Precision calculation of the pion electromagnetic form factor from lattice QCD

SFB443

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14.06.2011

References: arXiv:1010.2390 [hep-lat];
arXiv:1106.1554 [hep-lat]

Contents

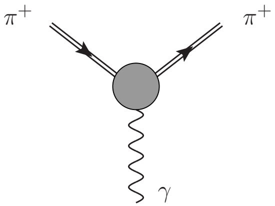
1. Introduction:

The electromagnetic vector form factor of the pion as a high precision lattice observable

Lattice QCD and experiment

- Recently lattice QCD has evolved in producing reliable results for a lot of physical quantities.
  Some of them collected in the reviews:
  [Colangelo *et al* (FLAG) arXiv:1011.4408]; Lunghi, Laiho, Van de Water arXiv:1102.3917
  (See plenary talk by Andreas Jüttner tomorrow)
- Nevertheless: Some quantities fail to reproduce physical results! (e.g. baryonic form factors, etc.)
- In general this might be due to the fact that simulations still lack full control of systematic effects.
- The charge radius of the pion, connected with the form factor, is the easiest case of an observable where these systematic effects enters and is thus well suited for a high precision benchmark of new techniques.

Vector form factor of the pion in the space-like regime

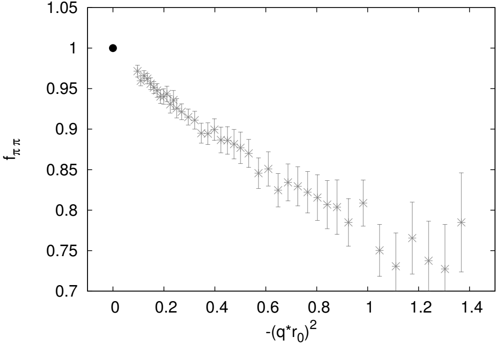


- Known from experiment with high precision.
- Relatively easy to extract from lattice simulations.
- Connected to the pion charge radius:

$$\langle r_\pi^2 \rangle = 6 \left. \frac{d\, f_\pi(q^2)}{dq^2} \right|_{q^2=0}$$

Vector form factor of the pion in the space-like regime: Experiment

Space-like regime with low q^2: [NA7 collaboration, Nucl. Phys. **B277** (1986)]



Charge radius: $\langle r_\pi^2 \rangle = 0.431(10)\ \mathrm{fm}^2$

Vector form factor of the pion in the space-like regime: Lattice

Related to the matrix element of the vector current by:

$$\left\langle \pi^+(\mathbf{p}_f) \middle| V_\mu(q^2) \middle| \pi^+(\mathbf{p}_i) \right\rangle = (p_f + p_i)_\mu \; f_\pi(q^2)$$

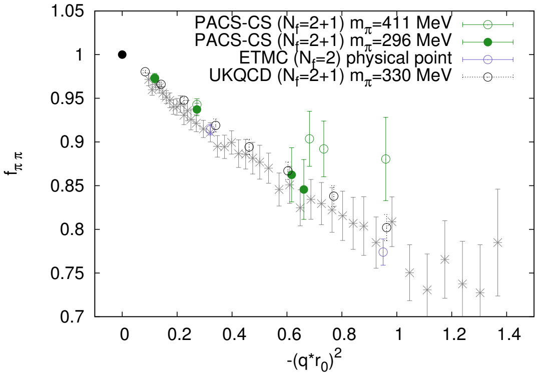
- no quark-disconnected diagrams contribute
- noise reduction techniques can be applied efficiently

  [Foster, Michael (1999); Boyle *et al* (2008)]

- in principle only space-like momenta are available due to euclidean signature of spacetime
  (recently extended also to time-like momenta) [Meyer (2011)]

⇒ Allows for a high precision simulation!

Vector form factor of the pion in the space-like regime: Lattice – results



r_0: Sommer scale ($\approx$ 0.5 fm) [Sommer (1994)]

2. Lattice regularisation:

Form factors from the lattice

The lattice

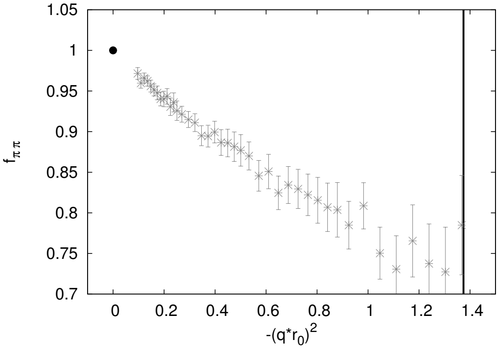
- Euclidean spacetime discretized on a 4d hypercubic lattice.
- Expectation values of observable (fermions integrated out):

$$\langle O \rangle = \frac{1}{Z} \int d[U]\, O[U] \prod_f \left[\det\left(D_f[U]\right)\right] \exp\left(-S[U]\right)$$

