Renormalization of Spin-Flavor Van der Waals Forces

Álvaro Calle Cordón

Theory Center @ JLab

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In collaboration with Enrique Ruiz Arriola (University of Granada)

Table of contents

Table of contents

NUCLEAR POTENTIALS AND SINGULARITIES

- OPE potential: Yukawa's meson theory (1935) & Proca, Kemmer, ... (1940's)

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$$\to \frac{g_{\pi NN}^2}{16\pi}\frac{1}{M_N^2}\frac{1}{r^3}\,S_{12}(\hat{r})\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \qquad \text{(singular potential)}$$

- OBE models (1960's): multipions $= \sigma,\ \rho,\ \omega$, ...

$1/r^3\ S_{12}(\hat{r})$	pseudo-scalar mesons $(\pi,\ \eta)$
$1/r^3\ \mathbf{L}\cdot\mathbf{S}$	scalar mesons $(\sigma,\ \delta)$
$1/r^3\ \mathbf{L}\cdot\mathbf{S},\ 1/r^3\ S_{12}(\hat{r})$	vector mesons $(\omega,\ \rho)$

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  OBE $+$ realistic couplings $+$ strong form factors to remove divergences
- Quark models (1980's): OGE + confinement $\to$ short-range repulsion (form factors)
  Hybrid models (meson exchanges) $\to$ account for medium and long-range attraction
- EFTs in nuclear physics (1990's): Chiral NN potential

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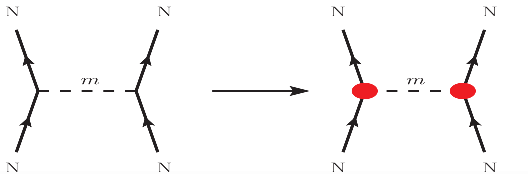

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In general, *nuclear potentials* present *singularities*

SINGULAR POTENTIALS AT SHORT DISTANCES

- By definition a singular potential satisfies $\lim_{r\to 0} r^2|U(r)| > \infty$ with $U(r) = 2\mu V(r)$
- Two-body scattering problem (for S-wave) with $U(r) = \pm\frac{1}{R_n^2}\left(\frac{R_n}{r}\right)^n$ and $n \geq 2$,

$$-u''(r) + U(r)u(r) = k^2u(r)$$

- At short distances ($r \to 0$) the WKB approximation is applicable

$$\lambda'(r) = \frac{d}{dr}\frac{1}{|p(r)|} = \frac{d}{dr}\frac{1}{\sqrt{k^2 - U(r)}} \ll 1 \qquad \Rightarrow \qquad r \ll \left(\frac{n}{2}\right)^{\frac{2}{2-n}} R_n$$

e.g. for a vdW potential $U(r) = -\frac{1}{R_6^2}\left(\frac{R_6}{r}\right)^6$ the applicability condition reads $r \lesssim R_6$.

- Semiclassical (WKB) short-distance wave functions
- Orthogonality condition between different energy states:

$$-\left[u_k'(r)u_0(r) - u_k(r)u_0'(r)\right]_0^\infty = k^2\int_0^\infty u_k(r)\,u_0(r)\,dr = \sin(\varphi_k - \varphi_0) = 0$$

imply the short-distance phase to be *common* to all eigenfunctions.

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- Semiclassical (WKB) short-distance wave functions

$$U(r) < k^2 \quad , \quad u_k^{\rm WKB}(r) = \frac{C}{\sqrt[4]{k^2 - U(r)}} \sin\left[\int_{r_0}^{r} \sqrt{k^2 - U(r')}\, \mathrm{d}r' + \varphi_k\right]$$

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with φ_k a *short-distances* phase which may depend on the energy.

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- Semiclassical (WKB) short-distance wave functions

$$U(r) \to -\frac{1}{R_n^2}\left(\frac{R_n}{r}\right)^n \quad , \quad u_k(r) \to C\left(\frac{r}{R_n}\right)^{n/4} \sin\left[-\frac{2}{n-2}\left(\frac{R_n}{r}\right)^{\frac{n}{2}-1} + \varphi_k\right]$$

$$U(r) \to +\frac{1}{R_n^2}\left(\frac{R_n}{r}\right)^n \quad , \quad u_k(r) \to C\left(\frac{r}{R_n}\right)^{n/4} \exp\left[-\frac{2}{n-2}\left(\frac{R_n}{r}\right)^{\frac{n}{2}-1}\right]$$

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Renormalization with Boundary Conditions

- Singular potentials: we have to specify a parameter (the phase φ_0).
- The short-distance phase φ_0 *encodes* the unknown short-distance physics:

Fix φ_0 from the experiment $\Leftrightarrow$ Fix the scattering length α_0

- Example: vdW case ($n = 6$)

$$\tan \varphi_0 = \frac{1.13214\,R - 0.69373\,\alpha_0}{1.67481\,\alpha_0 - 0.468947\,R}$$

