Spectra of Light and Heavy Mesons, Glueball and QCD Effective Coupling

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The spectroscopy of conventional mesons as well as the glueball ground state are investigated within a relativistic quantum-field model based on analytical confinement. Ladder Bethe-Salpeter-type equations are derived to define the spectra of quark-antiquark and two-gluon bound states. We provide a new analytic estimate of the lowest-state glueball mass and calculate the spectrum of light, intermediate and heavy mesons in a wide range of energy scale $\sim 0.1 - 10$ GeV. The QCD effective coupling α_s is studied by exploiting the conventional meson spectrum. A new, independent and specific infrared-finite behavior of QCD running coupling is found below ~ 1 GeV. By introducing only a minimal set of model parameters, we obtain results in reasonable agreement with recent experimental data.

1 Introduction

Understanding of a number of phenomena such as quark confinement, hadronization and nonvanishing vacuum expectation values, etc. requires a correct description of hadron dynamics in the infrared (IR) region at low energies below 1 GeV. Particularly, many quantities in hadron physics are affected by the IR behavior of the the QCD effective coupling α_s in different amounts. QCD predicts the functional form of the energy dependence of α_s on energy scale Q, but its actual values are determined only at relatively high energies [1]. The long-distance behavior of α_s is not well defined, it needs to be more specified [2, 3] and remains one of the actual problems in particle physics. On the other hand, the calculations of hadron mass characteristics on the level of experimental data precision still remain among the unsolved problems in QCD.

It represents a certain interest to investigate some low-energy physics problems, such as hadronisation, glueball states, QCD effective (running) charge within a simple relativistic

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model based on physically transparent hypotheses, which can be treated by simple analytic methods.

Below, we take into account the dependence of α_s on mass scale M and determine the QCD effective charge in the low-energy region by exploiting the hadron spectrum [4]. For the spectra of two-quark bound states we develop a relativistic quantum-field model based on analytic (or, IR) confinement and consider Lagrangian [5]:

$$\mathcal{L} = -\frac{1}{4}\left(\partial^\mu \mathcal{A}^A_\nu - \partial^\nu \mathcal{A}^A_\mu - g f^{ABC}\mathcal{A}^B_\mu \mathcal{A}^C_\nu\right)^2 + \sum_f \left(\bar{q}^a_f \left[\gamma_\alpha \partial^\alpha - m_f + g\Gamma^\alpha_C \mathcal{A}^C_\alpha\right]^{ab} q^b_f\right), \tag{1}$$

where $\mathcal{A}^C_\alpha$ is gluon adjoint representation, q^a_f - quark field and $\Gamma^\alpha_C = i\gamma_\alpha t^C$. The model parameters are the confinement scale Λ and the constituent quark masses $m_f=\{m_{ud}, m_s, m_c, m_b\}$.

Within the model the quark and gluon propagators $\tilde{S}(\hat{p})$ and $\tilde{D}(p)$ are entire analytic functions in the Euclidean space. The quark propagator reads:

$$\tilde{S}^{ab}_m(\hat{p}) = \delta^{ab}\frac{i\hat{p} + m_f[1 \pm \gamma_5\,(1+(m_f/\Lambda)^2/4)^{-1}]}{\Lambda m_f}\exp\left\{-\frac{p^2+m_f^2}{2\Lambda^2}\right\}. \tag{2}$$

Recent theoretical results predict an IR behavior of the gluon propagator [6,7]. We consider a gluon propagator exhibiting an IR-finite behavior in Feynman gauge as follows:

$$\tilde{D}^{AB}_{\mu\nu}(p) = \delta^{AB}\delta_{\mu\nu}\frac{1-\exp\left(-p^2/\Lambda^2\right)}{p^2} = \delta^{AB}\delta_{\mu\nu}\int\limits_0^{1/\Lambda^2} ds\, e^{-sp^2}. \tag{3}$$

