UNIVERSITÉ PARIS-SUD 11

IPN Orsay

Some properties of light scalar mesons in the complex plane

- Based on [*R. García-Martín, B.M., EPJ C70 (2010)*]
  Here:
  Applications to I=0 resonances $\sigma(600)$, $f_0(980)$

  → Couplings to simple operators

Motivations

- Interest in I=0 scalars motivated by the
  Glueball

Motivations

- Interest in I=0 scalars motivated by the

  Glueball

  → Lightest particle in QCD with heavy m_q ($\gtrsim 1$ GeV) is 0^{++} meson: $M_G = 1.730 \pm 0.030 \pm 0.080$ GeV

  [*Morningstar,Peardon PR D60 (1999)*]

Motivations

- Interest in I=0 scalars motivated by the Glueball
  - → Lightest particle in QCD with heavy m_q ($\gtrsim 1$ GeV) is 0^{++} meson: $M_G = 1.730 \pm 0.030 \pm 0.080$ GeV [*Morningstar,Peardon PR D60 (1999)*]
  - → Real QCD ($m_{u,d,s}$ << 1 GeV) ???
    LQCD not (yet) operative
    Laplace sum rules[*Novikov et al. N.P. B165 (1979), Narison, Veneziano I.J.M.P. A4 (1989)*]
    Two glueballs ? one very light ?
  - → Here: alternative determination of $\langle 0|\alpha_s G^2|S\rangle$

- Scalar multiplet:
  - → Flavour structure: $q\bar{q}$, $[qq][\bar{q}\bar{q}]$?
  - → Quantify with couplings. I=1,1/2:

    $\langle 0|[qq][\bar{q}\bar{q}]|S\rangle$: LQCD [*Prelovsek, PR D82 (2010)*]

    $\langle 0|q\bar{q}|S\rangle$: [*Maltman, PL B462 (1999)*]
  - → Couplings to two photons: $\langle 0|j_\mu(x)j_\nu(0)|S\rangle$,

Here definitions + update using:

analyticity in QCD + simple unitarity

experimental measurements ($\pi\pi \to \pi\pi$, $\gamma\gamma \to \pi\pi$)

chiral symmetry.

Roy eq. solution up to $K\overline{K}$ threshold

- Analyticity +crossing [*Roy P.L. B36 (1971)*]

$$\mathrm{Re}\, t_0^0(s) = a_0^0 + \frac{s-4m_\pi^2}{12m_\pi^2}(2a_0^0 - 5a_0^2)$$
$$+\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \left[\mathrm{Im}\, t_0^0(s')\left(\frac{1}{s'-s} + K_0(s',s)\right)\right.$$
$$\left. +\mathrm{Im}\, t_1^1(s')K_1(s',s) + \mathrm{Im}\, t_0^2(s')K_2(s',s)\right] + d_0^0(s)$$

Unitarity: (elastic effectively up to $4m_K^2$)

- Reconsided:
  [*Anant. et al (ACGL), P. Rep. 353 (2001)*] Solutions in: $\boxed{s < s_A = (0.8\ \text{GeV})^2}$

- Extended range: $s \leq (1.1\ \text{GeV})^2$
  [*GKPRY, PR D83 (2011)*] (within exp. error bars)
  [*CGL, work in progress*]

- Here:
  exact (numerically) solutions $\boxed{s < s_K = 4m_K^2}$
  (cover both σ, $f_0(980)$ regions)

- Theorems on solving one Roy eq. as BV problem [*Pomponiu,Wanders (1976)*]

  with matching point: $s_m = s_K = 4m_K^2$:

  Inputs:

  → above s_m: inelasticities, phase-shifts

  → below s_m: a_0^0, a_0^2 + phase-shift at one point (e.g. $\delta_A = \delta_0^0((0.8\text{ GeV})^2)$)