Three ingredients:

1. Fix α_0 and $u_0(r) \to 1 - \frac{r}{\alpha_0}$
2. Relate u_0 and u_k by orthogonality

$$\int_0^\infty u_k(r)\, u_0(r)\, dr = 0$$

3. Obtain phase shifts

$$u_k(r) \to \frac{\sin(kr + \delta(k))}{\sin \delta(k)}$$

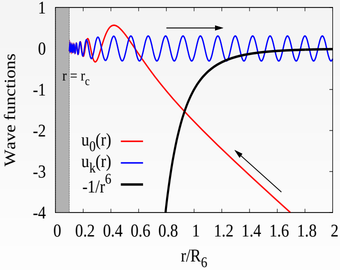
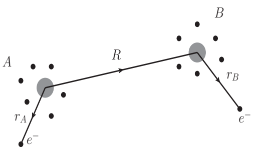


Table of contents

Long-range vdW interaction between atoms

- In molecular systems, constituents interact through Coulomb forces.
- At long-distances, $|\mathbf{R}| \gg |\mathbf{r}_A|, |\mathbf{r}_B|$, a dipole-dipole interaction appears,

$$\begin{aligned} H &= H_0 + \mathcal{V}_{dd} \\ H_0 &= \sum_{A,B} \left[-\frac{\hbar^2}{2\mu} \left(\nabla_A^2 + \nabla_B^2 \right) - \frac{e^2}{r_A} - \frac{e^2}{r_B} \right] \\ \mathcal{V}_{dd}(R) &= e^2 \sum_{A,B} \left[\frac{\mathbf{r}_A \cdot \mathbf{r}_B}{R^3} - 3 \frac{(\mathbf{r}_A \cdot \mathbf{R})(\mathbf{r}_B \cdot \mathbf{R})}{R^5} \right] \end{aligned}$$



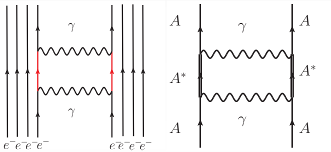
- To second order in perturbation theory

$$V_{AA} = \underbrace{\langle AA|\mathcal{V}_{dd}|AA\rangle}_{\text{null if no permanent dipoles}} + \sum_{AA \neq A^*A^*} \frac{|\langle AA|\mathcal{V}_{dd}|A^*A^*\rangle|^2}{E_{AA} - E_{A^*A^*}} + \cdots = -\frac{C_6}{R^6}$$

⇒ Relativistic corrections: retardation [Casimir and Polder, 1946]

⇒ Quantum field theory: 2γ-exchange [Feinberg and Sucher, 1970]

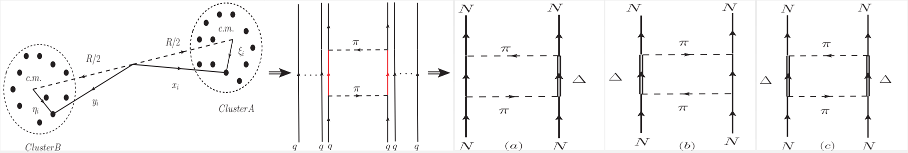
$$V_{AA}^{2\gamma} = -\frac{D}{R^7}$$



SPIN-FLAVOR VAN DER WAALS FORCES

⇒ Nonlinear sigma model ($m_\sigma \to 0$) at quark level, π-exchange between quarks

⇒ Hadrons as clusters of N_c quaks with pairwise interactions $V_{\rm int} = \sum_{i,j} V^{\pi}_{ij}(\vec{x}_i - \vec{y}_j) \to V_{\rm OPE}$



⇒ *Born-Oppenheimer approximation* to 2nd order (OPE-transition potentials):

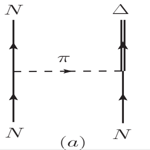
$$V_{NN} = \langle NN|V_{\rm OPE}|HH\rangle + \sum_{NN\neq HH'} \frac{|\langle NN|V_{\rm OPE}|HH'\rangle|^2}{E_{NN} - E_{HH'}} + \cdots$$

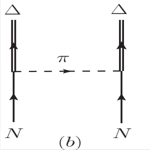
with $|HH'\rangle = |N\Delta\rangle, |\Delta\Delta\rangle$ arbitrary intermediate states.

