Effective Q-Q Interactions in Heavy Baryons

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Outline

Introduction

What has been learned

- For light baryons the relativistic constituent quark model provides an efficient and intuitive tool.
- Describes both spectroscopy and electroweak structures (at least for baryon ground states).

See, e.g.,

W. Plessas: PoS(LC2010)017

K.-S. Choi: PhD Thesis, University of Graz, 2011

Formalism of Poincaré-Invariant QM

Relativistic quantum mechanics (RQM)

i.e. quantum theory respecting Poincaré invariance

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P,J,\Sigma\rangle = M\,|P,J,\Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^{\mu}\hat{P}_{\mu}$$

$$\hat{P}^{\mu}\,|P,J,\Sigma\rangle = P^{\mu}\,|P,J,\Sigma\rangle \quad , \qquad \hat{P}^{\mu} = \hat{M}\hat{V}^{\mu}$$

Relativistic Constituent Quark Model (RCQM)

Interacting mass operator

$$\begin{aligned}\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\ \hat{M}_{free} &= \sqrt{\hat{H}_0^2 - \hat{\vec{P}}^2_{free}} \\ \hat{M}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j} [\hat{V}_{ij}^{conf} + \hat{V}_{ij}^{hf}]\end{aligned}$$

fulfilling the Poincaré algebra

$$\begin{array}{lll} [\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\ [\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\ [\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H} \end{array}$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Q-Q Interaction

Confinement

- Generally assumed as linearly rising potential, as following from lattice QCD.

What is the hyperfine interaction

- Phenomenology: It cannot be only one-gluon exchange
- Flavor independent potentials cannot produce correct level orderings, particularly in the N and Λ spectra.

Low-Energy QCD

Goldstone Bosons

- $SU(3)_L \times SU(3)_R \to SU(3)_V$
- Spontaneous breaking of chiral symmetry ($SB_\chi S$)
- Generation of $3^2 - 1 = 8$ Goldstone bosons
- $U(1)_A$ creates the ninth by explicit symmetry breaking

Constituent Quarks

- Constituent quarks appear as quasiparticles with dynamical mass

Effective Lagrangian

$$L \sim ig\bar{\psi}\gamma_5\vec{\lambda}^F \cdot \vec{\phi}\psi \quad (1)$$

$SU(3)_F$ GBE Model

Hyperfine Interaction (spin-spin part only)

$$V_{\text{hf}} = V_\chi^{\text{octet}} + V^{\text{singlet}} \tag{2}$$

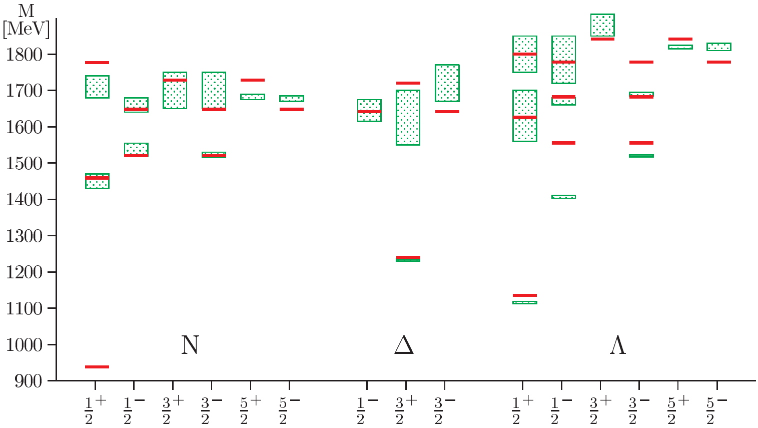
$$V_{\text{hf}} = \left[V_\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a + V_K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a + V_\eta \lambda_i^8 \lambda_j^8 + V_{\eta'} \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{3}$$