  → Output: unique solution in $[4m_\pi^2, 4m_K^2]$

- Assumption: $\delta_0^0(s_K)$ in $[180°, 225°]$

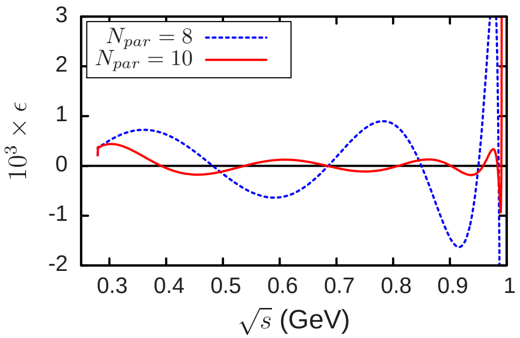
- Simple parametrization (Schenk):

$$\tan \delta_0^0(s) = \sigma_\pi(s) \left[a_0^0 + \sum_1^N \alpha_i \left(\frac{s}{s_\pi} - 1 \right)^i \right] \frac{s_\pi - s_0}{s - s_0} \frac{\sigma^K(s_\pi) + \beta}{\sigma^K(s) + \beta}$$

- Simple parametrization (Schenk):

$$\tan\delta_0^0(s) = \sigma_\pi(s)\left[a_0^0 + \sum_1^N \alpha_i\left(\frac{s}{s_\pi} - 1\right)^i\right]\frac{s_\pi - s_0}{s - s_0}\,\frac{\sigma^K(s_\pi)+\beta}{\sigma^K(s)+\beta}$$

- Convergence: $\epsilon(s) = \mathcal{R}[t_0^0](s) - \mathrm{Re}\, t_0^0(s)$



Inputs above matching point:

Inelasticity: [*Garcia-Martin et al.,Phys.Rev.D83 (2011)*]

→ It can be determined two different ways:

(a) From sum over inelastic channels: $K\overline{K}, \eta\eta, 4\pi$

(b) From elastic channel $\eta_0^0 = |S_0^0|$

Inputs above matching point:

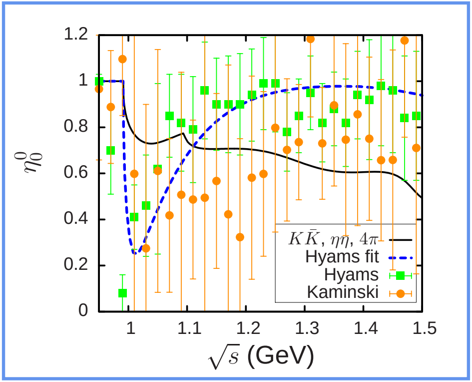
Inelasticity: [*Garcia-Martin et al.,Phys.Rev.D83 (2011)*]

→ It can be determined two different ways:

(a) From sum over inelastic channels: $K\bar{K}, \eta\eta, 4\pi$

(b) From elastic channel $\eta_0^0 = |S_0^0|$

→ Central determinations:



Inputs below matching point:

- New results from K decays and from pionium.
  Latest from NA48/2 [*Batley et al. EPJ C70 (2010)*]

$$a_0^0 = 0.2196 \pm 0.0028_{\text{stat}} \pm 0.0020_{\text{syst}}$$
$$a_0^2 = -0.0444 \pm 0.0007_{\text{stat}} \pm 0.0005_{\text{syst}} \pm 0.0008_{\text{ChPT}}$$

Inputs below matching point:

- New results from K decays and from pionium.
  Latest from NA48/2 [*Batley et al. EPJ C70 (2010)*]

$$a_0^0 = 0.2196 \pm 0.0028_{\mathrm{stat}} \pm 0.0020_{\mathrm{syst}}$$
$$a_0^2 = -0.0444 \pm 0.0007_{\mathrm{stat}} \pm 0.0005_{\mathrm{syst}} \pm 0.0008_{\mathrm{ChPT}}$$

- Phase-shifts from $\pi N \to \pi\pi N, \pi\pi\Delta$
  we include:

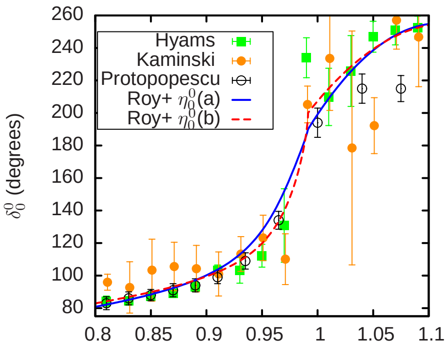
  1) CERN-Cracow-Munich [*Becker et al. NP B150 (1979), Kaminski et al ZP C74 (1997)*]

  2) CERN-Munich [*Hyams et al., NP B64 (1973)*]:
  with weight factor $\frac{1}{4}$

- We fit $\delta_A \equiv \delta_0^0(\sqrt{s} = 0.8)$ (also $\delta_K \equiv \delta_0^0(\sqrt{s} = 2m_K)$):