⇒ We look at the elastic NN channel with $T_{\rm CM} = m_\pi < \Delta \equiv M_\Delta - M_N = 293{\rm MeV}$

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\vec{r}) = V^{1\pi}_{NN,NN}(\vec{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\vec{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\vec{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)$$

OPE TRANSITION POTENTIALS





$$V^{\pi}_{NN,N\Delta}(\vec{r}) = \Big\{ \vec{\sigma}_1 \cdot \vec{S}_2 \, \big[W^{\pi}_S(r)\big]_{NN,N\Delta} + \big[S_{12}(\hat{r})\big]_{NN,N\Delta} \, \big[W^{\pi}_T(r)\big]_{NN,N\Delta} \Big\} \vec{T}_1 \cdot \vec{\tau}_2 \,,$$

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with the tensor operators,

$$\big[S_{12}(\hat{r})\big]_{NN,N\Delta} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{S}_2 \,,$$

$$\big[S_{12}(\hat{r})\big]_{NN,\Delta\Delta} = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \,,$$

and the radial functions

$$\Big[W^{\pi}_{S,T}(r)\Big]_{NN,N\Delta} = \frac{m_\pi}{3} \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \, Y_{0,2}(m_\pi r) \,,$$

$$\Big[W^{\pi}_{S,T}(r)\Big]_{NN,\Delta\Delta} = \frac{m_\pi}{3} \frac{f^2_{\pi N\Delta}}{4\pi} \, Y_{0,2}(m_\pi r) \,,$$

SPIN-FLAVOR VAN DER WAALS FORCES

⇒ The potential can be reduced to the form

$$\begin{aligned} \bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\vec{r}) &= [V_C(r) + V_S(r)\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + V_T(r) S_{12}] \\ &+ [W_C(r) + W_S(r)\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + W_T(r) S_{12}]\,(\vec{\tau}_1\cdot\vec{\tau}_2) \end{aligned}$$

⇒ Short distances behavior ($r \to 0$)

$$V_C(r) = -\frac{f^2_{\pi N\Delta}\left(9f^2_{\pi NN} + f^2_{\pi N\Delta}\right)}{9m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots \qquad W_C(r) = -\frac{f^2_{\pi N\Delta}\left(18f^2_{\pi NN} - f^2_{\pi N\Delta}\right)}{54m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots$$

$$V_S(r) = \frac{f^2_{\pi N\Delta}\left(18f^2_{\pi NN} - f^2_{\pi N\Delta}\right)}{108m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots \qquad W_S(r) = \frac{f^2_{\pi N\Delta}\left(36f^2_{\pi NN} + f^2_{\pi N\Delta}\right)}{648m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots$$

$$V_T(r) = -\frac{f^2_{\pi N\Delta}\left(18f^2_{\pi NN} - f^2_{\pi N\Delta}\right)}{108m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots \qquad W_T(r) = -\frac{f^2_{\pi N\Delta}\left(36f^2_{\pi NN} + f^2_{\pi N\Delta}\right)}{648m^4_\pi \pi^2 \Delta}\,\frac{1}{r^6} + \ldots$$

⇒ The potential is identical to Walet-Amado NN potential in the Skyrme soliton model.
⇒ Short distance is identical to one of ChTPE NLO-Δ potential if $h_A/g_A = f_{\pi N\Delta}/(2f_{\pi NN})$
⇒ Notice that some πN background (triangles, crossed box, football) is not *explicitly* included.

MESON-BARYON-BARYON COUPLING CONSTANTS

$f_{\pi NN}$ COUPLING CONSTANT $\Leftrightarrow$ AXIAL COUPLING g_A

Goldberger-Treiman relation $f_{\pi NN} = g_A m_\pi/(2f_\pi)$, $f_{\pi NN}/m_\pi = g_{\pi NN}/2M_N$
Pion decay constant $f_\pi = 92.4\mathrm{MeV}$ (weak leptonic decays $\pi^\pm \to \mu^\pm \nu_\mu$)
Axial coupling constant coming from β-decay ($g_{\pi NN} = 12.8$):

$$g_A = \begin{cases} 1.249(6) & \text{if } n \text{ decay rate is included,} \\ 1.257(9) & \text{if only angular distribution is used.} \end{cases}$$

Phase shift analysis of NN scattering yields $g_{\pi NN} = 13.1$ compatible with $g_A = 1.29$

Admissible values: $g_A = 1.25 - 1.29$

$f_{\pi N\Delta}$ COUPLING CONSTANT

Adkins, Nappi and Witten (Skyrme model): $f_{\pi N\Delta}/f_{\pi NN} = 3/\sqrt{2}$.
Dashen, Jenkin and Manohar (large N_c SU(4) spin-flavor symmetry): $f_{\pi N\Delta}/f_{\pi NN} = 3/\sqrt{2}$.
Karl and Paton & Jackson *et al.* naive $SU(N_c)$ quark model predicts

$$\frac{f_{\pi N\Delta}}{f_{\pi NN}} = \frac{3}{\sqrt{2}} \frac{\sqrt{(N_c-1)(N_c+5)}}{N_c+2} = \begin{cases} 3/\sqrt{2} & \text{for } N_c \to \infty \\ 6\sqrt{2}/5 = \sqrt{72/25} & \text{for } N_c = 3 \end{cases}$$

$$\lim_{N_c\to\infty} \text{Skyrme Model} \underbrace{=}_{\text{[Manohar]}} \lim_{N_c\to\infty} \text{Quark Models} \underbrace{\Longleftrightarrow}_{\text{[Dashen Jenkin Manohar]}} \text{QCD SU(4) spin-flavor}$$