Regularized Goldstone-Boson Exchange

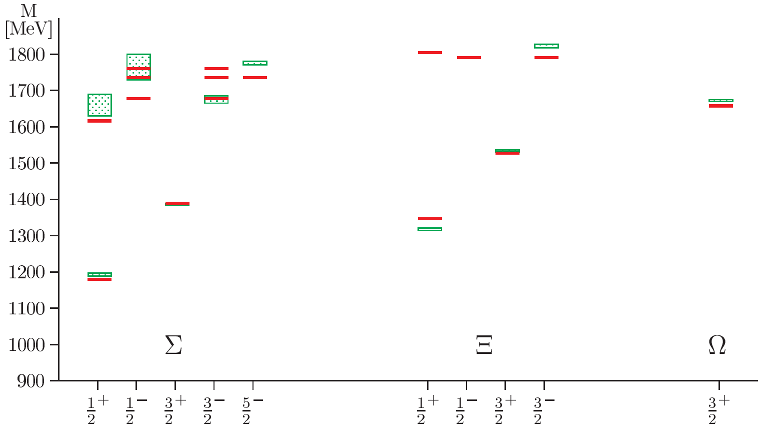
$$V_\gamma(r_{ij}) = \frac{g_\gamma^2}{2\pi} \frac{1}{12 m_i m_j} \left[\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}} \right] \tag{4}$$

$$\gamma = \pi, K, \eta, \eta'$$

N, Δ, Λ Spectra



Σ, Ξ, Ω Spectra


M
[MeV]
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
Σ
Ξ
Ω
$\frac{1}{2}^+$ $\frac{1}{2}^-$ $\frac{3}{2}^+$ $\frac{3}{2}^-$ $\frac{5}{2}^-$
$\frac{1}{2}^+$ $\frac{1}{2}^-$ $\frac{3}{2}^+$ $\frac{3}{2}^-$
$\frac{3}{2}^+$

Open Problem

Q: For heavy baryons,

- is GBE still present
- or is one-gluon exchange better suited?

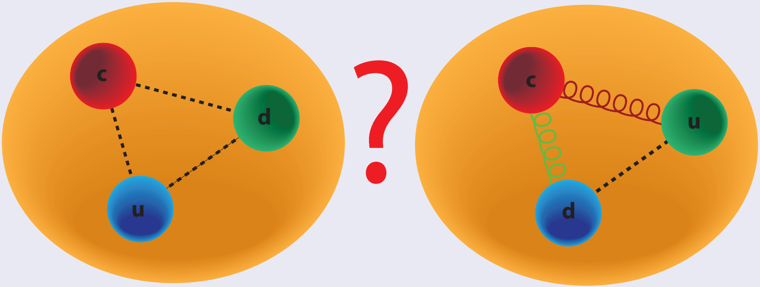
A: Let us see!

Baryons of $SU(5)_F$

One has to know Q-Q interactions

- Light-Light
- Light-Heavy
- Heavy-Heavy

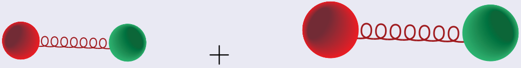
How should they be modeled?