The leading-order contributions to the $(q\bar{q})$ and two-gluon bound states read:

$$Z_{q\bar{q}} = \iint \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{-(\bar{q}S^{-1}q) + \frac{g^2}{2}\left\langle(\bar{q}\Gamma\mathcal{A}q)^2\right\rangle_D\right\},$$
$$Z_{\mathcal{A}\mathcal{A}} = \left\langle \exp\left\{-\frac{g}{2}\,(f\mathcal{A}\mathcal{A}F)\right\}\right\rangle_D, \qquad \langle(\bullet)\rangle_D \doteq \int \mathcal{D}\mathcal{A}\, e^{-\frac{1}{2}(\mathcal{A}D^{-1}\mathcal{A})}(\bullet).$$

First, we allocate the one-gluon exchange between colored biquark currents and isolate the color-singlet combinations. Then, perform a Fierz transformation and introduce a system of orthonormalized functions $\{U_Q(x)\}$, where $Q = \{n_r, l, \mu\}$ are quantum numbers. By involving a Gaussian path-integral representation defined on auxiliary fields $B_\mathcal{N}$ for the exponential in $Z_{q\bar{q}}$ we take explicit path integration over quark variables. Let us introduce a *Hadronization Ansatz* and identify $B_\mathcal{N}(x)$ with meson fields carrying quantum numbers $\mathcal{N} = \{Q, J, f_1, f_2\}$. We isolate all quadratic field configurations ($\sim B^2_\mathcal{N}$) in the 'kinetic' term and rewrite the partition function for mesons [4]:

$$Z_{q\bar{q}} \to Z = \int \prod_\mathcal{N} \mathcal{D}B_\mathcal{N}\, \exp\left\{-\frac{1}{2}\sum_{\mathcal{N}\mathcal{N}'}(B_\mathcal{N}\,[\delta^{\mathcal{N}\mathcal{N}'} + \alpha_s\lambda_{\mathcal{N}\mathcal{N}'}]\,B_{\mathcal{N}'}) - W_{res}[B_\mathcal{N}]\right\}, \tag{4}$$

$J^{PC}=0^{-+}$	$M_{\mathbf{P}}$	$J^{PC}=0^{-+}$	$M_{\mathbf{P}}$	$J^{PC}=1^{--}$	$M_{\mathbf{V}}$	$J^{PC}=1^{--}$	$M_{\mathbf{V}}$
$\pi(138)$	138	$\eta_c(2980)$	3039	$\rho(770)$	770	$D_s^*(2112)$	2112
$K(495)$	495	$B(5279)$	5339	$\omega(782)$	785	$J/\psi(3097)$	3097
$\eta(547)$	547	$B_s(5370)$	5439	$K^*(892)$	892	$B^*(5325)$	5357
$D(1870)$	1941	$B_c(6286)$	6489	$\Phi(1019)$	1022	$\Upsilon(9460)$	9460
$D_s(1970)$	2039	$\eta_b(9389)$	9442	$D^*(2010)$	2010		

Table 1: Estimated masses M of conventional mesons (in MeV) at $\Lambda = 345$ MeV [4].

where the residual part $W_{res}[B_{\mathcal{N}}] \sim 0(B_{\mathcal{N}}^3)$ describes interaction between mesons.

The Fourier transform of the leading-order term of the polarization operator reads

$$\lambda_{JJ'}(p,x,y) = \frac{16\pi}{9}\sqrt{C_J\,C_{J'}D(x)D(y)}\int\frac{d^4k}{(2\pi)^4}\,e^{-ik(x-y)}\mathrm{Tr}\left[O_J\tilde{S}_{m_1}\!\left(\hat{k}+\xi_1\hat{p}\right)O_{J'}\tilde{S}_{m_2}\!\left(\hat{k}-\xi_2\hat{p}\right)\right].$$

We diagonalize the polarization kernel on the orthonormal basis $\{U_{\mathcal{N}}\}$:

$$\iint dxdy\,U_{\mathcal{N}}(x)\lambda_{JJ'}(p,x,y)U_{\mathcal{N}'}(y) = \delta^{\mathcal{N}\mathcal{N}'}\,\lambda_{\mathcal{N}}(-p^2)$$

that is equivalent to the solution of the corresponding ladder Bethe-Salpeter equation.