COMPARISON WITH CHTPE-Δ POTENTIAL

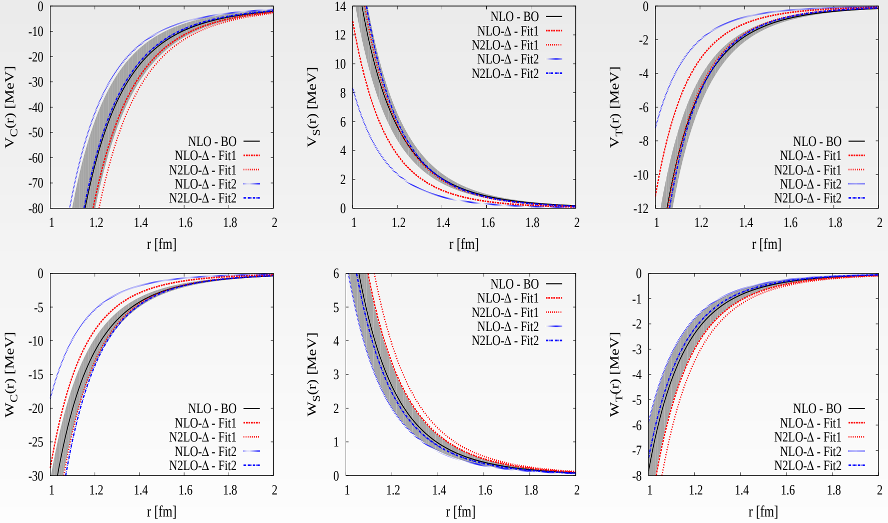


TABLE OF CONTENTS

UNCOUPLED CHANNELS

Reduced Schrödinger equation in the *pn* center-of-mass (c.m.) system

$$-u''_{k,l}(r) + \left[U(r) + \frac{l(l+1)}{r^2}\right]u_{k,l}(r) = k^2 u_{k,l}(r)$$

Reduced potential

$$\begin{aligned} U(r) &= M\Big(V_C(r) + \sigma\, V_S(r) + S_{12}(\hat{r})\, V_T(r) + \tau W_C(r) + \tau\,\sigma\, W_S(r) + \tau\, S_{12}(\hat{r})\, W_T(r)\Big) \\ &\to -\frac{R_6^4}{r^6} \quad \text{for } r \to 0 \end{aligned}$$

Short distance solution,

$$u_{k,l}(r) \quad \to \quad A_l \left(\frac{r}{R_6}\right)^{3/2} \sin\left[\frac{1}{2}\left(\frac{R_6}{r}\right)^2 + \varphi_l(k)\right]$$

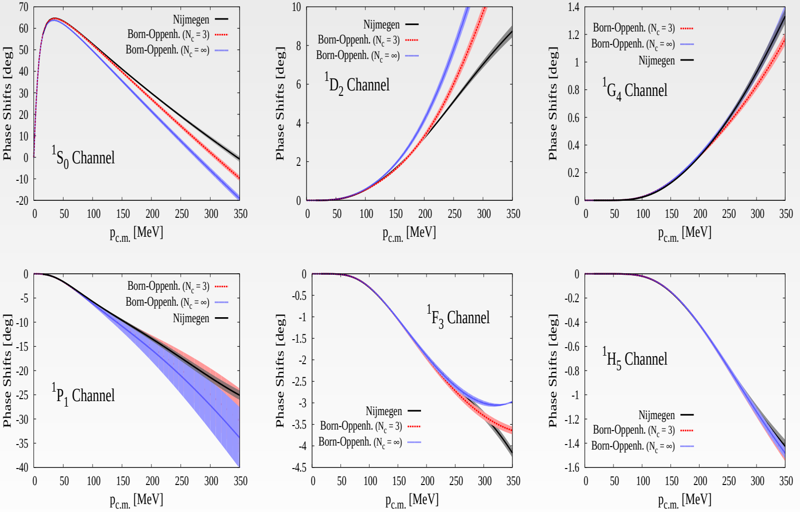
For $r_c < r < R_6$ with $r_c \to 0$ singularity dominates centrifugal barrier

$$\varphi_l(k_1) = \varphi_l(k_2) \qquad \text{and} \qquad \varphi_{l_1}(k) = \varphi_{l_2}(k)$$