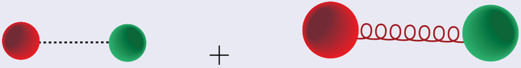
Q-Q Interaction Models

Q-Q Interactions

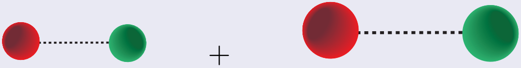
- OGE + OGE



- GBE + OGE



- GBE + GBE

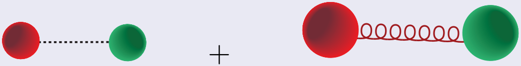


Q-Q Interaction Models

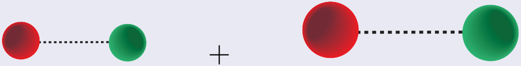
Q-Q Interactions

- OGE + OGE

- GBE + OGE



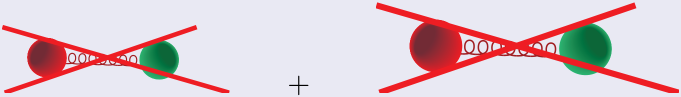
- GBE + GBE



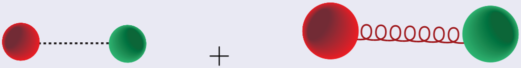
Q-Q Interaction Models

Q-Q Interactions

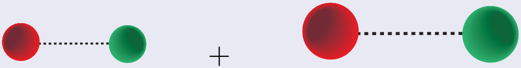
- OGE + OGE



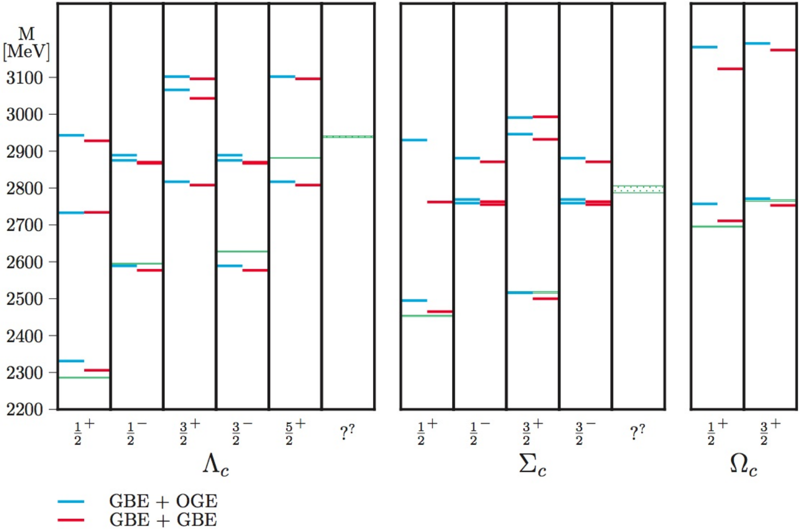
- GBE + OGE



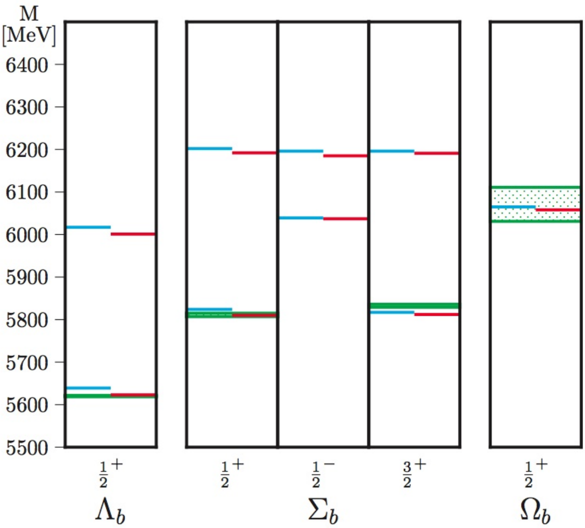
- GBE + GBE



Comparison in Charm Spectra



Comparison in Bottom Spectra



More Experimental Data is Needed!

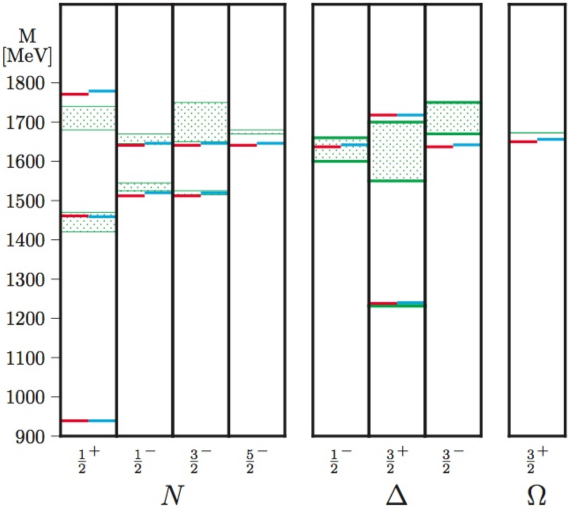
- Preliminary investigation appears to show that both the GBE and Hybrid models work well. More experimental data is necessary.

