \right) \]

- At large time separation: only a single mass in each eigenvalue.
- Eigenvectors can serve as a fingerprint.
Charmonium results I

- We use the low-lying Charmonium spectrum as a benchmark

Noticeable discretization effects expected for Spin-dependent quantities

Spin-averaged quantities agree nicely with experiment
Charmonium results II

<table>
<thead>
<tr>
<th>Mass difference</th>
<th>Our results [MeV]</th>
<th>Experiment [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S hyperfine</td>
<td>$97.8 \pm 0.5 \pm 1.4$</td>
<td>$116.6 \pm 1.2$</td>
</tr>
<tr>
<td>1P spin-orbit</td>
<td>$37.5 \pm 2.4 \pm 0.5$</td>
<td>$46.6 \pm 0.1$</td>
</tr>
<tr>
<td>1P tensor</td>
<td>$10.44 \pm 1.13 \pm 0.15$</td>
<td>$16.25 \pm 0.07$</td>
</tr>
<tr>
<td>2S hyperfine</td>
<td>$48 \pm 18 \pm 1$</td>
<td>$49 \pm 4$</td>
</tr>
</tbody>
</table>
**$D_s$ - Mixing for the $J^P = 1^+$ states**

- In the $1^+$ channel we consider mixing between interpolating fields that correspond to positive and negative charge conjugation in the mass-degenerate limit.
- Neglecting the mixing leads to mass splitting much smaller than in experiment.

**Effective masses diagonal correlators**

**Effective masses from Eigenvalues**

- $\chi^2$/d.o.f. = 6.03/6
- $\chi^2$/d.o.f. = 4.72/6
For $D_s$ mesons we also determine the 2S states

We obtain a reasonable hyperfine splitting (within somewhat large errors)
Results for the $D_{s0}$ and $D_{s1}$ ground states differs significantly from experiment

The results from $D$ and $D_s$ mesons differ significantly
Energy levels are very close to non-interacting scattering states

The energy of $D^0 K$ and $D^* K$ states is slightly unphysical

Future studies will have to include these states in the variational basis
Conclusions

- We determined the spectrum of low-lying charmonium and heavy-light states on configurations with 2+1 flavors of dynamical quarks.
- The charmonium spectrum below the $D\bar{D}$ threshold can be extracted with small discretization effects and agrees favorably with experiment.
- In some cases excited states can also be extracted.
- For P-wave charmed and charmed-strange mesons substantial differences with regard to experiment remain.
- In future studies effects of nearby scattering thresholds and/or the lattice discretization will have to be investigated.

Thanks to ...  
The PACS-CS collaboration for their gauge configurations  
Martin Lüscher for making his DD-HMC code available