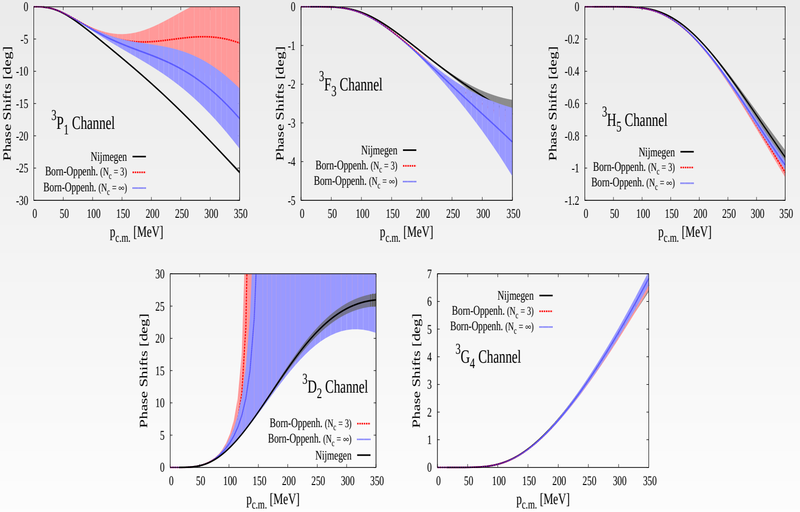
We would have the following correlations [Pavon and Arriola, PRC 83:044002,2011],

(I) Singlet isovector ($s = 0$, $t = 1$, $\sigma = -3$ and $\tau = 1$): ($^1S_0$, $^1D_2$, $^1G_4$, ...)
(II) Singlet isoscalar ($s = 0$, $t = 0$, $\sigma = -3$ and $\tau = -3$): ($^1P_1$, $^1F_3$, $^1H_5$, ...)
(III) Triplet isovector ($s = 1$, $t = 1$, $\sigma = 1$ and $\tau = 1$): ($^3P_1$, $^3F_3$, $^3H_5$, ...)
(IV) Triplet isoscalar ($s = 1$, $t = 0$, $\sigma = 1$ and $\tau = -3$): ($^3D_2$, $^3G_4$, ...)

Singlet channel phase shifts ($s = 0, S_{12}(\hat{r}) = 0$)



TRIPLET UNCOUPLED PHASE SHIFTS ($s = 1, S_{12}(\hat{r}) = 2$)



Triplet coupled channels

Coupled channel Schrödinger equation

We have to solve the Schrödinger equation

$$-\mathbf{u}''(r) + \left[\mathbf{U}(r) + \frac{\mathbf{L}^2}{r^2}\right]\mathbf{u}(r) = k^2\,\mathbf{u}(r)\,,$$

with

$$\mathbf{U}(r) = \begin{pmatrix} U_{j-1,j-1} & U_{j-1,j+1} \\ U_{j-1,j+1} & U_{j+1,j+1} \end{pmatrix}\,,\quad \mathbf{L}^2 = \begin{pmatrix} j(j-1) & 0 \\ 0 & (j+1)(j+2) \end{pmatrix}\,,\quad \mathbf{u}(r) = \begin{pmatrix} u(r) \\ w(r) \end{pmatrix}\,.$$

TRIPLET COUPLED CHANNELS

COUPLED CHANNEL SCHRÖDINGER EQUATION

We have to solve the Schrödinger equation

$$-\mathbf{u}''(r) + \left[\mathbf{U}(r) + \frac{\mathbf{L}^2}{r^2}\right]\mathbf{u}(r) = k^2\,\mathbf{u}(r)\,,$$

with

$$\mathbf{U}(r) = \begin{pmatrix} U_{j-1,j-1} & U_{j-1,j+1} \\ U_{j-1,j+1} & U_{j+1,j+1} \end{pmatrix}\,,\quad \mathbf{L}^2 = \begin{pmatrix} j(j-1) & 0 \\ 0 & (j+1)(j+2) \end{pmatrix}\,,\quad \mathbf{u}(r) = \begin{pmatrix} u(r) \\ w(r) \end{pmatrix}\,.$$

POTENTIAL DIAGONALIZATION

The potential can be split into $\mathbf{U} = \mathbf{1}U_{NT}(r) + \mathbf{S}^j_{12}U_T(r)$ with,

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\,,$$

$$\mathbf{S}^j_{12} = \frac{1}{2j+1}\begin{pmatrix} -2(j-1) & 6\sqrt{j(j+1)} \\ 6\sqrt{j(j+1)} & -2(j+2) \end{pmatrix}\,.$$

Triplet coupled channels

Coupled channel Schrödinger equation

We have to solve the Schrödinger equation

$$-\mathbf{u}''(r) + \left[\mathbf{U}(r) + \frac{\mathbf{L}^2}{r^2}\right]\mathbf{u}(r) = k^2\,\mathbf{u}(r)\,,$$

with

$$\mathbf{U}(r) = \begin{pmatrix} U_{j-1,j-1} & U_{j-1,j+1} \\ U_{j-1,j+1} & U_{j+1,j+1} \end{pmatrix}\,,\quad \mathbf{L}^2 = \begin{pmatrix} j(j-1) & 0 \\ 0 & (j+1)(j+2) \end{pmatrix}\,,\quad \mathbf{u}(r) = \begin{pmatrix} u(r) \\ w(r) \end{pmatrix}\,.$$