- What must we now consider when creating a full $SU(5)_F$ GBE potential?

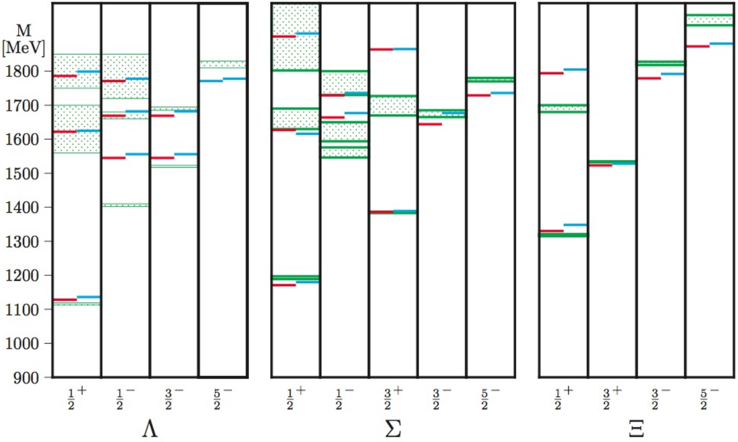
$SU(5)_F$ Potential Model

$$V_\chi = \Bigg[V_\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a + V_K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a + V_\eta \lambda_i^8 \lambda_j^8 + \frac{2}{5} V_{\eta'}$$
$$+ V_D \sum_{a=9}^{12} \lambda_i^a \lambda_j^a + V_{D_s} \sum_{a=13}^{14} \lambda_i^a \lambda_j^a + V_{\eta_c} \lambda_i^{15} \lambda_j^{15}$$
$$+ V_B \sum_{a=16}^{19} \lambda_i^a \lambda_j^a + V_{B_s} \sum_{a=20}^{21} \lambda_i^a \lambda_j^a + V_{B_c} \sum_{a=22}^{23} \lambda_i^a \lambda_j^a$$
$$+ V_{\eta_b} \lambda_i^{24} \lambda_j^{24} \Bigg] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

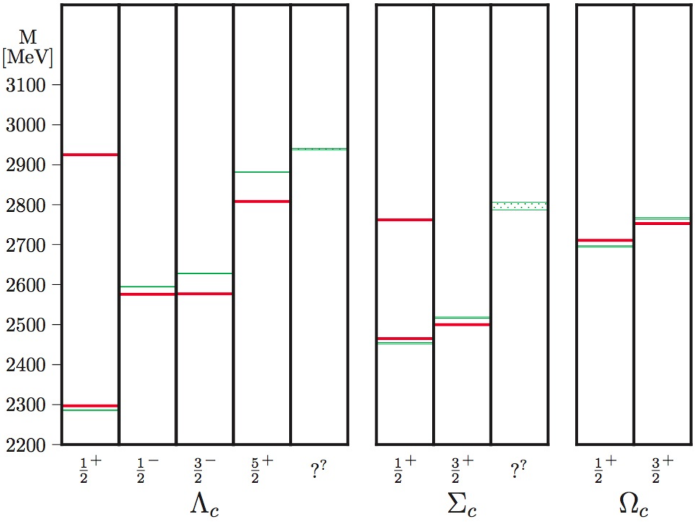
$SU(5)_F$ vs. $SU(3)_F$ - N, Δ, Λ Spectra Comparison