  Is measured on stochastically generated *representative* ensembles of gauge field configurations
- Physical results via an extrapolation to the continuum.
- Cost of the simulation growths when lowering the quark mass.
  $\Rightarrow$ An extrapolation to the physical point is needed in most cases.
- Problem: Momentum is usually introduced by Fourier transformation.
  $\Rightarrow$ Lower momentum cut-off due to finite volume!

Fourier momenta

Example: $a = 0.07$ fm; $L = 32\,a = 2.3$ fm



solid line: Minimal q^2 from Fourier momenta

Twisted boundary conditions

This problem can be cured by the use of partially twisted boundary conditions

[Bedaque (2004); Divitiis, Petronzio, Tantalo (2004)]

- Change of the boundary conditions of the quark fields leads to a shift in the quark momenta:

$$p_i = \frac{2\,\pi}{L}\,n_i + \frac{\theta_i}{L} \qquad \theta_i : \text{twist angles}$$

- Suitable tuning allows for arbitrarily small (space-like) momentum transfers. [Boyle *et al* (2007)]
- Additional finite volume effects are exponentially suppressed for matrix elements with at most one hadronic state in the initial and/or final state (like $f_\pi,\ f_{\pi\pi},\ldots$) [Sachrajda, Villadoro (2005)]

3. Results:

Form factor and charge radius

CLS ensembles – Lattice parameters

We use the ensembles generated within the CLS framework
(CLS: Coordinated Lattice Simulations)

[https://twiki.cern.ch/twiki/bin/view/CLS/WebHome]

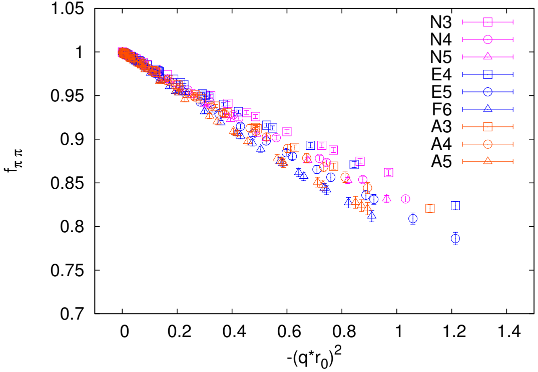
Discretisation: $N_f = 2$; non-perturbatively $\mathcal{O}(a)$-improved Wilson

Algorithm: deflation accelerated DD-HMC [Lüscher (2004-2007)]

Lattice parameters:

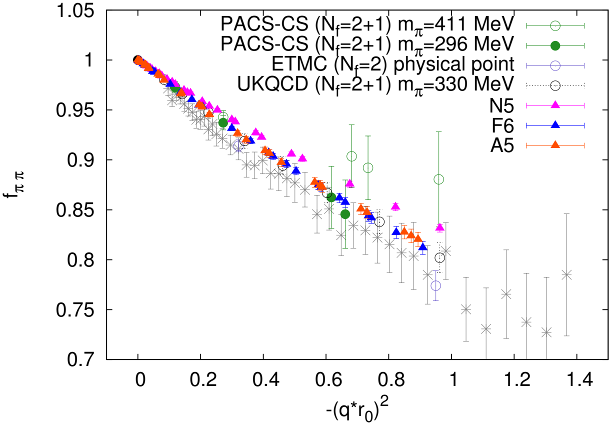
β	a[fm]	lattice	# masses	$m_\pi L$	Labels	Statistic
5.20	0.08	64×32^3	3	6.0 – 4.0	A3 – A5	$\mathcal{O}(100)$
5.30	0.07	64×32^3	2	6.2, 4.7	E4, E5	$\mathcal{O}(100)$
5.30	0.07	96×48^3	1	5.0	F6	233
5.50	0.05	96×48^3	3	7.7 – 5.3	N3 – N5	$\mathcal{O}(100)$

The pion form factor

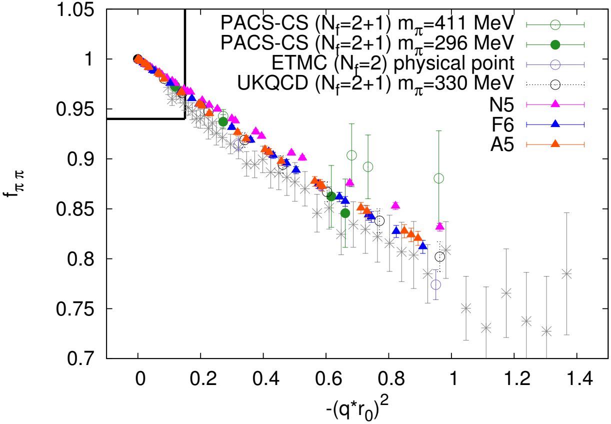


Highest precision and lowest momenta ever attained!