In relativistic quantum-field theory a stable bound state of n massive particles shows up as a pole in the S matrix with a center of mass energy. Accordingly, the meson masses may be derived from the equation [4]:

$$1+\hat{\alpha}_s\cdot\lambda_{\mathcal{N}}(M_{\mathcal{N}}^2) = 0\,, \qquad -p^2 = M_{\mathcal{N}}^2\,. \tag{5}$$

2 Meson Spectrum and QCD Effective Coupling

Further we exploit Eq. (5) in different ways, by solving either for $\hat{\alpha}_s$ at given masses, or for M_J at known values of $\hat{\alpha}_s$.

1) First, we adjust the model parameters by fitting heavy meson masses ($M \geqslant 2$ GeV) and newest data for α_s. Then, we fix model parameters as follows (in units of MeV) [4]:

$$\Lambda = 345\,,\quad m_{ud} = 192.56\,,\quad m_s = 293.45\,,\quad m_c = 1447.59\,,\quad m_b = 4692.51\,.$$

2) Having adjusted model parameters, we estimate $\hat{\alpha}_s(M)$ in the low-energy domain corresponding to light meson masses below ~ 1 GeV.

3) We calculate some meson masses in a wide range of energy $1 < M < 9.5$ GeV with relative error less than 3.5 percent (shown in Table 1). The estimated meson masses do not change considerably (less than 0.5 percent) under changes of $330 < \Lambda < 360$ MeV.

4) We perform global evaluation of $\hat{\alpha}_s(M)$ at the mass scale of conventional mesons by using formula $\hat{\alpha}_s(M_J) = -1/\lambda_J(M_J,\Lambda,m_1,m_2)$ and plot the resulting curves at different Λ in Figure 1 in comparison with recent low- and high-energy data of $\alpha_s(Q)$ [8].

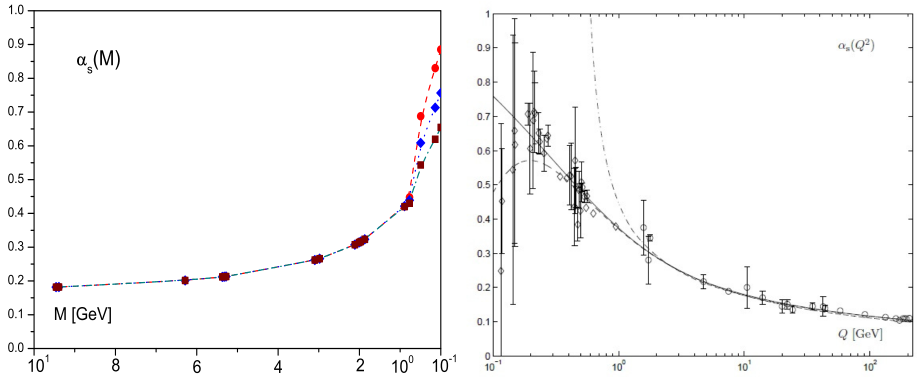

Figure 1: Our estimates of $\hat{\alpha}_s(M)$ in interval from 0 to 10 GeV at different values of confinement scale: $\Lambda = 330$ MeV (dots), $\Lambda = 345$ MeV (rhombs) and $\Lambda = 360$ MeV (squares) compared with $\alpha_s(Q)$ defined in low- (diamonds) and high-energy (circles) experiments. Also shown are the three-loop analytic coupling (solid curve), its perturbative counterpart (dot-dashed) and the massive one-loop analytic coupling (dashed) (see [8]).

3 IR-finite Behaviour of the QCD Running Coupling

The possibility that the QCD coupling features an IR-finite behavior has been extensively studied (e.g., [9]). By deriving the coupling at origin $\hat{\alpha}_s^0 \doteq \hat{\alpha}_s(0)$ for zero meson mass $M = 0$ and at particular values $m = m_{ud} = 192.56$ MeV and $\Lambda = 345$ MeV we estimate it

$$\hat{\alpha}_s^0 = 0.757\,, \qquad \text{or} \qquad \hat{\alpha}_s^0/\pi = 0.241\,. \tag{6}$$