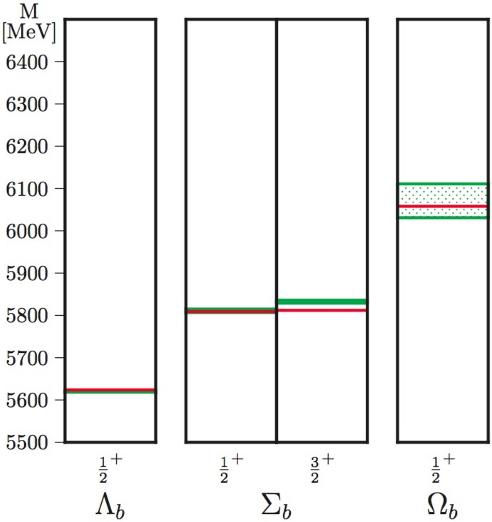
$SU(5)_F$ vs. $SU(3)_F$ - Σ, Ξ, Ω Spectra Comparison



Charm Baryon Spectra



Bottom Baryon Spectra



Conclusions

- We propose a universal RCQM based on GBE that works in all sectors
- While pure OGE is ruled out in the light and strange sector, there is a possibility that a hybrid model consisting of GBE + OGE survives in the heavy sector
- In order to concretely discern the correct Q-Q interaction for heavy-light and heavy-heavy interactions more data from heavy baryon spectroscopy is needed!

Thanks

Contributors

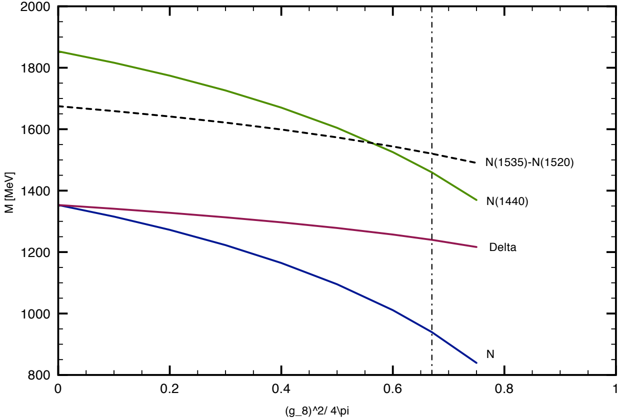
- Special thanks to my collaborators who made important contributions to this work!
- Ki-Seok Choi
- Dr. W. Plessas

References

- Melde, Plessas, Sengl - Phys. Rev. D 77, 114002 (2008)
- K.S. Choi - PhD Seminar Nov. 3 2010
- Glozman, Plessas, Varga, Wagenbrunn - Phys. Rev. D 58, 094030 (1998)

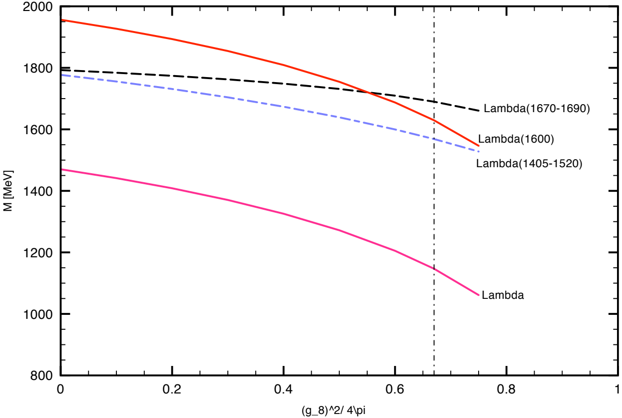
APPENDIX

Level Crossings as Calculated by the Faddeev Method



Plot previously appeared in Phys. Rev. D **58** 094030 (1998) calculated by the SVM.

Level Crossings as Calculated by the Faddeev Method



Plot previously appeared in Phys. Rev. D **58** 094030 (1998) calculated by the SVM.