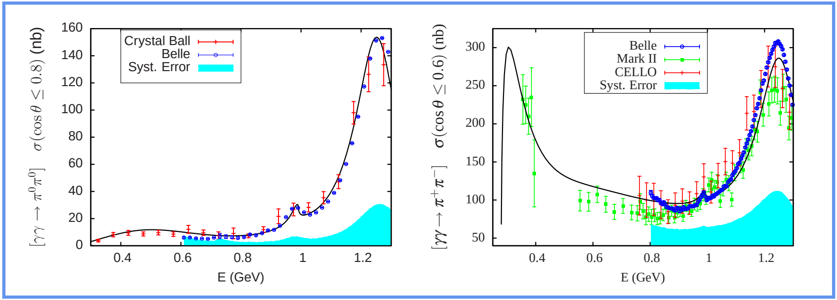
η_0^0	δ_A	δ_K	$\hat{\chi}^2_{hyams}$	$\hat{\chi}^2_{kaminski}$
no-dip	$(80.9 \pm 1.4)^\circ$	$\left(190^{+5}_{-10}\right)^\circ$	2.7	1.9
dip	$(82.9 \pm 1.7)^\circ$	$\left(200^{+5}_{-10}\right)^\circ$	2.2	1.3

Better χ^2 with "dip" inelasticity



Photon-photon amplitudes

- New $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-$ data from *[Belle, PR D75 (2007), D78 (2008)]*



→ High statistics, $f_0(980)$ clearly seen

→ Amplitude analysis, $S \to 2\gamma$:
Analyticity, unitarity: Omnès method
[Gourdin,Martin NC 17 (1960)]

- $f_0(980)$ region, 2-channel unitarity: Omnès matrix [*Mao et al., (2009), RGM +BM, (2010)*]

  → $\Omega_{ij}(s)$:computed from δ_0^0, η_0^0 (attributed to $K\overline{K}$)

- $f_0(980)$ region, 2-channel unitarity: Omnès matrix [*Mao et al., (2009), RGM +BM, (2010)*]

→ $\Omega_{ij}(s)$:computed from δ_0^0, η_0^0 (attributed to $K\overline{K}$)

$$\begin{pmatrix} h^0_{0,++}(s) \\ k^0_{0,++}(s) \end{pmatrix} = \begin{pmatrix} \bar{h}^{0,Born}_{0,++}(s) \\ \bar{k}^{0,Born}_{0,++}(s) \end{pmatrix} + \overline{\overline{\Omega}}(s) \times \Bigg[\begin{pmatrix} b^{(0)}s + b'^{(0)}s^2 \\ b_K^{(0)}s + b_K'^{(0)}s^2 \end{pmatrix} + \frac{s^3}{\pi}\int_{-\infty}^{-s_0} \frac{ds'}{(s')^3(s'-s)} \overline{\overline{\Omega}}^{-1}(s') \mathrm{Im} \begin{pmatrix} \bar{h}^{0,Res}_{0,++}(s') \\ \bar{k}^{0,Res}_{0,++}(s') \end{pmatrix} - \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^3(s'-s)} \mathrm{Im}\, \overline{\overline{\Omega}}^{-1}(s') \begin{pmatrix} \bar{h}^{0,Born}_{0,++}(s') \\ \bar{k}^{0,Born}_{0,++}(s') \end{pmatrix} \Bigg]$$

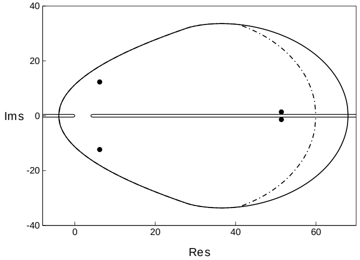
→ Parameters: chiral constraints + fit to data

→ Fit: 6 free parameters, 1783 data points

Complex plane

- $t_0^0(z)$ from Roy representation

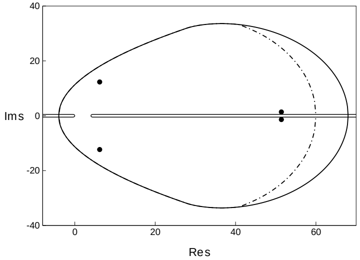
  Domain of validity:
  [*Caprini, Colangelo, Leutwyler PRL 96 (2006)*]