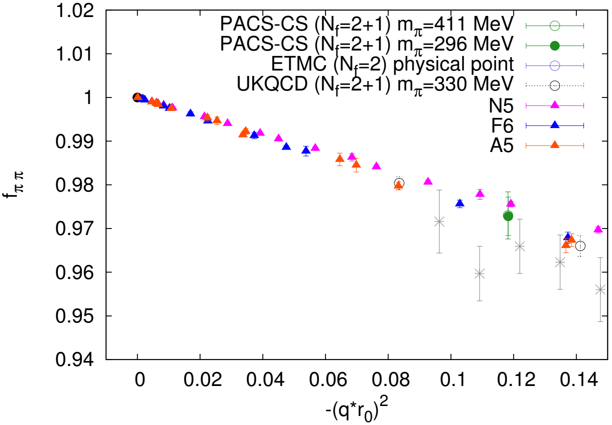
The pion form factor



The pion form factor



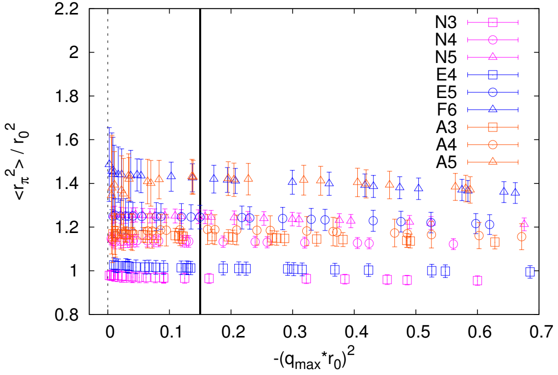
The pion form factor



Extraction of the charge radius

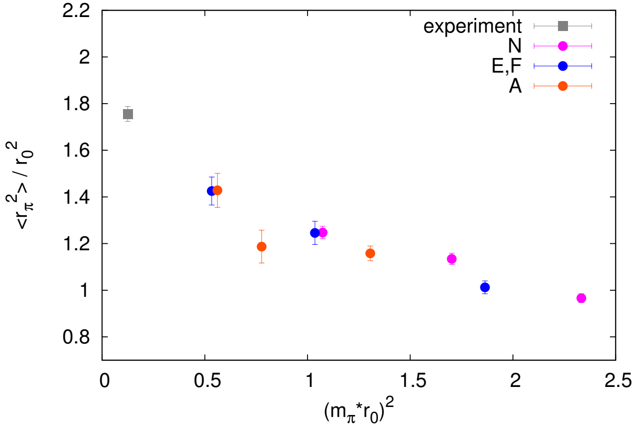
- High density of points close to $q^2 = 0$ allows the direct extraction of the slope using a linear fit.
  $\Rightarrow$ No scheme or model dependence!
- Can be checked against polynomial fits.
- Other crosschecks:
  - Vector pole dominance
  - χPt to NLO and NNLO

Charge radius:
q^2 dependence of linear fits



Solid line: $-(q\,r_0)^2 = 0.15$, maximum q^2 used for the linear fits

Charge radius:
chiral behavior



Charge radius:
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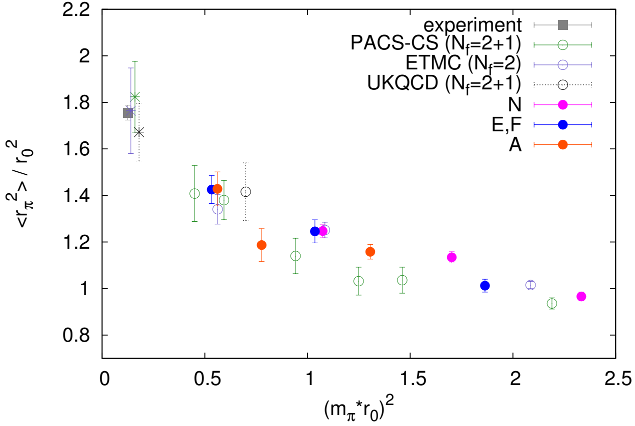