$SU(3)_F$ Light Baryon Results

Baryon	J^P	Theory GBE	Theory OGE	Experiment PDG
N(939)	$\frac{1}{2}^+$	939	939	938-940
N(1440)	$\frac{1}{2}^+$	1459	1577	1420-1470
N(1520)	$\frac{3}{2}^-$	1519	1521	1515-1525
N(1675)	$\frac{5}{2}^-$	1647	1690	1670-1680
N(1710)	$\frac{1}{2}^+$	1776	1859	1680-1740
Δ(1232)	$\frac{3}{2}^+$	1240	1231	1231-1233
Δ(1600)	$\frac{3}{2}^+$	1718	1854	1550-1700
Δ(1620)	$\frac{1}{2}^-$	1642	1642	1600-1660

All values in MeV

Melde, Plessas, Sengl - Phys. Rev. D 77, 114002 (2008)

$SU(3)_F$ Strange Baryon Results

Baryon	J^P	Theory		Experiment
		GBE	OGE	PDG
$\Lambda(1116)$	$\frac{1}{2}^+$	1136	1113	1116
$\Lambda(1690)$	$\frac{1}{2}^-$	1682	1734	1685-1695
$\Lambda(1800)$	$\frac{1}{2}^-$	1778	1844	1720-1850
$\Sigma(1193)$	$\frac{1}{2}^+$	1180	1213	1189-1197
$\Sigma(1385)$	$\frac{3}{2}^+$	1389	1373	1383-1387
$\Sigma(1560)$	$\frac{1}{2}^1$	1677	1732	1546-1576
$\Xi(1318)$	$\frac{1}{2}^+$	1348	1346	1315-1321
$\Xi(1820)$	$\frac{3}{2}^-$	1792	1894	1818-1828
$\Xi(1950)$	$\frac{5}{2}^-$	1881	1993	1935-1965

Melde, Plessas, Sengl - Phys. Rev. D 77, 114002 (2008)

$SU(5)_F$ Light Baryon Results

Baryon	J^P	Theory		Experiment
		SU(3)	SU(5)	PDG
N(939)	$\frac{1}{2}^+$	939	939	938-940
N(1440)	$\frac{1}{2}^+$	1459	1461	1420-1470
N(1520)	$\frac{3}{2}^-$	1519	1512	1515-1525
N(1675)	$\frac{5}{2}^-$	1647	1641	1670-1680
N(1710)	$\frac{1}{2}^+$	1776	1771	1680-1740
Δ(1232)	$\frac{3}{2}^+$	1240	1238	1231-1233
Δ(1600)	$\frac{3}{2}^+$	1718	1718	1550-1700
Δ(1620)	$\frac{1}{2}^-$	1642	1637	1600-1660

All values in MeV

$SU(5)_F$ Strange Baryon Results

Baryon	J^P	Theory		Experiment
		SU(3)	SU(5)	PDG
$\Lambda(1116)$	$\frac{1}{2}^+$	1136	1128	1116
$\Lambda(1690)$	$\frac{3}{2}^-$	1683	1669	1685-1695
$\Lambda(1800)$	$\frac{1}{2}^-$	1778	1771	1720-1850
$\Sigma(1193)$	$\frac{1}{2}^+$	1180	1171	1189-1197
$\Sigma(1385)$	$\frac{3}{2}^+$	1389	1386	1383-1387
$\Sigma(1660)$	$\frac{1}{2}^+$	1616	1627	1630-1690
$\Xi(1318)$	$\frac{1}{2}^+$	1348	1330	1315-1321
$\Xi(1530)$	$\frac{3}{2}^-$	1528	1523	1532-1535
$\Xi[1690]$	$\frac{1}{2}^+$	1805	1794	1680-1700