Complex plane

- $t_0^0(z)$ from Roy representation

  Domain of validity:
  [*Caprini, Colangelo, Leutwyler PRL 96 (2006)*]



- Resonances = poles on 2nd Riemann sheet
  $t_0^{0,II}(z)$: analytical cont. of $t_0^0(z)$ on $[4m_\pi^2, 16m_\pi^2]$

- Elastic unitarity eq. (+real analyticity)

$$t_0^{0,II}(z) = \frac{t_0^0(z)}{1 - 2\sigma^\pi(z) t_0^0(z)}$$

Poles and residues from extended Roy solution:

	$\sqrt{z_0}$ (MeV)	$\dot{S}_0^0(z_0)$ (GeV^{-2})
$\sigma(600)$	$\left(442^{+5}_{-8}\right) - i\left(274^{+6}_{-5}\right)$	$-\left(0.75^{+0.10}_{-0.15}\right) - i\left(2.20^{+0.14}_{-0.10}\right)$
$f_0(980)$	$\left(996^{+4}_{-14}\right) - i\left(24^{+11}_{-3}\right)$	$-\left(1.1^{+3.0}_{-0.4}\right) - i\left(6.6^{+0.8}_{-1.0}\right)$

→ Central values: η_0^0 "dip"

→ Errors: a_0^0, a_0^2, δ_A, δ_K, η_0^0 $\longrightarrow$ effect on $f_0(980)$

→ CCL: $m_\sigma = 441^{+16}_{-8}$ $\Gamma_\sigma/2 = 272^{+9}_{-13}$

PDG: $m_{f_0} = 980 \pm 10$ $\Gamma_{f_0}/2 = [20 - 50]$

- Photon-photon amplitude on second sheet:

$$h^{0,II}_{0++}(z) = \frac{h^0_{0++}(z)}{1 - 2\sigma^\pi(z) t^0_0(z)}$$

- Residues identified with (complex) couplings [*Pennington PRL 97 (2006)*]

$$32\pi t^{0,II}_0(z)\Big|_{pole} = \frac{g^2_{S\pi\pi}}{z - z_0}, \quad h^{0,II}_{0,++}(z)\Big|_{pole} = \frac{g_{S\pi\pi} g_{S\gamma\gamma}}{z - z_0} .$$

- Definition of width:

$$\Gamma_{S\to 2\gamma} \equiv \frac{|g_{S\gamma\gamma}|^2}{16\pi m_S} .$$

- Results for 2γ couplings

$$\Gamma_{\sigma(600)\to 2\gamma} = \left(2.08 \pm 0.20\,^{+0.07}_{-0.04}\right) \quad \text{(KeV)}$$
$$\Gamma_{f_0(980)\to 2\gamma} = \left(0.29 \pm 0.21\,^{+0.02}_{-0.07}\right) \quad \text{(KeV)}\ .$$

First error: polynomial parameters ($\gamma\gamma$ data)

Second error: $\pi\pi$ amplitude

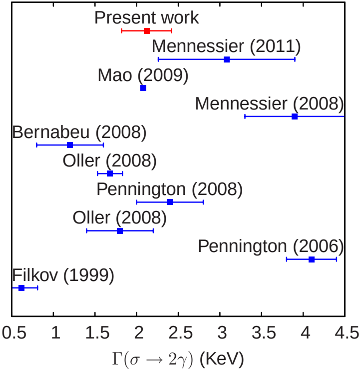
- Results for 2γ couplings

$$\Gamma_{\sigma(600)\to 2\gamma} = \left(2.08 \pm 0.20\,^{+0.07}_{-0.04}\right) \quad \text{(KeV)}$$
$$\Gamma_{f_0(980)\to 2\gamma} = \left(0.29 \pm 0.21\,^{+0.02}_{-0.07}\right) \quad \text{(KeV)}\,.$$

First error: polynomial parameters ($\gamma\gamma$ data)

Second error: $\pi\pi$ amplitude

- Comparison with others for σ width:



Couplings of scalar mesons to operators

- $\bar{q}q$ couplings

$$\langle 0|\bar{u}u+\bar{d}d|S\rangle = \sqrt{2}B_0\, C_S^{uu}$$
$$\langle 0|\bar{s}s|S\rangle = B_0\, C_S^{ss}$$

- Gluon couplings

$$\langle 0|\theta^\mu_\mu|S\rangle = m_S^2 C_S^\theta$$
$$\langle 0|\alpha_s G^{a\mu\nu}G^a_{\mu\nu}|S\rangle = m_S^2\, C_S^G$$