NLO χPt:

Formulae

χPt for $f_\pi(q^2)$: [Gasser, Leutwyler (1984); Bijnens, Colangelo, Talavera (1998)]

$$f_{\pi\pi}(q^2) = 1 + \frac{m_\pi^2}{f_\pi^2}\left[\frac{1}{6}\left(\frac{q^2}{m_\pi^2} - 4\right)\bar{J}\left(\frac{q^2}{m_\pi^2}\right) + \frac{q^2}{m_\pi^2}\left(-\ell_6^r - \frac{1}{6}L\left(\frac{m_\pi^2}{\mu^2}\right) - \frac{1}{288\pi^2}\right)\right]$$

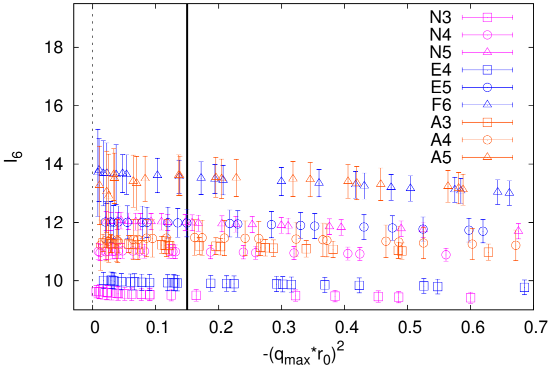
$$\text{with } \ell_6^r = -\frac{1}{96\,\pi^2}\left[\bar{\ell}_6 + 16\,\pi^2\,L\left(\frac{m_\pi^2}{\mu^2}\right)\right]$$

χPt for $\langle r_\pi^2 \rangle$:

$$\langle r_\pi^2 \rangle = \frac{m_\pi^2}{f_\pi^2}\left(-6\,\ell_6^r - L\left(\frac{m_\pi^2}{\mu^2}\right) - \frac{1}{16\pi^2}\right)$$

- only free parameter: $\bar{\ell}_6$
- at the moment f_π fixed to experimental value
- renormalisation scale: $\mu = m_\rho$:

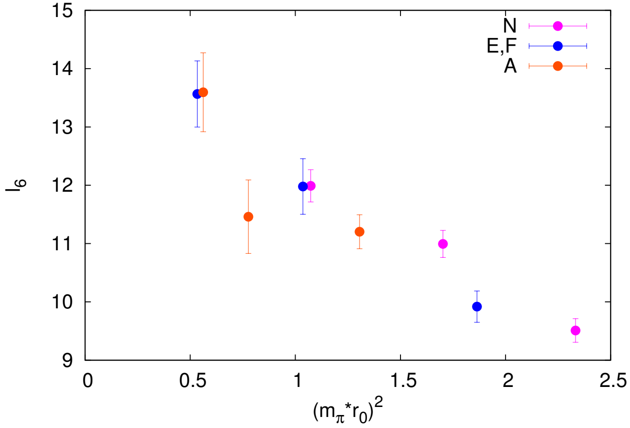
NLO χPt:
q^2 dependence of $\bar{\ell}_6$



Solid line: $-(q\, r_0)^2 = 0.15$, maximum q^2 used for the fits

NLO χPt:

pion mass dependence of $\bar{\ell}_6$



4. Conclusions and outlook:

Conclusions

- State of the art lattice simulations of mesonic and baryonic form factors are still affected by systematic uncertainties.
- Improved simulation techniques are needed to compare lattice results and experiment to high precision to verify the conjectured agreement between the two.
- One interesting case is the charge radius of the pion that, despite other systematic effects, suffers from a model dependent extraction in almost all previous calculations.
- I presented our $N_f = 2$ study of the efficiency of novel techniques for the pion form factor and the extraction of the charge radius.
- Partially twisted boundary conditions are the main ingredient to reduce the model dependence in $\langle r_\pi^2 \rangle$, due to a large number of measurements at low q^2.
  (here we can even do better as experiment)

Outlook

- The accuracy of our data is good enough for lattice artefacts to become visible (eventually) and a more systematic analysis is under way.
- χPt to NLO does not work accurately as can be seen by the behavior of $\bar{\ell}_6$ with m_π^2.
  $\Rightarrow$ To perform the extrapolation to the physical point we are going to utilise χPt to NNLO.
- The chiral extrapolation will be assisted by additional measurements at smaller pion masses.
- In the end we aim at a model-independent result for the charge radius of the pion from first principles to compare to the experimental value.

Thank you for your attention!