Charm Baryon Spectra

Baryon	J^P	GBE	PDG	Status
Λ_c	$\frac{1}{2}^+$	2297	2286.46 ± 0.14	****
Λ_c	$\frac{1}{2}^+$	2925	$2939.3^{+1.4}_{-1.5}$	***
Λ_c	$\frac{1}{2}^-$	2576	2595.4 ± 0.6	***
Λ_c	$\frac{3}{2}^-$	2577	2628.1 ± 0.6	***
Λ_c	$\frac{5}{2}^+$	2808	2881.53 ± 0.35	***
Σ_c	$\frac{1}{2}^+$	2465	2454.02 ± 0.18	****
Σ_c	$\frac{1}{2}^-$	2749	2801^{+4}_{-6}	***
Σ_c	$\frac{3}{2}^+$	2500	2518.4 ± 0.6	***
Ω_c	$\frac{1}{2}^+$	2711	2695.2 ± 1.7	***
Ω_c	$\frac{3}{2}^+$	2753	2765.9 ± 2.0	***

New Meson Exchanges

In $SU(4)_F$

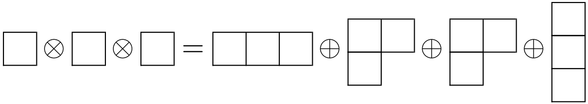
$$4 \otimes \bar{4} = \ydiagram{1} \otimes \ydiagram{1,1,1} = \ydiagram{2,1,1} \oplus \ydiagram{1,1,1,1} = 15 \oplus 1 \qquad (5)$$

In $SU(5)_F$

$$5 \otimes \bar{5} = \ydiagram{1} \otimes \ydiagram{1,1,1,1} = \ydiagram{2,1,1,1} \oplus \ydiagram{1,1,1,1,1} = 24 \oplus 1 \qquad (6)$$

$$U(5)_L \times U(5)_R = SU(5)_L \times SU(5)_R \times U(1)_V \times U(1)_A$$

Baryon Multiplets



(7)

In $SU(4)_F$

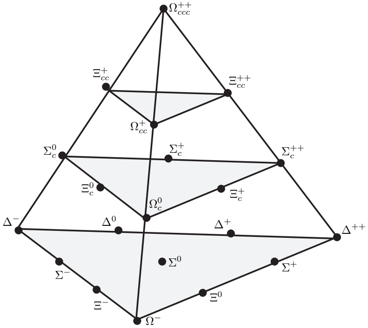
$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4} \tag{8}$$

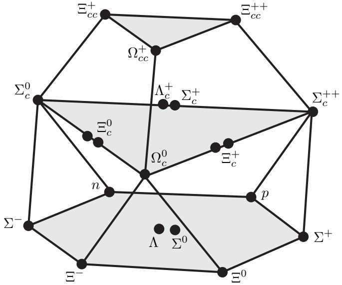
In $SU(5)_F$

$$5 \otimes 5 \otimes 5 = 35 \oplus 40 \oplus 40 \oplus \overline{10} \tag{9}$$

Of course $SU(3)_F \subset SU(4)_F \subset SU(5)_F$ so the the new potential should produce results consistent with the old potential

Baryon Multiplet Examples





$SU(5)_F$ Potential Model

$$V_\chi = \Big[V_\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a + V_K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a + V_\eta \lambda_i^8 \lambda_j^8 + \frac{2}{5} V_{\eta'}$$

$$+ V_D \sum_{a=9}^{12} \lambda_i^a \lambda_j^a + V_{D_s} \sum_{a=13}^{14} \lambda_i^a \lambda_j^a + V_{\eta_c} \lambda_i^{15} \lambda_j^{15}$$

$$+ V_B \sum_{a=16}^{19} \lambda_i^a \lambda_j^a + V_{B_s} \sum_{a=20}^{21} \lambda_i^a \lambda_j^a + V_{B_c} \sum_{a=22}^{23} \lambda_i^a \lambda_j^a$$

$$+ V_{\eta_b} \lambda_i^{24} \lambda_j^{24} \Big] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Consequences in the Light and Strange sector

Diagonal Elements in λ^{15}

$$\lambda^{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Diagonal Elements in λ^{24}

$$\lambda^{24} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}$$