→ Definitions from complex poles

- Consider correlator

$$\Pi_{jj}(p^2) = i\int d^4x e^{ipx}\langle 0|Tj_S(x)j_S(0)|0\rangle$$

(j_S one of the scalar operators)

- Discontinuity from Källen-Lehman representation

$$\Pi_{jj}^{II}(z) = \Pi_{jj}(z) + \frac{3}{16\pi}\frac{\sigma^\pi(z)\left(F_j(z)\right)^2}{1-2\sigma^\pi(z)t_0^0(z)}\ .$$

$F_j(t)\delta^{ab} = \langle\pi^a|j_S|\pi^b\rangle$ is the $\pi\pi$ form-factor

Main result: C_S^j proportional $F_j(z_0)$

- $F_j(t)$ obey Omnès representation
  [like $\gamma\gamma$ but no LH cut]

→ subtraction constants: chiral symmetry to $O(p^2)$
[*Donoghue, Gasser, Leutwyler NP B343 (1990)*]

→ θ_μ^μ : $\quad \theta^\pi(t) = t + 2m_\pi^2 + O(p^4)$

$$\theta^K(t) = t + 2m_K^2 + O(p^4)$$

→ $O(p^4)$ corrections in errors

- $F_j(t)$ obey Omnès representation
  [like $\gamma\gamma$ but no LH cut]

  → subtraction constants: chiral symmetry to $O(p^2)$
  [*Donoghue, Gasser, Leutwyler NP B343 (1990)*]

  → θ^μ_μ : $\theta^\pi(t) = t + 2m_\pi^2 + O(p^4)$
  $\theta^K(t) = t + 2m_K^2 + O(p^4)$

  → $O(p^4)$ corrections in errors

- $\alpha_s G^2$ from θ^μ_μ:

$$\theta^\mu_\mu = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a\mu\nu} + (1 + \gamma_m(g)) \sum_{q=u,d,s} m_q \bar{q} q$$

  We will use lowest order $\beta(g)$ and $\gamma_m(g)$

- Results for couplings to $\bar{q}q$ operators

	$\sigma(600)$	$f_0(980)$
$\lvert C_S^{uu} \rvert$ (MeV)	$206 \pm 4^{+4}_{-6}$	$41 \pm 15^{+7}_{-4}$
$\lvert C_S^{ss} \rvert$ (MeV)	$17 \pm 5^{+1}_{-7}$	$139 \pm 42^{+13}_{-6}$

First error: $O(p^4)$, Second error $\pi\pi$

- $I = 1/2$, $I = 1$ scalars (MeV)

$C^{us}_{\kappa(800)} \simeq 156$ (from RS equation)

$C^{ud}_{a_0} = 197 \pm 37$ [*Maltman, PL B462 (1999)*]

Higher mass scalar:

$C^{us}_{K_0^*(1430)} = 370 \pm 20$: Larger coupling !

- Results for couplings to gluonic operators

	$\sigma(600)$	$f_0(980)$
$\lvert C_S^{\theta}\rvert$ (MeV)	$197 \pm 15^{+21}_{-6}$	$114 \pm 44^{+22}_{-7}$
$\lvert C_S^{G}\rvert$	$472 \pm 15^{+26}_{-16}$	$227 \pm 41^{+51}_{-16}$

- Note: significant size of $C^{\theta}_{f_0(980)}$

- For comparison $\boxed{C^{\theta}_{\sigma} = [272 - 329]}$ MeV (Laplace sum rule)[*Narison, Veneziano IJMP A4 (1989)*]

Conclusions

- $q\bar{q}$: not negligible, results in agreement with $\sigma,\ \kappa,\ a_0,\ f_0$ forming a nonet.
- $\alpha_s G^2$: both σ, $f_0(980)$ couple, qualitative agreement with Laplace QCD sum-rule. No naive glueball !
- $[qq][\bar{q}\bar{q}]$? chiral transformation

$$(3_L \times \bar{3}_L, 1_R) + (3_L, \bar{3}_R) \quad + (L \longleftrightarrow R)$$

need input: $\dot{F}_j(0)$ $F_j(0)$

- 2γ: improved $f_0(980)$ from Belle, new measurements $E < 0.8$ GeV (KLOE2, BESIII ?) very useful! (for σ, pion polarizabilities)