Potential diagonalization

$$\mathbf{R}_j = \frac{1}{2j+1}\begin{pmatrix} \sqrt{j+1} & \sqrt{j} \\ -\sqrt{j} & \sqrt{j+1} \end{pmatrix} = \begin{pmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{pmatrix}$$

$$\mathbf{S}^j_{12,D} = \mathbf{R}_j \mathbf{S}^j_{12} \mathbf{R}^T_j = \begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix}\,,\quad \mathbf{L}^2_D = \mathbf{R}_j \mathbf{L}^2 \mathbf{R}^T_j = \begin{pmatrix} j(j+1) & 2\sqrt{j(j+1)} \\ 2\sqrt{j(j+1)} & j(j+1)-2 \end{pmatrix}\,.$$

Triplet coupled channels

Short distance problem

At very short distances $\mathbf{U} \to \frac{M\mathbf{C}_6}{r^6}$ with $M\mathbf{C}_6$ a short distance diagonalizable matrix (attractive-attractive potential case)

$$\begin{pmatrix} MC_{6,{}^3L_j^{j-1}} & MC_{6,E_j} \\ MC_{6,E_j} & MC_{6,{}^3L_j^{j+1}} \end{pmatrix} = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \\ \sin\theta_j & \cos\theta_j \end{pmatrix} \begin{pmatrix} -R_+^4 & 0 \\ 0 & -R_-^4 \end{pmatrix} \begin{pmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{pmatrix},$$

with $-R_+^4 = M(C_{NT} + 2C_T)$ and $-R_-^4 = M(C_{NT} - 4C_T)$.
In the diagonal basis $\mathbf{v}_j = \mathbf{R}_j \mathbf{u}_j$ at short distances the singularity dominates

$$\begin{pmatrix} v_+ \\ v_- \end{pmatrix} = \begin{pmatrix} \cos_j\theta & \sin_j\theta \\ -\sin_j\theta & \cos_j\theta \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix},$$

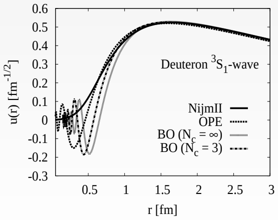
and the system decouples

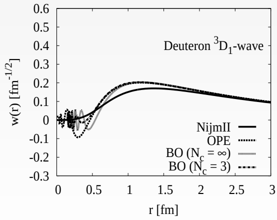
$$-v_\pm'' - \frac{R_\pm^4}{r^6} v_\pm = k^2\, v_\pm\,, \quad v_\pm(r) = \left(\frac{r}{R_\pm}\right)^{\frac{3}{2}} C_\pm \sin\left[\frac{1}{2}\frac{R_\pm^2}{r^2} + \varphi_\pm(k)\right].$$

DEUTERON $^3S_1 - ^3D_1$

Input parameters:
Binding energy $B_d = 2.224575$ MeV, D/S ratio $\eta = 0.0256$ and $a_{^3S_1} = 5.419$ fm
Solve the diagonalized Schrödinger equation for negative energy $k^2 = -\gamma^2 = -M B_d$

Set		γ (fm^{-1})	η	A_S (fm$^{-1/2}$)	r_m (fm)	Q_d(fm^2)	P_D (%)	$\langle r^{-1} \rangle$
OPE		Input	0.02633	0.8681	1.9351	0.2762	7.88	0.476
BO ($N_c = 3$)	$g_A = 1.25$	Input	Input	0.8674	1.9340	0.2711	8.19	0.473
	$g_A = 1.29$	Input	Input	0.8783	1.9549	0.2712	6.46	0.462
BO ($N_c = \infty$)	$g_A = 1.25$	Input	Input	0.8801	1.9605	0.2781	7.76	0.448
	$g_A = 1.29$	Input	Input	0.8931	1.9857	0.2798	5.74	0.433
NLO-Δ ($h_A = 1.34$)		Input	Input	0.884(3)	1.963(7)	0.274(9)	5.9(4)	0.446(10)
NLO-Δ ($h_A = 1.05$)		Input	Input	0.84(4)	1.86(8)	0.24(3)	12(5)	0.62(15)
NijmII		0.231605	0.02521	0.8845	1.9675	0.2707	5.635	0.4502
Reid93		0.231605	0.02514	0.8845	1.9686	0.2703	5.699	0.4515
Exp.		0.231605	0.0256(4)	0.8846(9)	1.9754(9)	0.2859(3)	5.67(7)	-