Note, QCD running coupling behaves different in Euclidean and Minkowskian domains, but at origin they coincide $\alpha_s(0) = \hat{\alpha}_s(0)$ [4]. We may conclude that our result (6) is in a reasonable agreement with often quoted estimates

$$\left\{ \begin{array}{llll} \alpha_s^0/\pi \simeq 0.19 - 0.25 & [10]\,, & \alpha_s^0/\pi \simeq 0.265 & [11]\,, \\ \alpha_s^0/\pi \simeq 0.26 & [12]\,, & \left\langle \alpha_s^0/\pi \right\rangle_{1\,GeV} \simeq 0.2 & [13]\;. \end{array} \right. \tag{7}$$

4 Glueball Lowest State

Because of the confinement, gluons are not observed, they may only come in bound states called *glueballs*. Glueballs are the most unusual particles predicted by the QCD but not found experimentally yet [14]. There are predictions expecting non-$q\bar{q}$ scalar objects, like glueballs and multiquark states in the mass range $\sim 1500 \div 1800$ MeV [15–17].

The glueball spectrum has been studied by using effective approaches like the QCD sum rules [18], Coulomb gauge QCD [19], various potential [20] and string models [21] as well as lattice QCD simulations [22,23].

Below we consider a two-gluon scalar bound state. First, we isolate the color-singlet term in the bi-gluon current in $Z_{\mathcal{A}\mathcal{A}}$. The second-order matrix element contains color-singlet two-gluon current and consists of spin-zero (scalar) and spin-two (tensor) components. We consider the scalar component. By omitting details of intermediate calculations (similar to those represented in the previous section) we define the Bethe-Salpeter kernel:

$$\Pi(z) \doteq \iint dtds \, U_n(t)\sqrt{W(t)} \, D\left(\frac{t+s}{2}+z\right) \cdot D\left(\frac{t+s}{2}-z\right) \sqrt{W(s)} \, U_n(s) \, .$$

Here we consider the gluon propagator (in Feynman gauge):

$$\tilde{D}^{AB}_{\mu\nu}(p) = \delta^{AB}\frac{\delta_{\mu\nu}}{p^2}\exp\left(-p^2/4\Lambda^2\right) \, .$$

The glueball mass M_G is defined from equation:

$$1 - \frac{8\,g^2}{3}\int dz \, e^{izp} \, \Pi(z) = 0 \, , \qquad p^2 = -M_G^2 \, . \tag{8}$$

The final analytic result for the lowest-state glueball mass reads

$$M_G = 2\Lambda\left[\ln\left(\frac{\alpha_{crit}}{\alpha_s}\right)\right]^{1/2} \, , \qquad \alpha_{crit} \doteq \frac{3\pi(3+2\sqrt{2})^2}{4} \, . \tag{9}$$

Particularly, for $\Lambda_{QCD} \approx 360$ MeV and $\alpha_s(M_\tau) = 0.343$ we estimate $M_G \approx 1710$ MeV.

Else, with values $\alpha_s = 1.5023$ and $\Lambda = 416.4$ MeV obtained by fitting the meson masses and weak decay constants we calculate the scalar glueball mass as follows [5]

$$M_G = 1661 \text{ MeV} \, . \tag{10}$$

Our estimate is in reasonable agreement with other predictions [15, 18, 22, 23]. The recent quenched lattice estimate favors a scalar glueball mass $M_G = 1710 \pm 50 \pm 58$ MeV [24].

To conclude, we demonstrated that global properties of some low-energy phenomena may be explained reasonably in the framework of a simple relativistic quantum-field model if one guesses correct symmetry structure of the quark-gluon interaction in the confinement region and uses simple forms of propagators in the hadronization regime. We do not aim to obtain the behavior of the coupling constant at all scales. At moderate $M^2 = -p^2$ we obtain α_s in coincidence with the QCD predictions. However, at large mass scale (above 10 GeV) $\hat{\alpha}_s$ decreases much faster than expected by QCD prediction. The reason is the use of confined propagators in the form of entire (exponential) functions. As an application, we performed estimates on conventional meson spectrum and the lowest glueball mass and, the result was in reasonable agreement with experimental data.

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