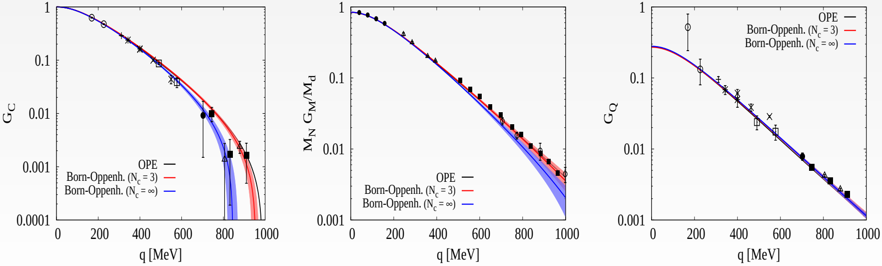
DEUTERON EM FORM FACTORS (IA)

Elastic electron-deuteron differential cross section in the lab-frame

$$\frac{d\sigma}{d\Omega_e}\left(q^2,\theta_e\right) = \left(\frac{d\sigma}{d\Omega_e}\right)_{\rm Mott}\left[A(q^2) + B(q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right] .$$

Deuteron structure functions A and B:

$$\begin{aligned} A(q^2) &= G_C^2(q^2) + \frac{2}{3}\,\eta\, G_M^2(q^2) + \frac{8}{9}\,\eta^2\, G_Q^2(q^2)\,, \\ B(q^2) &= \frac{4}{3}\,\eta\,(1+\eta)\,G_M^2(q^2)\,, \end{aligned}$$



HIGHER PARTIAL WAVES (CORRELATIONS)

- For higher partial waves one solves the coupled channel Schrödinger eq.

$$-\mathbf{u}''_{k,j}(r) + \left[\mathbf{U}(r) + \frac{\mathbf{L}^2}{r^2}\right] \mathbf{u}_{k,j}(r) = k^2\, \mathbf{u}_{k,j}(r)$$

- We fix the scattering lengths and integrate downwards from $r \to \infty$ to $r = r_c$.
- Defining $\mathbf{L}_{k,j}(r) = \mathbf{u}'_{k,j}(r)\, \mathbf{u}_{k,j}{}^{-1}(r)$ the finite energy solution is constructed from

$$\mathbf{L}_{k,j}(r_c) = \mathbf{L}_{0,j}(r_c)\,.$$

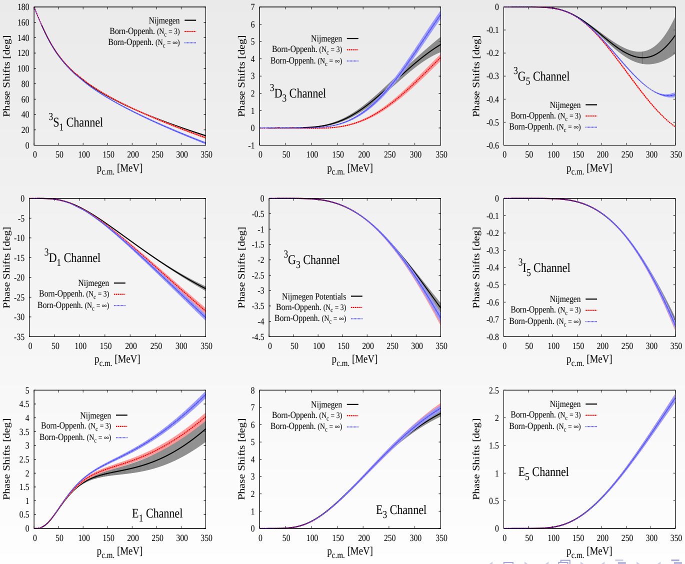
- In the rotated basis $\mathbf{v}_j = \mathbf{R}_j \mathbf{u}_j$ the tensor $\mathbf{S}^j_{12,D}$ does not depend on j and isoscalar ($^3C_1$, $^3C_3$, $^3C_5$) and isovector ($^3C_2$, $^3C_4$) channels possess the same short distance potential in the rotated basis,

$$\begin{aligned}
\mathbf{R}_1\, \mathbf{V}_{^3C_1}(r)\, \mathbf{R}_1^{\mathrm{T}} &= \mathbf{R}_3\, \mathbf{V}_{^3C_3}(r)\, \mathbf{R}_3^{\mathrm{T}} = \mathbf{R}_5\, \mathbf{V}_{^3C_5}(r)\, \mathbf{R}_5^{\mathrm{T}} = \dots\,, \\
\mathbf{R}_2\, \mathbf{V}_{^3C_2}(r)\, \mathbf{R}_2^{\mathrm{T}} &= \mathbf{R}_4\, \mathbf{V}_{^3C_4}(r)\, \mathbf{R}_4^{\mathrm{T}} = \dots\,,
\end{aligned}$$

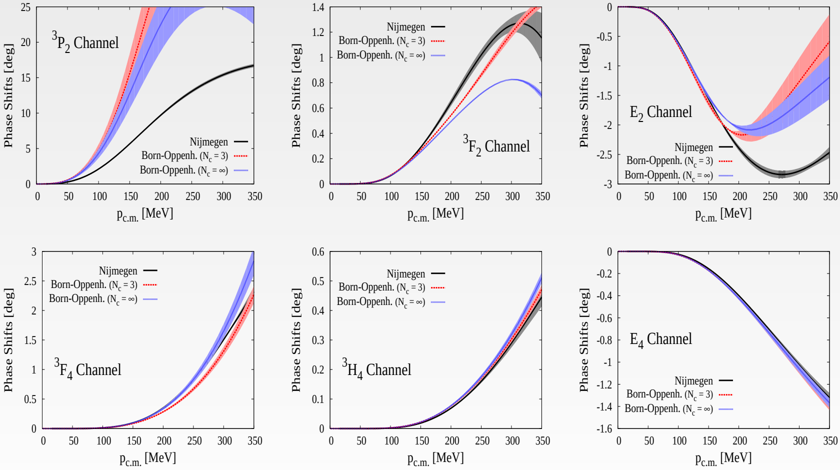
- At very short distances the singularity of the potential dominates the centrifugal barrier and independence with j is achieved in the rotated basis [Pavon and Arriola, PRC 83:044002,2011]

$$\begin{aligned}
\mathbf{R_1}\, \mathbf{L}_{k,1}(r_c)\, \mathbf{R_1}^{\mathrm{T}} &= \mathbf{R_3}\, \mathbf{L}_{k,3}(r_c)\, \mathbf{R_3}^{\mathrm{T}} = \mathbf{R_5}\, \mathbf{L}_{k,5}(r_c)\, \mathbf{R_5}^{\mathrm{T}} = \dots\,, \\
\mathbf{R_2}\, \mathbf{L}_{k,2}(r_c)\, \mathbf{R_2}^{\mathrm{T}} &= \mathbf{R_4}\, \mathbf{L}_{k,4}(r_c)\, \mathbf{R_4}^{\mathrm{T}} = \dots\,.
\end{aligned}$$

CORRELATED TRIPLET COUPLED ISOSCALAR PHASES

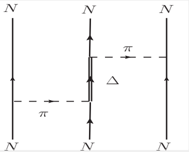


CORRELATED TRIPLET COUPLED ISOVECTOR PHASES



3NF IN THE BORN-OPPENHEIMER APPROXIMATION

$$V_{3N}(\boldsymbol{r}) = \langle NNN|V_{OPE}|NNN\rangle + \sum_{NNN\neq HH'H''} \frac{|\langle NNN|V_{OPE}|HH'H''\rangle|^2}{E_{NNN} - E_{HH'H''}}$$



At the 2π-exchange level: $V_{OPE} = V_{OPE}^{(12)} + V_{OPE}^{(13)} + V_{OPE}^{(23)}$

$$\begin{aligned}
V_{3N}(\boldsymbol{r}) &= \langle NNN|V_{OPE}|NNN\rangle + \frac{|\langle NNN|V_{OPE}|N\Delta N\rangle|^2}{M_N - M_\Delta} \\
\langle NNN|V_{OPE}|NNN\rangle &= 3\left\langle NN|V_{OPE}^{(12)}|NN\right\rangle \\
|\langle NNN|V_{OPE}|NDN\rangle|^2 &= |V_{NN,N\Delta}^{1\pi}(r_{12})|^2 + |V_{NN,N\Delta}^{1\pi}(r_{13})|^2 + |V_{NN,N\Delta}^{1\pi}(r_{23})|^2 \\
&+ 2V_{NN,N\Delta}^{1\pi}(r_{12})V_{NN,N\Delta}^{1\pi}(r_{23}) \\
&+ 2V_{NN,N\Delta}^{1\pi}(r_{12})V_{NN,N\Delta}^{1\pi}(r_{13}) \\
&+ 2V_{NN,N\Delta}^{1\pi}(r_{23})V_{NN,N\Delta}^{1\pi}(r_{13}) \\
&= \ldots \text{(complicated structures)} \ldots
\end{aligned}$$

Table of contents

Conclusions and outlook

- We have used the Born-Oppenheimer approximation to obtain a NN potential starting from OPE at the quark level and using second order perturbation theory.
- This potential is identical to the one obtained by Walet and Amado using the Skyrme model.
- The short distance behavior is identical to the ChTPE NLO-Δ if we identify $h_A/g_A = f_{\pi N\Delta}/(2f_{\pi NN})$, with a short-distance singularity of vdW type. The mid-range behavior looks very similar to each other.
- We have used renormalization with boundary conditions to deal with that singularity where the number of counter-terms needed has been reduced by applying correlations between partial waves.
- By varying the coupling constants $f_{\pi NN}$ and $f_{\pi N\Delta}$ within admissible values we have obtained good phenomenology (deuteron, phase shifts, EM form factors).
- In this formalism the extension to 3NF is straightforward ...